

Candidate Name: _____

Class: _____



JC2 PRELIMINARY EXAM
Higher 2

MATHEMATICS

Paper 1

9758/01
13 Sept 2017
3 hours

Additional Materials: Cover page
 Answer papers
 List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your full name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

- 1** The first three terms of a sequence are given by $u_1 = 70$, $u_2 = 136$, $u_3 = 198$. Given that u_n is a quadratic polynomial in n , find u_n in terms of n . [4]

- 2** A sequence u_0, u_1, u_2, \dots is given by $u_0 = \frac{3}{2}$ and $u_n = u_{n-1} + 2^n - n$ for $n \geq 1$.

(i) Find u_1, u_2 and u_3 . [3]

(ii) By considering $\sum_{r=1}^n (u_r - u_{r-1})$, find a formula for u_n in terms of n . [5]

- 3** By sketching the graphs of $y = e^{2x}$ and $y = 2e^{-x} - 1$, solve the inequality

$$e^{2x} \geq 2e^{-x} - 1. \quad [3]$$

Hence, without using a calculator, find

$$\int_{-1}^2 |e^{2x} - 2e^{-x} + 1| dx,$$

giving your answer in terms of e . [4]

- 4** The function f is defined by

$$f : x \mapsto \left| \frac{2x+6}{4-x} \right|, \quad x \in \mathbb{R}, \quad x \neq 4.$$

- (i) Sketch the graph of $y = f(x)$, giving the equations of any asymptotes and the coordinates of the points where the curve crosses the axes. Hence state the range of f . [3]
- (ii) Determine whether the function f^2 exists, justifying your answer. [1]
- (iii) The function f^{-1} exists if the domain of f is further restricted to $x \leq k$. State the greatest value of k . [1]
- (iv) Using the domain in (iii), find $y = f^{-1}(x)$ and state the domain of f^{-1} . [4]

- 5** A curve is given parametrically by the equations

$$x = 2t - 1, \quad y = \frac{1}{2t + 1},$$

where $t \in \mathbb{R}$, $t \neq -\frac{1}{2}$.

- (i) Sketch the curve, labelling the axial intercepts and asymptotes. [2]
- (ii) Find the equation of the tangent to the curve at the point $P(-1, 1)$. [3]
- (iii) State the range of values of m for which the line $y = mx$ does not intersect the curve. [1]
- (iv) The normal to the curve at P meets the curve again at Q . Find the coordinates of Q . [4]

- 6** Two expedition teams are to climb a vertical distance of 8500 m from the foot to the peak of a mountain over a period of time.

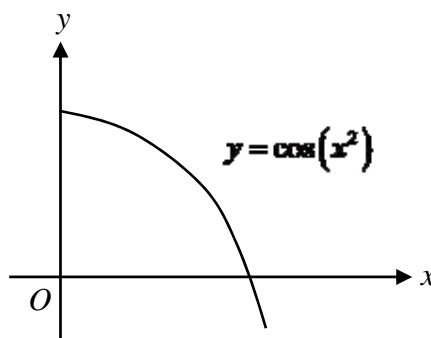
- (i) Team A plans to cover a vertical distance of 400 m on the first day. On each subsequent day, the vertical distance covered is 5 m less than the vertical distance covered in the previous day. Find the number of days required for Team A to reach the peak. [2]
- (ii) Team B plans to cover a vertical distance of 800 m on the first day. On each subsequent day, the vertical distance covered is 90% of the vertical distance covered in the previous day. On which day will Team A overtake Team B? [3]
- (iii) Explain why Team B will never be able to reach the peak. [2]
- (iv) At the end of the 15th day, Team B decided to modify their plan, such that on each subsequent day, the vertical distance covered is 95% of the vertical distance covered in the previous day. Which team will be the first to reach the peak of the mountain? Justify your answer. [5]

7 The curve C has equation $y = 2 + \frac{x-3}{(x-2)(x+1)}$.

- (i) Find algebraically the set of values that y can take. [5]
- (ii) Sketch C , giving the coordinates of the axial intercepts, turning points and equations of any asymptotes. [3]
- (iii) By adding an appropriate graph to the sketch of C , determine the range of values of k such that the equation $(x-2)^2 + \frac{(x-3)^2}{(x-2)^2(x+1)^2} = k^2$ has at least one negative real root. [4]

8 (a) Find $\int \sqrt{\frac{1-x}{x}} dx$ by using the substitution $x = \sin^2 \theta$, where $0 < \theta < \frac{\pi}{2}$. [6]

(b) The diagram below shows a sketch of part of the curve $y = \cos(x^2)$.



