

Candidate Name: _____

Class: _____



JC2 PRELIMINARY EXAM
Higher 2

MATHEMATICS

Paper 2

9758/02
18 Sept 2017
3 hours

Additional Materials: Cover page
 Answer papers
 List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your full name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

- 1** Referred to origin O , points A and B have position vectors \mathbf{a} and \mathbf{b} respectively. Point P lies on OA produced such that $OA:AP = 1:\lambda$. Point Q lies on OB , between O and B , such that $OQ:QB = 3:1$. The mid-point of PB is M . Show that the ratio of the area of triangle OPM to the area of triangle OQM is independent of λ . [5]

- 2** By differentiating $\cos x \frac{dy}{dx}$ with respect to x , solve the differential equation

$$\cos x \frac{d^2 y}{dx^2} - \sin x \frac{dy}{dx} = \sec^2 x + \cos 2x,$$

giving y in terms of x . [6]

- 3** (a) State a sequence of transformations which transform the graph of $y = \ln(2x+1)$ to the graph of $y = \ln\left(\frac{3}{2x-1}\right)$. [3]

(b) It is given that

$$f(x) = \begin{cases} ax & 0 \leq x < 1, \\ a & 1 \leq x \leq 2, \\ 3a - ax & 2 < x \leq 3, \end{cases}$$

and that $f(x+3) = \frac{1}{2}f(x)$, for all real values of x , where a is a positive constant.

- (i) Sketch the graph of $y = f(x)$ for $-2 \leq x \leq 8$. [3]
- (ii) Find, in terms of a , $\int_0^2 f(-x) dx$. [1]
- (iii) Find the value of the constant a for which $\int_0^\infty f(x) dx = 16$. [2]

4 Do not use a graphic calculator in answering this question.

The complex number z is given by $z = -1 + ic$, where c is a non-zero real number.

Given that $\frac{z^n}{z^*}$ is purely real, find

(i) the possible values of c when $n = 2$, [4]

(ii) the three smallest positive integer values of n when $c = \sqrt{3}$. [5]

5 It is given that $y = \sec 2x$.

(i) Show that $\left(\frac{dy}{dx}\right)^2 = 4y^2(y^2 - 1)$. [3]

(ii) By further differentiation, find the Maclaurin's series for y up to and including the term in x^4 . [5]

(iii) By considering $\sec 2x = \frac{1}{\cos 2x}$, check on the correctness of your answer in part (ii). [3]

Section B: Statistics [60 marks]

6 An unbiased disc has a single dot marked on one side and two dots marked on the other side. A tetrahedral die has faces marked with score of 1, 2, 3, and 4. The probability of getting a score of 1, 2, 3, and 4 is $\frac{1}{5}$, p , $\frac{1}{5}$ and q respectively, where $p, q \in [0, 1]$.

A game is played by throwing the disc and the die together. The random variable S is the sum of the score showing on the die and twice the number of dots showing on the disc.

(i) Find $P(S = 6)$. [2]

Given that $P(S = 4) = \frac{1}{6}$,

(ii) calculate the values of p and q , [2]

(iii) and find the probability distribution of S . [2]

- 7 The masses, in kilograms, of black sea bass fish and red tilapia fish sold in a supermarket are normally distributed. The means and standard deviations of these distributions, and the selling prices, in \$ per kilogram, are given in the following table.

	Mean Mass (kg)	Standard Deviation (kg)	Selling Price (\$ per kg)
Black sea bass fish	1.10	0.20	12
Red tilapia fish	0.55	0.05	9

