Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) 9758/02

Paper 2

September 2017
3 hours

Additional Materials: Answer Paper
Graph Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Total
Score											
Max Score	**5**	**8**	**13**	**14**	**7**	**12**	**8**	**8**	**9**	**16**	**100**

Section A: Pure Mathematics [40 marks]

1 **(i)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\tan^2 x$. Hence evaluate $\int_0^{\frac{1}{4}\pi} \sec^2 x \tan x \, \mathrm{e}^{\tan^2 x} \, \mathrm{d}x$, leaving your answer in exact form. [3]

(ii) By expressing $1+72x-32x^3$ as $1+mx(9-4x^2)$ where m is a constant, find $\int \frac{1+72x-32x^3}{\sqrt{(9-4x^2)}} \, \mathrm{d}x$. [2]

2 The curve C with equation $y=\frac{x^2+(a-1)x-a-1}{x-1}$, where a is a constant, has the oblique asymptote $y=x+1$.

(i) Show that a = 1. Hence sketch C, giving the equations of any asymptotes and the exact coordinates of any points of intersection with the axes. [3]

(ii) The region bounded by C for $x>1$ and the lines $y=x+1$, $y=2$ and $y=4$ is rotated through 2π radians about the line $x=1$. By considering a translation of C, or otherwise, find the volume of revolution formed. [5]

3 The variables y and x satisfy the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1-\ln x}{x\ln x+2x^2}.$$

(i) Show that the substitution $u=\frac{\ln x}{x}$ reduces the differential equation to $\frac{\mathrm{d}u}{\mathrm{d}y}=u+2$.

Given that $y=0$ when $x=1$, show that $y=\ln\left(\frac{\ln x}{2x}+1\right)$. [6]

The curve C has equation $y=\ln\left(\frac{\ln x}{2x}+1\right)$. It is given that C has a maximum point and two asymptotes $y=a$ and $x=b$.

(ii) Find the exact coordinates of the maximum point. [2]

(iii) Explain why $a=0$. [You may assume that as $x\to\infty$, $\frac{\ln x}{x}\to 0$.] [1]

(iv) Determine the value of b, giving your answer correct to 4 decimal places. [2]

(v) Sketch C. [2]

4 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel. The point C lies on OB produced such that $3OC = 5OB$. It is given that $|\mathbf{a}| = 2|\mathbf{b}|$ and $\cos\angle AOB = -\frac{1}{4}$.

(a) **(i)** Show that a vector equation of the line AC is $\mathbf{r} = \mathbf{a} + \lambda(3\mathbf{a} - 5\mathbf{b})$, where λ is a real parameter. [2]

The line l lies in the plane containing O, A and B.

(ii) Explain why the direction vector of l can be expressed as $s\mathbf{a} + t\mathbf{b}$, where s and t are real numbers. [1]

Given that l is perpendicular to AB, show that $t = 3s$. [4]

Given further that l passes through B, write down a vector equation of l, in a similar form as part **(i)**. [1]

(iii) Find the position vector of the point of intersection of line AC and l, in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(b) Explain why, for any constant k, $|(\mathbf{a} + k\,\mathbf{b}) \times \mathbf{b}|$ gives the area of the parallelogram with sides OA and OB. Find the area of the parallelogram, leaving your answer in terms of $|\mathbf{a}|$. [4]

Section B: Statistics [60 marks]

5 A new game has been designed for a particular casino using two fair die. In each round of the game, a player places a bet of $2 before proceeding to roll the two die. The player's score is the sum of the results from both die. For the scores in the following table, the player keeps his bet and receives a payout as indicated.

Score	Payout
9 or 10	$1
2 or 4	$5
11	$8

For any other scores, the player loses his bet.

Let X be the random variable denoting the winnings of the casino from each round of the game.

(i) Show that $\mathrm{E}(X) = \frac{1}{12}$ and find $\mathrm{Var}(X)$. [4]

(ii) $\overline{X}$ is the mean winnings of the casino from n rounds of this game. Find $\mathrm{P}(\overline{X} > 0)$ when $n = 30$ and $n = 50\,000$. Make a comparison of these probabilities and comment in context of the question. [3]

6 The students in a college are separated into two groups of comparable sizes, Group X and Group Y. The marks for their Mathematics examination are normally distributed with means and variances as shown in the following table.

	Mean	Variance
Group X	55	20
Group Y	34	25

(i) Explain why it may not be appropriate for the mark of a randomly chosen student from the college population to be modelled by a normal distribution. [1]

(ii) In order to pass the examination, students from Group Y must obtain at least d marks. Find, correct to 1 decimal place, the maximum value of d if at least 60% of them pass. [3]

(iii) Find the probability that the total marks of 4 students from Group Y is less than three times the mark of a student from Group X. State clearly the mean and variance of the distribution you use in your calculation. [3]

(iv) The marks of 40 students, with 20 each randomly selected from Group X and Group Y, are used to compute a new mean mark, $\overline{M}$. Given that $\mathrm{P}(|\overline{M}-44.5|<k)=0.9545$, find the value of k. [4]

State a necessary assumption for your calculations to hold in parts **(iii)** and **(iv)**. [1]

7 The company Snatch provides a ride-hailing service comprising taxis and private cars in Singapore. Snatch claims that the mean waiting time for a passenger from the booking time to the time of the vehicle's arrival is 7 minutes.