Find the exact volume of the solid generated when the region bounded by the curve $y = \cos(x^2)$, the axes and the line $x = \frac{\sqrt{\pi}}{2}$ is rotated through 2π radians about the y -axis. [7]

9

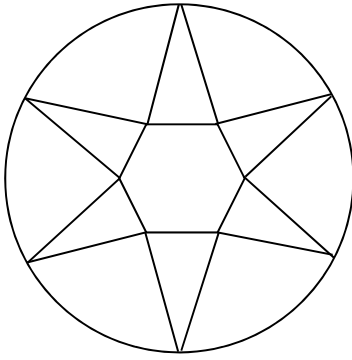


Fig. 1

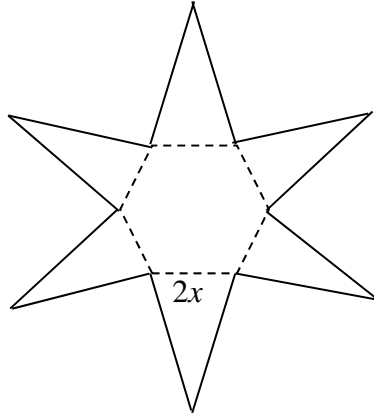


Fig. 2

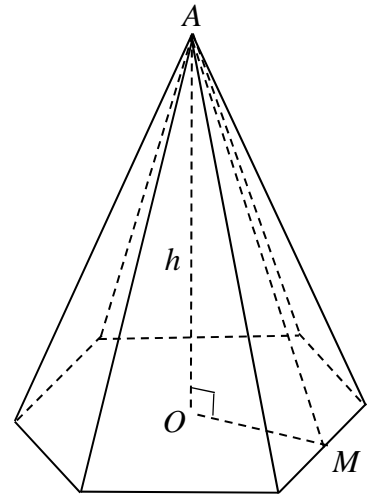


Fig. 3

Fig. 1 shows a piece of circular card of radius 15 cm. A star shape, which consists of a regular hexagon of side $2x$ cm and 6 isosceles triangles, is cut out from the card to give the shape shown in Fig. 2. The remaining card shown in Fig. 2 is folded along the dotted lines to form a pyramid of height h cm as shown in Fig. 3. (The diagrams are not drawn to scale).

- (i) By considering triangle AOM as shown in Fig. 3, where O is the centre of the hexagon and M is the midpoint of a side of the hexagon, show that

$$h^2 = 225 - 30\sqrt{3}x. \quad [3]$$

- (ii) Hence show that the volume V of the pyramid is given by

$$V^2 = 180x^4(15 - 2\sqrt{3}x). \quad [3]$$

- (iii) Use differentiation to find the maximum value of V , proving that it is a maximum. [5]
- (iv) Determine the value of h for which V is maximum. [1]

- 10** The plane p contains the point A with coordinates $(-3, 4, -2)$ and the line l with equation $x + 2 = \frac{4 - y}{3}, z = 0$.

(i) Find a cartesian equation of p . [3]

(ii) Find a vector equation of the line which is a reflection of l in the y -axis. [4]

The line m passes through A and the point $(-9, 9, -6)$.

(iii) Find the acute angle between l and m . [2]

(iv) Find the coordinates of the points on m that are equidistant from p and the x - y plane. [4]

Pioneer Junior College
H2 Mathematics
JC2 H2 Preliminary Examination Paper 1 (Solution)

JC2 2017

Q1

$$u_n = an^2 + bn + c$$

$$u_1 = a(1)^2 + b(1) + c = 70 \quad \Rightarrow \quad a + b + c = 70 \quad (1)$$

$$u_2 = a(2)^2 + b(2) + c = 136 \quad \Rightarrow \quad 4a + 2b + c = 136 \quad (2)$$

$$u_3 = a(3)^2 + b(3) + c = 198 \quad \Rightarrow \quad 9a + 3b + c = 198 \quad (3)$$

Using GC

$$a = -2, \quad b = 72, \quad c = 0$$

$$u_n = -2n^2 + 72n$$

Q2

(i)

$$\begin{array}{lll} u_1 = u_0 + 2 - 1 & u_2 = u_1 + 2^2 - 2 & u_3 = u_2 + 2^3 - 3 \\ = \frac{3}{2} + 2 - 1 & = \frac{5}{2} + 4 - 2 & = \frac{9}{2} + 8 - 3 \\ = \frac{5}{2} & = \frac{9}{2} & = \frac{19}{2} \end{array}$$

