

YISHUN JUNIOR COLLEGE
2017 JC2 Preliminary Examination

MATHEMATICS
HIGHER 2

9758/02

12 SEPTEMBER 2017
TUESDAY 0800h – 1100h

Additional materials :
Answer paper
List of Formulae (MF26)



TIME 3 hours

READ THESE INSTRUCTIONS FIRST

Write your CTG and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise. Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, write down the question number of the questions attempted, model of calculator used on the spaces provided on the cover page. Tie your cover page on top of the answer scripts before submission.

The number of marks is given in brackets [] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

- 1 (i) Show that if $a_r = T_r - T_{r-1}$ for $r = 1, 2, 3, \dots$, and $T_0 = 0$, then

$$\sum_{r=1}^n a_r = T_n. \quad [1]$$

- (ii) Deduce that $\sum_{r=1}^n \pi^{-r} [(1-\pi)r^2 + 2\pi r - \pi] = n^2 \pi^{-n}.$ [3]

- (iii) Hence, find the exact value of $\sum_{r=4}^{20} \pi^{-r} [(1-\pi)r^2 + 2\pi r - \pi].$ [2]

- 2 Functions g and h are defined by

$$g: x \mapsto x^2 + 6x + 8, \quad x \in \mathbb{R}, \quad x \leq \alpha,$$

$$h: x \mapsto -e^x, \quad x \in \mathbb{R}, \quad x > -2.$$

- (i) Given that the function g^{-1} exists, write down the largest value of α and define g^{-1} in similar form. State a transformation which will transform the curve $y = g(x)$ onto the curve $y = g^{-1}(x).$ [5]

- (ii) Given instead that $\alpha = -2$, explain why the composite function hg exists and find the exact range of $hg.$ [2]

- 3 **Do not use a calculator in answering this question.**

Given that $z = 1 + i$ is a root of the equation $2z^4 + az^3 + 7z^2 + bz + 2 = 0$, find the values of the real numbers a and b and the other roots. [5]

Deduce the roots of the equation $2z^4 + bz^3 + 7z^2 + az + 2 = 0.$ [2]

- 4 A curve C has parametric equations

$$x = t^2, \quad y = t - t^3, \quad t \leq 0.$$

- (i) The point P on the curve has parameter p . Show that the equation of the tangent at P is $2py = x(1 - 3p^2) + p^2 + p^4.$ [3]
- (ii) If the tangent at P passes through the point $(6, 5)$, find the possible coordinates of $P.$ [3]
- (iii) Find the area of the region bounded by C and the x -axis. [3]

- 5 The planes p_1 and p_2 have equations $x - 4y + 8z = 4$ and $mx + ny + 2z = 1$ respectively, where m and n are constants.

- (i) If p_1 and p_2 meet at a line that has equation $\mathbf{r} = 2\mathbf{i} - 0.5\mathbf{j} + \lambda(-4\mathbf{i} + \mathbf{j} + \mathbf{k})$, where $\lambda \in \mathbb{R}$, find the values of m and n . [3]

It is given instead that $m = 1$ and $n = 2$.

- (ii) Find the acute angle between p_1 and p_2 . [2]
 (iii) The point $(1, b, 5)$ is equidistant from p_1 and p_2 . Calculate the possible value(s) of b . [6]

Section B: Statistics [60 marks]

- 6 (a) Find the number of ways to arrange the letters of the word **TOTORO** such that
- (i) all the 'O's are together, [1]
 - (ii) all the 'O's are separated, [2]
 - (iii) the last letter is a consonant. [3]
- (b) Tontoro soft toys are sold in four different colours, of which each varies in three sizes, small, medium and large. Each set of Tontoro soft toys consists of a small, a medium and a large sized soft toy and exactly two are of the same colour. Find the number of different possible sets of Tontoro soft toys. [2]
- 7 A game is played with a set of 4 cards, each distinctly numbered 1, 2, 3 and 4. A player randomly picks a pair of cards without replacement. If the sum of the cards' numbers is an odd number, the sum is the player's score. If the sum of the two cards' number is an even number, the player randomly picks a third card from the remaining cards. The square of the third card's number is the player's score.
- (i) Find the probability that a player obtains a score of 4. [2]
 - (ii) Find the probability distribution of a player's score, S . Hence, find the expected score of a player. [4]
 - (iii) Find the probability that a player obtains a score lower than 5, given that he draws three cards. [3]

- 8 An archaeologist examines rocks to look for fossils. On average, 10% of the rocks selected from a particular area with a large number of rocks contain fossils. The archaeologist selects a random sample of 25 rocks from this area. The number of rocks that contain fossils is denoted by X .

