<table>
<tr><td>1</td><td>

(i)

$w^5 = 243$

$\quad = 243e^{i0}$

$\quad = 243e^{i(0+2k\pi)}$

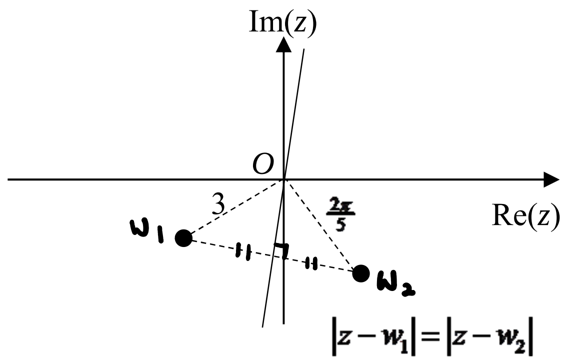
$\quad = 243e^{i(2k\pi)}$

$w = 3e^{i\left(\frac{2k\pi}{5}\right)}$, where $k = -2, -1, 0, 1, 2$

$\quad = 3e^{-i\frac{4\pi}{5}}, 3e^{-i\frac{2\pi}{5}}, 3, 3e^{i\frac{2\pi}{5}}, 3e^{i\frac{4\pi}{5}}$

(ii)

$w_1 = 3e^{-i\frac{4\pi}{5}}$ and $w_2 = 3e^{-i\frac{2\pi}{5}}$



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<tr><td>2</td><td>

(i)

$f : x \mapsto 3 + \frac{1}{x-2}, \; x \in \mathbb{R}, \; x > 2$

Let $y = f(x)$.

$y = 3 + \frac{1}{x-2}$

$x - 2 = \frac{1}{y-3}$

$x = 2 + \frac{1}{y-3}$

$\therefore \; f^{-1}(x) = 2 + \frac{1}{x-3}, \; x \in \mathbb{R}, \; x > 3$

(ii)

$D_f = (2, \infty)$

$R_f = (3, \infty)$

Since $R_f \subseteq D_f$, the composite function f^2 exists.

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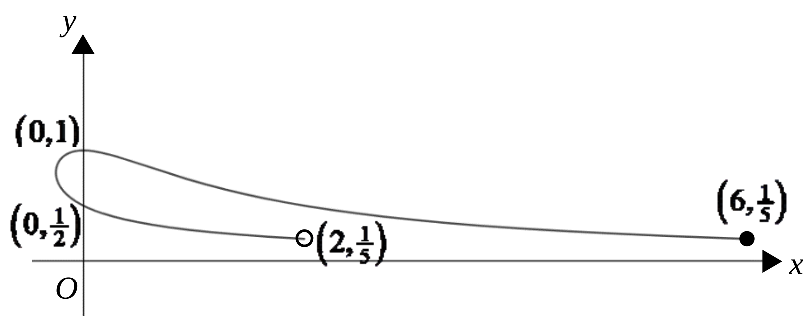
	(iii) $$\mathrm{f}^2(x) = x$$ $$\mathrm{f}\left(3+\frac{1}{x-2}\right) = x$$ $$3+\frac{1}{3+\frac{1}{x-2}-2} = x$$ $$3+\frac{1}{\left(\frac{x-1}{x-2}\right)} = x$$ $$\frac{3(x-1)+(x-2)}{x-1} = x$$ $$4x-5 = x(x-1)$$ $$x^2-5x+5=0$$ Using GC, $x = 1.38$ (rej $\because 1.38 \notin \mathrm{D_f}$) or $x = 3.62$ $\mathrm{ff}(x) = x$ $\mathrm{f}^{-1}\mathrm{ff}(x) = \mathrm{f}^{-1}(x)$ $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ Therefore $x = 3.62$ satisfies $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$.
3	(i)  When $x = 0$, $t(t-1) = 0$ $\Rightarrow t = 0$ or $t = 1$ $\Rightarrow y = 1$ or $y = \frac{1}{2}$ Coordinates are $(0,1)$ and $\left(0, \frac{1}{2}\right)$.

