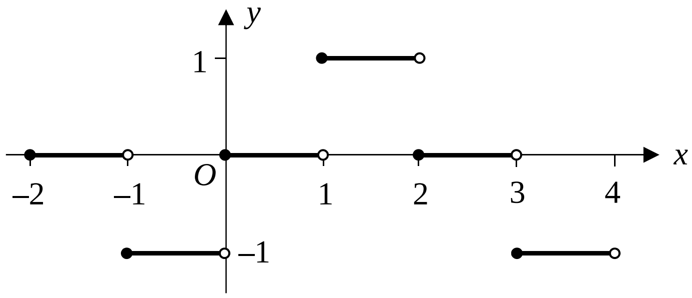
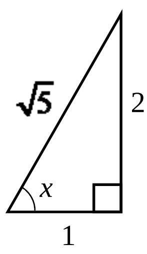
2017 HCI H2 Prelim Paper 1 Suggested Solutions

S/N	Solution
1 (i)	$\mathrm{f}(-1.2)=\mathrm{f}(2.8)=0$ $\mathrm{f}(3.6)=\mathrm{f}(-0.4)=-1$
1 (ii)	
1 (iii)	$\int_{-2}^{4}\mathrm{f}(x)\mathrm{d}x=-1+1-1=-1$
2	$u=\sec x \Rightarrow u'=\sec x\tan x$ $v'=\sec^2 x \Rightarrow v=\tan x$ $\int \sec^3 x \ \mathrm{d}x$ $=\int \sec x \sec^2 x \ \mathrm{d}x$ $=\sec x\tan x-\int \sec x\tan^2 x \ \mathrm{d}x$ $=\sec x\tan x-\int \sec x(\sec^2 x-1) \ \mathrm{d}x$ $=\sec x\tan x-\int \sec^3 x-\sec x \ \mathrm{d}x$ $=\sec x\tan x-\int \sec^3 x \ \mathrm{d}x+\ln\lvert\sec x+\tan x\rvert$ $2\int \sec^3 x \ \mathrm{d}x=\sec x\tan x+\ln\lvert\sec x+\tan x\rvert$ $\int \sec^3 x \ \mathrm{d}x=\frac{1}{2}(\sec x\tan x+\ln\lvert\sec x+\tan x\rvert)+C$
	$\int_0^{\tan^{-1}2}\sec^3 x \ \mathrm{d}x$ $=\frac{1}{2}\left[\sec x\tan x+\ln\lvert\sec x+\tan x\rvert\right]_0^{\tan^{-1}2}$ $=\frac{1}{2}\left[\sqrt{5}\times 2+\ln(\sqrt{5}+2)\right]$ $=\sqrt{5}+\frac{1}{2}\ln(\sqrt{5}+2)$ 

<table>
<tr><th>S/N</th><th>Solution</th></tr>
<tr><td>3 (i)</td><td>

$$\begin{aligned} 3x - x^2 - 4 &= -\left(x^2 - 3x + 4\right) \\ &= -\left(\left(x - \frac{3}{2}\right)^2 + \frac{7}{4}\right) \\ &= -\left(x - \frac{3}{2}\right)^2 - \frac{7}{4} \end{aligned}$$

Since $\left(x - \frac{3}{2}\right)^2 \geq 0$ for all $x \in \mathbb{R}$, $-\left(x - \frac{3}{2}\right)^2 \leq 0$

Hence $3x - x^2 - 4 = -\left(x - \frac{3}{2}\right)^2 - \frac{7}{4} \leq -\frac{7}{4} < 0$

$\therefore$ $3x - x^2 - 4$ is always negative for all values of x.

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<tr><td>3 (ii)</td><td>

$$\frac{\left(3x - x^2 - 4\right)\left(x - 1\right)^2}{x^2 - 2x - 5} \leq 0$$

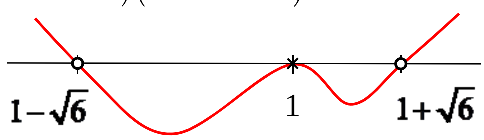
Since $3x - x^2 - 4$ is always negative, $\frac{\left(x-1\right)^2}{x^2 - 2x - 5} \geq 0$

<u>Method 1</u> (Quadratic formula)

Let $x^2 - 2x - 5 = 0$

$\therefore$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)} = \frac{2 \pm \sqrt{24}}{2} = 1 \pm \sqrt{6}$

Hence $\frac{\left(x-1\right)^2}{\left(x - (1 - \sqrt{6})\right)\left(x - (1 + \sqrt{6})\right)} \geq 0$

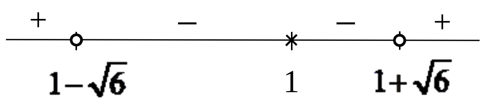


$\therefore$ $x < 1 - \sqrt{6}$ or $x > 1 + \sqrt{6}$ or $x = 1$

<u>Method 2</u> (Complete the square)

$$\frac{\left(x-1\right)^2}{\left(x-1\right)^2 - 6} \geq 0$$

$$\frac{\left(x-1\right)^2}{\left(x - (1 - \sqrt{6})\right)\left(x - (1 + \sqrt{6})\right)} \geq 0$$