(ii)

$$u_n - u_{n-1} = 2^n - n$$

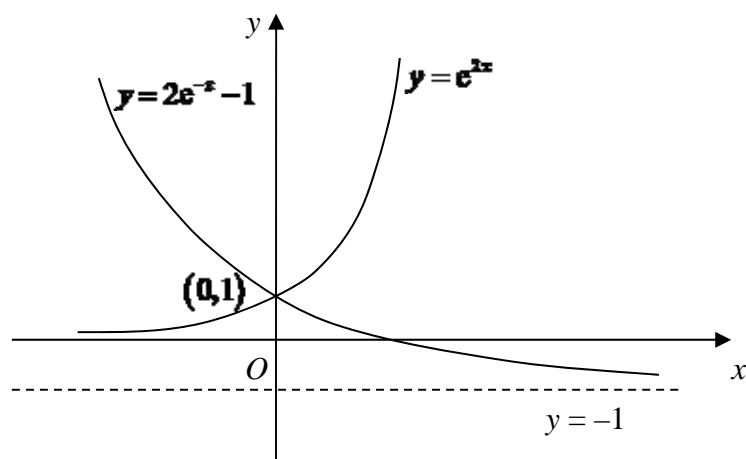
$$\begin{aligned} \sum_{r=1}^n (u_r - u_{r-1}) &= \sum_{r=1}^n 2^r - r \\ &= \sum_{r=1}^n 2^r - \sum_{r=1}^n r \end{aligned}$$

$$\begin{array}{l} u_1 - u_0 \\ + u_2 - u_1 \\ + u_3 - u_2 \\ + \\ \vdots \\ + u_{n-2} - u_{n-3} \\ + u_{n-1} - u_{n-2} \\ + u_n - u_{n-1} \end{array} = \frac{2(1-2^n)}{1-2} - \frac{n(n+1)}{2}$$

$$u_n - u_0 = \frac{2(1-2^n)}{1-2} - \frac{n(n+1)}{2}$$

$$\begin{aligned} u_n &= -2(1-2^n) - \frac{n(n+1)}{2} + \frac{3}{2} \\ &= 2^{n+1} - \frac{1}{2} - \frac{n(n+1)}{2} \end{aligned}$$

Q3



$$\begin{aligned} e^{2x} &\geq 2e^{-x} - 1 \\ x &\geq 0 \end{aligned}$$

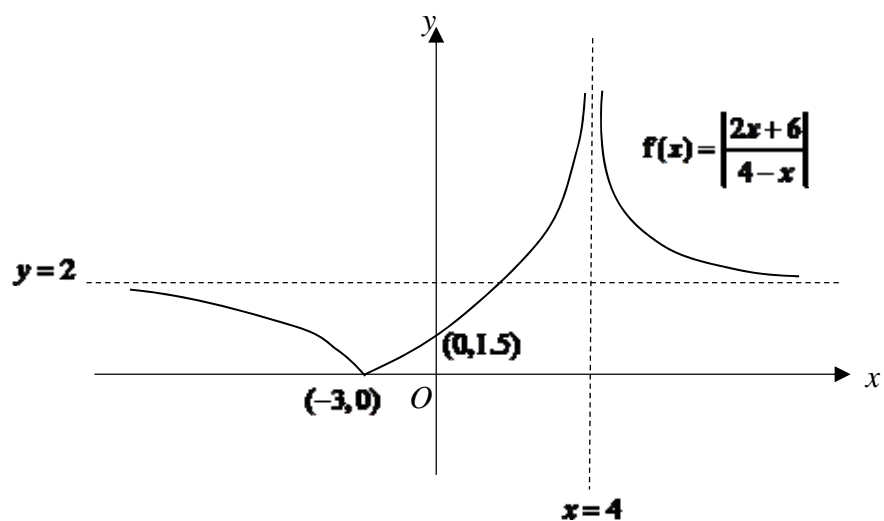
$$\text{For } x \geq 0, \quad e^{2x} \geq 2e^{-x} - 1 \Rightarrow e^{2x} - 2e^{-x} + 1 \geq 0$$

$$\text{For } x < 0, \quad e^{2x} - 2e^{-x} + 1 < 0.$$

$$\begin{aligned} \int_{-1}^2 |e^{2x} - 2e^{-x} + 1| dx &= \int_{-1}^0 -(e^{2x} - 2e^{-x} + 1) dx + \int_0^2 (e^{2x} - 2e^{-x} + 1) dx \\ &= -\left[\frac{1}{2}e^{2x} + 2e^{-x} + x\right]_{-1}^0 + \left[\frac{1}{2}e^{2x} + 2e^{-x} + x\right]_0^2 \\ &= -\left[\left(\frac{1}{2} + 2\right) - \left(\frac{1}{2}e^{-2} + 2e - 1\right)\right] + \left[\left(\frac{1}{2}e^4 + 2e^{-2} + 2\right) - \left(\frac{1}{2} + 2\right)\right] \\ &= \frac{1}{2}e^4 + 2e + \frac{5}{2}e^{-2} - 4 \end{aligned}$$

Q4**(i)**

$$R_f = [0, \infty)$$

**(ii)**

$$R_f = [0, \infty)$$

$$D_f = (-\infty, 4) \cup (4, \infty) \text{ or } D_f = \mathbb{R} \setminus \{4\}$$

$$R_f \not\subset D_f$$

f^2 does not exist.

