ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

MATHEMATICS 9758/01

Paper 1 **16 August 2017**

3 hours

Additional Materials: Cover Sheet
Answer Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT
JC2 Preliminary Examination 2017

MATHEMATICS 9758
Higher 2
Paper 1

/ 100

Index No: | | | | |

Form Class: ____________

Name: ____________________________

Calculator model: ____________________

Arrange your answers in the same numerical order.

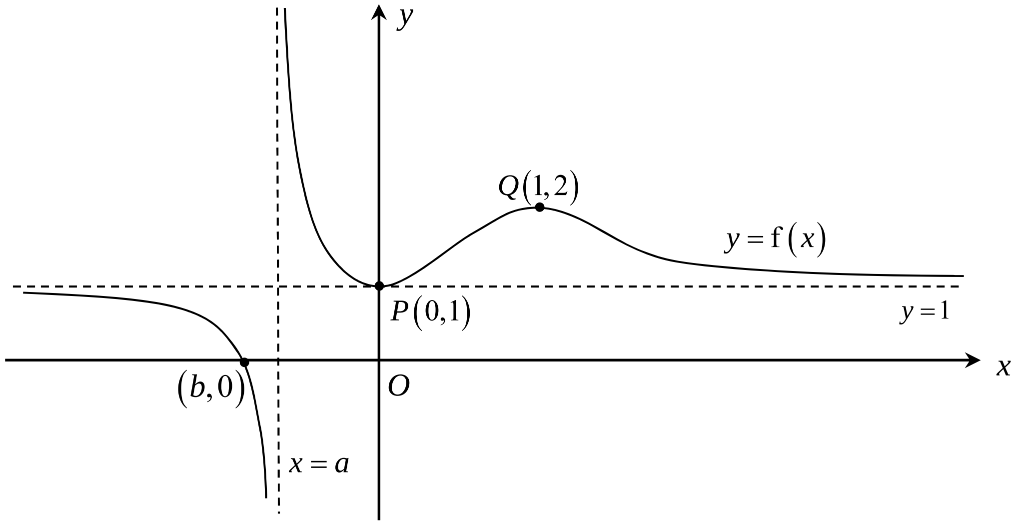
Place this cover sheet on top of them and tie them together with the string provided.

Question No.	Marks
1	/5
2	/6
3	/7
4	/7
5	/8
6	/8
7	/9
8	/12
9	/13
10	/12
11	/13

Summary of Areas for Improvement			
Knowledge (K)	Careless Mistakes (C)	Read/Interpret Qn wrongly (R)	Presentation (P)

1 The graph of $y=\dfrac{x-1}{ax^2+bx+c}$, where a, b and c are non-zero constants, has a turning point at $(-1,1)$, and an asymptote with equation $x=-\dfrac{1}{3}$. Find the values of a, b and c. [5]

2 The diagram below shows the graph of $y=\mathrm{f}(x)$.



The graph passes through the point $(b,0)$ and has turning points at $P(0,1)$ and $Q(1,2)$. The lines $y=1$ and $x=a$, where $b<a<-\dfrac{1}{2}$, are asymptotes to the curve.

On separate diagrams, sketch the graphs of

(i) $y=\mathrm{f}\left(\dfrac{x-1}{2}\right)$, [3]

(ii) $y=\mathrm{f}'(x)$, [3]

labelling, in terms of a and b where applicable, the exact coordinates of the points corresponding to P and Q, and the equations of any asymptotes.

3 Solve the inequality $\dfrac{1}{x+a}\le\dfrac{2a}{x^2-a^2}$, leaving your answer in terms of a, where a is a positive real number. [3]

Hence or otherwise, find $\displaystyle\int_{2a}^{4a}\left|\frac{1}{x+a}-\frac{2a}{x^2-a^2}\right|\mathrm{d}x$ exactly. [4]

4 **(i)** Expand $\left(k+x\right)^{n}$, in ascending powers of x, up to and including the term in x^2, where k is a non-zero real constant and n is a negative integer. [3]

(ii) State the range of values of x for which the expansion is valid. [1]

(iii) In the expansion of $\left(k+y+3y^2\right)^{-3}$, the coefficient of y^2 is 2. By using the expansion in **(i)**, find the value of k. [3]

5 The points *O*, *A* and *B* are on a plane such that relative to the point *O*, the points *A* and *B* have non-parallel position vectors **a** and **b** respectively.
The point *C* with position vector **c** is on the plane *OAB* such that *OC* bisects the angle *AOB*.
Show that $\left(\frac{\mathbf{a}}{|\mathbf{a}|}-\frac{\mathbf{b}}{|\mathbf{b}|}\right)\cdot\mathbf{c}=0$. [2]

The lines *AB* and *OC* intersect at *P*. By first verifying that $\overrightarrow{OC}$ is parallel to $\frac{\mathbf{a}}{|\mathbf{a}|}+\frac{\mathbf{b}}{|\mathbf{b}|}$, show that the ratio of $AP:PB=|\mathbf{a}|:|\mathbf{b}|$. [6]

6 It is given that $\mathrm{e}^{y}=\left(1+\sin x\right)^2$.

(i) Show that

$$\mathrm{e}^{y}\left[\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]=2\left(\cos 2x-\sin x\right).$$

By repeated differentiation, find the series expansion of y in ascending powers of x, up to and including the term in x^3, simplifying your answer. [5]

