Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) 9758/01

Paper 1

12 September 2017

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Score												
Max Score	3	5	5	5	5	9	13	15	16	12	12	100

1 Given that $\sum_{k=1}^{n} k!(k^2+1)=(n+1)!\,n$, find $\sum_{k=1}^{n-1}(k+1)!(k^2+2k+2)$. [3]

2 A geometric sequence $T_1, T_2, T_3, \ldots$ has a common ratio of e. Another sequence $U_1, U_2, U_3, \ldots$ is such that $U_1 = 1$ and

$$U_r = \ln T_r - 3 \quad \text{for all } r \geq 1.$$

(i) Prove that the sequence $U_1, U_2, U_3, \ldots$ is arithmetic. [2]

A third sequence $W_1, W_2, W_3, \ldots$ is such that $W_1 = \frac{1}{2}$ and $W_{r+1} = W_r + U_r$ for all $r \geq 1$.

(ii) By considering $\sum_{r=1}^{n-1}(W_{r+1} - W_r)$, show that $W_n = \frac{1}{2}(n^2 - n + 1)$. [3]

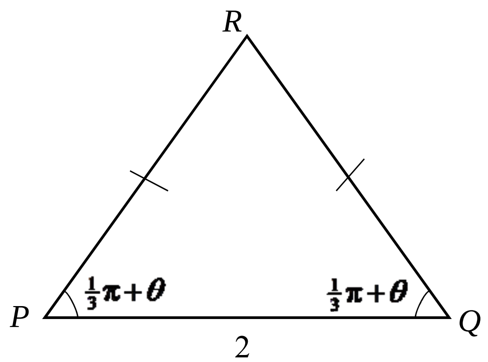
3 Using an algebraic method, find the set of values of x that satisfies the inequality

$$2 - x \leq \frac{x}{2-x}.$$

[3]

Hence solve $2 - x^2 \leq \dfrac{x^2}{2-x^2}$. [2]

4



In the isosceles triangle PQR, $PQ = 2$ and the angle QPR = angle $PQR = \left(\frac{1}{3}\pi + \theta\right)$ radians. The area of triangle PQR is denoted by A.

Given that θ is a sufficiently small angle, show that

$$A = \frac{\sqrt{3} + \tan\theta}{1 - \sqrt{3}\tan\theta} \approx a + b\theta + c\theta^2,$$

for constants a, b and c to be determined in exact form. [5]

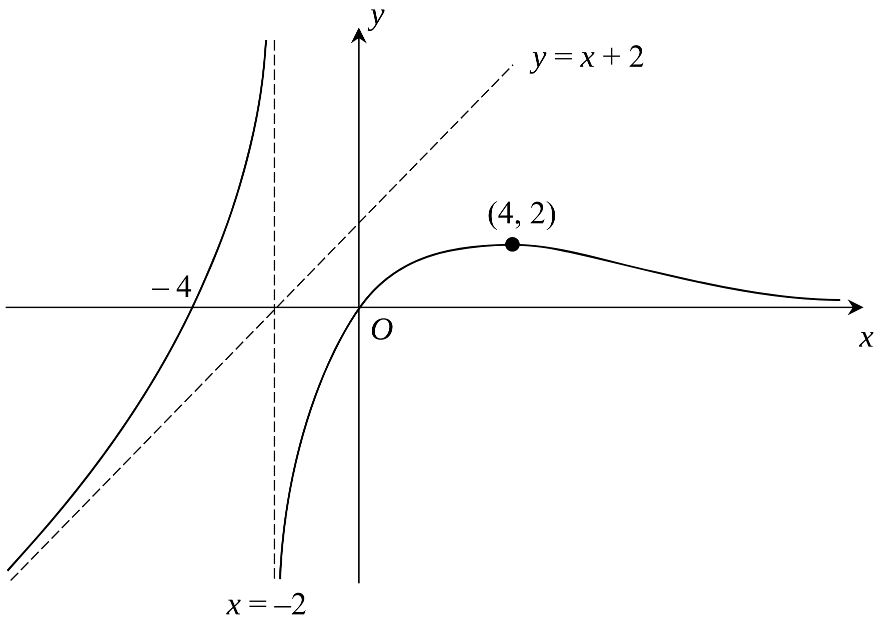
5 **(a)** Given that $\operatorname{cosec} y = x$ for $0 < y < \frac{1}{2}\pi$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y. Deduce that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\operatorname{cosec}^{-1} x) = -\dfrac{1}{x\sqrt{(x^2-1)}}$ for $x > 1$. [3]

