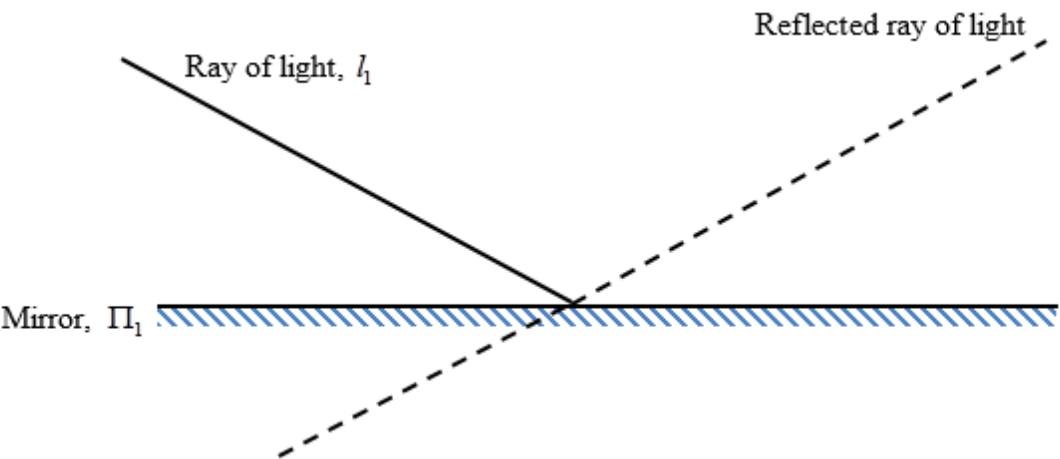


H2 Mathematics 2017 Prelim Exam Paper 2 Question

Answer all questions [100 marks].

1	<p>(i) Prove that $\frac{\sin(A-B)}{\cos A \cos B} = \tan A - \tan B$. [1]</p> <p>(ii) Hence, by considering a suitable expression of A and B, find $\sum_{r=1}^N \frac{\sin x}{\cos[(r+1)x] \cos(rx)}. \quad [3]$</p> <p>(iii) Using your answer to part (ii), find $\sum_{r=1}^N \left(\frac{\sqrt{3}}{2 \cos \frac{r\pi}{3} \cos \frac{(r+1)\pi}{3}} \right)$, leaving your answer in terms of N. [2]</p>
2	<p>(i) Find $\int_2^n \frac{9x}{(x^2-1)^3} dx$, where $n \geq 2$ and hence evaluate $\int_2^\infty \frac{9x}{(x^2-1)^3} dx$. [3]</p> <p>(ii) Sketch the curve $y = \frac{9x}{(x^2-1)^3}$ for $x \geq 0$. [2]</p> <p>(iii) The region R is bounded by the curve, the line $y = \frac{2}{3}$ and the line $x = 5$. Write down the equation of the curve when it is translated by $\frac{2}{3}$ units in the negative y-direction. [1] Hence or otherwise, find the volume of the solid formed when R is rotated completely about the line $y = \frac{2}{3}$, leaving your answer correct to 3 decimal places. [2]</p>
3	<p>(a) (i) Show that $\frac{d}{d\theta} \left(\sin \theta - \frac{1}{3} \sin^3 \theta \right) = \cos^3 \theta$. [1]</p> <p>(ii) Find the solution to the differential equation $\operatorname{cosec} x \frac{d^2 y}{dx^2} = -\cos^2 x$ in the form $y = f(x)$, given that $y = 0$ and $\frac{dy}{dx} = \frac{1}{3} + \frac{2}{\pi}$ when $x = 0$. [4]</p> <p>(b) Show, by means of the substitution $v = x^2 y$, that the differential equation $x \frac{dy}{dx} + 2y + 4x^2 y = 0$</p>