(iii)

$$k = -3$$

(iv)

$$\text{For } D_f = (-\infty, -3]$$

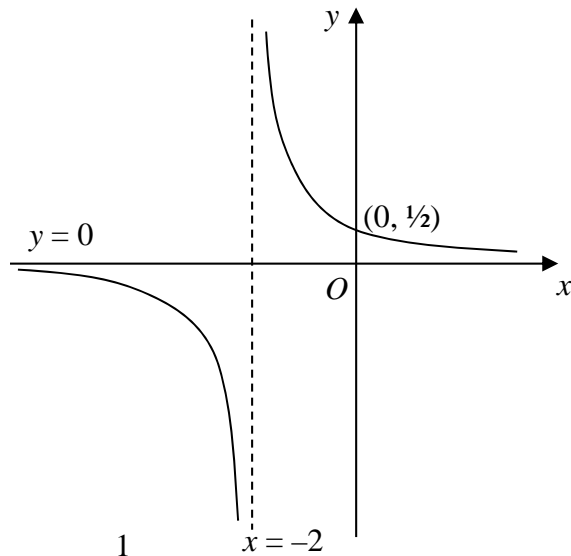
$$y = -\left(\frac{2x+6}{4-x}\right)$$

$$y = \frac{2x+6}{x-4}$$

$$yx - 2x = 6 + 4y$$

$$x = \frac{6+4y}{y-2}$$

$$f^{-1}: x \mapsto \frac{6+4x}{x-2}, \quad x \in \mathbb{R}, \quad 0 \leq x < 2$$

Q5**(i)****(ii)**

$$x = 2t - 1 \quad y = \frac{1}{2t + 1}$$

$$\frac{dx}{dt} = 2 \quad \frac{dy}{dt} = -\frac{2}{(2t + 1)^2}$$

$$\frac{dy}{dx} = -\frac{1}{(2t + 1)^2}$$

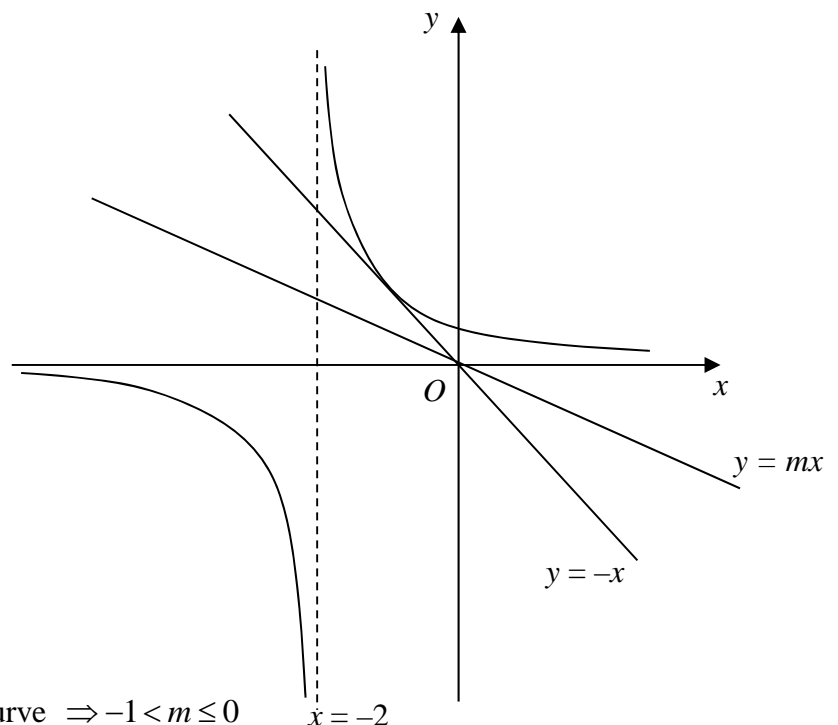
At the point $P(-1, 1)$, $t = 0$

$$\frac{dy}{dx} = -1$$

Equation of tangent at P is

$$y - 1 = -1(x + 1)$$

$$y = -x$$

(iii)The line $y = mx$ does not cut the curve $\Rightarrow -1 < m \leq 0$

(iv)

Gradient of normal at $P = 1$ Equation of normal at P is

$$y - 1 = x - (-1)$$

$$y = x + 2$$

Subst $x = 2t - 1$, $y = \frac{1}{2t+1}$ into $y = x + 2$

$$\frac{1}{2t+1} = 2t - 1 + 2 = 2t + 1$$

$$(2t+1)^2 = 1$$

$$2t+1 = \pm 1$$

$$t = 0 \quad \text{or} \quad t = -1$$

At the point Q , $t = -1$

$$x = 2(-1) - 1 = -3, \quad y = \frac{1}{2(-1)+1} = -1$$

Coordinates of Q are $(-3, -1)$ **Q6**

(i)