(b) The function f is such that $\mathrm{f}(x)$ and $\mathrm{f}'(x)$ exist for all real x. Sketch a possible graph of f which illustrates that the following statement is not necessarily true:

"If the equation $\mathrm{f}'(x) = 0$ has exactly one root $x = 0$ and $\mathrm{f}''(0) > 0$, then $\mathrm{f}(x) \to \infty$ as $x \to \pm\infty$." [2]

6 **(a)** State a sequence of transformations that transform the graph of $x^2 + \frac{1}{3}(y-2)^2 = 1$ to the graph of $(x-2)^2 + y^2 = 1$. [3]

(b) The diagram below shows the curve $y = \mathrm{f}(x)$. It has a maximum point at $(4, 2)$ and intersects the x-axis at $(-4, 0)$ and the origin. The curve has asymptotes $x = -2$, $y = 0$ and $y = x + 2$.



Sketch on separate diagrams, the graphs of

(i) $y = \mathrm{f}'(x)$, [3]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

including the coordinates of the points where the graphs cross the axes, the turning points and the equations of any asymptotes, where appropriate.

7 **(i)** Express $\sin x + \sqrt{3}\cos x$ as $R\sin(x+\alpha)$, where $R>0$ and α is an acute angle. [1]

The function f is defined by

$$\mathrm{f}: x \mapsto \sin x + \sqrt{3}\cos x, \quad x \in \mathbb{R},\ -\tfrac{1}{3}\pi \le x \le \tfrac{1}{6}\pi.$$

(ii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iii) Find $\mathrm{f}^{-1}(x)$, stating the domain of f^{-1}. On the same diagram as in part **(ii)**, sketch the graph of $y = \mathrm{f}^{-1}(x)$, indicating the equation of the line of symmetry. [4]

(iv) Using integration, find the area of the region bounded by the graph of f^{-1} and the axes. [3]

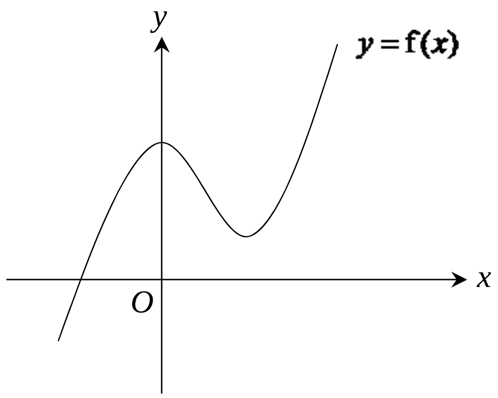
The function g is defined by

$$\mathrm{g}: x \mapsto |\ln(x+2)|, \text{ for } x \in \mathbb{R},\ x > -2.$$

(v) Show that the composite function gf^{-1} exists, and find the range of gf^{-1}. [3]

8 **Do not use a graphic calculator in answering this question.**

(a)



It is given that $\mathrm{f}(x)$ is a cubic polynomial with real coefficients. The diagram shows the curve with equation $y = \mathrm{f}(x)$. What can be said about all the roots of the equation $\mathrm{f}(x) = 0$? [2]

(b) The equation $2z^2 - (7+6\mathrm{i})z + 11 + \mathrm{i}c = 0$, where c is a non-zero real number, has a root $z = 3+4\mathrm{i}$. Show that $c = -2$. Determine the other root of the equation in cartesian form. Hence find the roots of the equation $2w^2 + (-6+7\mathrm{i})w - 11 + 2\mathrm{i} = 0$. [6]

(c) The complex number z is given by $z = 1 + \mathrm{e}^{\mathrm{i}\alpha}$.

(i) Show that z can be expressed as $2\cos\left(\tfrac{1}{2}\alpha\right)\mathrm{e}^{\mathrm{i}\left(\frac{1}{2}\alpha\right)}$. [2]

(ii) Given $\alpha = \tfrac{1}{3}\pi$ and $w = -1-\sqrt{3}\mathrm{i}$, find the exact modulus and argument of $\left(\dfrac{z}{w^3}\right)^*$. [5]

9 The line l_1 passes through the point A, whose position vector is $3\mathbf{i}+7\mathbf{j}+3\mathbf{k}$, and is parallel to the vector $3\mathbf{i}+4\mathbf{j}+\mathbf{k}$. The line l_2 is given by the cartesian equation $x-2=\dfrac{3-y}{2}=\dfrac{z-5}{2}$.

