ANDERSON JUNIOR COLLEGE
2017 Preliminary Examination
H2 Mathematics Paper 2 (9758/02)

Duration: 3 hours

	Section A: Pure Mathematics [40 marks]
1	At the intensive care unit of a hospital, patients of a particular condition receive a certain treatment drug through an intravenous drip at a constant rate of 30mg per hour. Due to the limited capacity for absorption by the body, the drug is lost from a patient's body at a rate proportional to x , where x is the amount of drug (in mg) present in the body at time t (in hours). It is assumed that there is no presence of the drug in any patient prior to admission to the hospital. (i) Form a differential equation involving x and t and show that $x=\frac{30}{k}\left(1-\mathrm{e}^{-kt}\right)$ where k is a positive constant. [4] (ii) If there is more than 1000mg of drug present in a patient's body, it is considered an overdose. Suppose the drug continues to be administered, determine the range of values of k such that a patient will have an overdose. [2] For a particular patient, $k=\frac{1}{50}$. (iii) Find the time required for the amount of the drug present in the patient's body to be 200mg. [3]
2	The polynomial P(z) has real coefficients. The equation P(z) = 0 has a root $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $0<\theta<\pi$. Write down a second root in terms of r and θ, and hence show that a quadratic factor of P(z) is $z^2-2rz\cos\theta+r^2$. [2] Let P(z) = $z^3+az^2+15z+18$ where a is a real number. One of the roots of the equation P(z) = 0 is $3\mathrm{e}^{\mathrm{i}\left(\frac{2\pi}{3}\right)}$. By expressing P(z) as a product of two factors with real coefficients, find a and the other roots of P(z) = 0. [4] Deduce the roots of the equation $18z^3+15z^2+az+1=0$. [2]

3 Planes Π_1 and Π_2 are defined by

$$\Pi_1 : x - 2y + 2z = 7, \quad \Pi_2 : \mathbf{r} \cdot \begin{pmatrix} a \\ 3 \\ -1 \end{pmatrix} = 8.$$

where a is a constant.

(i) The point P has position vector $-2\mathbf{i} + \mathbf{j} + \mathbf{k}$. Find the position vector of F, the foot of the perpendicular from P to plane Π_1.
Hence, or otherwise, find the shortest distance from P to plane Π_1. [5]

(ii) Line m passes through the point F and is parallel to both planes Π_1 and Π_2. Find the vector equation of line m. [2]

(iii) It is given that the point $Q(1, -4, -1)$ lies on line m. Find the value of a. [3]

(iv) Find the length of projection of $\overrightarrow{PQ}$ on the x-y plane. [3]

4 The function f is defined by

$$\mathrm{f} : x \mapsto \frac{\mathrm{e}^x - 1}{\mathrm{e} - 1} \quad \text{for} \quad x \in \mathbb{R}.$$

Sketch the graph of $y = \mathrm{f}(x)$ and state the range of f. [3]

Another function h is defined by

$$\mathrm{h} : x \mapsto \begin{cases} (x-1)^2 + 1 & \text{for} \quad x \le 1 \\ 1 - \dfrac{|1-x|}{2} & \text{for} \quad 1 < x \le 4 \end{cases}$$

Sketch the graph of $y = \mathrm{h}(x)$ for $x \le 4$ and explain why the composite function $\mathrm{f}^{-1}\mathrm{h}$ exists. Hence find the exact value of $\left(\mathrm{f}^{-1}\mathrm{h}\right)^{-1}(3)$. [7]

Section B: Probability and Statistics [60 marks]

5 A vehicle insurance company classifies the drivers it insures as class A, B and C according to whether they are of low risk, medium risk or high risk with regard to having an accident. The company estimates that 30% of the drivers who are insured are class A and 50% are class B. The probability that a class A driver will have at least one accident in any 12 month period is 0.01, the corresponding probabilities for class B and C are 0.03 and 0.06 respectively.

(i) Find the probability that a randomly chosen driver will have at least one accident in a 12-month period. [2]

(ii) The company sold a policy to a driver and within 12 months, the driver had at least one accident. Find the probability that the driver is of class C. [2]

(iii) Three drivers insured by the company are chosen randomly. Find the probability that all three drivers are of class C and exactly one of them had at least one accident in a 12-month period. [3]

6 In an experiment to investigate the decay of organic material over time, a bag of leaf litter was allowed to sit for a 20-week period in a moderately forested area.

The table below shows the weight of the remaining leaf litter (y kg) when x number of weeks have passed.

x	1	2	4	6	8	9	11	15	20
y	60.9	51.8	34.7	26.2	14.0	12.3	8.2	3.1	1.4

(i) Draw a scatter diagram of these data. [1]

(ii) Find the equation of the regression line of y on x and calculate the corresponding estimated value of y when $x = 17$.
Comment on the suitability of the linear model for these data. [3]

The variable W is defined as $W = \ln y$.