(ii) Show how you can use the standard series expansion(s) to verify that the terms up to x^3 for your series expansion of y in **(i)** are correct. [3]

7 **(a)** Given that $2z+1=|w|$ and $2w-z=4+8\mathrm{i}$, solve for *w* and *z*. [5]

(b) Find the exact values of x and y, where $x, y \in \mathbb{R}$, such that $2\mathrm{e}^{-\left(\frac{3+x+\mathrm{i}y}{\mathrm{i}}\right)}=1-\mathrm{i}$. [4]

8 The curve C and the line L have equations $y = x^2$ and $y = \frac{1}{2}x - 2$ respectively.

(i) The point A on C and the point B on L are such that they have the same x-coordinate. Find the coordinates of A and B that gives the shortest distance AB. [3]

(ii) The point P on C and the point Q on L are such that they have the same y-coordinate. Find the coordinates of P and Q that gives the shortest distance PQ. [3]

(iii) Find the exact area of the polygon formed by joining the points found in **(i)** and **(ii)**. [2]

(iv) A variable point on the curve C with coordinates $\left(s, s^2\right)$ starts from the origin O and moves along the curve with s increasing at a rate of 2 units/s. Find the rate of change of the area bounded by the curve, the y-axis and the line $y = s^2$, at the instant when $s = \sqrt{2}$. [4]

9 **(a)** By writing

$$\sin\left(x+\frac{1}{4}\right)\pi - \sin\left(x-\frac{3}{4}\right)\pi$$

in terms of a single trigonometric function, find $\sum_{x=1}^{n}\cos\left(x-\frac{1}{4}\right)\pi$, leaving your answer in terms of n. [4]

(b) The function f is defined by

$$\mathrm{f} : x \mapsto \sin\left(x+\frac{1}{4}\right)\pi - \sin\left(x-\frac{3}{4}\right)\pi,\ x \in \mathbb{R},\ \ a \le x \le 1.$$

(i) State the range of f and sketch the curve when $a = -1$, labelling the exact coordinates of the points where the curve crosses the x- and y- axes. [3]

(ii) State the least value of a such that f^{-1} exists, and define f^{-1} in similar form. [3]

The function g is defined by

$$\mathrm{g} : x \mapsto \frac{2x}{1-x},\ x \in \mathbb{R},\ \ x \ge \frac{13}{5}.$$

(iii) Given that fg exists, find the greatest value of a, and the corresponding range of fg. [3]

10 Abbie and Benny each take a \$50 000 study loan for their 3-year undergraduate program, disbursed on the first day of the program. The terms of the loan are such that during the 3-year period of their studies, interest is charged at 0.1% of the outstanding amount at the end of each month. Upon graduation, interest is charged at 0.375% of the outstanding amount at the end of each month.

(a) Since the interest rate is lower during her studies, Abbie decides that she will make a constant payment at the beginning of each month from the start of the program for its entire duration.

(i) Find the amount, correct to the nearest cent, Abbie needs to pay at the beginning of each month so that the outstanding amount after interest is charged remains at \$50 000 at the end of every month. [2]

(ii) After graduating, Abbie intends to increase her payment to a constant \$$k$ at the beginning of every month. Show that the outstanding amount Abbie owes the bank at the end of n months after graduation, and after interest is charged, is

$$\$\left[1.00375^n\left(50000\right)-\frac{803}{3}k\left(1.00375^n-1\right)\right].$$ [2]

(iii) Abbie plans to repay her loan within 10 years after graduation. Determine if she can do this with a monthly instalment of \$500, justifying your answer. [1]
Find the amount she needs to pay so that she fully repays her loan at the end of exactly 10 years after graduation, leaving your answer to the nearest cent. [2]

(b) Benny wishes to begin his loan repayment only after graduation. Like Abbie, he aims to repay the loan at the end of exactly 10 years after graduation.
Leaving your answer to the nearest cent, find

(i) the constant amount Benny needs to pay each month in order to do this, [3]

(ii) the amount of interest Benny pays altogether. [2]

11 **(i)** Show that for any real constant k,

$$\int t^2\,\mathrm{e}^{-kt}\,\mathrm{d}t=-\mathrm{e}^{-kt}\left(\frac{a}{k}t^2+\frac{b}{k^2}t+\frac{c}{k^3}\right)+D,$$

where D is an arbitrary constant, and a, b, and c are constants to be determined. [3]

On the day of the launch of a new mobile game, there were 100,000 players. After t months, the number of players on the game is x, in hundred thousands, where x and t are continuous quantities. It is known that, on average, one player recruits 0.75 players into the game per month, while the number of players who leave the game per month is proportional to t^2.

(ii) Write down a differential equation relating x and t. [1]

(iii) Using the substitution $x=u\,\mathrm{e}^{\frac{3}{4}t}$, show that the differential equation in **(ii)** can be reduced to

$$\frac{\mathrm{d}u}{\mathrm{d}t}=-pt^2\,\mathrm{e}^{-\frac{3}{4}t},$$

where p is a positive constant.
Hence solve the differential equation in **(ii)**, leaving your answer in terms of p. [5]

(iv) For $p=\frac{1}{3}$, find the maximum number of players on the game, and determine if there will be a time when there are no players on the game. [2]

(v) Find the range of values of p such that the game will have no more players after some time. [2]