



VICTORIA JUNIOR COLLEGE

JC 2 PRELIMINARY EXAMINATION 2017

H2 MATHEMATICS

9758/01

Paper 1

3 hours

Additional Materials: Answer Paper
 Graph Paper
 List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and CT group on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **6** printed pages

[Turn over

1. Without using a calculator, solve the inequality $\frac{6x-13}{x^2-4} \dots 1$. [4]
2. The Singapore Utility Board charges the residential users based on the usage for electricity, water and gas. Electricity and gas are charged by kilowatt hour (kWh) used while water usage is charged by cubic meters (CuM). Below are the monthly utility statements for Mr Pandey from May to August 2017.

| | |
|---|--|
| <p>SP Utility Bill (May 2017)</p> <p>Mr Pandey Blk 20 Marine ...</p> <p>Current month charges</p> <p>Electricity 514 kWh ***.**</p> <p>Water 18.8 CuM **.**</p> <p>Gas 134 kWh **.**</p> <p>Total \$155.54</p> | <p>SP Utility Bill (June 2017)</p> <p>Mr Pandey Blk 20 Marine ...</p> <p>Current month charges</p> <p>Electricity 309 kWh ***.**</p> <p>Water 11.3 CuM **.**</p> <p>Gas 89 kWh **.**</p> <p>Total \$ 94.99</p> |
| <p>SP Utility Bill (July 2017)</p> <p>Mr Pandey Blk 20 Marine ...</p> <p>Current month charges</p> <p>Electricity 639 kWh ***.**</p> <p>Water 21.7 CuM **.**</p> <p>Gas 108 kWh **.**</p> <p>Total \$ 208.40</p> | <p>SP Utility Bill (August 2017)</p> <p>Mr Pandey Blk 20 Marine ...</p> <p>Current month charges</p> <p>Electricity 555 kWh ***.**</p> <p>Water ??? CuM **.**</p> <p>Gas 128 kWh **.**</p> <p>Total \$ 184.84</p> |

It is known that the unit costs for electricity, water and gas remain unchanged for May and June. The unit cost for electricity was increased by 20% with effect from July 2017, while the unit cost for gas and water remain unchanged.

- (i) Calculate the unit cost for electricity, water and gas for June 2017, giving your answers correct to the nearest 4 decimal places. [3]
- (ii) The water usage for August 2017 was not clearly printed on the bill. Using your answers in part (i), calculate the water usage for August 2017 to the nearest CuM. [2]

3. It is given that

$$f(x) = \begin{cases} (x-2)^2 - 1, & \text{for } 0 < x \leq 3, \\ x-3, & \text{for } 3 < x \leq 6, \end{cases}$$

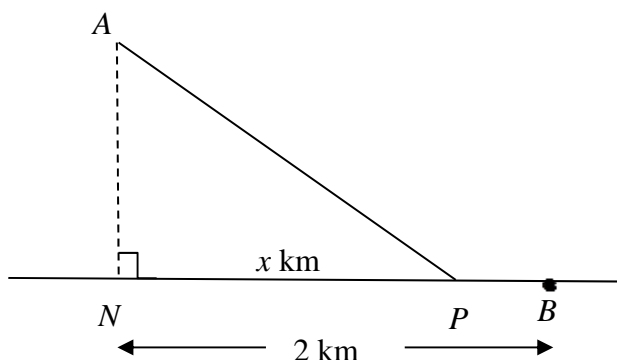
and that $f(x) = f(x+6)$ for all real values of x .

- (i) Sketch the graph of $y = f(x)$ for $0 < x \leq 10$. [3]
- (ii) On a separate diagram, sketch the graph of $y = 1 + f\left(\frac{1}{2}x\right)$ for $0 < x \leq 10$. [2]

4. The curve C has equation $(y+4)^2 - (x+3)^2 = 4$. Sketch C , giving the coordinates of any turning points and the equations of any asymptotes. [3]

Hence find the set of values of m such that the straight line with gradient m that passes through the point $(-3, -4)$ intersects C at least once. [2]

5.



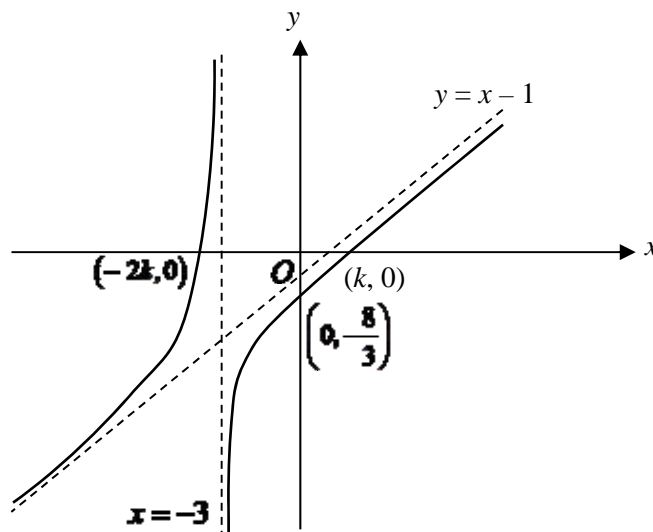
Alvin is at the point A on a floating platform in the sea. He wants to reach point B located on a straight stretch of beach. N is the point on the beach nearest to A and $NB = 2$ km. Alvin swims at a constant speed in a straight line from A to P and then runs at a constant speed from P to B , where P is a point on the straight stretch of beach from N to B . $NP = x$ km and T minutes is the time taken for Alvin to complete the journey.

