

NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 2

MATHEMATICS

9758/02

Paper 2

12 Sep 2017
3 hours

Additional Materials: Answer Paper
 Graph Paper
 List of Formulae (MF26)
 Cover Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, registration number, subject tutorial group, on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in the brackets [] at the end of each question or part question.

This document consists of **6** printed pages.



National Junior College

[Turn Over

Section A: Pure Mathematics [40 marks]

- 1 There are 3 bike-sharing companies in the current market. For each ride, α -bike charges a certain amount per 5 min block or part thereof, β -bike charges a certain amount per 10 min block or part thereof and μ -bike charges a certain amount per 15 min block or part thereof. Rebecca rode each of the bike-sharing companies' bikes once in each month. The table below shows the amount of time Rebecca clocked for each ride and her total spending for each month. In celebration of the company's first anniversary, the pricings in February and March 2017 of μ -bikes are a 5% discount off the immediate previous month's pricing.

	January 2017	February 2017	March 2017
α -bike	25 min	17 min	36 min
β -bike	30 min	10 min	39 min
μ -bike	15 min	44 min	33 min
Total spending	\$5.70	\$5.72	\$9.71

Determine which bike-sharing company offers the cheapest rate (without any discount) for a 40-min ride. Justify your answer clearly. [4]

- 2 A function f is said to self-inverse if $f(x) = f^{-1}(x)$ for all x in the domain of f .

The functions f and g are defined by

$$f : x \mapsto \frac{7-3x}{3-x}, \quad x \in \mathbb{R}, x \neq 3,$$

$$g : x \mapsto |(2-x)(1+x)|, \quad x \in \mathbb{R}, x \in (-\infty, -1].$$

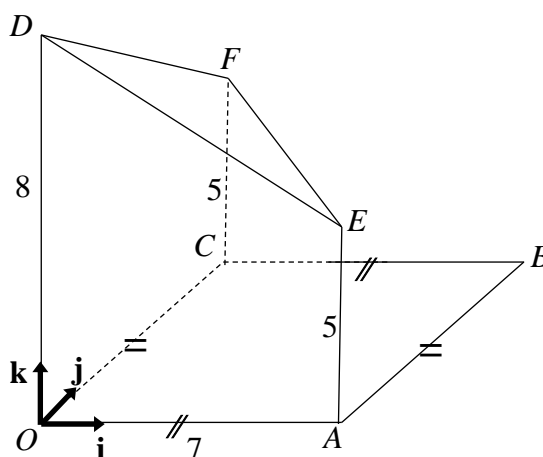
- (i) Explain why f^{-1} exists and show that f is self-inverse. Hence, or otherwise, evaluate $f^{2003}(5)$. [4]
- (ii) Find an expression for $g^{-1}(x)$. [3]
- (iii) Sketch, on the same diagram, the graphs of $y = g(x)$ and $y = g^{-1}(x)$, illustrating clearly the relationship between the two graphs, and labelling the axial intercept(s), if any. Write down the set of values of x that satisfies the equation $g g^{-1}(x) = x$. [3]
- (iv) Show that $f g^{-1}$ exists. Find the exact range of $f g^{-1}$. [3]

- 3 Using differentiation, find the Maclaurin's series of $\frac{e^{2x}}{1+x^2}$, in ascending powers of x up to and including x^3 . [6]

Let $h(x) = \frac{e^{2x}}{1+x^2}$ and the cubic polynomial obtained above be $f(x)$.

Find, for $-2 \leq x \leq 2$, the set of values of x for which the value of $f(x)$ is within ± 0.5 of the value of $h(x)$. [3]

- 4 The diagram (not drawn to scale) shows the structure of a partially constructed building that is built on a horizontal ground. The building has a square base foundation of 7 m in length. Points O, A, B and C are the corners of the foundation of the building. The building currently consists of three vertical pillars OD, AE and CF of heights 8 m, 5 m and 5 m respectively. A canvas is currently attached at D, E and F , forming a temporary shelter for the building. O is taken as the origin and vectors \mathbf{i}, \mathbf{j} , and \mathbf{k} , each of length 1 m, are taken along OA, OC and OD respectively.