$\therefore$ $x < 1 - \sqrt{6}$ or $x > 1 + \sqrt{6}$ or $x = 1$

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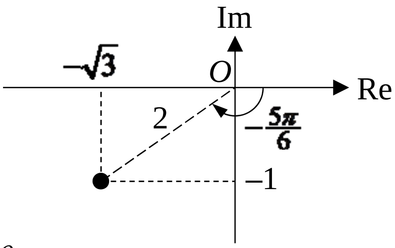
S/N	Solution
4 (i)	<u>Method 1</u> $w=\left(-\sqrt{3}-\mathrm{i}\right)z$ $=[2\,\mathrm{e}^{\mathrm{i}\left(-\frac{5\pi}{6}\right)}]r\,\mathrm{e}^{\mathrm{i}\theta}$ $=2r\,\mathrm{e}^{\mathrm{i}\left(-\frac{5\pi}{6}+\theta\right)}$ $\therefore \lvert w\rvert=2r\,,\ \arg w=-\frac{5\pi}{6}+\theta$  <u>Method 2</u> $\lvert w\rvert=\left\lvert\left(-\sqrt{3}-\mathrm{i}\right)z\right\rvert$ $=\left\lvert\left(-\sqrt{3}-\mathrm{i}\right)\right\rvert\lvert z\rvert$ $=2r$ $\arg w=\arg\left(\left[-\sqrt{3}-\mathrm{i}\right]z\right)$ $=\arg\left(-\sqrt{3}-\mathrm{i}\right)+\arg z$ $=-\frac{5\pi}{6}+\theta$
4 (ii)	<u>Method 1</u> $\arg\left(\frac{z^8}{w^*}\right)=\arg(z^8)-\arg(w^*)$ ← $\arg(w^*)=-\arg(w)$ $=8\theta+\arg w$ $=8\theta+\left(-\frac{5\pi}{6}+\theta\right)$ ← From **(ii)** $=9\theta-\frac{5\pi}{6}$ For $\frac{z^8}{w^*}$ to be purely imaginary, $\arg\left(\frac{z^8}{w^*}\right)=\ \ldots,-\frac{3\pi}{2},\ -\frac{\pi}{2},\ \frac{\pi}{2},\ \frac{3\pi}{2},\ \ldots$ $\therefore\ 9\theta-\frac{5\pi}{6}=\ \ldots,-\frac{3\pi}{2},\ -\frac{\pi}{2},\ \frac{\pi}{2},\ \frac{3\pi}{2},\ \ldots$ $9\theta=\ldots,-\frac{2\pi}{3},\ \frac{\pi}{3},\ \frac{4\pi}{3},\ \frac{7\pi}{3},\ \ldots$ $\theta=\ldots,-\frac{2\pi}{27},\ \frac{\pi}{27},\ \frac{4\pi}{27},\ \frac{7\pi}{27},\ \ldots$ $\therefore$ the three smallest values of θ are $\frac{\pi}{27}$, $\frac{4\pi}{27}$ and $\frac{7\pi}{27}$.

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<tr><th>S/N</th><th>Solution</th></tr>
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<u>Method 2</u>
$\dfrac{z^8}{w^*} = \dfrac{(r\,\mathrm{e}^{\mathrm{i}\theta})^8}{2r\,\mathrm{e}^{\mathrm{i}\left[-\left(-\frac{5\pi}{6}+\theta\right)\right]}} = \dfrac{r^8\,\mathrm{e}^{\mathrm{i}(8\theta)}}{2r\,\mathrm{e}^{\mathrm{i}\left(\frac{5\pi}{6}-\theta\right)}}$
$= \dfrac{r^7}{2}\mathrm{e}^{\mathrm{i}\left[8\theta-\left(\frac{5\pi}{6}-\theta\right)\right]}$
$= \dfrac{r^7}{2}\mathrm{e}^{\mathrm{i}\left(9\theta-\frac{5\pi}{6}\right)}$
For $\dfrac{z^8}{w^*}$ to be purely imaginary,
$\arg\left(\dfrac{z^8}{w^*}\right) = \dfrac{\pi}{2}+k\pi$, where $k \in \mathbb{Z}$
$\therefore\ 9\theta - \dfrac{5\pi}{6} = \dfrac{\pi}{2}+k\pi$
$9\theta = \dfrac{4\pi}{3}+k\pi$
$\theta = \dfrac{4\pi}{27}+\dfrac{k\pi}{9}$
When $k=-2$, $\theta = -\frac{2\pi}{27}$
When $k=-1$, $\theta = \frac{\pi}{27}$
When $k=0$, $\theta = \frac{4\pi}{27}$
When $k=1$, $\theta = \frac{7\pi}{27}$
$\therefore$ the three smallest values of θ are $\dfrac{\pi}{27}$, $\dfrac{4\pi}{27}$ and $\dfrac{7\pi}{27}$.
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<tr><td>5 (a)</td><td>
$(\underset{\sim}{a}+\underset{\sim}{b})\times(\underset{\sim}{a}+\underset{\sim}{c}) = \underset{\sim}{b}\times\underset{\sim}{c}$
$(\underset{\sim}{a}\times\underset{\sim}{a})+(\underset{\sim}{a}\times\underset{\sim}{c})+(\underset{\sim}{b}\times\underset{\sim}{a})+(\underset{\sim}{b}\times\underset{\sim}{c}) = \underset{\sim}{b}\times\underset{\sim}{c}$
$(\underset{\sim}{a}\times\underset{\sim}{c})+(\underset{\sim}{b}\times\underset{\sim}{a}) = \underset{\sim}{0}$
$(\underset{\sim}{a}\times\underset{\sim}{c})-(\underset{\sim}{a}\times\underset{\sim}{b}) = \underset{\sim}{0}$
$\underset{\sim}{a}\times(\underset{\sim}{c}-\underset{\sim}{b}) = \underset{\sim}{0}$
Since $\underset{\sim}{a}$ is non-zero and $\underset{\sim}{b}\neq\underset{\sim}{c}$,
$\therefore$ $\underset{\sim}{a}$ is parallel to $(\underset{\sim}{c}-\underset{\sim}{b})$.
$\therefore$ $\underset{\sim}{a} = k(\underset{\sim}{c}-\underset{\sim}{b})$, $k \in \mathbb{R}$.
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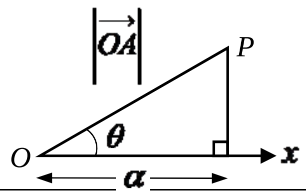
<tr><td>5 (b) (i)</td><td>
$\left|\overrightarrow{OA}\right| = \sqrt{\alpha^2+\beta^2+\gamma^2}$
$\therefore\ \cos\theta = \dfrac{\alpha}{\sqrt{\alpha^2+\beta^2+\gamma^2}}$