	(ii) $\frac{dx}{dt} = 2t - 1, \quad \frac{dy}{dt} = \frac{-2t}{\left(t^2+1\right)^2}$ $\therefore \ \frac{dy}{dx} = \frac{-2t}{\left(t^2+1\right)^2} \times \frac{1}{2t-1}$ $= \frac{-2t}{\left(t^2+1\right)^2 (2t-1)}$ When tangent is parallel to y-axis, $\left(t^2+1\right)^2 (2t-1) = 0 \Rightarrow t = \frac{1}{2} \qquad \left(\because \ \left(t^2+1\right)^2 > 0\right)$ Equation of tangent: $x = -\frac{1}{4}$ (iii) Area of the required region $= \int_{-1/4}^{0} y \ dx$ $= \int_{1/2}^{1} \frac{1}{t^2+1}(2t-1) \, dt$ $= \int_{1/2}^{1} \frac{2t}{t^2+1} - \frac{1}{t^2+1} \, dt$ $= \left[\ln\left(t^2+1\right) - \tan^{-1} t\right]_{1/2}^{1}$ $= \left[\left(\ln 2 - \frac{\pi}{4}\right) - \left(\ln\frac{5}{4} - \tan^{-1}\frac{1}{2}\right)\right]$ $= \ln\frac{8}{5} - \frac{\pi}{4} + \tan^{-1}\frac{1}{2}$ When $x = -\frac{1}{4}, \ t = \frac{1}{2}$ When $x = 0, \ t = 1$
4	(a)(i) Area of **unsown** ploughed land $= 0.4\left[0.4(300) + 100\right]$ $= 88 \text{ m}^2$

(a)(ii)

n	Beginning of week	End of week
1	300	$0.4(300)$
2	$0.4(300)+100$	$0.4\left[0.4(300)+100\right]$ $=0.4^2(300)+0.4(100)$
3	$0.4^2(300)+0.4(100)$ $+100$	$0.4\left[0.4^2(300)+0.4(100)+100\right]$ $=0.4^3(300)+0.4^2(100)+0.4(100)$
..	…	…
n	…	$0.4^n(300)+0.4^{n-1}(100)+...$ $+0.4^2(100)+0.4^1(100)$

Area of land **unsown** ploughed land at the end of nth week

$$=0.4^n(300)+100\left[\frac{0.4\left(1-0.4^{n-1}\right)}{1-0.4}\right]$$

$$=\left[0.4^n(300)+\frac{200}{3}\left(1-0.4^{n-1}\right)\right]\ \text{m}^2$$

$\therefore$ the value of k is $\dfrac{200}{3}$.

(a)(iii)

Method 1

$$0.4^n(300)+\frac{200}{3}\left(1-0.4^{n-1}\right)<70$$

$$0.4^n(300)+\frac{200}{3}-\frac{200}{3}(0.4)^{-1}0.4^n<70$$

$$\frac{400}{3}\left(0.4^n\right)<\frac{10}{3}$$

$$0.4^n<\frac{1}{40}$$

$$n>\ln\left(\tfrac{1}{40}\right)\Big/\ln 0.4$$

$$n>4.02588$$

Hence the number of complete weeks required is 5.

Method 2

$$0.4^n(300)+\frac{200}{3}\left(1-0.4^{n-1}\right)<70$$

Using GC,
when $n = 4$, unsown ploughed land = 70.08 (> 70)
when $n = 5$, unsown ploughed land = 68.032 (< 70)
when $n = 6$, unsown ploughed land = 67.213 (< 70)

Hence the number of complete weeks required is 5.

(b)(i)

n	Beginning of week	End of week
1	300	$300-80$
2	$300+(100)-80$	$300+(100)-80-100$
3	$300+2(100)-80-100$	$300+2(100)-80-100-120$
..	$\dots$	$\dots$
n	$\dots$	$300+(n-1)(100)-80-100$ $-\ \cdots\ -\left[80+20(n-1)\right]$

Area of **unsown** ploughed land at the end of nth week

$=300+100(n-1)-\frac{n}{2}\left[2(80)+20(n-1)\right]$

$=300+100n-100-\frac{n}{2}(140+20n)$

$=300+100n-100-70n-10n^2$

$=-10n^2+30n+200$

(b)(ii)
For the farmer to finish sowing all the ploughed farmland,
$-10n^2+30n+200\le 0$

Method 1:
Solving the inequality,
$n\ge 6.21699$ or $n\le -3.21699$ (rejected)
Hence the number of complete weeks is 7.

Method 2:
Using GC to set up a table,
When $n=6$, area unsown $=20$
When $n=7$, area unsown $=-80$
When $n=8$, area unsown $=-200$
Hence the number of complete weeks is 7.

