Name	( )	Class	

RIVER VALLEY HIGH SCHOOL
2017 Year 6 Preliminary Examination II
Higher 2

FURTHER MATHEMATICS 9649/02

Paper 2 **20 Sep 2017**

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)
Cover Page

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.
Write your name, class and index number in the space at the top of this page.

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, place the cover page on top of your answer paper and fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [50 marks]

1. Show that the equation

$$|x|\left(x^2-3\right)-x-3=0$$

has a negative root in the interval $(n, n+1)$ where n is an integer to be determined. [2]

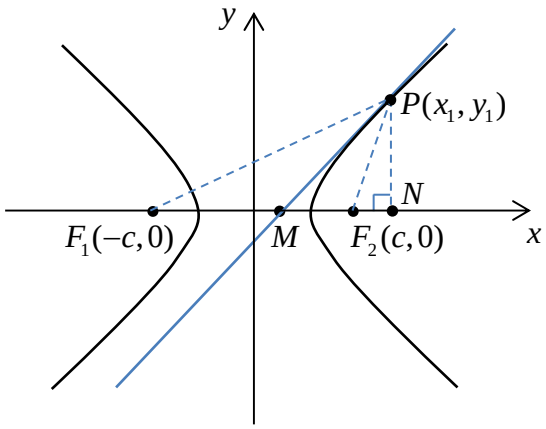
Explain why using $n + 1$ as the first approximation in the Newton-Raphson method is not an appropriate choice. [2]

Suggest a more appropriate first approximation and use it to find the root of the equation $|x|\left(x^2-3\right)-x-3=0$, correct to 3 decimal places, using the Newton-Raphson method. [2]

2. The diagram shows a hyperbola with equation

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,$$

where a and b are positive constants, and foci $F_1(-c,0)$ and $F_2(c,0)$.



Given that $P(x_1, y_1)$ is a point on the hyperbola, find the gradient of the tangent to the hyperbola at point P. [2]

The tangent to the hyperbola at P cuts the x-axis at point M and N is the foot of perpendicular from P to the x-axis. Show that $\tan\angle MPN=\frac{a^2y_1}{b^2x_1}$. [1]

Find, in terms of x_1, y_1 and c, expressions for $\tan\angle F_1PN$ and $\tan\angle F_2PN$. [2]

By finding expressions for $\tan\angle F_1PM$ and $\tan\angle F_2PM$, deduce the relationship between $\angle F_1PM$ and $\angle F_2PM$. [5]

3. (i) The matrix **A** has eigenvalue λ with corresponding eigenvector **x**. The matrix **B** has eigenvalue μ with corresponding eigenvector **x**. Find a eigenvalue and corresponding eigenvector of matrix **AB**. [2]

(ii) Find the eigenvalues and the corresponding eigenvectors of the matrix **A**, where

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 2 \\ 1 & 2 & 1 \end{pmatrix}.$$

Hence write down a matrix **Q** and a diagonal matrix **D** such that $\mathbf{A} = \mathbf{Q}^{-1}\mathbf{D}\mathbf{Q}$. [6]

(iii) Find the eigenvalues and corresponding eigenvectors of the matrix **N** where

$$\mathbf{N} = (\mathbf{A} + \mathbf{I})(\mathbf{A} - \mathbf{I})^{-1}. \quad [3]$$

4. The curve *C* has parametric equations

$$x = \frac{1}{2}(t-1)^2, \quad y = \frac{4}{3}t^{\frac{3}{2}} \quad \text{for } t \geq 1.$$

(i) *R* is the region bounded by the curve *C*, the line $y = \frac{32}{3}$ and the *y*-axis. Find the exact volume of the solid of revolution formed when *R* is rotated through 360^o about the *x*-axis. [4]

(ii) *S* is the region bounded by the curve *C*, the line $x = \frac{9}{2}$, the *x*-axis and *y*-axis. Find the volume of the solid of revolution formed when *S* is rotated through 360^o about the line $x = \frac{9}{2}$, giving your answer correct to 3 decimal places. [3]

(iii) The curve *D* is obtained by scaling the curve *C* parallel to the *y*-axis, with scale factor $\frac{1}{\sqrt{2}}$. *A* denotes the length of arc for the section of the curve *D* from the point where $t = 1$ to the point where $t = 4$.
Use the Simpson's rule with 7 ordinates to estimate the value of *A*, giving your answer correct to 4 decimal places. [4]

5. A damped vibrating spring-mass system has an external force acting on the object. The motion of the object in the system may be described by the differential equation

$$5\ddot{x} + c\dot{x} + 2cx = 4\sin 2t$$

where x is the displacement of the object from the equilibrium position in centimetres at time t seconds, and c is a positive constant.