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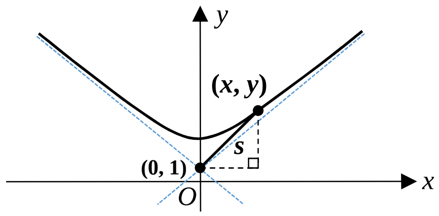
S/N	Solution
	$\cos\phi = \dfrac{\beta}{\sqrt{\alpha^2+\beta^2+\gamma^2}}$ $\cos\omega = \dfrac{\gamma}{\sqrt{\alpha^2+\beta^2+\gamma^2}}$ $\cos^2\theta+\cos^2\phi+\cos^2\omega$ $=\left(\dfrac{\alpha}{\sqrt{\alpha^2+\beta^2+\gamma^2}}\right)^2+\left(\dfrac{\beta}{\sqrt{\alpha^2+\beta^2+\gamma^2}}\right)^2+\left(\dfrac{\gamma}{\sqrt{\alpha^2+\beta^2+\gamma^2}}\right)^2$ $=\dfrac{\alpha^2+\beta^2+\gamma^2}{\alpha^2+\beta^2+\gamma^2}$ $=1$
(ii)	$\cos 2\theta+\cos 2\phi+\cos 2\omega$ $=2\cos^2\theta-1+2\cos^2\phi-1+2\cos^2\omega-1$ $=2\left(\cos^2\theta+\cos^2\phi+\cos^2\omega\right)-3$ $=2(1)-3$ $=-1$ (shown)
6 (a)	$x=\cot 3t \Rightarrow \dfrac{dx}{dt}=-3\operatorname{cosec}^2 3t$ $y=2\operatorname{cosec}3t+1 \Rightarrow \dfrac{dy}{dt}=-6\operatorname{cosec}3t\cot 3t$ $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{-6\operatorname{cosec}3t\cot 3t}{-3\operatorname{cosec}^2 3t}$ $=\dfrac{2\cot 3t}{\operatorname{cosec}3t}=2\cos 3t$ At point P, $\dfrac{dy}{dx}\Big\vert_{t=p}=2\cos 3p$ Equation of tangent at P: $y-\left(2\operatorname{cosec}3p+1\right)=2\cos 3p\left(x-\cot 3p\right)$ When tangent meets y-axis, $x=0$. Hence $y=-\left(2\cos 3p\right)\left(\cot 3p\right)+\left(2\operatorname{cosec}3p+1\right)$ $y=\dfrac{-2\left(\cos^2 3p\right)}{\sin 3p}+\dfrac{2}{\sin 3p}+1$ $y=\dfrac{-2(\cos^2 3p-1)}{\sin 3p}+1$ $y=\dfrac{-2(-\sin^2 3p)}{\sin 3p}+1$ $y=2\sin 3p+1$ Hence the coordinates of Q is $\left(0,\ 2\sin 3p+1\right)$. (shown)

S/N	Solution
6 (b)	



<u>Method 1</u>

$$s^2 = x^2 + \left(y-1\right)^2$$
$$= \cot^2 3t + \left(2\operatorname{cosec}3t + 1 - 1\right)^2$$
$$= (\operatorname{cosec}^2 3t - 1) + 4\operatorname{cosec}^2 3t$$
$$= 5\operatorname{cosec}^2 3t - 1$$