- (i) Ayden bought 2 black sea bass fish and 3 red tilapia fish. Find the probability that he pays more than \$40. State an assumption needed in your calculation. [4]
- (ii) Five red tilapia fish are randomly chosen. Find the probability that the fifth red tilapia fish is the third red tilapia fish weighing less than half a kilogram. [3]
- 8 The average amount of cholesterol in one standard fillet of raw red snapper from a fish farm is w mg. To lower the cost of operations, the farmer decides to use a cheaper mixture of fish feed. The farmer conducts a test to check if the average amount of cholesterol in one standard fillet of raw red snapper is affected by the change of fish feed. 50 standard fillets of raw red snapper from 50 different fish were taken and the average amount of cholesterol in these fillets is found to be 78.5 mg, with a standard deviation of 2 mg.
- (i) Given that at 5% level of significance, there is insufficient evidence to conclude that the mean amount of cholesterol in one standard fillet of raw red snapper is affected, find the range of possible values of w . [5]
- (ii) State, giving a reason, whether any assumptions about the population are needed in order for the test to be valid. [2]
- 9 A student working on a coding project studies 11-digit quaternary sequences. A quaternary sequence is a sequence formed using the digits 0, 1, 2 or 3. Examples of such sequences are 12030201131, 01122211100, 12321232123 and 00000000000. Find the number of ways that 11-digit quaternary sequences can be formed with
- (i) no restriction, [1]
- (ii) exactly four 0s and four 2s, [3]
- (iii) at least two consecutive digits that are the same. [3]

- 10** For events A and B , it is given that $P(A) = \frac{11}{20}$ and $P(B) = \frac{1}{2}$.

(i) Find the greatest and least possible values of $P(A \cap B)$. [2]

It is given in addition that $P(B | A') = \frac{7}{9}$.

(ii) Find $P(A \cup B)$. [2]

(iii) Determine if A and B are independent events. Justify your answer. [2]

(iv) Given another event C such that $P(C) = \frac{2}{5}$, $P(A \cup B \cup C) = \frac{19}{20}$,
 $P(A \cap B \cap C) = \frac{1}{10}$ and $P(A \cap C) = 2P(B \cap C)$, find $P(A \cap C)$. [3]

- 11** Based on past statistical data, there is a 7% chance that a passenger with reservation for a flight will not show up. In order to maximise revenue, airline companies accept more reservations than the passenger capacity of its planes. State 2 assumptions needed such that the number of passengers who do not show up for a flight may be well modelled by a Binomial distribution. [2]

An airline company operates a flight from Singapore to Maluku on Boeing 737-200 planes, which has capacity of 232 passengers each.

- (i) Find the probability that when 245 reservations are accepted, the flight is overbooked, i.e. there is not enough seats available for the passengers who show up. [2]
- (ii) Find the maximum number of reservations that should be accepted in order to ensure that the probability of overbooking is less than 1%. [3]

This flight operates once daily throughout the year and 245 reservations are accepted for each flight.

- (iii) Find the probability that no flight is overbooked in a week. [2]
- (iv) Taking a year as 52 weeks, estimate the probability that the mean number of flights that is overbooked in each week for the year is not more than 1. [3]

- 12** (i) Sketch a scatter diagram that might be expected when x and y are related approximately as given in each of the cases (A), (B) and (C) below. In each case your diagram should include 5 points, approximately equally spaced with respect to x , and with all x - and y - values positive. The letters p , q , r , s , t and u represent constants.

(A) $y = p + qx^2$, where p is positive and q is negative,

(B) $y = r + se^x$, where r is negative and s is positive,

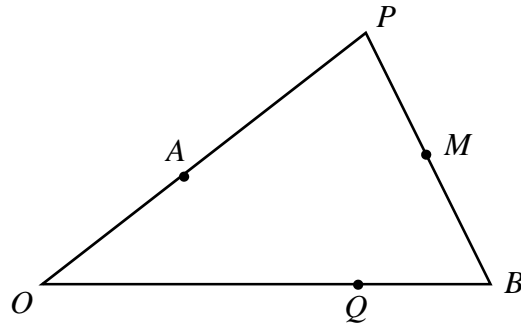
(C) $y = t + \frac{u}{x}$, where t is positive and u is positive. [3]

Daisy enrolled in a weight management programme to reduce her weight. Her weight, y kg at the end of week x of the programme are given in the table.

x	1	2	3	4	5	6	7
y	74.9	72.9	71.6	70.8	70.4	70.2	70.1

- (ii) Draw a scatter diagram to illustrate the data. [2]
- (iii) Explain which of the three cases in part (i) is the most appropriate for modelling these values, and calculate the product moment correlation coefficient for this case. [2]
- (iv) Use the case that you identified in part (iii) to find the equation of a suitable regression line, and use your equation to estimate Daisy's weight at the end of week 10. [3]
- (v) Given that 1 week = 7 days, re-write your equation from part (iv) so that it can be used to estimate the weight when the time period of the programme is given in days. [2]