In week 6, the area of **unsown** ploughed land
$=-10(6)^2+30(6)+200=20\ \text{m}^2$
$\therefore$ area of ploughed land to be **sown** in week 7 (the final week)
$=20+100=120\ \text{m}^2$

5	(i) Number of arrangements = $6!\times 2^6 = 46080$ (ii) Required probability $=\dfrac{{}^6C_5\times(5-1)!\times 2}{{}^{12}C_{10}\times(10-1)!}$ $=\dfrac{288}{23950080}$ $=0.0000120$ (3 sig fig)
6	(i) P(Clark wins in 3rd draw) $=\dfrac{7}{9}\times\dfrac{7}{9}\times\dfrac{7}{9}\times\dfrac{7}{9}\times\dfrac{2}{9}$ $=0.081322$ $=0.0813$ (ii) P(Kara wins) $=\dfrac{7}{9}\times\dfrac{2}{9}+\left(\dfrac{7}{9}\right)^3\times\dfrac{2}{9}+\left(\dfrac{7}{9}\right)^5\times\dfrac{2}{9}+....$ $=\dfrac{2}{9}\left[\dfrac{7}{9}+\left(\dfrac{7}{9}\right)^3+\left(\dfrac{7}{9}\right)^5+...\right]$ $=\dfrac{2}{9}\left(\dfrac{\frac{7}{9}}{1-\left(\frac{7}{9}\right)^2}\right)$ $=0.4375$ *or* $\dfrac{7}{16}$
7	(i) Let Y be the number of calls received by the office in a t-minute period. $Y \sim \text{Po}(0.4t)$ Given: $\text{P}(Y=0)=0.1$ Using GC, $t=6$ (nearest minute) (ii) Let T be the number of calls received by the office in a 2-hour period. $T \sim \text{Po}(48)$ Since $\text{E}(T)=48>10$, therefore $T \sim \text{N}(48, 48)$ approximately. $\text{P}(T\le 50)=\text{P}(T\le 50.5)$ (with continuity corrections) $=0.641$ (3 sig fig)

	(iii) The average number of calls may not increase at a constant rate over longer periods of time interval. The calls that arrived may not be independent of one another, because the calls might be made by the people who witness the same accident at a particular location.
8	(i) Whether a randomly chosen patient turns up for an appointment is independent of any other patient. (ii) Let X be the number of patients who turn up for their appointments, out of 20 appointments. $X \sim \mathrm{B}(20, 0.845)$ $\mathrm{P}(X > 15)$ $= 1 - \mathrm{P}(X \le 15)$ $= 0.812$ (3 sig fig) (iii) Required probability $= \mathrm{P}(X \le 17 \mid X \ge 12)$ $= \dfrac{\mathrm{P}(12 \le X \le 17)}{\mathrm{P}(X \ge 12)}$ $= \dfrac{\mathrm{P}(X \le 17) - \mathrm{P}(X \le 11)}{1 - \mathrm{P}(X \le 11)}$ $= 0.618$ (3 sig fig) (iv) Let A be the number of appointments for which the patients fail to turn up, out of 300 appointments. $A \sim \mathrm{B}(300, 0.155)$ Since n = 300 is large, $np = 46.5 > 5$ and $nq = 253.5 > 5$, therefore $A \sim \mathrm{N}(46.5, 39.2925)$ approximately. $\mathrm{P}(40 \le A \le 50)$ $= \mathrm{P}(39.5 \le A \le 50.5)$ (by continuity corrections) $= 0.606$ (3 sig fig)
9	(i)(a) Given: $L \sim \mathrm{N}(35.2, 5.2^2)$ $P \sim \mathrm{N}(24.6, 3.8^2)$ $C \sim \mathrm{N}(29.3, 4.3^2)$ Let $T = 3L + 2P$. $\mathrm{E}(T) = 3 \times 35.2 + 2 \times 24.6 = 154.8$ $\mathrm{Var}(T) = 3^2 \times 5.2^2 + 2^2 \times 3.8^2 = 301.12$ $\therefore T \sim \mathrm{N}(154.8, 301.12)$

Let a be the required score exceed by 1% of the candidates.

$\mathrm{P}(T > a) = 0.01$

$\Rightarrow \ \mathrm{P}(T \le a) = 0.99$

Using GC, $a = 195.2$ (1 dec pl)

(i)(b)

Required probability

$= [\mathrm{P}(T > 150)]^3 \, [\mathrm{P}(T < 140)]^2 \times \left(\dfrac{5!}{2!3!}\right)$

$= 0.0875$ (3 sig fig)

(ii)