Differentiate w.r.t. t ,

$$2s\frac{\mathrm{d}s}{\mathrm{d}t} = 10\operatorname{cosec}3t(-\operatorname{cosec}3t\cot 3t)(3)$$
$$= -30\operatorname{cosec}^2 3t\cot 3t$$
$$s\frac{\mathrm{d}s}{\mathrm{d}t} = -15\operatorname{cosec}^2 3t\cot 3t$$

When $t = \frac{\pi}{4}$, $s^2 = (2\sqrt{2} + 1 - 1)^2 + (-1)^2 = 9$

$\therefore\ s = 3$ (since $s > 0$)

$$\therefore\ \frac{\mathrm{d}s}{\mathrm{d}t} = -5\operatorname{cosec}^2 3\left(\tfrac{\pi}{4}\right)\cot 3\left(\tfrac{\pi}{4}\right)$$
$$= -5(2)(-1)$$
$$= 10 \text{ unit/s}$$

<u>Method 2</u>

$$s^2 = x^2 + \left(y-1\right)^2$$
$$= \cot^2 3t + \left(2\operatorname{cosec}3t + 1 - 1\right)^2$$
$$= \cot^2 3t + 4\operatorname{cosec}^2 3t$$

Differentiate w.r.t. t ,

$$2s\frac{\mathrm{d}s}{\mathrm{d}t} = 2\cot 3t(-\operatorname{cosec}^2 3t)(3) + 8\operatorname{cosec}3t(-\operatorname{cosec}3t\cot 3t)(3)$$
$$= -6\operatorname{cosec}^2 3t\cot 3t - 24\operatorname{cosec}^2 3t\cot 3t$$
$$= -30\operatorname{cosec}^2 3t\cot 3t$$
$$s\frac{\mathrm{d}s}{\mathrm{d}t} = -15\operatorname{cosec}^2 3t\cot 3t$$

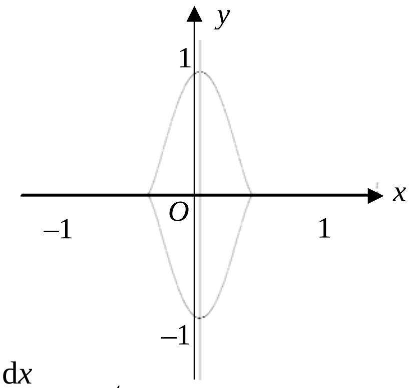
When $t = \frac{\pi}{4}$, $s^2 = (2\sqrt{2} + 1 - 1)^2 + (-1)^2 = 9$

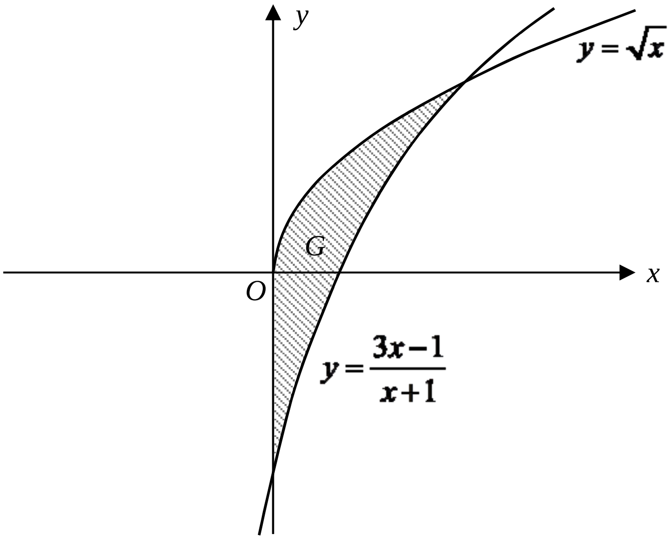
$\therefore\ s = 3$ (since $s > 0$)

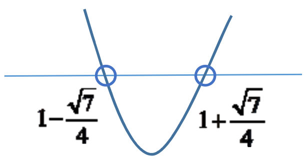
$$\therefore\ \frac{\mathrm{d}s}{\mathrm{d}t} = -5\operatorname{cosec}^2 3\left(\tfrac{\pi}{4}\right)\cot 3\left(\tfrac{\pi}{4}\right)$$
$$= -5(2)(-1)$$
$$= 10 \text{ unit/s}$$