Q1



$$\overrightarrow{OP} = (\lambda + 1)\mathbf{a}$$

$$\begin{aligned}\overrightarrow{OM} &= \frac{\overrightarrow{OP} + \overrightarrow{OB}}{2} \\ &= \frac{(\lambda + 1)\mathbf{a} + \mathbf{b}}{2}\end{aligned}$$

$$\begin{aligned}\text{area of triangle } OPM &= \frac{1}{2} |\overrightarrow{OP} \times \overrightarrow{OM}| \\ &= \frac{1}{2} \left| (\lambda + 1)\mathbf{a} \times \frac{(\lambda + 1)\mathbf{a} + \mathbf{b}}{2} \right| \\ &= \frac{(\lambda + 1)}{4} |\mathbf{a} \times \mathbf{b}|\end{aligned}$$

$$\begin{aligned}\text{area of triangle } OQM &= \frac{1}{2} |\overrightarrow{OQ} \times \overrightarrow{OM}| \\ &= \frac{1}{2} \left| \frac{3}{4} \mathbf{b} \times \frac{(\lambda + 1)\mathbf{a} + \mathbf{b}}{2} \right| \\ &= \frac{3(\lambda + 1)}{16} |\mathbf{a} \times \mathbf{b}|\end{aligned}$$

Ratio of the area of triangle OPM to the area of triangle OQM is

$$\frac{(\lambda + 1)}{4} : \frac{3(\lambda + 1)}{16} = 4 : 3 \text{ (Shown)}$$

Q2

$$\frac{d}{dx} \left[\cos x \frac{dy}{dx} \right] = \cos x \frac{d^2 y}{dx^2} - \sin x \frac{dy}{dx}$$

$$\frac{d}{dx} \left[\cos x \frac{dy}{dx} \right] = \sec^2 x + \cos 2x$$

$$\cos x \frac{dy}{dx} = \int \sec^2 x + \cos 2x \, dx$$

$$\cos x \frac{dy}{dx} = \tan x + \frac{1}{2} \sin 2x + C$$

$$\frac{dy}{dx} = \sec x \tan x + \sin x + C \sec x$$

$$y = \int \sec x \tan x + \sin x + C \sec x \, dx$$

$$y = \sec x - \cos x + C \ln |\sec x + \tan x| + D$$

Q3**(a)****Method 1**Step 1 : Translate by 1 unit in the direction of the x -axis.Step 2 : reflection about the x -axis.Step 3 : Translate by $\ln 3$ units in the direction of the y -axis.

$$y = \ln(2x+1) \rightarrow y = \ln[2(x-1)+1] \rightarrow y = -\ln(2x-1) \rightarrow y = \ln 3 - \ln(2x-1) = \ln\left(\frac{3}{2x-1}\right)$$

Method 2Step 1 : reflection about the x -axis.Step 2 : Translate by 1 unit in the direction of the x -axis.Step 3 : Translate by $\ln 3$ units in the direction of the y -axis.

$$y = \ln(2x+1) \rightarrow y = -\ln(2x+1) \rightarrow y = -\ln[2(x-1)+1] \rightarrow y = \ln 3 - \ln(2x-1) = \ln\left(\frac{3}{2x-1}\right)$$

Method 3Step 1 : reflection about the x -axis.Step 2 : Translate by $\ln 3$ units in the direction of the y -axis.Step 3 : Translate by 1 unit in the direction of the x -axis.

$$y = \ln(2x+1) \rightarrow y = -\ln(2x+1) \rightarrow y = \ln 3 - \ln(2x+1) \rightarrow y = \ln 3 - \ln[2(x-1)+1] = \ln\left(\frac{3}{2x-1}\right)$$

Method 4

Step 1 : Translate by 1 unit in the direction of the x -axis.

Step 2 : Translate by $-\ln 3$ units in the direction of the y -axis.

Step 3 : reflection about the x -axis.

$$y = \ln(2x+1) \rightarrow y = \ln[2(x-1)+1] \rightarrow y = -\ln 3 + \ln(2x-1) \rightarrow y = \ln 3 - \ln(2x-1) = \ln\left(\frac{3}{2x-1}\right)$$

Method 5

Step 1 : Translate by $-\ln 3$ units in the direction of the y -axis.