	<p>can be reduced to the form</p> $\frac{dv}{dx} = -4vx.$ <p>Hence find y in terms of x, given that $y = \frac{1}{3}$ when $x = -3$. [6]</p>
4	<p>In the study of light, we may model a ray of light as a straight line.</p> <p>A ray of light, l_1, is known to be parallel to the vector $2\mathbf{i} + \mathbf{k}$ and passes through the point P with coordinates $(1, 1, 0)$. The ray of light hits a mirror, and is reflected by the mirror which may be modelled by a plane Π_1 containing the points A, B and C with coordinates $(-1, 1, 0)$, $(0, 0, 2)$ and $(0, 3, -3)$ respectively. This scenario is depicted in the diagram below:</p>  <p>(i) Show that an equation for plane Π_1 is given by $-x + 5y + 3z = 6$. [3]</p> <p>(ii) Find the coordinates of the point where the ray of light meets the mirror. [2]</p> <p>(iii) Determine the position vector of the foot of the perpendicular from the point P to the mirror and hence, find an equation of the line that may be used to model the reflected ray of light. [6]</p> <p>A second ray of light which is parallel to the mirror may be modelled by the line l_2, with Cartesian equation $\frac{x-1}{2} = \frac{z-2}{\alpha}, y = \beta$. Given that the distance between l_2 and the mirror is $\frac{14}{\sqrt{35}}$ units, find the values of the positive constants α and β. [4]</p>
5	<p>A random variable X has the probability distribution given in the following table.</p>

	<table><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$P(X = x)$</td><td>0.2</td><td>a</td><td>b</td><td>0.45</td></tr></table>	x	2	3	4	5	$P(X = x)$	0.2	a	b	0.45
x	2	3	4	5							
$P(X = x)$	0.2	a	b	0.45							
	<p>Given that $E(X - 4) = \frac{11}{10}$, find the values of a and b. [3]</p> <p>Two independent observations of X are taken. Find the probability that one of them is 2 and the other is at most 4. [2]</p>										
6	<p>In a large consignment of mangoes, 4.5% of the mangoes are damaged.</p> <p>(i) A total of 21 mangoes are selected at random. Calculate the probability that not more than 3 mangoes are damaged. [2]</p> <p>(ii) The mangoes are randomly selected and packed into boxes of 21. For shipping purposes, the boxes are packed into cartons, with each carton containing 12 boxes. A box containing more than 3 damaged mangoes is considered low standard. Calculate the probability that, in a randomly selected carton, there are at least 2 boxes which are of low standard. [3]</p> <p>(iii) Find the probability that a randomly chosen box that is of low standard contains no more than five damaged mangoes. [3]</p>										
7	<p>(a) Seven boys and five girls formed a group in a school orientation. During one of the game segments, they are required to arrange themselves in a row. Find the exact probability that</p> <p>(i) the girls are separated from one another, [2]</p> <p>(ii) there will be exactly one boy between any two girls. [2]</p> <p>In another game segment, they are required to sit at a round table with twelve identical chairs. Find the exact probability that one particular boy is seated between two particular girls. [2]</p> <p>(b) The events A and B are such that $P(A) = \frac{7}{10}$, $P(B) = \frac{2}{5}$ and $P(A B) = \frac{13}{20}$.</p> <p>(i) Find $P(A \cup B)$, [3]</p> <p>(ii) State, with a reason, whether the events A and B are independent. [1]</p> <p>(c) A man plays a game in which he draws balls, with replacement, from a bag containing 3 yellow balls, 2 red balls and 4 black balls. If he draws a black ball, he loses the game and if he draws a red ball he wins the game. If he draws a yellow ball, the ball is replaced and he draws again. He continues drawing until he either wins or loses the game. Find the probability that he wins the game. [2]</p>										
8	<p>A company manufactures bottles of iced coffee. Machines A and B are used to fill the bottles with iced coffee.</p> <p>(i) Machine A is set to fill the bottles with 500 ml of iced coffee. A random sample of 50 filled bottles was taken and the volume of iced coffee (x ml) in each bottle was measured. The following data was obtained</p>										