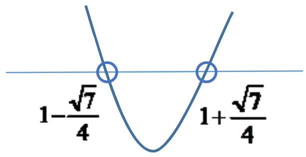
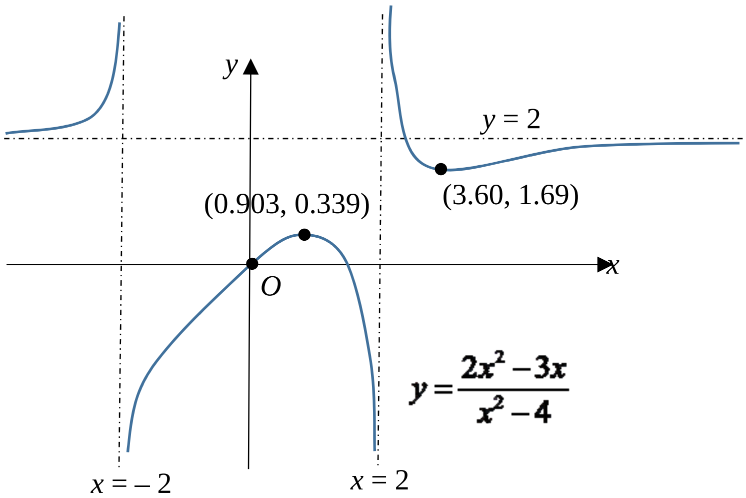
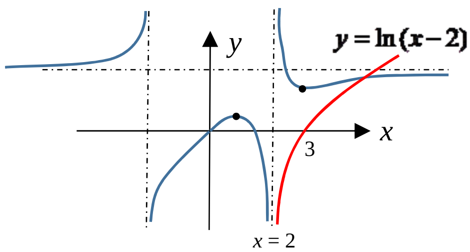
S/N	Solution
	<u>Method 3</u> $s^2 = x^2 + (y-1)^2$ Differentiate w.r.t. t , $2s\frac{\mathrm{d}s}{\mathrm{d}t} = 2x\frac{\mathrm{d}x}{\mathrm{d}t} + 2(y-1)\frac{\mathrm{d}y}{\mathrm{d}t}$ $s\frac{\mathrm{d}s}{\mathrm{d}t} = x\frac{\mathrm{d}x}{\mathrm{d}t} + (y-1)\frac{\mathrm{d}y}{\mathrm{d}t}$ When $t = \frac{\pi}{4}$, $x = \frac{1}{\tan\left(\frac{3\pi}{4}\right)} = -1, \quad y = \frac{2}{\sin\left(\frac{3\pi}{4}\right)} + 1 = 2\sqrt{2} + 1$ $\frac{\mathrm{d}x}{\mathrm{d}t} = -3\operatorname{cosec}^2 3t = \frac{-3}{\sin^2\left(\frac{3\pi}{4}\right)} = -6$ $\frac{\mathrm{d}y}{\mathrm{d}t} = -6\cot 3t \operatorname{cosec} 3t = \frac{-6}{\tan\left(\frac{3\pi}{4}\right)} \times \frac{1}{\sin\left(\frac{3\pi}{4}\right)} = 6\sqrt{2}$ $s^2 = (2\sqrt{2}+1-1)^2 + (-1)^2 = 9$ $\therefore\ s = 3 \quad (\text{since } s > 0)$ Hence $\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{1}{s}\left[x\frac{\mathrm{d}x}{\mathrm{d}t} + (y-1)\frac{\mathrm{d}y}{\mathrm{d}t}\right]$ $= \frac{1}{3}\left[(-1)(-6) + (2\sqrt{2})(6\sqrt{2})\right]$ $= 10$ unit/s
7 (i)	<u>Method 1</u> $\ln y = 2\sin x$ $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 2\cos x$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 2y\cos x$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -2y\sin x + 2\cos x\frac{\mathrm{d}y}{\mathrm{d}x} = -y\ln y + \frac{1}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ (shown) <u>Method 2</u> $y = \mathrm{e}^{2\sin x}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = (2\cos x)\mathrm{e}^{2\sin x}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = (2\cos x)y$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -2y\sin x + 2\cos x\frac{\mathrm{d}y}{\mathrm{d}x}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -y\ln y + \frac{1}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ (shown)

S/N	Solution
7 (ii)	$\frac{d^3y}{dx^3} = -y\left(\frac{1}{y}\frac{dy}{dx}\right) - \ln y\frac{dy}{dx} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^3 + \frac{2}{y}\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)$ When $x=0$, $y=1$, $\frac{dy}{dx}=2$, $\frac{d^2y}{dx^2}=4$, $\frac{d^3y}{dx^3}=6$ $y = 1+2x+\frac{4x^2}{2!}+\frac{6x^3}{3!}+...$ $y = 1+2x+2x^2+x^3+...$
7 (iii)	Method 1 $y = e^{2\sin x}$ $= 1+(2\sin x)+\frac{(2\sin x)^2}{2}+\frac{(2\sin x)^3}{6}+...$ $= 1+2(x-\frac{x^3}{6}+...)+\frac{[2(x+...)]^2}{2}+\frac{[2(x+...)]^3}{6}+...$ $= 1+2x-\frac{x^3}{3}+2x^2+\frac{4x^3}{3}+...$ $= 1+2x+2x^2+x^3+...$ Method 2 $y = e^{2(x-\frac{x^3}{3!})}$ $= 1+2(x-\frac{x^3}{6})+\frac{[2(x-\frac{x^3}{6})]^2}{2}+\frac{[2(x-\frac{x^3}{6})]^3}{6}+...$ $= 1+2x-\frac{2x^3}{6}+\frac{4x^2}{2}+\frac{8x^3}{6}+...$ $= 1+2x+2x^2+x^3+...$
7 (iv)	$e^{(2\sin x)-\ln(\sec x)} = e^{(2\sin x)}e^{-\ln\sec x} = e^{(2\sin x)}e^{\ln\cos x}$ $= e^{(2\sin x)}\cos x$ Method 1 $e^{(2\sin x)}\cos x \approx (1+2x+2x^2+x^3)(1-\frac{x^2}{2})$ $= 1-\frac{x^2}{2}+2x-\frac{2x^3}{2}+2x^2+x^3+...$ $= 1+2x+\frac{3}{2}x^2+...$ Method 2 $y = e^{2\sin x}$ $\frac{dy}{dx} = (2\cos x)e^{2\sin x}$ $\therefore\ \cos x e^{2\sin x} = \frac{1}{2}\frac{dy}{dx}$ $= \frac{1}{2}\frac{d}{dx}(1+2x+2x^2+x^3+...)$ ← From **(iii)**