- (i) Find a Cartesian equation of the plane that represents the canvas DEF . [3]
 (ii) Find the acute angle which the canvas DEF makes with the horizontal ground. [2]
 (iii) Given that the canvas is to be extended along the plane DEF till it touches the horizontal ground, explain why point B will lie beneath the canvas. [2]

A cement roof is to be built to replace the extended canvas. A vertical partition wall is also to be built such that it is d m away from and parallel to the plane $ODFC$, where $0 < d < 7$.

- (iv) Find the exact vector equation of the line where the roof meets the partition wall. Show your working clearly, leaving your answer in terms of d . [4]
 (v) A lighting point, P , is to be placed on the roof such that it is closest to B . Find the position vector of P . [3]

Section B: Statistics [60 marks]

- 5** A delegation of four students is to be selected from five badminton players, m floorball players, where $m > 3$, and six swimmers to attend the opening ceremony of the 2017 National Games. A pair of twins is among the floorball players. The delegation is to consist of at least one player from each sport.

- (i) Show that the number of ways to select the delegation in which neither of the twins is selected is $k(m-2)(m+6)$, where k is an integer to be determined. [3]
- (ii) Given that the number of ways to select a delegation in which neither of the twins is selected is more than twice the number of ways to select a delegation which includes exactly one of the twins, find the least value of m . [2]

The pair of twins, one badminton player, one swimmer and two teachers, have been selected to attend a welcome lunch at the opening ceremony. Find the number of ways in which the group can be seated at a round table with distinguishable seats if the pair of twins is to be seated together and the teachers are separated. [3]

- 6** In the fishery sciences, researchers often need to determine the length of a fish as a function of its age. The table below shows the average length, L inches, at age, t years, of a kind of fish called the North Sea Sole.

t	1	2	3	4	5	6	7	8
L	3.6	7.5	10.1	11.7	12.7	13.4	14.0	14.4

- (i) Draw a scatter diagram of these data, and explain how you know from your diagram that the relationship between L and t should not be modelled by an equation of the form $L = at + b$. [3]
- (ii) Which of the formulae $L = a\sqrt{t} + b$ and $L = c \ln t + d$, where a , b , c and d are constants, is the better model for the relationship between L and t ? Explain fully how you decided, and find the constants for the better formula. [3]
- (iii) Use the formula you chose from part (ii) to estimate the average length of a six-month old Sole. Explain whether your estimate is reliable. [2]

A popular approach to determine the average length of a fish as a function of its age is the von Bertalanffy model. The model shows the relationship between the average length that is yet to be grown, G inches, at age, t years. The maximum average length attained by the Sole is 14.8 inches.

- (iv) The product moment correlation between L and t is given as r_1 while that between G and t is given as r_2 . State the relationship between r_1 and r_2 . [1]

- 7 There are three identically shaped balls, numbered from 1 to 3, in a bag. Balls are drawn one by one at random and with replacement. The random variable X is the number of draws needed for any ball to be drawn a second time. The two draws of the same ball do not need to be consecutive.

(i) Show that $P(X = 4) = \frac{2}{9}$ and find the probability distribution of X . [3]

(ii) Show that $E(X) = \frac{26}{9}$ and find the exact value of $\text{Var}(X)$. [3]

(iii) The mean for forty-four independent observations of X is denoted by \bar{X} . Using a suitable approximation, find the probability that \bar{X} exceeds 3. [3]

- 8 Heart rate, also known as pulse, is the number of times a person's heart beats per minute. The normal heart rate of teenagers has a mean of 75 at the resting state.

