Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

FURTHER MATHEMATICS (Higher 2) 9649/02

Paper 2

September 2017

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Score												
Max Score	**5**	**6**	**6**	**8**	**11**	**14**	**4**	**11**	**11**	**12**	**12**	**100**

Section A: Pure Mathematics [50 marks]

1 Determine, with reasons, whether each of the following transformations is a linear transformation:

(a) The transformation $S : \mathbb{R}^3 \to \mathbb{R}$ is given by $S(\mathbf{x}) = \sqrt{\mathbf{x}^T \mathbf{x}}$. [2]

(b) The transformation $T : P_n(x) \to P_{n-1}(x)$ is given by $T(\mathrm{f}(x)) = \mathrm{f}'(x)$ where $P_n(x)$ is the set of polynomials with real coefficients of the form $a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$. [3]

2 The linear transformation $V : \mathbb{R}^5 \to \mathbb{R}^3$ is represented by the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 & -1 & 7 \\ -3 & 4 & 1 & -7 & -1 \\ 5 & -6 & -1 & 11 & 3 \end{pmatrix}.$$

(i) Find the basis of the null space of V and the rank of **A**. [3]

(ii) Show that the range space of V is a plane in $\mathbb{R}^3$ and find the cartesian equation of this plane. [3]

3 The equation $z^4 = a + \mathrm{i}b$, where a and b are real constants, has a root $z = 2\mathrm{e}^{i\frac{\pi}{3}}$.

(i) Show the roots of the equation on the Argand diagram. [3]

(ii) Two of the roots, z_1 and z_2, where $0 < \arg(z_1) < \arg(z_2) < \pi$, are represented by the points P and Q respectively. Given that the point R is such that the triangle PQR is an equilateral triangle, find a possible complex number represented by the point R in the form $x + \mathrm{i}y$. [3]

4 The sequence of numbers u_n, where n = 0, 1, 2, 3, …, is such that $u_0 = -2$ and

$$u_n = \frac{(n+2)u_{n-1}}{n+1+2u_{n-1}}.$$

(i) Write down the values of u_1, u_2 and u_3. [2]

(ii) Given further that $u_4 = \frac{6}{7}$ and $u_5 = \frac{7}{9}$, make a conjecture for u_n in terms of n. Hence prove your conjecture using mathematical induction. [6]

5 A differential equation is given by $\frac{\mathrm{d}y}{\mathrm{d}x}+2xy-x^3=0$.

(i) Find the general solution of the differential equation, giving y explicitly in terms of x. [4]

(ii) Write down the line of symmetry that each solution curve possesses. [1]

(iii) Determine the number of stationary point(s) for the different solution curves with different values of the arbitrary constant. [3]

(iv) Sketch the family of solution curves. [3]

6 **(i)** Show algebraically that the equation $x^4-x-10=0$ has exactly two real roots α and β where $\alpha \in (-1.7,-1.6)$ and $\beta \in (1.8,1.9)$. [4]

(ii) Iterations of the form $x_{n+1}=\mathrm{F}(x_n)$ are based on each of the following rearrangements of the equation in part **(i)**:

(A) $x=\frac{10}{x^3-1}$,

(B) $x=(x+10)^{\frac{1}{4}}$,

(C) $x=\frac{\sqrt{(x+10)}}{x}$.

Using an initial value of $x_1=1.8100$, a student attempts to find the value of β correct to 4 decimal places by performing five iterations for each arrangement. Replicate his results in a table. [3]

(a) Identify the arrangement that fails to converge to β and illustrate this on a graph. [3]

(b) State which arrangement will converge faster to β. Justify your answer. [2]
Hence find the value of β correct to 4 decimal places and explain how you can verify its correctness. [2]

Section B: Statistics [50 marks]

7 It is thought that there is an association between the colour of a person's eyes and the reaction of the person's skin to ultraviolet light. In order to investigate this, each of a random sample of 120 people were subjected to a standard dose of ultraviolet light. The degree of their reaction was noted and shown in the table below.

		Eye colour		
		Blue	Grey or green	Brown
Reaction	no reaction	7	8	18
	slight reaction	29	10	16
	strong reaction	21	9	2

Perform an appropriate test at the 1% significance level, stating your null and alternative hypothesis clearly. [4]

8 The Society for the Prevention of Cruelty to Animals (SPCA) is investigating the survival times (in days) of guinea pigs reared by a particular farm. The farmer, having some statistical knowledge, decides to conduct a test on his own to show that the average survival times of guinea pigs in his farm is at least 60 days. He picks a random sample of 20 guinea pigs, which yielded the following survival times (in days).

40	42	42	43	43	44	44	48	48	48
52	53	55	55	58	60	65	65	75	120

With the assumption that the survival times of guinea pigs in his farm is normally distributed, the farmer conducted a significance test at 10% level and reported his findings to SPCA.