AP with $a = 400$, $d = -5$

$$S_n = 8500$$

$$\frac{n}{2} [2(400) + (n-1)(-5)] = 8500$$

$$5n^2 - 805n + 17000 = 0$$

$$n = 25 \text{ or } n = 136 \text{ (rejected as already reached peak when } n = 25 \text{)}$$

(ii)

GP with $a = 800$, $r = 0.9$

$$S_{n(AP)} > S_{n(GP)}$$

$$\frac{n}{2} [2(400) + (n-1)(-5)] > \frac{800(1-0.9^n)}{1-0.9}$$

$$805n - 5n^2 > 16000(1 - 0.9^n)$$

Using GC,

$$n \geq 20$$

A will overtake B on the 20th day.

NORMAL FLOAT AUTO REAL RADIAN MP				
PRESS + FOR Δ Tb1				
X	Y1	Y2		
18	6435	6799.2		
19	6745	6919.3		
20	7050	7027.4		
21	7350	7124.6		
22	7645	7212.2		
23	7935	7291		
24	8220	7361.9		
25	8500	7425.7		
26	8775	7483.1		
27	9045	7534.8		
28	9310	7581.3		
X=18				

(iii)

$$S_{\infty} = \frac{800}{1-0.9} = 8000 (< 8500)$$

Hence, Team B will never be able to reach the peak.

(iv)

$$T_{15} = 800(0.9^{15-1}) = 183.014$$

$$S_{15} = \frac{800(1-0.9^{15})}{1-0.9} = 6352.871$$

$$\text{Remaining distance} = 8500 - 6352.871 = 2147.129$$

$$\text{First term of new GP} = 183.014 \times 0.95 = 173.864$$

$$S_{n(\text{New GP})} = 2147.129$$

$$\frac{173.864(1-0.95^n)}{1-0.95} = 2147.129$$

$$0.95^n = 0.38253$$

$$n = 18.7$$

Team B will take $15 + 19 = 34$ days

Hence, Team A will reach the peak first.

Q7

(i)

Consider the graph of $y = 2 + \frac{x-3}{(x-2)(x+1)}$ and $y = p$ intersecting.

$$p = 2 + \frac{x-3}{(x-2)(x+1)}$$

$$p-2 = \frac{x-3}{x^2-x-2}$$

$$px^2 - px - 2p - 2x^2 + 2x + 4 = x - 3$$

$$(p-2)x^2 + (1-p)x + (7-2p) = 0$$

Discriminant ≥ 0

$$(1-p)^2 - 4(p-2)(7-2p) \geq 0$$

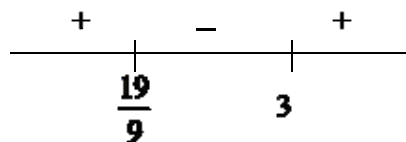
$$1 - 2p + p^2 - 28p + 8p^2 + 56 - 16p \geq 0$$