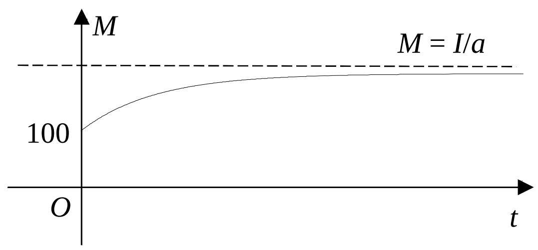
S/N	Solution
	$=\frac{1}{2}(2+4x+3x^2+...)$ $=1+2x+\frac{3}{2}x^2+...$
8 (a) (i)	 $x=\sin t \Rightarrow \frac{dx}{dt}=\cos t$ When $x=0$, $t=0$. When $x=1$, $t=\frac{\pi}{2}$. Area $=4\int_0^1 y\ dx$ $=4\int_0^{\frac{\pi}{2}}(\cos^3 t)\cos t\ dt$ $=4\int_0^{\frac{\pi}{2}}\cos^4 t\ dt$ (shown) $\therefore\ k=4$
(ii)	Area $=4\int_0^{\frac{\pi}{2}}\cos^4 t\ dt$ $=\int_0^{\frac{\pi}{2}}(2\cos^2 t)^2\ dt$ $=\int_0^{\frac{\pi}{2}}(1+\cos 2t)^2\ dt$ $=\int_0^{\frac{\pi}{2}}1+2\cos 2t+\cos^2 2t\ dt$ $=\int_0^{\frac{\pi}{2}}1+2\cos 2t+\frac{1+\cos 4t}{2}\ dt$ $=\int_0^{\frac{\pi}{2}}\frac{3}{2}+2\cos 2t+\frac{\cos 4t}{2}\ dt$ $=\left[\frac{3t}{2}+\sin 2t+\frac{\sin 4t}{8}\right]_0^{\frac{\pi}{2}}$ $=\frac{3\pi}{4}\ \text{unit}^2$

S/N	Solution
8 (b)	 From GC, coordinates of intersection = (1, 1) <u>Method 1</u> $y=\frac{3x-1}{x+1} \Rightarrow xy+y=3x-1 \Rightarrow x=\frac{1+y}{3-y}$ Required volume $=\pi\int_{-1}^{1}\left(\frac{1+y}{3-y}\right)^2 \text{ d}y-\pi\int_{0}^{1}(y^2)^2 \text{ d}y$ $=\pi\int_{-1}^{1}\left(\frac{4}{3-y}-1\right)^2 \text{ d}y-\pi\int_{0}^{1}y^4 \text{ d}y$ $=\pi\int_{-1}^{1}\left(\frac{16}{(3-y)^2}-\frac{8}{3-y}+1\right)\text{d}y-\pi\left[\frac{y^5}{5}\right]_0^1$ $=\pi\left[\frac{16}{3-y}+8\ln\lvert 3-y\rvert+y\right]_{-1}^{1}-\frac{\pi}{5}$ $=\pi\left[8+8\ln 2+1-(4+8\ln 4-1)\right]-\frac{\pi}{5}$ $=\pi\left[6+8\ln 2-16\ln 2\right]-\frac{\pi}{5}$ $=\frac{29\pi}{5}-8\pi\ln 2 \quad \text{unit}^3$ <u>Method 2</u> $y=\frac{3x-1}{x+1} \Rightarrow xy+y=3x-1 \Rightarrow x=\frac{1+y}{3-y}$ Required volume $=\pi\int_{-1}^{1}\left(\frac{1+y}{3-y}\right)^2 \text{ d}y-\pi\int_{0}^{1}(y^2)^2 \text{ d}y$ $=\pi\int_{-1}^{1}\frac{y^2+2y+1}{y^2-6y+9} \text{ d}y-\pi\int_{0}^{1}y^4 \text{ d}y$