The plane p_1 contains l_1 and is parallel to l_2. Another plane p_2 also contains l_1 and is perpendicular to p_1.

(i) Find the cartesian equation of p_1. [3]

(ii) Find the distance of l_2 to p_1. [2]

(iii) Find the equation of p_2 in the scalar product form. [2]

A particle P moves along a straight line c which lies in the plane p_2 and c passes through a point $(5, \frac{1}{2}, -3)$. P hits the plane p_1 at A and rebounds to move along another straight line d in p_2. The angle between d and l_1 is the same as the angle between c and l_1.

(iv) Find the direction cosines of d. [6]

(v) Another particle, Q, is placed at the point $(\frac{25}{2}, \frac{21}{2}, -\frac{1}{2})$. Find the shortest distance PQ as P moves along d. [3]

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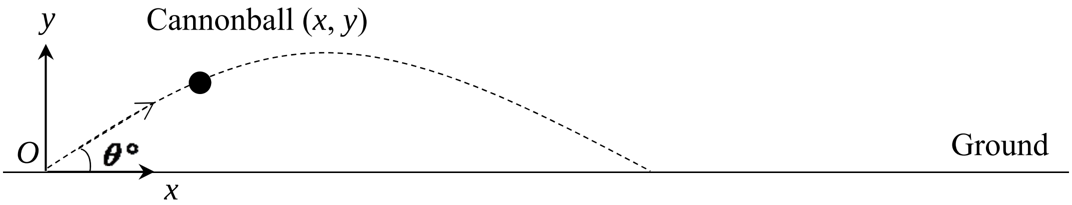

The diagram shows the trajectory of a cannonball fired off from an origin O with an initial speed of v ms^{-1} and at an angle of $\theta°$ above the ground. At time t seconds, the position of the cannonball can be modelled by the parametric equations

$$x=(v\cos\theta)t,\ \ y=(v\sin\theta)t-5t^2,$$

where x m is the horizontal distance of the cannonball with respect to O and y m is the vertical distance of the cannonball with respect to ground level.

(i) Find the horizontal distance, d m, that a cannonball would have travelled by the time it hits the ground. Leave your answer in terms of v and θ. [4]

Use $v=200$ to answer the remaining parts of the question.

An approaching target is travelling at a constant speed of 10 ms^{-1} along the ground. A cannonball is fired towards the target when it is 3000 m away. You may assume the height of the moving target is negligible.

(ii) Show that in order to hit the target, the possible angles at which the cannonball should be fired are $22.7°$ and $69.5°$. [2]

(iii) Explain at which angle the cannonball should be fired in order to hit the target earlier. [2]

(iv) Given that $\theta=22.7$, find the angle that the tangent to the trajectory makes with the horizontal when $x=370$. [4]

11 For this question, you may leave your answers to the nearest dollar.

(a) Mr Foo invested \$25,000 in three different stocks A, B and C. After a year, the value of the stocks A and B grew by 2% and 6% respectively, while the value of stock C fell by 2%. Mr Foo did not gain or lose any money. Let a, b and c denote the amount of money he invested in stocks A, B and C respectively.

(i) Find expressions for a and b, in terms of c. [2]

(ii) Find the values between which c must lie. [2]

(b) Mr Lee is interested in growing his savings amount of \$55,000 and is considering the Singapore Savings Bonds. He is able to enjoy a higher average return per year when he invests over a longer period of time as shown in the following table.

Number of years invested	1	2	3	4	5	6	7	8	9	10
Average return per year, %	1.04	1.21	1.35	1.48	1.60	1.71	1.82	1.92	2.02	2.12

For example, if Mr Lee invests for two years, he is able to enjoy compound interest at a rate of 1.21% per year.

(i) Calculate the compound interest earned by Mr Lee if he were to invest \$55,000 in this bond for a period of five years. [2]

A bank offers a dual-savings account with the following scheme:

“ For every \$1,000 deposited into the normal savings account, an individual can deposit \$10,000 into the special savings account to enjoy a higher interest rate. The annual compound interest rates for the normal savings account and the special savings account are 0.19% and 1.8% respectively.”

Mr Lee is interested in setting up this dual-savings account and considers an n-year investment plan as such:

At the start of each year, he will place \$1,000 in the normal savings account and \$10,000 in the special savings account.

(ii) Find the respective amount of money in the normal savings account and special savings account at the end of n years. Leave your answers in terms of n. [4]

(iii) Find the least value of n such that the compound interest earned in dual-savings account is more than the compound interest earned in part **(i)**. [2]