To test whether the claim is true, a random sample of 30 passengers' waiting times is obtained. The standard deviation of the sample is 2 minutes. A hypothesis test conducted concludes that there is sufficient evidence at the 1% significance level to reject the claim.

(i) State appropriate hypotheses and the distribution of the test statistic used. [3]

(ii) Find the range of values of the sample mean waiting time, $\overline{t}$. [3]

(iii) A hypothesis test is conducted at the 1% significance level whether the mean waiting time of passengers is more than 7 minutes. Using the existing sample, deduce the conclusion of this test if the sample mean waiting time is more than 7 minutes. [2]

8 A retail manager of a large electrical appliances store wants to investigate the relationship between the monthly advertising expenditure, x hundred dollars, and the monthly sales of their refrigerators, y thousand dollars. The table below shows the results of the investigation.

x	5	8	12	16	18	20	23
y	12.5	12.9	13.6	14.8	17.0	19.3	25.1

(i) The manager concludes that an increase in monthly advertising expenditure will result in an increase in the monthly sales of refrigerators. State, with a reason, whether you agree with his conclusion. [1]

(ii) Draw a scatter diagram to illustrate the above data. Explain why a linear model is not likely to be appropriate. [2]

It is thought that the monthly sales y thousand dollars can be modelled by one of the formulae

$$y = a + b\mathrm{e}^{\sqrt{x}} \quad \text{or} \quad y = a + b\,x^2$$

where a and b are constants.

(iii) Find, correct to 4 decimal places, the value of the product moment correlation coefficient between

(A) $\mathrm{e}^{\sqrt{x}}$ and y,

(B) x^2 and y.

Explain which of $y = a + b\mathrm{e}^{\sqrt{x}}$ or $y = a + b\,x^2$ is the better model. [2]

Assume that the better model in part **(iii)** holds for part **(iv)**.

(iv) The manager forgot to record the monthly advertising expenditure when the monthly sales of refrigerators was \$11300. Combining this with the above data set, it is found that $a = 10.876$ and $b = 0.09906$ for the model. Find the monthly advertising expenditure that the manager forgot to record, leaving your answer to the nearest hundred. [3]

9 A sample of 5 people is chosen from a village of large population.

(i) The number of people in the sample who are underweight is denoted by X. State, in context, the assumption required for X to be well modelled by a binomial distribution. [1]

(ii) On average, the proportion of people in the village who are underweight is p. It is known that the mode of X is 2. Use this information to show that $\frac{1}{3} < p < \frac{1}{2}$. [3]

1000 samples of 5 people are chosen at random from the village and the results are shown in the table below.

x	0	1	2	3	4	5
Number of groups	93	252	349	220	75	11

(iii) Using the above results, find $\overline{x}$. Hence estimate the value of p. [2]

You may now use your estimate in part **(iii)** as the value of p.

(iv) Two random samples of 5 people are chosen. Find the probability that the first sample has at least 4 people who are underweight and has more people who are underweight than the second sample. [3]

10 **(a)** The word DISTRIBUTION has 12 letters.

(i) Find the number of different arrangements of the 12 letters that can be made. [1]

(ii) Find the number of different arrangements which can be made if there are exactly 8 letters between the two Ts. [3]

One of the Is is removed from the word and the remaining letters are arranged randomly.

(iii) Find the probability that no adjacent letters are the same. [4]

(b) The insurance company Adiva classifies 10% of their car policy holders as ‘low risk’, 60% as ‘average risk’ and 30% as ‘high risk’. Its statistical database has shown that of those classified as ‘low risk’, ‘average risk’ and ‘high risk’, 1%, 15% and 25% are involved in at least one accident respectively.

Find the probability that

(i) a randomly chosen policy holder is not involved in any accident if the holder is classified as ‘average risk’, [1]

(ii) a randomly chosen policy holder is not involved in any accident, [2]

(iii) a randomly chosen policy holder is classified as ‘low risk’ if the holder is involved in at least one accident. [2]

It is known that the cost of repairing a car when it meets with an accident has the following probability distribution.

Cost incurred (in thousand dollars)	5	10	50	100
Probability	0.75	0.15	0.08	0.02

It is known that a ‘low risk’ policy holder will not be involved in more than one accident in a year. You may assume that there will be no cost incurred by the company in insuring a holder whose car is not involved in any accident.

(iv) Construct the probability distribution table of the cost incurred by Adiva in insuring a ‘low risk’ policy holder assuming that the cost of repairing a car is independent of a ‘low risk’ policy holder meeting an accident. [1]

(v) In order to have an expected profit of \$200 from each policy holder, find the amount that Adiva should charge a ‘low risk’ policy holder when he renews his annual policy. [2]