$$9p^2 - 46p + 57 \geq 0$$

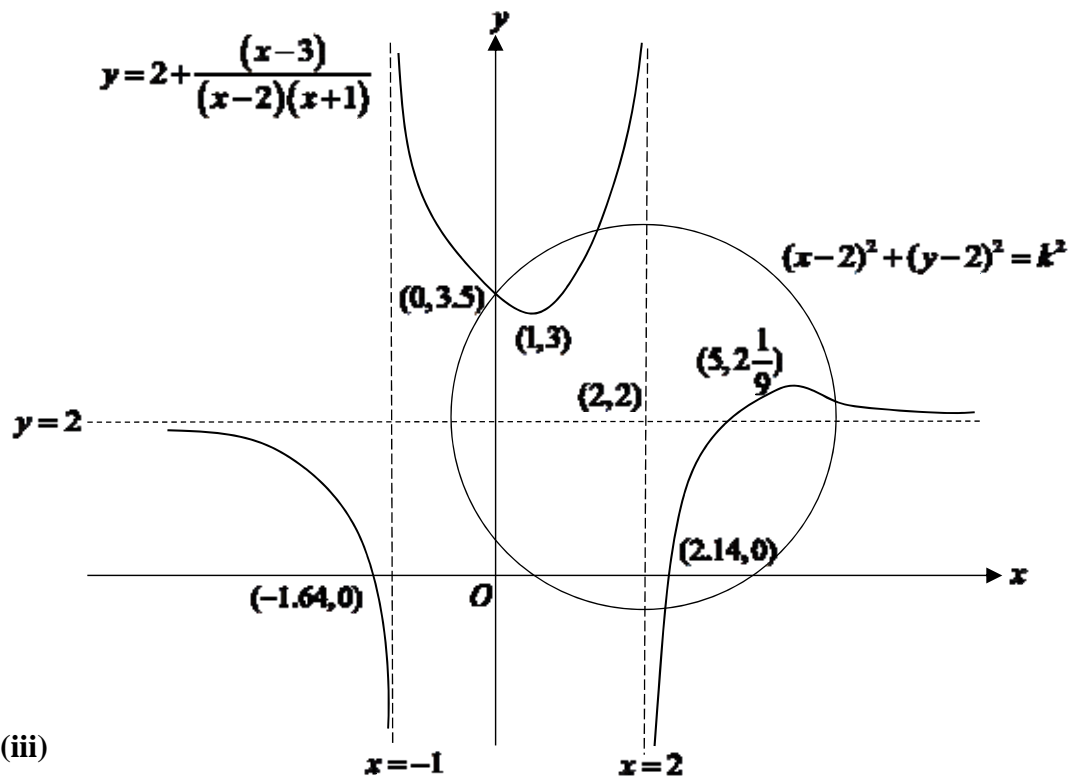
$$(9p-19)(p-3) \geq 0$$

$$p \leq \frac{19}{9} \quad \text{or} \quad p \geq 3$$

$$y \leq 2\frac{1}{9} \quad \text{or} \quad y \geq 3$$



(ii)



(iii)

$$(x-2)^2 + \frac{(x-3)^2}{(x-2)^2(x+1)^2} = k^2$$

$$(x-2)^2 + (y-2)^2 = k^2$$

Distance from centre of circle to the y -intercept of $y = 2 + \frac{(x-3)}{(x-2)(x+1)}$

$$= \sqrt{2^2 + \left(\frac{7}{2} - 2\right)^2} = \frac{5}{2}$$

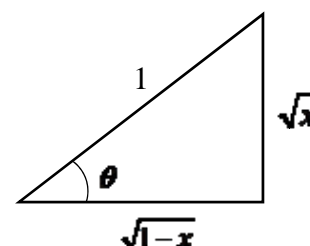
$$k < -2.5 \quad \text{or} \quad k > 2.5$$

Q8**(a)**

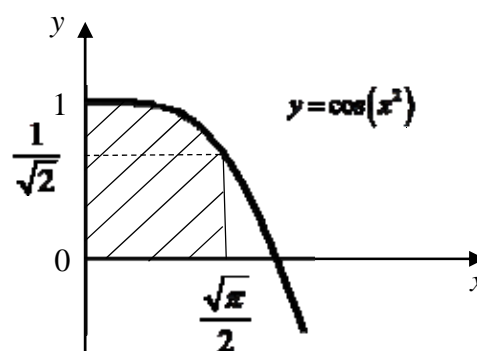
$$\begin{aligned}
& \int \sqrt{\frac{1-x}{x}} dx \\
&= \int \sqrt{\frac{1-\sin^2 \theta}{\sin^2 \theta}} 2 \sin \theta \cos \theta d\theta \\
&= \int 2 \cos^2 \theta d\theta \\
&= \int (1 + \cos 2\theta) d\theta \\
&= \theta + \frac{1}{2} \sin 2\theta + C \\
&= \theta + \sin \theta \cos \theta + C \\
&= \sin^{-1}(\sqrt{x}) + \sqrt{x(1-x)} + C
\end{aligned}$$

$$\begin{aligned}
x &= \sin^2 \theta \\
\frac{dx}{d\theta} &= 2 \sin \theta \cos \theta
\end{aligned}$$

Since $\sqrt{x} = \sin \theta$
 Consider a right angle triangle
 or use trigo identity
 $\cos^2 \theta + \sin^2 \theta = 1$

**(b)**

$$\begin{aligned}
y &= \cos(x^2) \\
x=0 &\Rightarrow y=1 \\
x=\frac{\sqrt{\pi}}{2} &\Rightarrow y=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}} \\
\text{Required volume} &= \pi \left(\frac{\sqrt{\pi}}{2} \right)^2 \left(\frac{1}{\sqrt{2}} \right) + \pi \int_{\frac{1}{\sqrt{2}}}^1 \cos^{-1} y dy \\
&= \frac{\pi^2}{4\sqrt{2}} + \pi \int_{\frac{1}{\sqrt{2}}}^1 \cos^{-1} y dy
\end{aligned}$$



$$\begin{aligned}
& \int \cos^{-1} y dy \\
&= y \cos^{-1} y - \int -\frac{y}{\sqrt{1-y^2}} dy \\
&= y \cos^{-1} y - \frac{1}{2} \int \frac{-2y}{\sqrt{1-y^2}} dy \\
&= y \cos^{-1} y - \frac{1}{2} [2(1-y^2)^{\frac{1}{2}}] + c \\
&= y \cos^{-1} y - \sqrt{1-y^2} + c
\end{aligned}$$