	$\sum x = 24965 \quad \sum (x - \bar{x})^2 = 365$ <p>Calculate unbiased estimates of the population mean and variance. Test at the 2% level of significance, whether the mean volume of iced coffee per bottle is 500 ml. [6]</p> <p>(ii) The company claims that Machine <i>B</i> filled the bottles with μ_0 ml of iced coffee. A random sample of 70 filled bottles was taken and the mean is 489.1 ml with standard deviation 4 ml. Find the range of values of μ_0 for which there is sufficient evidence for the company to have overstated the mean volume at the 2% level of significance. [5]</p>														
9	<p>An online survey revealed that 34.1% of junior college students spent between 3 to 3.8 hours on their mobile phones daily. Assuming that the amount of time a randomly chosen junior college student spends on mobile phones daily follows a normal distribution with mean 3.4 hours and standard deviation σ hours, show that $\sigma = 0.906$, correct to 3 decimal places. [3]</p> <p>Find the probability that</p> <p>(i) four randomly chosen students each spend between 3 to 3.8 hours daily on their mobile phones. [1]</p> <p>(ii) the total time spent on their mobile phones daily by the three randomly chosen junior college students is less than twice that of another randomly chosen junior college student. [3]</p> <p>(iii) State an assumption required for your calculations in (i) and (ii) to be valid. [1]</p> <p><i>N</i> samples, each consisting of 50 randomly selected junior college students, are selected. It is expected that 15 of these samples will have a mean daily time spent on mobile phones greater than 3.5 hours.</p> <p>(iv) Estimate the value of <i>N</i>. [4]</p>														
10	<p>In a medical study, researchers investigated the effect of varying amounts of calcium intake on the bone density of Singaporean women of age 50 years. A random sample of eighty 50-year-old Singaporean women was taken.</p> <p>(i) Explain, in the context of this question, the meaning of the phrase ‘random sample’. [1]</p> <p>The daily calcium intake (<i>x</i> mg) of the women was varied and the average percentage increase in bone density (<i>y</i>%) was measured. The data is as shown in the table below.</p> <table><tr><td><i>x</i> (in mg)</td><td>700</td><td>800</td><td>900</td><td>1000</td><td>1050</td><td>1100</td></tr><tr><td><i>y</i> (%)</td><td>0.13</td><td>0.78</td><td>1.38</td><td>1.88</td><td>2.07</td><td>2.10</td></tr></table>	<i>x</i> (in mg)	700	800	900	1000	1050	1100	<i>y</i> (%)	0.13	0.78	1.38	1.88	2.07	2.10
<i>x</i> (in mg)	700	800	900	1000	1050	1100									
<i>y</i> (%)	0.13	0.78	1.38	1.88	2.07	2.10									

- (ii) Calculate the product moment correlation coefficient and suggest why its value does not necessarily mean that the best model for the relationship between x and y is $y = a + bx$. [2]

- (iii) Draw a scatter diagram representing the data above. [2]

The researchers suggest that the change in bone density can instead be modelled by the equation $\ln(P - y) = a + bx$.

The product moment correlation coefficient between x and $\ln(P - y)$ is denoted by r . The following table gives values of r for some possible values of P .

P	3	5	10
r		-0.993803	-0.991142

- (iv) Calculate the value of r for $P = 3$, giving your answer correct to 6 decimal places. Use the table and your answer to suggest with reason, which of 3, 5 or 10 is the most appropriate value of P . [2]

The Healthy Society wants to recommend a daily calcium intake that would ensure an average of 1.8% increase in bone density.

- (v) Using the value of P found in part (iv), calculate the values of a and b and use your answer to estimate the daily calcium intake that the Health Society should recommend. Comment on the reliability of the estimate obtained. [4]
- (vi) Give an interpretation, in the context of the question, of the meaning of the value of P . [1]

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