(iii) Find the product moment correlation coefficient between W and x. [1]

(iv) It is given that the weight of the leaf litter in the bag was 75.0 kg initially. Using an appropriate regression line, estimate how long it takes for the weight of the leaf litter to drop to half its initial value, giving your answer to one decimal place. [3]
Give two reasons why you would expect this estimate to be reliable. [2]

7 **(a)** The Health Promotion Board of a certain country claims that the average number of hours of sleep of working adults is at most 6 hours per day. To investigate this claim, the editor of a magazine plans to conduct a survey on a sample of adults travelling to work by train.

(i) Explain why this method of sampling will not give a random sample for the purpose of the investigation. [1]

The editor of another magazine interviewed a random sample of 50 working adults and their number of hours of sleep per day, x, are summarised as follows:

$$\sum x = 320, \ \sum x^2 = 2308.5$$

(ii) Test at the 5% level of significance whether there is any evidence to doubt the Health Promotion Board's claim. State with a reason, whether it is necessary to assume that the number of hours of sleep per day follows a normal distribution. [5]

(b) The Health Promotion Board carried out their own survey on another random sample of 50 working adults. The sample yielded an average of 6.14 hours of sleep per day and a standard deviation of 2.1 hours.

If the sample does not provide significant evidence at the 5% level of significance that the mean number of hours of sleep per day of working adults differs from μ_o hours, find the range of values of μ_o [4]

8 A biscuit manufacturer produces both cream and chocolate biscuits. Biscuits are chosen randomly and packed into boxes of 10. The number of cream biscuits in a box is denoted by X.

(a) On average, the proportion of cream biscuits is p. Given that $\mathrm{P}(X = 1 \text{ or } 2) = 0.15$, write down an equation for the value of p. Hence find the value(s) of p numerically. [3]

(b) It is given instead that the biscuit manufacturer produces 3 times as many cream biscuits as chocolate biscuits.

(i) Find the most likely value of X. [2]

(ii) A random sample of 18 boxes is taken. Find the probability that at least 3 but fewer than 7 boxes have equal numbers of cream and chocolate biscuits. [3]

A box of biscuits is sold at \$10. The manufacturer gives a discount of \$2 per box to its premium customers. The mean and variance of the number of boxes sold per day to each type of customers (assuming independence) are as follows:

	Mean	Variance
Number of boxes sold at usual price	180	64
Number of boxes sold at discounted price	840	169

Find the approximate probability that the total amount collected per month from the sales of biscuits is not less than \$255,000, assuming that there are 30 days in a month. [4]

9 Four families arrive at Science Centre together. Mr and Mrs *A* brought their 2 children while Mr *B* brought his 2 children. Mr and Mrs *C* brought their 3 children while Mrs *D* brought her only child. All these 14 people have to go through a gate one at a time to enter the centre.

(i) In how many different ways can they go through the gate if each family goes in one after another? [2]

There are two experiments at the ***Science Magic Experience*** station.

(ii) In one experiment, participants are to be in groups of twos or threes. In how many different ways can the 8 children from the four families be grouped among themselves? [3]

(iii) In another experiment, the four families have to hold hands to form two separate circles of equal size to experience a science phenomenon. Each circle must have exactly four children and members of the same family must be in the same circle. Find the number of ways of arranging these 14 people in the two circles such that there is no more than one adult between any two children. [3]

10 Males and females visiting an amusement park have heights, in centimetres, which are normally distributed with means and standard deviations as shown in the following table:

	Mean (cm)	Standard deviation (cm)
Male	165	12
Female	155	σ

It is found that 38.29% of the females have heights between 150 cm and 160 cm.

(i) Show that $\sigma = 10.0$ cm, correct to 3 significant figures. [2]

(ii) Find the probability that the height of a randomly chosen female is within 20 cm of three-quarter the height of a randomly chosen male. State an assumption that is necessary for the calculation to be valid. [4]

The amount, $\$X$, a visitor has to pay for a popular ride in the park is \$10 if the visitor's height is at least 120 cm but less than 150 cm, and $\$m$ if the visitor's height is 150 cm and above. If the visitor's height is less than 120 cm, he/she does not need to pay for the ride.

(iii) Assuming that a visitor purchasing a ticket for the ride is equally likely to be a male or female, find in terms of m, the probability distribution of X. [3]

Given that the expected amount a visitor will pay for a ride is \$17.93, show that $m = 20.00$, correct to 2 decimal places. [1]

(iv) Three visitors were randomly chosen. Find the probability that the total amount they paid for a ride together is more than \$40. [3]

End of paper