Obesity is a leading preventable cause of death worldwide. It is most commonly caused by a combination of excessive food intake, lack of physical activity and genetic susceptibility. To examine the effect of obesity on heart rate, 70 obese teenagers are randomly selected and their heart rates h are measured in a resting state. The results are summarised as follows.

$$n = 70 \qquad \sum h = 5411 \qquad \sum h^2 = 426433$$

The Health Promotion Board (HPB) wishes to test whether the mean heart rate for obese teenagers differs from the normal heart rate by carrying out a hypothesis test.

- (i) Explain whether HPB should use a 1-tail test or a 2-tail test. [1]
- (ii) Explain why HPB is able to carry out a hypothesis test without knowing anything about the distribution and variance of the heart rates. [2]
- (iii) Find the unbiased estimates of the population mean and variance, and carry out the test at the 10% level of significance for the HPB. [6]

A researcher wishes to test whether obese teenagers have a **higher** mean heart rate. He finds that the mean heart rate for 80 randomly obese teenagers is 79.4, then carries out a hypothesis test at the 10% level of significance.

- (iv) Explain, with justification, how the population variance of the heart rates will affect the conclusion made by the researcher. [3]
- (v) Show that the probability of any normal variable lying within one standard deviation from its mean is approximately 0.683. [1]

By considering (iv) and (v), explain why it is likely for the researcher to reject the null hypothesis in his test if it is assumed that heart rates follow a normal distribution at the resting state. [1]

- 9** The number of days of gestation for a Dutch Belted cow is normally distributed, with a mean of μ days and a standard deviation of σ days. 8.08% of this cattle breed has a gestation period shorter than 278 days whereas 21.2% has a gestation period longer than 289 days. Find the values of μ and σ , giving your answers correct to 3 significant figures. [3]

- (i) Find the probability that the mean gestation period for thirty-two randomly chosen Dutch Belted cows is more than 287 days. State a necessary assumption for your calculation to be valid. [3]

For another cattle breed, the Jersey cow, the number of days of gestation is normally distributed with a mean of 278 days and a standard deviation of 2.5 days.

During gestation, a randomly chosen pregnant Dutch Belted cow eats 29 kg of feed daily while a randomly chosen pregnant Jersey cow eats 26 kg of feed daily.

- (ii) Find the value of a such that during their respective gestation periods, there is a probability of 0.35 that the amount of feed consumed by a randomly chosen pregnant Jersey cow exceeds half of the amount consumed by a randomly chosen pregnant Dutch Belted cow by less than a kg. Express your answer to the nearest kg. [2]
- (iii) Calculate the probability that during their respective gestation periods, the difference between the amount of feed consumed by three randomly chosen pregnant Dutch Belted cows and four randomly chosen pregnant Jersey cows is more than 4000 kg. State clearly the parameters of the distribution used in the calculation. [3]

- 10** Factory A manufactures a large batch of light bulbs. It is known that on average, 1 out of 200 light bulbs manufactured by Factory A, is defective. A random sample of 180 light bulbs is inspected. The batch is accepted if the sample contains less than r defective light bulbs.

- (i) Explain why the context above may not be well-modelled by a binomial distribution. [1]
Assume now that the context above is well-modelled by a binomial distribution.

- (ii) Determine the value of r such that the probability of accepting the batch is 0.998. [1]

In Factory B, a random sample of 30 light bulbs is taken from a large batch. If the sample contains no defective light bulbs, the batch is accepted. The batch is rejected if the sample contains more than two defective light bulbs. If the sample contains one or two defective light bulbs, a second random sample of 30 light bulbs is chosen and the batch is accepted only if this second sample contains no defectives. It is known that Factory B produces $(100p)\%$ defective light bulbs.

- (iii) Find the probability that the batch is accepted. Leave your answer in terms of p . [3]

Forty random samples of 30 light bulbs are taken from each of the two factories A and B.

- (iv) Given that $p = 0.007$ and there is exactly one defective bulb, find the probability that it is from Factory B. [4]

End of Paper