Name	( )	Class	

RIVER VALLEY HIGH SCHOOL
2017 Year 6 Preliminary Examination II
Higher 2

MATHEMATICS 9758/02

Paper 2

18 September 2017

3 hours

Additional Materials: Answer Paper
Graph Paper
List of Formulae (MF26)
Cover Page

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.
Write your name, class and index number in the space at the top of this page.

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphic calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages and **1** blank page.

Section A: Pure Mathematics [40 Marks]

1. The curve C is defined parametrically by equations

$$x = \cos(p), \quad y = \sin^3(p), \quad 0 \le p \le 2\pi$$

The point P on C has parameter p. Given that p is increasing at a rate of 0.5 units per second, find the rate at which $\frac{\mathrm{d}y}{\mathrm{d}x}$ is increasing when $p = \frac{\pi}{3}$. [4]

2. An arithmetic sequence $u_1,\ u_2,\ u_3, \ldots$ is such that the difference between the fourteenth term and the fifth term is equal to the sum of the terms between the fifth term and the fourteenth term (both inclusive). Given further that that sum of the third, fifth and fourteenth terms is 19, find the common difference of the sequence. [4]

Hence, or otherwise, find the largest of value of n such that the sum of the first n terms is positive. [2]

3. **(i)** Express $\dfrac{4r+6}{(r+1)(r+2)(r+3)}$ as partial fractions. [1]

(ii) Hence find $\displaystyle\sum_{r=1}^{n} \frac{4r+6}{(r+1)(r+2)(r+3)}$ in terms of n. [3]

(iii) Use your answer in part **(ii)** to find the sum of the infinite series

$$\frac{3}{1\times2\times3} + \frac{5}{2\times3\times4} + \frac{7}{3\times4\times5} + \frac{9}{4\times5\times6} + \cdots.$$ [3]

4. Let $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \mathrm{e}^{\sqrt{(1-x)^3}}$ for $x \le 1$.

Show that $4\sqrt{1-x}\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 6(1-x)\dfrac{\mathrm{d}y}{\mathrm{d}x} - 3y = 0$. [4]

Hence find the Maclaurin series for f(x) up to and including the term in x^2. [3]

Using the standard series of e^x and $(1+x)^n$ given in the List of Formulae (MF26), show how you could verify the correctness of the series of $\mathrm{f}(x)$ above. [4]

5. The functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x^2 - x, \quad x \in \mathbb{R}, \quad x \ge 0,$$

$$\mathrm{g} : x \mapsto -3 + \frac{1}{\sqrt{2x + \frac{1}{2}}}, \quad x \in \mathbb{R}, \quad x > -\frac{1}{4}.$$

(i) Give a reason why f does not have an inverse. [1]

(ii) If the domain of f is restricted to $x \ge k$, state the least value of k for which the function f^{-1} exists, and find f^{-1} in similar form for this domain. [3]

(iii) Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same diagram if the domain of f is restricted to $x \ge k$, where k is the value found in part **(ii)**. Your diagram should show clearly the relationship between the two graphs. [3]

(iv) Solve algebraically the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ for the restricted domain of f in part **(ii)**. [2]

(v) For f defined for $x \ge 0$, show that the composite function gf exists and find its range. [3]

Section B: Statistics [60 Marks]

6. A restaurant is setting up a spinning wheel for its customers to try and win vouchers. The wheel is split into 8 identical segments, comprising of \$0, \$5, \$10, \$15, \$20, \$25, \$30 and \$50.

Find the number of ways the segments can be arranged on the wheel if

(i) there are no restrictions. [1]

(ii) the \$0 segment cannot be next to the \$5 segment [2]

(iii) there must be at least two segments between the \$30 and \$50 segments. [2]

The restaurant decides to replace the \$30 and \$50 segments with another two \$0 segments.

(iv) Find the number of possible arrangements of the 8 segments. [1]

(v) Find the number of possible arrangements if the \$0 segments must be separated. [2]

7. A board game simulates players attacking each other by throwing tetrahedral (8-sided) dice. When attacking, the player throws an attack die once. An attack die has 5 of the sides printed with the number "0", 2 of the sides printed with the number "1", and 1 of the sides printed with the number "2". After the attacking player has thrown the attack die, the defending player throws a defence die once. A defence die has 2 of the sides printed with the number "0", 4 of the sides printed with the number "1" and 2 of the sides printed with the number "2". The damage dealt during a round is equal to the score shown on the attack die minus the score shown on the defence die. If the score on the defence die is more than the score on the attack die, the damage dealt will be zero.