Step 2 : Translate by 1 unit in the direction of the x -axis.

Step 3 : reflection about the x -axis.

$$y = \ln(2x+1) \rightarrow y = -\ln 3 + \ln(2x+1) \rightarrow y = -\ln 3 + \ln[2(x-1)+1] \rightarrow y = \ln 3 - \ln(2x-1) = \ln\left(\frac{3}{2x-1}\right)$$

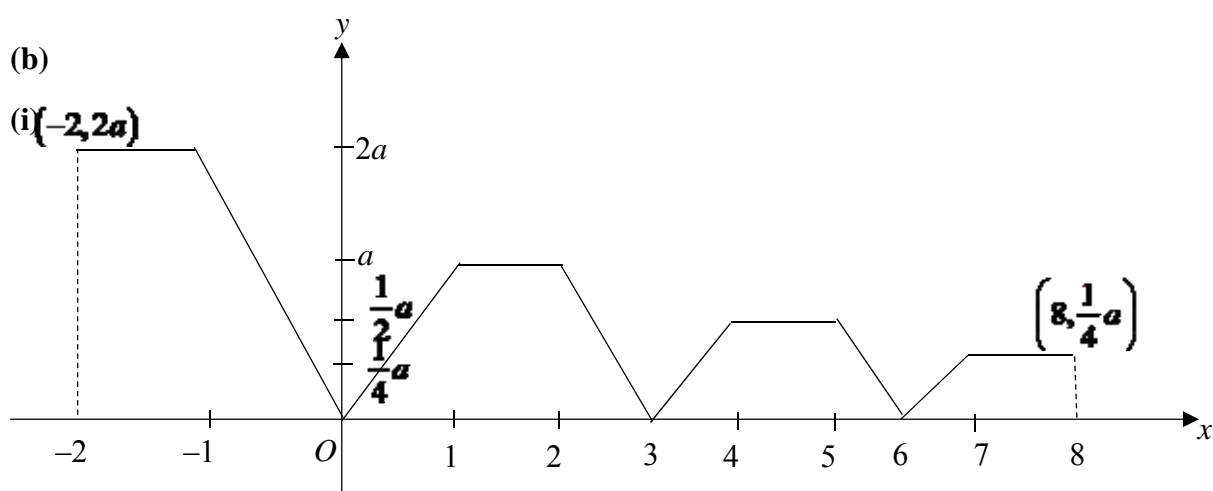
Method 6

Step 1 : Translate by $-\ln 3$ units in the direction of the y -axis.

Step 2 : reflection about the x -axis.

Step 3 : Translate by 1 unit in the direction of the x -axis.

$$y = \ln(2x+1) \rightarrow y = -\ln 3 + \ln(2x+1) \rightarrow y = \ln 3 - \ln(2x+1) \rightarrow y = \ln 3 - \ln[2(x-1)+1] = \ln\left(\frac{3}{2x-1}\right)$$



(ii)

$$\int_0^2 f(-x) dx = \frac{1}{2}(2+1)(2a) = 3a$$

(iii)

$$\int_0^\infty f(x) dx = 16$$

$$2\left(a + \frac{1}{2}a + \frac{1}{4}a + \dots\right) = 16$$

$$2a\left(\frac{1}{1 - \frac{1}{2}}\right) = 16$$

$$4a = 16$$

$$a = 4$$

Q4

(i)

$$\begin{aligned}\frac{z^2}{z^*} &= \frac{(-1+ic)^2}{(-1-ic)} \\ &= \frac{1-i2c-c^2}{(-1-ic)} \\ &= \frac{1-i2c-c^2}{(-1-ic)} \times \frac{(-1+ic)}{(-1+ic)} \\ &= \frac{-1+ic+i2c+2c^2+c^2-ic^3}{1+c^2}\end{aligned}$$