T and x satisfy the differential equation

$$\frac{dT}{dx} = \frac{5\sqrt{5}x}{\sqrt{x^2 + 4}} - 5.$$

- (i) Solve the differential equation. [3]
 - (ii) Given that the minimum time taken for Alvin to complete this journey is 30 minutes, find T in terms of x . [3]
 - (iii) Using your answer in part (ii), find the longest time taken by Alvin to complete the journey. [2]
6. The function h is defined by
- $$h : x \mapsto e^{x-2} - 1, \quad \text{for } x \in \mathbb{R}.$$
- (i) Find $h^{-1}(x)$ and state the domain of h^{-1} . [3]
 - (ii) Sketch, on the same diagram, the graphs of $y = h(x)$ and $y = h^{-1}(x)$, giving the equations of any asymptotes and the exact coordinates of any points where the curves cross the x - and y -axes. [3]
 - (iii) Find the set of values of x such that $h^{-1}(x) > h(x)$. [2]

7. The diagram below shows the curve with equation $y = f(x)$. The curve crosses the x - and y -axes at the points $(-2k, 0)$, $(k, 0)$ and $(0, -\frac{8}{3})$ where $k > 0$. The curve has an oblique asymptote $y = x - 1$ and vertical asymptote $x = -3$.



- (i) On separate diagram, sketch the graph of $y = \frac{1}{f(x)}$, including the coordinates of the points where the graph crosses the axes and the equations of any asymptotes. [3]
- (ii) It is further known that $f(x) = \frac{x^2 + ax + b}{x + c}$ where a , b and c are constants. Find the values of a , b and c . [4]
8. It is given that $\sum_{r=1}^n \frac{r^2}{3^r} = \frac{3}{2} - \frac{n^2 + 3n + 3}{2(3^n)}$.
- (i) Find $\sum_{r=1}^{\infty} \frac{r^2 + (-1)^r}{3^r}$. [3]
- (ii) Show that $\sum_{r=4}^n \frac{(r-2)^2}{3^{r-2}} = \frac{p}{q} - \frac{an^2 - an + a}{2(3^{n-2})}$, where a , p and q are integers to be determined. [5]
9. (a) Given that $\int_0^a x \sin x \, dx = 0.5$, where $0 < a < 2$, find an equation that is satisfied by a and use it to find the value of a . [5]
- (b) Write down a definite integral that represents the area of the region bounded by the curve with equation $y = \frac{\sqrt{x}}{3 - \sqrt{x}}$, the two axes and the line $x = 4$. Use the substitution $u = 3 - \sqrt{x}$ to find the exact value of the area. [6]

10. It is given that z_1 , z_2 and z_3 are the roots of the equation

$$2z^3 + pz^2 + qz - 4 = 0$$

such that $\arg z_1 < \arg z_2 < \arg z_3$ and $z_1 = 1 - i\sqrt{3}$. Find the values of the real numbers p and q . [3]

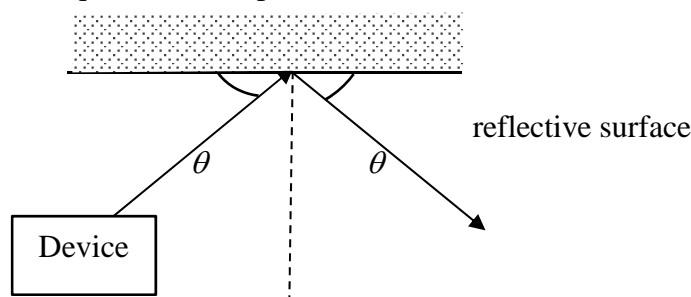
- (i) Without using the calculator, find z_2 and z_3 . [3]

In an Argand diagram, points P , Q and R represent the complex numbers z_1 , $w = \sqrt{2} + i\sqrt{2}$ and $z_1 + w$ respectively and O is the origin.

- (ii) Express each of z_1 and w in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$.
Give r and θ in exact form. [2]
- (iii) Indicate P , Q and R on the Argand diagram and identify the type of the quadrilateral $OPRQ$. [3]
- (iv) Find the exact value of $\arg(z_1^4 w^*)$. [3]

11. Physicists are investigating the reflective property of a particular reflective surface. The diagram below shows the set-up of a particular experiment, where a laser emitting device was placed at the point with coordinates $(1, 2, 3)$. A laser beam was emitted in the direction parallel to $\mathbf{i} + \mathbf{k}$. The path of the emitted laser beam and its reflected path make the same angle θ with the reflective surface. The plane containing these two paths is perpendicular to the reflective surface.

Write down the vector equation of the path of the emitted laser beam. [1]



It is known that the reflective surface has equation $x + y + z = 4$.

- (i) Find θ . [3]
- (ii) Show that the laser beam meets the reflective surface at the point $(0, 2, 2)$. [3]
- (iii) Find the vector equation of the path of the reflected laser beam. [5]

12. A curve C has equation $y = \ln(x^2)$, $x \neq 0$.
- (i) Sketch C . [2]
 - (ii) The part of C from the point $A(e^{-1}, -2)$ to the point $B(e^{\frac{k}{2}}, k)$, $k > 4$, and the line $y = -2$ is rotated about the y -axis to form the curved surface and the circular base of an open vase. Find the volume of the vase, giving your answer in terms of π and k , in exact form. [2]
 - (iii) Water flows into the vase at a constant rate of 2 cm^3 per second. By first showing that the volume of water in the vase is given by $V = \pi(x^2 - e^{-2})$ when the radius of the water surface is $x \text{ cm}$, find the rate at which x is increasing, giving your answer in terms of x . [4]
 - (iv) An insect lands on the inner surface of the vase at the point $(e, 2)$ just as the incoming water reaches the depth of 2 cm . It immediately starts to crawl along C such that the x -coordinate of its location increases by a constant value of 0.03 cm per second. Find the coordinates of the point on C at which the insect will first come into contact with water. [5]