ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

FURTHER MATHEMATICS 9649/02

Paper 2 **28 August 2017**

3 hours

Additional Materials: Cover Sheet
Answer Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT
JC2 Preliminary Examination 2017

FURTHER MATHEMATICS 9649
Higher 2
Paper 2

/ 100

Index No: | | | | | Form Class: _____

Name: _____

Calculator model: _____

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

Question No.	Marks
1	/6
2	/7
3	/8
4	/14
5	/15
6	/6
7	/7
8	/9
9	/12
10	/16

Summary of Areas for Improvement			
Knowledge (K)	**Careless Mistakes (C)**	**Read/Interpret Qn wrongly (R)**	**Presentation (P)**

Section A: Pure Mathematics [50 marks]

1 Solve the recurrence relation $x_{n+2} = 6x_{n+1} - 12x_n$, $n \geq 0$, with initial conditions $x_0 = 1$, $x_1 = 6$, for positive integers n. [6]

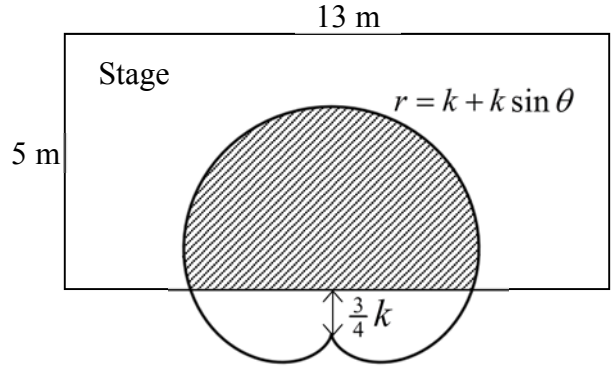
2 The complex number z satisfies the following inequalities:

$$|z - 2 - 2\mathrm{i}| \leq 1,$$
$$|z - 1 - 3\mathrm{i}| - |z - 3 + \mathrm{i}| \leq 0.$$

(i) On an Argand diagram, sketch the region R in which z must lie. [4]

(ii) Find the maximum and minimum possible value of $\arg(z)$, correct to 3 significant figures. [3]

3 Jenny is holding her vocal live performance on a rectangular stage of length 13 m and breadth 5 m. The technical crew uses a microphone with a cardioid pickup pattern so that it minimizes the pickup of noise from the audience. The crew places the microphone at a distance of $\frac{3}{4}k$, $k \geq 0$, from the front of the stage as shown in the figure.

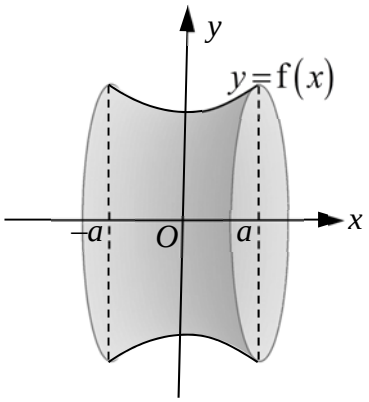


The boundary of the optimal pickup region is given by the cardioid $r = k + k\sin\theta$, where r is measured in meters and the microphone is at the pole. The optimal pickup region on the stage is indicated by the shaded area in the figure. Find

(a) the furthest distance, in terms of k, that Jenny can be on the stage from the microphone so that she is within the optimal pickup region, and [2]

(b) the minimum value of k if the optimal pickup region that Jenny has on stage is at least 75% of the stage area. [6]

4 If a frame consisting of two parallel rings with the same radius is dipped into soap solution, the soap film formed between the rings can be modelled as the surface of revolution of a curve around the *x*-axis (see figure). The frame, which is the boundary of the curve, is located at $x=a$ and $x=-a$, where *a* is positive.



(Adapted from: http://facstaff.susqu.edu/brakke/evolver/examples/cat/catenoids.html)

The curve has equation

$$y=\frac{k}{2}\left(\mathrm{e}^{\frac{x}{k}}+\mathrm{e}^{-\frac{x}{k}}\right),$$

where *k* is positive.

Let *S* be the curved surface area of the soap film. Show that

$$S=\frac{k\pi}{2}\int_{-a}^{a}\left(\mathrm{e}^{\frac{x}{k}}+\mathrm{e}^{-\frac{x}{k}}\right)^2\,\mathrm{d}x,$$

and evaluate this integral in terms of *a* and *k*. [7]

Let S_1 be the curved surface area of a cylinder with height 2*a* and the same radius as the rings. Determine, to 3 significant figures, the value of $\frac{a}{k}$ such that $S=S_1$. [5]

If *V* is the volume of the region bounded by the soap film and the planes of the rings, show that $V=\frac{1}{2}Sk$. [2]

5 **(i)** By using the substitution $z = xy$, show that the differential equation

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + (2-4x)\frac{\mathrm{d}y}{\mathrm{d}x} + 4y(x-1) = 0$$

can be simplified into the differential equation

$$\frac{\mathrm{d}^2z}{\mathrm{d}x^2} - 4\frac{\mathrm{d}z}{\mathrm{d}x} + 4z = 0\,. \qquad [2]$$

Hence find the general solution for y in terms of x. [3]

(ii) Using a machine, a particle is accelerated from rest such that at a time t seconds after the machine is turned on, its displacement s from its initial starting point is modelled by the following differential equation:

$$\frac{\mathrm{d}^2s}{\mathrm{d}t^2} - 4\frac{\mathrm{d}s}{\mathrm{d}t} + 4s = \cos t\,.$$

Find s in terms of t. [8]