Since $\frac{z^2}{z^*}$ is purely real,

$$\frac{3c-c^3}{1+c^2} = 0$$

$$c(3-c^2) = 0$$

$$c = 0 \text{ (rej since } c \text{ is non-zero)} \quad c = \pm\sqrt{3}$$

(ii)

$$z = -1 + i\sqrt{3}$$

$$|z| = 2, \quad \arg(z) = \frac{2\pi}{3}$$

$$\frac{z^n}{z^*} = \frac{\left(2e^{i\frac{2\pi}{3}}\right)^n}{2e^{i\left(-\frac{2\pi}{3}\right)}} = 2^{n-1} e^{i\left(\frac{2n\pi}{3} + \frac{2\pi}{3}\right)}$$

Since $\frac{z^n}{z^*}$ is purely real,

$$\arg\left(\frac{z^n}{z^*}\right) = 0, \pm\pi, \pm2\pi, \pm3\pi, \pm4\pi, \pm5\pi, \pm6\pi \dots \text{ or } \sin\left[(n+1)\frac{2\pi}{3}\right] = 0$$

$$(n+1)\frac{2\pi}{3} = 0, \pm\pi, \pm2\pi, \pm3\pi, \pm4\pi, \pm5\pi, \pm6\pi \dots$$

$$n+1 = 0, \pm\frac{3}{2}, \pm3, \pm\frac{9}{2}, \pm6, \dots$$

Considering positive integer values only,

$$n+1 = 3, 6, 9 \dots$$

Three smallest positive integer values of n are 2, 5, 8

Q5

(i)

$$y = \sec 2x$$

$$\frac{dy}{dx} = 2 \sec 2x \tan 2x = 2y \tan 2x$$

$$\begin{aligned} \left(\frac{dy}{dx}\right)^2 &= 4y^2 \tan^2 2x \\ &= 4y^2(\sec^2 2x - 1) \\ &= 4y^2(y^2 - 1) \end{aligned}$$

(ii)

$$2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right) = 16y^3 \frac{dy}{dx} - 8y \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = 8y^3 - 4y$$

$$\frac{d^3y}{dx^3} = 24y^2 \frac{dy}{dx} - 4 \frac{dy}{dx}$$

$$\frac{d^4y}{dx^4} = 24y^2 \frac{d^2y}{dx^2} + 48y \left(\frac{dy}{dx}\right)^2 - 4 \frac{d^2y}{dx^2}$$

$$\text{When } x = 0, \quad y = 1, \quad \frac{dy}{dx} = 0, \quad \frac{d^2y}{dx^2} = 4, \quad \frac{d^3y}{dx^3} = 0, \quad \frac{d^4y}{dx^4} = 80$$

$$y = 1 + \frac{x^2}{2!}(4) + \frac{x^4}{4!}(80) + \dots$$

$$y = 1 + 2x^2 + \frac{10}{3}x^4 + \dots$$

(iii)

$$\sec 2x = \frac{1}{\cos 2x}$$

$$\approx \frac{1}{1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!}}$$

$$= (1 - 2x^2 + \frac{2}{3}x^4)^{-1}$$

$$= 1 + (-1)(-2x^2 + \frac{2}{3}x^4) + \frac{(-1)(-2)}{2!} \left(-2x^2 + \frac{2}{3}x^4 \right)^2 + \dots$$

$$= 1 + 2x^2 - \frac{2}{3}x^4 + 4x^4 + \dots$$

$$\approx 1 + 2x^2 + \frac{10}{3}x^4$$

Q6

(i)

		Score on the die			
Dots on the disc	S	1 ($\frac{1}{5}$)	2 (p)	3 ($\frac{1}{5}$)	4 (q)
	1 ($\frac{1}{2}$)	3 ($\frac{1}{10}$)	4 ($\frac{1}{2}p$)	5 ($\frac{1}{10}$)	6 ($\frac{1}{2}q$)
	2 ($\frac{1}{2}$)	5 ($\frac{1}{10}$)	6 ($\frac{1}{2}p$)	7 ($\frac{1}{10}$)	8 ($\frac{1}{2}q$)

Since total probability = 1, $\frac{1}{5} + p + \frac{1}{5} + q = 1 \Rightarrow p + q = \frac{3}{5}$

OR: $\left(\frac{1}{10}\right) + \left(\frac{1}{2}p\right) + \left(\frac{1}{5}\right) + \left(\frac{1}{2}p + \frac{1}{2}q\right) + \left(\frac{1}{10}\right) + \left(\frac{1}{2}q\right) = 1 \Rightarrow p + q = \frac{3}{5}$

Hence $P(S=6) = \frac{1}{2}p + \frac{1}{2}q = \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$.