Let A denotes the score on an attack die, and D denotes the score on a defence die.

(i) Write down the probability distributions for A and D. Hence find the expected value and variance of $A - D$. [4]

Let X denote the damage dealt during a round.

(ii) Find the probability distribution for X. Hence find the expected value and variance of X. [5]

(iii) Explain why, in the context of the question, $\mathrm{E}(X) > 0$ when $\mathrm{E}(A) < \mathrm{E}(D)$. [1]

8. A car park next to a small commercial building has a total of 12 parking lots. Land surveillance officers have been observing the usage of parking lots per day to determine if the land has been efficiently utilised. Each parking lot can be occupied by at most one vehicle per day.

(i) Denoting the number of occupied parking lots per day by X , state in context, two assumptions needed for X to be well modelled by a binomial distribution. [2]

(ii) It is further observed that for 80% of the days in the survey period, there are at least 4 occupied lots in the car park for each day. Find the probability that a parking lot is being occupied in a day. [2]

(iii) Given that at least one of the parking lots is occupied in a particular day, show that the probability that at least 2 but less than 4 lots are occupied in the particular day is given by

$$\mathrm{f}(p)=\frac{22p^2(1-p)^9(3+7p)}{1-(1-p)^{12}}$$

where p is the probability of a parking lot being occupied in a day. What can you say about this probability if p is approximately 0.185? [5]

9. In the study of how the population of a harmful bacteria varies with temperature, scientists conducted an experiment to collect the following set of data:

Temperature (x °C)	10	12	14	16	18	20	22	24	26	28
Population (y millions)	25.4	25.1	24.4	22.9	20.8	18.3	15.4	12.2	8.8	5.3

(i) Draw a scatter diagram for the above data, labelling the axes clearly. [2]

(ii) Calculate the value of the product moment correlation coefficient. Explain why a linear model is not appropriate. [2]

It is suggested the relationship between x and y can be modelled by one of the following formulae:

$$y=a+\frac{b}{x} \quad \text{or} \quad y=a-bx^2$$

where a and b are positive constants.

(iii) Explain which of the above two models is the better model and calculate the values of a and b for the chosen model. [3]

(iv) It is required to estimate the temperature when the population of the bacteria is 10 millions. By using an appropriate regression line, find an estimate of the value of x and comment on the reliability of your answer. [2]

10. Each month the amount of electricity, X measured in kilowatt-hours (kWh), used by a household in a particular city may be assumed to follow a normal distribution with mean 950 and standard deviation σ. The charge for electricity used per month is fixed at \$0.22 per kWh.

(i) Given that 65% of the households uses less than 960 kWh of electricity in a month, find the value of σ, correct to 1 decimal place. [2]

For the rest of the question, σ is the value found in part **(i)**.

(ii) Find the probability that the difference in the amount of electricity used among 2 randomly chosen households in a particular month is not more than 30 kWh. [3]

(iii) In the month of August, the mayor of the city decides to provide 50% and 30% subsidies for the electricity bills of households in the North and South districts of the city respectively. Find the probability that the total electricity bill of 2 randomly chosen North district households and 1 South district household is less than \$360. [4]

(iv) In December, a random sample of n households is chosen to study the mean monthly electricity usage per household in the city. Find the least value of n if the probability of the sample mean being less than 955 kWh is at least 0.9. [3]

11. Physicists are conducting an experiment involving collisions between protons and anti-protons. The mean amount of energy, $\bar{x}$ MeV, released in n collisions is found to be 1864 MeV.

One model predicts the energy released would be 1860 MeV with standard deviation 40 MeV. This is tested at a 1% level of significance against a newer model that claims a higher value.

(i) Find the least value of n such that the hypothesis that the mean amount of energy released is 1860 MeV is rejected. [5]

Given instead that $n = 600$.

(ii) Calculate the p-value and state its meaning in context of the question. [3]

(iii) State, with a reason, whether it is necessary to assume the amount of energy released in collisions to be normally distributed for this test to be valid. [1]

Two-sigma is an indicative of how confident researchers feel their results are. For researchers to feel confident, they must be able to produce a "two-sigma" result – that is the experimental result must be at least two standard deviations away from the predicted mean under the null hypothesis.

(iv) Calculate the level of significance that corresponds to a "two-sigma" test. Hence, using your answer from part **(ii)** determine whether the experiment has met the "two-sigma" threshold. [3]

END OF PAPER

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