(i) Calculate the unbiased estimates of the population mean and variances of the survival times of guinea pigs from the farm. [1]

(ii) Replicate the significance test that the farmer conducted, stating clearly the conclusion of his report to SPCA. [4]

While going through the farmer's report, SPCA noticed that the assumption of the survival times of guinea pigs in the farm being normally distributed may not be valid.

(iii) With reference to the data from the random sample of 20 guinea pigs, explain why SPCA disagreed with the farmer's normal distribution assumption. [1]

(iv) What test would be more appropriate than the one conducted in **(ii)**? [1]

(v) Conduct the test stated in **(iv)**, at 10% significance level. [4]

9 The random variable X has a Poisson distribution with mean λ. A random sample of n observations is taken and the largest of these observations is M.

(i) Show that $\mathrm{P}(M \leq 1) = \mathrm{e}^{-n\lambda}\left(1+\lambda\right)^{n}$. [2]

In a manufacturing process for a type of electrical component, the probability of a component being defective is p, independently of all others. Components are packed in boxes of 100 each.

Let W be the random variable denoting the number of defective components in a randomly chosen box. It is known that p is small (i.e. $p < 0.02$) such that W can be approximated by $W \sim \mathrm{Po}(100p)$.

A box is accepted if it contains not more than one defective component.

(ii) Estimate the probability, in terms of p, that not all the boxes are acceptable out of 8 randomly chosen boxes. [2]

A batch of components which are packed in boxes for a day costs \$100,000 to produce. For quality control, 8 boxes are randomly chosen from the batch and the batch will have to be destroyed if any box is not accepted.

(iii) If $p = 0.001$, discuss whether the company will make money if they charge their client \$110,000 for each batch of components delivered. You may assume that the client purchases a large number of batches. Explain what happens if the client purchases only a small number of batches. [3]

Once a batch is accepted, it will be delivered to the client. The time it takes to deliver the shipment follows a uniform distribution with a mean of 3 days and standard deviation of $\dfrac{1}{\sqrt{3}}$ day.

(iv) Find the probability that a randomly chosen shipment takes more than 3.5 days to reach the client. [4]

10 A statistician had been tasked to test the durability of two materials A and B used for soles of ladies' shoes. Each of the 200 volunteers wore a pair of shoes with one sole made of A and the other made of B. The difference in wear, denoted by $D = A - B$ (in suitable units), is summarized in the table below.

Difference in wear, $A-B$	Frequency
$-1.5 \le d < -1.2$	1
$-1.2 \le d < -0.9$	6
$-0.9 \le d < -0.6$	34
$-0.6 \le d < -0.3$	80
$-0.3 \le d < 0$	61
$0 \le d < 0.3$	14
$0.3 \le d < 0.6$	3
$0.6 \le d < 0.9$	0
$0.9 \le d < 1.2$	1

The statistician suspects that the data obtained follows a normal distribution.

(i) Show, at the 10% significance level, that there is evidence to suggest that the data support the statistician's suspicion. [5]

(ii) State the least value of k, to one decimal place, for which the null hypothesis in **(i)** can be rejected at $k\%$ significance level. [1]

The statistician then proceeds to test whether there is a difference in wear between the two materials. With the above evidence to support his suspicion, the statistician randomly picks the data corresponding to 10 volunteers to conduct the test.

Volunteer	1	2	3	4	5	6	7	8	9	10
A	14.4	9.2	10.8	14.6	12.1	6.1	9.7	11.4	9.1	13.2
B	14.7	9.7	11.3	14.9	11.9	7.2	9.6	11.7	9.7	14.0

(iii) State the parametric test that the statistician should conduct. State, with a reason, if the statistician requires any further assumption to conduct this parametric test. [2]

(iv) Perform the parametric test, at a 1% significance level, on whether there is a difference in wear between the two materials. [4]

11 A metalworker produces cubes of various sizes. The length, X cm, of the edge of a randomly chosen cube has probability density function given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{1}{4}x, & 1 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

The surface area of a randomly chosen cube is Y cm^2.

(i) A cube is deemed "large" if its surface area is more than 48 cm^2. Find the probability that a randomly chosen cube is "large". [2]

(ii) The supervisor for the metalworker has to examine a "large" cube. Cubes are randomly drawn one at a time until a "large" cube is drawn. Find the least number of cubes to draw such that the supervisor has at least a probability of 0.9 to draw a "large" cube. [2]

(iii) Find $\mathrm{E}(Y)$ and show that $\mathrm{Var}(Y) = 192$. [3]

(iv) A random sample of 50 cubes is drawn. Find the probability that sample mean of the cubes' surface area is less than 27 cm^2. [2]

(v) The worker went for a training course to learn methods to reduce the mean surface area of the cubes that he produces. It is known that the random sample drawn in **(iv)** was obtained after his training and its sample mean of the surface area of the cubes is 27 cm^2. Management wants to know whether his training was effective and proceeds to do a hypothesis test using this sample. Assuming that there is no change in the variance of the surface area of the cubes he produces before and after the training, state the hypotheses and deduce the result of the test at 5% significance level. [3]