(ii)

$$P(S=4) = \frac{1}{6} \Rightarrow \frac{1}{2}p = \frac{1}{6} \Rightarrow p = \frac{1}{3}$$

Since $p + q = \frac{3}{5}$, then $q = \frac{3}{5} - \frac{1}{3} = \frac{4}{15}$.

(iii)

s	3	4	5	6	7	8
P(S=s)	$\frac{1}{10}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{10}$	$\frac{2}{15}$

Q7

(i)

 $X \sim$ mass of a black sea bass fish. $X \sim N(1.1, 0.2^2)$ $Y \sim$ mass of a red tilapia fish. $Y \sim N(0.55, 0.05^2)$ Let T be the total cost of 2 black sea bass and 3 red tilapia. Then

$$T = 12(X_1 + X_2) + 9(Y_1 + Y_2 + Y_3)$$

$$E(T) = (12)(2)E(X) + (9)(3)E(Y)$$

$$= 26.4 + 14.85$$

$$= 41.25$$

$$\text{Var}(T) = (12)^2(2)\text{Var}(X) + (9)^2(3)\text{Var}(Y)$$

$$= 11.52 + 0.6075$$

$$= 12.1275$$

Thus $T \sim N(41.25, 12.1275)$.

$$P(T > 40) = 0.64018 \approx 0.640 \quad (3 \text{ s.f.})$$

An assumption needed is the price / mass of all fish are independent of one another.

(ii)

$$\text{Probability required} = \frac{4!}{2!2!} [P(Y > 0.5)]^2 [P(Y < 0.5)]^3 \approx 0.0170$$

Q8

(i)

 $X \sim$ amount of cholesterol in one standard fillet of raw red snapper.

$$\text{unbiased estimate of population variance } \sigma^2 \text{ is } s^2 = \frac{n}{n-1} \sigma_x^2 = \frac{50}{49} (2)^2 = \frac{200}{49}$$

$$\text{Test } H_0: \mu = w \quad \text{vs} \quad H_1: \mu \neq w$$

$$\text{Since } n = 50 \text{ is large, by Central Limit Theorem, } \bar{X} \sim N\left(w, \frac{200}{50}\right) \text{ approx.}$$

$$\bar{X} \sim N\left(w, \frac{4}{49}\right) \text{ approx.}$$

Level of significance: 5 %

Critical region is $z < -1.9600$ or $z > 1.9600$

$$\text{Standardised test statistic: } z = \frac{78.5 - w}{\frac{2}{7}}$$

Since H_0 is not rejected, z lies outside the critical region.

$$-1.9600 < \frac{78.5 - w}{\frac{2}{7}} < 1.9600$$

$$\begin{aligned}
-1.96\left(\frac{2}{7}\right) &< 78.5 - w < 1.96\left(\frac{2}{7}\right) \\
-1.96\left(\frac{2}{7}\right) - 78.5 &< -w < 1.96\left(\frac{2}{7}\right) - 78.5 \\
-79.06 &< -w < -77.94 \\
77.94 &< w < 79.06
\end{aligned}$$

(ii)

No. It is not necessary to assume that amount of cholesterol in a standard fillet follows a normal distribution since sample size is large, by Central Limit Theorem, sample mean is normally distributed approximately.

Q9

(i)

$$\text{No. of ways} = 4^{11} = 4194304$$

(ii)

Case 1: four 0s, four 2s, with one 1, two 3s **or** one 3, two 1s

$$\text{No. of ways} = \frac{11!}{4!4!2!} \times 2 = 69300$$

Case 2: four 0s, four 2s, with three 1s **or** three 3s

$$\text{No. of ways} = \frac{11!}{4!4!3!} \times 2 = 23100$$

Hence the total number of ways is
 $69300 + 23100 = 92400$

Alternative

No. of ways $= {}^{11}C_4 \times {}^7C_4 \times 2^3 = 92400$
 Choose 4 slots from 11 slots to place four 0s. Choose 4 slots from remaining 7 slots to place four 2s. 2 choices (digit 1 or 3) for each of the remaining 3 slots.