$$\text{Required volume} = \frac{\pi^2}{4\sqrt{2}} + \pi \left[y \cos^{-1} y - \sqrt{1-y^2} \right]_{\frac{1}{\sqrt{2}}}^1$$

$$\text{Let } u = \cos^{-1} y$$

$$\frac{dy}{dy} = 1$$

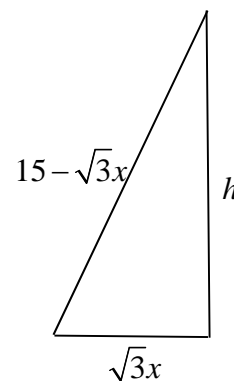
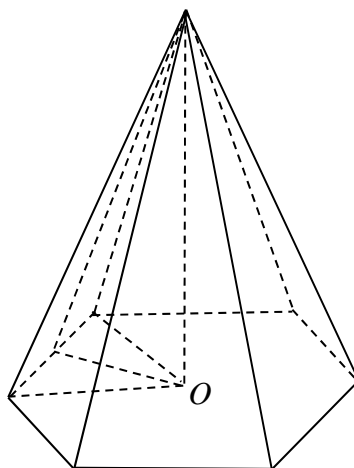
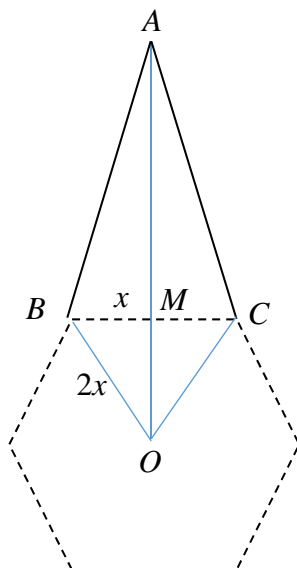
$$\frac{du}{dy} = -\frac{1}{\sqrt{1-y^2}}$$

$$v = y$$

$$= \frac{\pi^2}{4\sqrt{2}} + \pi \left[0 - \left(\frac{1}{\sqrt{2}} \frac{\pi}{4} - \frac{1}{\sqrt{2}} \right) \right]$$

$$= \frac{\pi}{\sqrt{2}}$$

Q9



$$OM = \sqrt{(2x)^2 - x^2} = \sqrt{3}x$$

$$\therefore AM = 15 - \sqrt{3}x$$

Let h cm be the height of the pyramid.

$$h^2 = (15 - \sqrt{3}x)^2 - (\sqrt{3}x)^2$$

$$= 225 - 30\sqrt{3}x + 3x^2 - 3x^2$$

$$= 225 - 30\sqrt{3}x \quad (\text{shown})$$

Area of hexagon = $6 \times$ area of triangle OBC

$$= 6 \left(\frac{1}{2} \right) (2x)(\sqrt{3}x)$$

$$= 6\sqrt{3}x^2$$

$$\therefore V = \frac{1}{3} (6\sqrt{3}x^2) \sqrt{225 - 30\sqrt{3}x}$$

$$V^2 = 180x^4 (15 - 2\sqrt{3}x) \quad (\text{shown})$$

$$V^2 = 180(15x^4 - 2\sqrt{3}x^5)$$

Differentiating wrt x ,

$$2V \frac{dV}{dx} = 180(60x^3 - 10\sqrt{3}x^4)$$

$$= 1800x^3(6 - \sqrt{3}x)$$

$$\frac{dV}{dx} = 0 \Rightarrow x = 0 \quad \text{or} \quad x = \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

(NA as $x > 0$)

Alternatively,

$$V = 6\sqrt{3}x^2 \sqrt{15 - 2\sqrt{3}x}$$

$$\frac{dV}{dx} = (6\sqrt{3}x^2) \frac{1}{2} (15 - 2\sqrt{3}x)^{-\frac{1}{2}} (-2\sqrt{3})$$

$$+ (15 - 2\sqrt{3}x)^{\frac{1}{2}} (12\sqrt{3}x)$$

$$= 12\sqrt{3}x(15 - 2\sqrt{3}x)^{-\frac{1}{2}} - 6\sqrt{15}x^2(15 - 2\sqrt{3}x)^{-\frac{1}{2}}$$

$$= 6\sqrt{3}x(15 - 2\sqrt{3}x)^{-\frac{1}{2}} [2(15 - 2\sqrt{3}x) - \sqrt{3}x]$$

$$= \frac{30\sqrt{3}x(6 - \sqrt{3}x)}{\sqrt{(15 - 2\sqrt{3}x)}}$$

$$\frac{dV}{dx} = 0 \Rightarrow x = 0 \quad \text{or} \quad x = \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