S/N	Solution
	$=\pi\int_{-1}^{1} 1+\dfrac{8y-8}{y^2-6y+9}\ \mathrm{d}y-\pi\left[\dfrac{y^5}{5}\right]_0^1$ $=\pi\left[y\right]_{-1}^{1}+4\pi\int_{-1}^{1}\dfrac{2y-6}{y^2-6y+9}\ \mathrm{d}y+\pi\int_{-1}^{1}\dfrac{16}{(y-3)^2}\ \mathrm{d}y-\dfrac{\pi}{5}$ $=2\pi+4\pi\left[\ln\left\|y^2-6y+9\right\|\right]_{-1}^{1}+16\pi\left[\dfrac{(y-3)^{-1}}{-1}\right]_{-1}^{1}-\dfrac{\pi}{5}$ $=\dfrac{9\pi}{5}+4\pi\left[\ln 4-\ln 16\right]+16\pi\left[\dfrac{1}{3-y}\right]_{-1}^{1}$ $=\dfrac{9\pi}{5}+4\pi\ln\dfrac{1}{4}+16\pi\left[\dfrac{1}{2}-\dfrac{1}{4}\right]$ $=\dfrac{9\pi}{5}-4\pi\ln 4+4\pi$ $=\dfrac{29\pi}{5}-8\pi\ln 2\quad \text{unit}^3$
9 **(i)**	$y=\dfrac{ax^2-bx}{x^2-c}$ Since $y=2$ is a horizontal asymptote, $a=2$. Since $x=-2$ is a vertical asymptote, $c=4$. $\left(3,\frac{9}{5}\right)$ lies on $y=\dfrac{2x^2-bx}{x^2-4}$ $\therefore\ \frac{9}{5}=\frac{2(3)^2-b(3)}{(3)^2-4}\quad\Rightarrow\quad b=3$
9 **(ii)**	$y=\dfrac{2x^2-3x}{x^2-4}$ $y\left(x^2-4\right)=2x^2-3x$ $\left(y-2\right)x^2+3x-4y=0$ For no real roots, $(3)^2-4\left(y-2\right)\left(-4y\right)<0$ $16y^2-32y+9<0$  <u>Method 1</u> $\therefore\ y=\dfrac{32\pm\sqrt{(32)^2-4(16)(9)}}{2(16)}=\dfrac{32\pm\sqrt{448}}{32}=1\pm\frac{\sqrt{7}}{4}$ $\therefore$ required set is $\left\{y\in\mathbb{R}:1-\frac{\sqrt{7}}{4}<y<1+\frac{\sqrt{7}}{4}\right\}$. <u>Method 2</u> (completing the square)

S/N	Solution
	$16y^2-32y+9<0$ $y^2-2y+\frac{9}{16}<0$ $(y-1)^2-\frac{7}{16}<0$ $\left(y-1+\frac{\sqrt{7}}{4}\right)\left(y-1-\frac{\sqrt{7}}{4}\right)<0$ $\therefore\ 1-\frac{\sqrt{7}}{4}<y<1+\frac{\sqrt{7}}{4}$ $\therefore$ required set is $\left\{y\in\mathbb{R}:1-\frac{\sqrt{7}}{4}<y<1+\frac{\sqrt{7}}{4}\right\}$ 
9 (iii)	
9 (iv)	$e^y=x-r$ $y=\ln(x-r)$ $r\geq 2$ 
9 (v)	$C_1: y=\frac{2x^2-3x}{x^2-4}=2+\frac{8-3x}{x^2-4}$ $C_2: y=2+\frac{3x+5}{x^2-2x-3}$ $=2+\frac{3x+5}{(x-1)^2-4}$ $=2+\frac{8-3(1-x)}{(1-x)^2-4}$ Long division: $x^2-4\,\overline{)\,2x^2-3x}$, quotient 2; subtract $\underline{2x^2-8}$; remainder $-3x+8$ <u>Method 1</u> Transformation: $x\to x+1\to -x+1$ 1. Translation of C_1 1 unit in the negative x-direction to get

S/N	Solution
	$y=2+\frac{8-3(x+1)}{(x+1)^2-4}=2+\frac{-3x+5}{x^2+2x-3}$ followed by 2. Reflection of $y=2+\frac{-3x+5}{x^2+2x-3}$ in the y-axis to get C_2. <u>Method 2</u> Transformation: $x \to -x \to -(x-1)=-x+1$ 1. Reflection of C_1 in the y-axis to get $y=2+\frac{8+3x}{x^2-4}$ followed by 2. Translation of $y=2+\frac{8+3x}{x^2-4}$ 1 unit in the positive x-direction to get C_2.
10 (i)	$\frac{\mathrm{d}M}{\mathrm{d}t} \propto I-kM$, where k is a positive constant. $\frac{\mathrm{d}M}{\mathrm{d}t}=b(I-kM)$ If $I=0$, $-\frac{1}{100}M=b(0-kM)$ $-\frac{M}{100}=-bkM$ $b=\frac{1}{100k}$ $\frac{\mathrm{d}M}{\mathrm{d}t}=\frac{1}{100k}(I-kM)=\frac{I-kM}{100k}$ $=\frac{I-aM}{100a}$, where $a=k$ (shown) Assumption (any 1 below): • The man does not exercise so that no food energy is used up through exercising. • The man does not fall sick so that no food energy is used up to help him recover from his illness. • The man does not consume weight enhancing/loss supplements that affect his food energy gain/loss other than maintaining the healthy functioning of his body and increasing body mass.
10 (ii)	For $\frac{\mathrm{d}M}{\mathrm{d}t}$ to be zero, $I=aM$
10 (iii)	$\int \frac{a}{I-aM}\,\mathrm{d}M=\int \frac{1}{100}\,\mathrm{d}t$ $-\ln\lvert I-aM\rvert=\frac{t}{100}+C$