(iii)

No. of ways = no. of ways without restriction –

$$\begin{aligned}
&\text{no. of ways with no consecutive digits that are the same} \\
&= 4194304 - 4 \times 3^{10} = 3958108
\end{aligned}$$

Q10

(i)

$$\text{Least value of } P(A \cap B) = P(A) + P(B) - 1 = \frac{11}{20} + \frac{1}{2} - 1 = \frac{1}{20}$$

$$\text{Greatest value of } P(A \cap B) = P(B) = \frac{1}{2}$$

(ii)

$$P(B \cap A') = P(B | A')P(A') = \frac{7}{9} \times \frac{9}{20} = \frac{7}{20}$$

$$P(A \cup B) = P(B \cap A') + P(A) = \frac{7}{20} + \frac{11}{20} = \frac{9}{10}$$

(iii)

$\because P(B|A') = \frac{7}{9} \neq \frac{1}{2} = P(B)$, then B and A' are not independent events.

Hence A and B are not independent events.

OR:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{9}{10} = \frac{11}{20} + \frac{1}{2} - P(A \cap B)$$

$$P(A \cap B) = \frac{3}{20}$$

$$P(A)P(B) = \frac{11}{20} \times \frac{1}{2} = \frac{11}{40}$$

$$P(A \cap B) \neq P(A)P(B)$$

Hence, A and B are not independent events.

(iv)

$$P[C \cap (A \cup B)'] = P(A \cup B \cup C) - P(A \cup B) = \frac{19}{20} - \frac{9}{10} = \frac{1}{20}$$

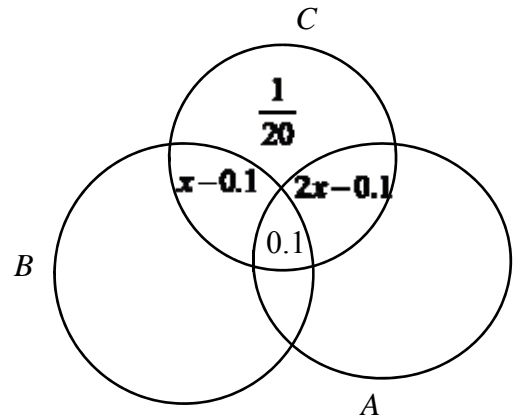
Let $P(B \cap C) = x$, then $P(A \cap C) = 2x$

$$P(C) = \frac{1}{20} + \frac{1}{10} + \left(x - \frac{1}{10}\right) + \left(2x - \frac{1}{10}\right)$$

$$\frac{2}{5} = 3x - \frac{1}{20}$$

$$x = 0.15$$

$$P(A \cap C) = 2x = 0.3$$

**Or**

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$\frac{19}{20} = \frac{11}{20} + \frac{1}{2} + \frac{2}{5} - \frac{3}{20} - 2P(B \cap C) - P(B \cap C) + \frac{1}{10}$$

$$3P(B \cap C) = \frac{9}{20}$$

$$P(B \cap C) = \frac{3}{20}$$

$$P(A \cap C) = \frac{6}{20} = \frac{3}{10}$$

Q11

2 assumptions:

- Occurrence of show / no show is independent among passengers.
- Probability that a passenger does not show up is constant.

(i)

$X \sim$ number of passengers with reservation, who show up, out of 245.

$X \sim B(245, 0.93)$

$P(X > 232) = 1 - P(X \leq 232) = 0.118761 \approx 0.119$ (3 s.f.)

(ii)

$W \sim$ number of passengers with reservation, who show up, out of n .

$W \sim B(n, 0.93)$

$P(W > 232) < 0.01$

$1 - P(W \leq 232) < 0.01$

$P(W \leq 232) > 0.99$

Using GC,

When $n = 239$, $P(W \leq 232) = 0.998 > 0.99$

When $n = 240$, $P(W \leq 232) = 0.995 > 0.99$

When $n = 241$, $P(W \leq 232) = 0.989 < 0.99$

When $n = 242$, $P(W \leq 232) = 0.977 < 0.99$

Hence the maximum reservations that should be accepted is 240.