(NA as $x > 0$)

To Prove Maximum

Method 1

$$2V \frac{d^2V}{dx^2} + 2 \left(\frac{dV}{dx} \right)^2 = 180 [180x^2 - 40\sqrt{3}x^3]$$

$$x = 2\sqrt{3}, \frac{d^2V}{dx^2} = \frac{180}{2V} [180(2\sqrt{3})^2 - 40\sqrt{3}(2\sqrt{3})^3] = -\frac{64800}{V} < 0 \text{ since } V > 0$$

Method 2

x	3.4	$2\sqrt{3} = 3.46$	3.5
$\frac{dV}{dx}$	$\approx \frac{7855}{2V} > 0$	0	$\approx -\frac{4799}{2V} < 0$

V is maximum when $x = 2\sqrt{3}$ cm.

$$\text{Max } V = 72\sqrt{15} \text{ cm}^3.$$

(iv)

$$\text{When } x = 2\sqrt{3}, h^2 = 225 - 30\sqrt{3}(2\sqrt{3}) = 45$$

$$h = 3\sqrt{5} \text{ cm (reject } h = -3\sqrt{5} \text{ as } h > 0)$$

Q10

(i)

$$l: x + 2 = \frac{4-y}{3}, z = 0$$

$$l: \mathbf{r} = \begin{pmatrix} -2 \\ 4 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} \quad \lambda \in \mathbb{R}$$

$$\begin{pmatrix} -2 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -6 \\ -2 \\ 3 \end{pmatrix}$$

$$r \cdot \begin{pmatrix} 6 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 2 \\ -3 \end{pmatrix} = -4$$

$$p: 6x + 2y - 3z = -4$$

(ii)To find intersection between y-axis and l , sub $x = 0$ into l

$$0 + 2 = \frac{4 - y}{3} \Rightarrow y = -2$$

Thus, point of intersection is $(0, -2, 0)$.Point of reflection of $(-2, 4, 0)$ about y-axis is $(2, 4, 0)$

$$\begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$$

$$\text{Line of reflection, } l': \mathbf{r} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} \quad s \in \mathbb{R}$$

(iii)

$$\begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -6 \\ 5 \\ -4 \end{pmatrix}$$

$$\left| \begin{pmatrix} -6 \\ 5 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} \right| = \left| \begin{pmatrix} -6 \\ 5 \\ -4 \end{pmatrix} \right| \left| \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} \right| \cos \theta$$

$$\cos \theta = \frac{21}{\sqrt{(-6)^2 + 5^2 + (-4)^2} \sqrt{1^2 + 3^2}} = \frac{21}{\sqrt{770}}$$

$$\theta = 40.8^\circ$$

(iv)Let the point that is equidistant from both planes be C .

$$\overrightarrow{OC} = \begin{pmatrix} -3 \\ 4 \\ -2 \end{pmatrix} + t \begin{pmatrix} 6 \\ -5 \\ 4 \end{pmatrix} \text{ for some } t \in \mathbb{R}$$

Distance of C from p = Distance of C from x -y plane

$$\frac{\left| \begin{bmatrix} -3+6t \\ 4-5t \\ -2+4t \end{bmatrix} - \begin{pmatrix} -3 \\ 4 \\ -2 \end{pmatrix} \right| \cdot \begin{pmatrix} 6 \\ 2 \\ -3 \end{pmatrix} \right|}{\sqrt{6^2 + 2^2 + 3^2}} = \frac{\left| \begin{bmatrix} -3+6t \\ 4-5t \\ -2+4t \end{bmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right| \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right|}{\sqrt{0^2 + 0^2 + 1^2}}$$

$$\frac{|36t - 10t - 12t|}{7} = |-2 + 4t|$$

$$|t| = |2t - 1|$$

$$t^2 = 4t^2 - 4t + 1$$

$$3t^2 - 4t + 1 = 0$$

$$t = 2t - 1 \quad \text{or} \quad t = -2t + 1$$

$$t = 1 \text{ or } t = \frac{1}{3}$$

$$\overrightarrow{OC} = \begin{pmatrix} -3 \\ 4 \\ -2 \end{pmatrix} + (1) \begin{pmatrix} 6 \\ -5 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \quad \text{or} \quad \overrightarrow{OC} = \begin{pmatrix} -3 \\ 4 \\ -2 \end{pmatrix} + \left(\frac{1}{3}\right) \begin{pmatrix} 6 \\ -5 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ 7 \\ -2 \end{pmatrix}$$

The 2 points are $(3, -1, 2)$ and $\left(-1, \frac{7}{3}, -\frac{2}{3}\right)$.