Consider $A = 3L + 2P - 5C$

$\mathrm{E}(A) = 154.8 - 5(29.3) = 8.3$

$\mathrm{Var}(A) = 301.12 + 5^2 \left(4.3^2\right) = 763.37$

$\therefore \ A \sim \mathrm{N}(8.3 \ , 763.37)$

Required probability

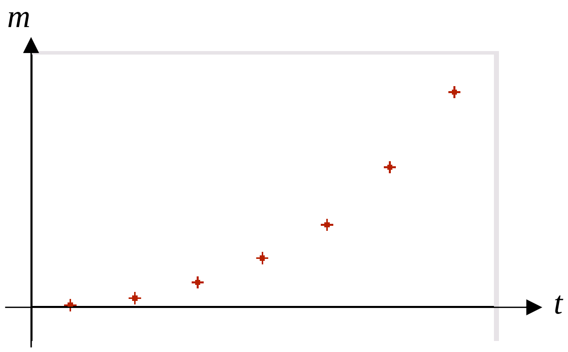
$= \mathrm{P}(|A| < 25)$

$= \mathrm{P}(-25 < A < 25)$

$= 0.613$ (3 sig fig)

Required percentage = 61.3%

10 (i)



(ii)

The product moment correlation coefficient between t and m is $r = 0.94597$ (5 d.p.).

A value of 0.94597 for r suggests that there is a strong positive linear correlation between t and m. However, the points on the scatter diagram **show a curvilinear relationship**. Therefore this value of r does not necessarily mean that the linear model is best model for the relationship between t and m.

	(iii) $m = at^{b}$ $\ln m = \ln\left(at^{b}\right)$ $\ln m = b\ln t + \ln a$ The product moment correlation coefficient between ln t and ln m is $r = 0.98967 = 0.990$ (3 sig fig) **Reason 1:** From the scatter diagram, as t increases, the **weight of the foetus increases at an increasing rate**. **Reason 2:** The value of r between ln t and ln m is 0.98967, which is closer to 1 as compared to that between t and m, hence indicating a **stronger positive linear correlation** between ln t and ln m. Hence $m = at^{b}$ is a better model. (iv) From GC, $\ln m = -8.3764 + 4.5938\ \ln t$ (5 sig fig) $\ln a = -8.3764$ and $b = 4.59$ $a = 2.30 \times 10^{-4}$ (v) When $t = 26$, $\ln m = -8.3764 + 4.5938 \ln 26$ $m = 728$ (nearest grams) Since the value of 26 is within the range of values of t and the value of r is close to 1, this estimate is reliable.
11	(i) Let X be the random variable denoting the mass of strawberry jam, in grams, in a randomly chosen jar. Unbiased estimate of population mean $\bar{x} = \frac{-66}{30} + 200 = 197.8$ Unbiased estimate of population variance $s^2 = \frac{1}{29}\left[958 - \frac{(-66)^2}{30}\right] = 28.02759$ $H_0 : \mu = 200$ $H_1 : \mu < 200$ Test at 2% significance level Assume H_0 is true. $\bar{X} \sim N\left(200, \frac{28.02759}{30}\right)$

Test statistic: $Z = \dfrac{\overline{X} - 200}{\sqrt{28.02759/30}} \sim \mathrm{N}(0,1)$

Using GC, p-value = 0.011420121 < 0.02

Reject H_0 and conclude that there is sufficient evidence at 2% level of significance that the mean mass of strawberry jam in each jar is overstated. Therefore the retailer's suspicion is justifiable.

(ii)

At 2% significance level means that there is a probability of 0.02 that **<u>the test will indicate</u>** that the mean mass of the strawberry jam in the jar is less than 200 g when in fact it is 200 g.

(iii)

$H_0 : \mu = 200$

$H_1 : \mu \neq 200$

For a two tailed test, the p-value will be twice of 0.0114 which is 0.0228. This value is now more than the 0.02 where we do not reject H_0 at 2% significance level. As such this will result in a different conclusion.

(iv)

$H_0 : \mu = 200$

$H_1 : \mu \neq 200$

Test at 2% significance level

Assume H_0 is true. $\overline{X} \sim \mathrm{N}\left(200, \dfrac{3.5^2}{20}\right)$.

Test statistic: $Z = \dfrac{\overline{X} - 200}{\sqrt{3.5^2/20}} \sim \mathrm{N}(0,1)$

For the retailer's suspicion that the mean mass differs from 200 g to be not justified, **do not reject H_0.**

$\Rightarrow$ z-value falls outside the critical region

$-2.32635 < z\text{-value} < 2.32635$

$$-2.32635 < \frac{k - 200}{3.5/\sqrt{20}} < 2.32635$$

$$-1.82066 < k - 200 < 1.82066$$

$$198.17934 < k < 201.82066$$

$\Rightarrow 198.2 < k < 201.8$ (to 1 d.p)