(iii)

$Y \sim$ number of flights which is overbooked, out of 7.

$Y \sim B(7, 0.118761)$

$P(Y = 0) = 0.41272 \approx 0.413$ (3 s.f.)

(iv)

Since $n = 52$ is large, by CLT,

$\bar{Y} \sim N(E(Y), \frac{\text{Var}(Y)}{52})$ approximately

$\bar{Y} \sim N\left((7)(0.118761), \frac{(7)(0.118761)(0.881239)}{52}\right)$ approximately

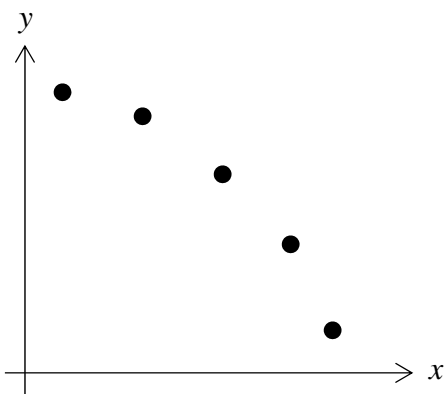
$\bar{Y} \sim N\left(0.831327, \frac{0.732598}{52}\right)$ approximately

$\bar{Y} \sim N(0.831327, 0.014088)$ approximately

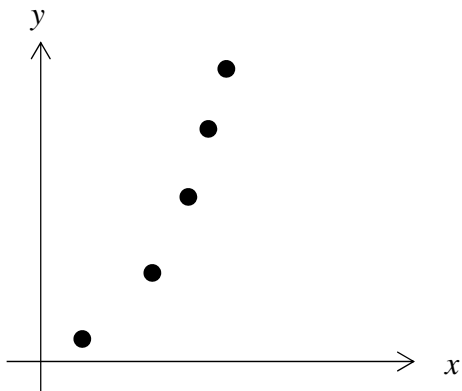
$P(\bar{Y} \leq 1) = 0.9223521 \approx 0.922$ (3 s.f.)

Q12**(i)**

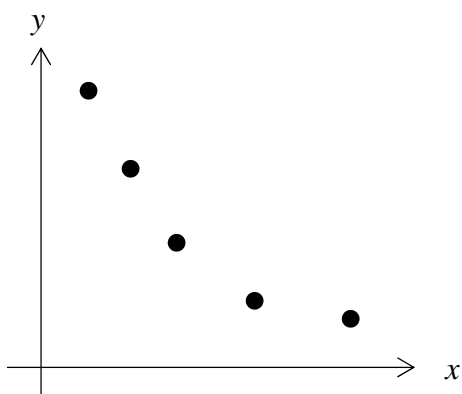
$$y = p + qx^2$$



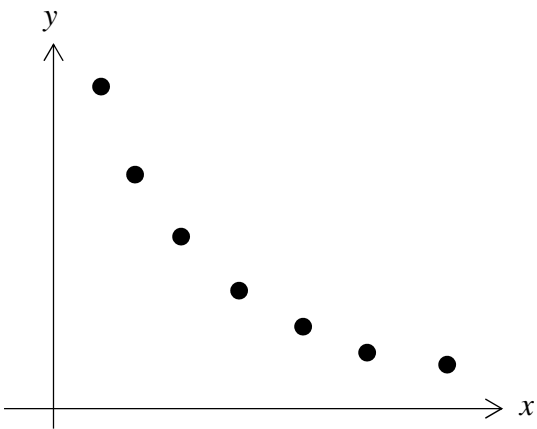
$$y = r + se^x$$



$$y = t + \frac{u}{x}$$



(ii)



(iii)

As x increases, y decreases at a decreasing rate. Hence, model (C) is the most appropriate.
Using GC, $r = 0.984$

(iv)

Equation of regression line: $y = 69.425 + \frac{5.75555}{x} \approx 69.4 + \frac{5.76}{x}$

When $x = 10$,

$$y = 69.425 + \frac{5.75555}{10} = 70.0$$

(iv)

Replace x with $\frac{x}{7}$,

New equation:

$$y = 69.425 + \frac{5.75555}{\frac{x}{7}} \approx 69.4 + \frac{40.3}{x}$$