Hence find the amount of time required for the particle's speed to exceed the speed of sound ($340\ \mathrm{m\,s^{-1}}$), giving your answer to the nearest hundredth of a second. [2]

Section B: Probability and Statistics [50 marks]

6 A gambler suspected that the slot machines in a nearby casino are rigged. Based on data supplied by the slot machines' manufacturer, the machines are supposed to pay out 5% of the time. The gambler frequented the casino over a period of 100 days, pulling the same machine 10 times each visit. He recorded the number of his pay-outs per visit in the table below:

No. of pay-outs	0	1	2	4	7
Frequency	72	25	1	1	1

Carry out a chi-squared goodness of fit test to determine whether the gambler's suspicions are merited (according to the data supplied by the slot machines' manufacturers), stating your hypotheses clearly. [4]

Discuss what your test indicates based on the p value for your test and the contributions of individual cells to the test statistic. [2]

7 The manager of a skincare company wishes to test the effectiveness of their online advertising. Each of the ten products, *A, B, C, ..., J* is given a week's intensive advertising, and the sales of the product during the week are compared with the sales in the preceding week. The following results are obtained:

Product	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Sales before	171	531	701	686	362	160	479	591	300	872
Sales after	203	534	783	674	388	168	451	633	370	995

Give a reason why a non-parametric test may be more appropriate than a parametric test in this case. Explain why a paired-sample test is being used in this context. [2]

Test whether the advertising has improved sales at the 5% level of significance using an appropriate test. State clearly the assumptions you made in carrying out this test. [5]

8 In a netball game, Jane and Claire take turns to shoot the netball into the net. A game ends when the ball enters the net. In every game, Claire starts first and has a probability of 0.4 of scoring on each throw while Jane has a probability of 0.35 of scoring on each throw. The number of total throws made by both Claire and Jane, up to and including the successful throw, is denoted by X.

(i) Find the probability that Claire wins a game on her r^{th} throw. Hence show that the probability that Claire wins a game is $\frac{40}{61}$. [3]

(ii) Given that Claire has won the first two games, find the probability that she wins the second game on her 6^{th} throw (The 6 throws that Claire makes include the throws she made in winning the first game). [2]

(iii) Show that $\mathrm{P}(X > a+b \mid X > a) = (0.39)^{\frac{b}{2}}$ where a and b are even integers.

Given that both Claire and Jane have not made two successful throws on their second throw, find the least even integer n such that the probability that more than n throws are required to achieve the first successful throw is less than 0.1. (The n throws include the first successful throw.) [4]

9 An observatory on the top floor of a 50-storey building is a tourist destination. The observatory is accessible via a lift P which carries a maximum of 4 passengers. The lift runs at every 15-minute interval. During the off-peak period from 3 pm to 6 pm, tourists to the observatory arrive at the lift at random at an average rate of 5 per 30 minutes and wait for places on the lift. After the departure of the lift at 3 pm, no one is waiting in the queue. Find the probability that no one is waiting in the queue after the next departure. [2]

Another lift Q of similar capacity is installed besides the lift P. Assuming that no one is waiting in the queue after the departure of lift P at 3 pm, find the probability that no one is waiting after the next departure of P if

(i) P and Q leave together at the same time at 15-minute intervals; [1]

(ii) P and Q take turn to leave alternately at 10-minute intervals (Q leaves at 3.10 pm, then P leaves at 3.20 pm, and so on). [4]

(You may assume that the lift carries up to the maximum capacity when in operation.)

On a particular occasion, a statistician models the time interval T, in minutes, between successive arrivals of passengers, by an exponential distribution with mean 6 min. By considering $\mathrm{P}(T > t),\ t \geq 0$, and hence the probability density function of T, explain how the statistician arrives at this model. State a necessary assumption and explain whether it is reasonable for such an assumption in the context of the question. [5]

10 Marine biologists have been studying the population of mussels at a particular region in the United States. The size (in mm) of 15 mussels caught at site *A* were as follows:

53	44	56	46	50	48	49	43	52	47	46	54	49	46	45

Over-hunting in the last three decades was known to have led to a decrease in the mean size of mussels caught. Conservation measures were introduced to prevent further reduction in the mean size of mussels. Following the conservation measures, the size (in mm) of 15 mussels caught at site *B* were as follows:

51	53	50	45	49	48	52	50	56	52	46	52	54	50	49

Conduct a *t*-test to examine whether the conservation measures are effective at the 5% level of significance, stating clearly any necessary assumptions. Suggest two ways with reasons to improve the accuracy of this statistical testing. [7]

The statutory minimum mean mussel size of 51 mm is now introduced to provide a benchmark to the conservation measures. At one particular site where the mussel sizes are normally distributed and the mean size had fallen to 40 mm, mussel fishing was banned for a period of 3 years. Following this period, a random sample of 18 mussels was obtained at this site and used to produce a 95% confidence interval for the population mean size of mussels. The resulting confidence interval was (45.7, 49.3).

(i) Find unbiased estimates for the population mean size and variance of the mussels. [3]

(ii) Explain what a 95% confidence interval for the population mean size of mussels means. Discuss how the limits of this confidence interval may be used to support the view that the conservation measures are working. Comment on whether the fishing ban should be lifted for this site. [3]

(iii) Following a further 3-year period, a random sample of 12 mussels was obtained at this site and the size of each mussel recorded such that the mean was 52.4 mm and the variance was k. The Environment agency carries out a test at the 10% level of significance to test whether the conservation measures are working. Explain how the values of k will affect the conclusion made by the agency. [3]