(i) State the meanings of the following terms in the given differential equation: $-c\dot{x}$, $-2cx$ and $4\sin 2t$. [3]

(ii) Find the value of c for the vibrating system to be critically damped. [2]

(iii) Given that $c = 1$, find the general solution of the differential equation and describe the the motion of the object as $t \to \infty$. [7]

Section B: Probability And Statistics [50 marks]

6. A bike-sharing company observed a number of mis-use of its bicycles in a neighbourhood. The number of cases reported in a week over a few weeks are recorded below.

30 11 13 25 24 19 27 16

Calculate a 90% confidence interval for the mean number of bicycle mis-use cases in a week. [2]

Explain the meaning of "90% confidence interval" in the context of the question. [1]

To tackle the problem, the company implemented some preventive measures. The number of cases of mis-use in the same neighbourhood per week after implementation are recorded over a few weeks below.

14 8 9 32 19 24 16 28

A test is conducted at the 5% level of significance to determine if the preventive measures are effective in reducing the number of mis-use.

(i) Find the value of the test statistic obtained by these samples. State clearly any assumptions for the calculations to be valid. [4]

(ii) Find the probability that another samples of size 8 each before measures implementation and after measures implementation give a test statistic as extreme as that found in part (i) if the null hypothesis is in fact true. [2]

7. At a sports complex, there are 3 badminton courts available for booking each day. The demand for courts per day is assumed to have a Poisson distribution with mean 2.3. The demand each day is independent of other days.

(i) Show that the probability of a successful booking each day is 0.799, correct to 3 significant figures. [1]

(ii) Find the expected number of courts booked each day. [2]

(iii) The sports complex decides to build more courts to meet greater demand. What is the least number of courts to be built so that 99% of the demand per day could be met? [2]

(iv) Explain why a Poisson distribution with mean demand of 2.3 per week is not appropriate to model the demand over a year period? [1]

Alvin and Ben take turns to call the sports complex once a day to book a badminton court. For example, Alvin calls once on day 1, Ben calls once on day 2, Alvin calls once on day 3, and so on. The probability of a successful booking each day is 0.799, independent of other days. Each successful booking allows them to use one court for one day. Alvin calls first.

(v) If calling stops when either Alvin or Ben gets a successful booking, find the probability that Ben succeeded in booking a court. [3]

(vi) Nearing a competition, Alvin and Ben need to book more courts for practice. They continue booking till a certain number of days are booked. What is the probability that by the 20^{th} day of calling, they have booked courts for 16 days. [3]

8. A local taxi company recently launched a flat-fare system which is based on distance travelled and is computed based on existing surcharge. The table below shows the fares over 9 journeys using the flat-fare system and the traditional metered-fare system.

Start Point	Jurong Point	IMM	Jurong Point	Junction 8	IMM	Ion Orchard	Junction 8	Marina Square	Tiong Bahru Plaza
End Point	Tiong Bahru Plaza	Tiong Bahru Plaza	Ion Orchard	Jurong Point	Marina Square	Tiong Bahru Plaza	IMM	Jurong Point	Marina Square
Flat-fare $	17	12	26	22	18	20	20	21	11
Metered-fare $	16	21	33	27	15	20	18	25	17

(i) State two precautions, with reasons, that should be taken when collecting information about the fares. [3]

(ii) Carry out a Wilcoxon matched-pair signed rank test, at the 5% level of significance, to investigate if the flat-fare is cheaper than the metered-fare. [5]

(iii) Instead of carrying out a hypothesis test at the 5% level of significance to determine if there is a difference between the fares of the two systems, suggest another method that could be used. Carry out this method and state the conclusion. State any assumptions made. [4]

9. The interest rate charged by a bank on its personal loan is denoted by X %. It claims that the probability that it charges a rate of more than x% is given by $\frac{(2-k)x+31}{2x+31}$, for $x \in \mathbb{R}$, $0 < x \leq 1.5$. Show that $k = \frac{68}{3}$. [2]

A sample of 100 loans taken at this bank is recorded. The table shows the frequencies of interest rate charged.

Interest rate x%	$0 < x \leq 0.3$	$0.3 < x \leq 0.6$	$0.6 < x \leq 0.9$	$0.9 < x \leq 1.2$	$1.2 < x \leq 1.5$
Frequency f	28	20	16	25	11

Determine, at the 5% level of significance, if the data supports the claim that the probability that the bank charges interest rate of more than x% is $\frac{(2-k)x+31}{2x+31}$, where $k = \frac{68}{3}$. [7]

The interest \$$I$ payable for a loan of \$$L$ from the bank is $I = L\mathrm{e}^{-X}$, for $0 < x \leq 1.5$.

(i) Find the cumulative distribution function of I. Hence, find the median interest payable on a loan of \$$L$. [5]

(ii) Find the expected interest payable on a loan of \$$L$. [3]

– End of Paper –