S/N	Solution
	$\ln\lvert I-aM\rvert=\dfrac{-t}{100}-C$ $I-aM=\pm e^{\frac{-t}{100}}e^{-C}=Ae^{\frac{-t}{100}}$, where $A=\pm e^{-C}$ When t = 0, M = 100 $\Rightarrow A=I-100a$ $I-aM=(I-100a)e^{\frac{-t}{100}}$ $aM=I-(I-100a)e^{\frac{-t}{100}}$ $M=\dfrac{I}{a}-\left(\dfrac{I}{a}-100\right)e^{\frac{-t}{100}}$
10 (iv)	 Explanation (any 1 below): • The man consumes more food than is necessary for maintaining a healthy functioning body. Therefore the graph shows that his body mass will increase. • Since $I>100a$, hence $\frac{I}{a}>100$. The man's body mass is always less than $\frac{I}{a}$. • In the long run, the man's body mass will approach $\frac{I}{a}$.
10 (v)	Given $I=50a$, $90=50-(50-100)e^{\frac{-t}{100}}$ ← Using equation found in **(iii)** $50e^{\frac{-t}{100}}=40$ $e^{\frac{-t}{100}}=\dfrac{4}{5}$ $\dfrac{-t}{100}=\ln\dfrac{4}{5}$ $\therefore\ t=-100\ln\dfrac{4}{5}=22.3$ days (3 s.f.)
11(i)	<u>Method 1</u> Distance covered at the n^{th} pull $=45+(n-1)(-1.6)$ $=46.6-1.6n$ $46.6-1.6n\geq 0$ $n\leq 29.125$ Hence number of pulls needed to achieve maximum total height is 29.

S/N	Solution
	Maximum total height $=\frac{29}{2}\left[2(45)+(29-1)(-1.6)\right]$ $=655.4$ cm <u>Method 2</u> Distance covered at the n^{th} pull, $u_n=45+(n-1)(-1.6)$ $=46.6-1.6n$ Using GC, <table><tr><td>n</td><td>u_n</td></tr><tr><td>29</td><td>0.2</td></tr><tr><td>30</td><td>–1.4</td></tr></table>Hence number of pulls needed to achieve maximum total height is 29. Maximum total height $=\frac{29}{2}\left(45+0.2\right)=655.4$ cm <u>Method 3</u> Distance covered at the n^{th} pull $=45+(n-1)(-1.6)=0$ $\Rightarrow n=29.125$ <table><tr><td>n</td><td>u_n</td></tr><tr><td>29</td><td>0.2</td></tr><tr><td>30</td><td>–1.4</td></tr></table>Hence number of pulls needed to achieve maximum total height is 29. Maximum total height $=\frac{29}{2}\left(45+0.2\right)=655.4$ cm <u>Method 4</u> Total height after n pulls, $S_n=\frac{n}{2}\left[2(45)+(n-1)(-1.6)\right]=45.8n-0.8n^2$ Using GC, <table><tr><td>n</td><td>S_n</td></tr><tr><td>28</td><td>655.2</td></tr><tr><td>29</td><td>655.4</td></tr><tr><td>30</td><td>654</td></tr></table>Hence the number of pulls needed to achieve maximum total height is 29, and the maximum total height covered is 655.4 cm.
11 (ii)	Since $r=0.95<1$, sum to infinity of G.P. exists. $\therefore$ maximum total height $=\dfrac{45}{1-0.95}=900$ cm

<table>
<tr><th>S/N</th><th>Solution</th></tr>
<tr><td>11 (iii)</td><td>
<table>
<tr><td></td><td>Total height reached</td></tr>
<tr><td>Before 2nd pull</td><td>$0.98(45)$</td></tr>
<tr><td>Before 3rd pull</td><td>$0.98\left(0.98(45)+45\right)$
$=0.98^2(45)+0.98(45)$</td></tr>
<tr><td>Before 4th pull</td><td>$0.98\left(0.98^2(45)+0.98(45)+45\right)$
$=0.98^3(45)+0.98^2(45)+0.98\left(45\right)$</td></tr>
<tr><td>$\vdots$</td><td>$\vdots$</td></tr>
<tr><td>Before $(n+1)^{\text{th}}$ pull</td><td>$0.98^n(45)+0.98^{n-1}(45)+\ldots+0.98\left(45\right)$
$=\dfrac{0.98(45)(1-0.98^n)}{1-0.98}$
[sum of G.P. with $a=45$, $r=0.98$]</td></tr>
</table>
$\therefore$ before 4^{th} pull, total height reached
$=\dfrac{0.98(45)(1-0.98^3)}{1-0.98}$
$=129.67164$
$=130$ cm (3 s.f.)
Before $(n+1)^{\text{th}}$ pull, total height reached
$=\dfrac{0.98(45)(1-0.98^n)}{1-0.98}$
$=2205-2250(0.98)^{n+1}$, where $X=2205$, $Y=-2250$
</td></tr>
<tr><td>11 (iv)</td><td>From (iii),
Total height reached by load using hoist C $=2205-2250(0.98)^{n+1}$
As $n\to\infty$, $(0.98)^{n+1}\to 0$.
Hence maximum total height $\to 2205$.
Therefore maximum total height reached by load using hoist C will approach 2205 cm. Therefore the hoist C cannot be used to lift the load up the building of 2500 cm</td></tr>
</table>