(i) Find the probability that more than 4 but at most 10 rocks contain fossils. [2]

(ii) Show that $\frac{P(X = k + 1)}{P(X = k)} = \frac{25 - k}{9(k + 1)}$, for $k = 0, 1, 2, 3, \dots, 24$. Hence, by considering $P(X = k + 1) > P(X = k)$, find the most probable value of X . [4]

The archaeologist explores a new area. On average, $p\%$ ($p > 10$) of the rocks in the new area contain fossils. A random sample of 20 rocks from the new area is selected. Given that the probability that there are two rocks that contain fossils is 0.17, find the value of p , giving your answer correct to 2 decimal places. [3]

- 9 A researcher investigates the relationship between the population of a particular species of bacteria in millions (b) and the surrounding temperature in $^{\circ}\text{C}$ (t). The researcher keeps records so that she can estimate the population of the bacteria at a certain temperature. Observations at different temperatures give the data as shown in the following table.

t	26.5	27.5	28.5	29.5	30.5	31.5	32.5
b	1.31	2.10	3.65	5.80	α	19.56	31.20

- (i) Given that the regression line of b on t is $b = -129.368 + 4.75214t$, show that $\alpha = 12.12$, correct to 2 decimal places. [2]
- (ii) Sketch a scatter diagram for the data. [1]
- (iii) Explain which of $b = ct + d$ or $b = kt^3 + l$ is the more appropriate model for the relationship between b and t and find the equation of a suitable regression line for this model. [2]
- (iv) Use the model you chose in part (iii) to estimate the population of the bacteria when the temperature is 33°C . Comment on the reliability of the estimate obtained. [2]
- (v) It is given that the temperature T , in $^{\circ}\text{F}$, is related to the temperature t , in $^{\circ}\text{C}$, by the equation $T = 1.8t + 32$. Rewrite your equation from part (iii) so that it can be used to estimate the population of bacteria when the temperature is given in $^{\circ}\text{F}$. [2]

- 10** In a factory, the average time taken by a machine to assemble a smartphone is 53 minutes. A new assembly process is trialled and the time taken to assemble a smartphone, x minutes, is recorded for a random sample of 60 smartphones. The total time taken was found to be 3129 minutes and the variance of the time was 18.35 minutes².

The engineer wants to test whether the average time taken by a machine to assemble a smartphone has decreased, by carrying out a hypothesis test.

- (i) Explain why the engineer is able to carry out a hypothesis test without assuming anything about the distribution of the times taken to assemble a smartphone. [1]
- (ii) Find unbiased estimates of the population mean and variance and carry out the test at the 10% level of significance. [6]
- (iii) Explain, in the context of the question, the meaning of ‘at 10% level of significance’. [1]

After several trials, the engineer claims that the average time taken by a machine to assemble a smartphone is 45 minutes using the new assembly process. The internal control manager wishes to test whether the engineer’s claim is valid. The population variance of the time taken to assemble a smartphone using the new assembly process may be assumed to be 9 minutes². A random sample of 50 smartphones is taken.

- (iv) Find the range of values of the mean time of this sample for which the engineer’s claim would be rejected at the 10% significance level. [4]

[Question 11 is printed on the next page.]

- 11** In the manufacture of child car seats, a resin made up of three ingredients is used. The ingredients are polymer *A*, polymer *B* and an impact modifier. The resin is prepared in batches and each ingredient is supplied by a separate feeder. The masses, in kg, of polymer *A*, polymer *B* and the impact modifier in each batch of resin are assumed to be normally distributed with means and standard deviations as shown in the table. The three feeders are also assumed to operate independently of each other.

	Mean	Standard deviation
Polymer <i>A</i>	2030	44.8
Polymer <i>B</i>	1563	22.7
Impact modifier	μ	σ

It is known that 3% of the batches of resin have less than 1350 kg of impact modifier and 30% of the batches of resin have more than 1414 kg of impact modifier.

- (i) Show that $\mu \approx 1400$ and $\sigma \approx 26.6$. [3]
- (ii) Given that polymer *A* costs \$2.20 per kg, polymer *B* costs \$2.80 per kg and the impact modifier costs \$1.50 per kg, find the probability that the total cost of 2 batches of resin exceeds \$22,000. [4]
- (iii) A random sample of n batches of resin is chosen. If the probability that at most 6 batches of resin has more than 1414 kg of impact modifier is less than 0.001, find the least value of n . [3]
- (iv) Each batch of resin is used to make a large number of car seats. It is found that the tensile strength (N/m^2) of resin for a car seat has mean 125 and standard deviation 17. A random sample of 50 car seats is selected. Find the probability that the average tensile strength of resin for these 50 car seats is less than 130 N/m^2 . [3]

~ ~ *End of Paper* ~ ~