RI H2 Mathematics 2017 Prelim Exam Paper 1 Question

1 A local wholesaler sells Pikachi plushies in two sizes, small and large. The number of Pikachi plushies bought by three particular retailers and the total amount they paid are shown in the following table.

Retailer	Small	Large	Total Amount paid
A	30	50	\$1375
B	k	$2k$	\$2704
C	$2k$	k	\$2522

Find the price of each small and each large Pikachi plushy and determine the value of k. [4]

2 A right circular cone has base radius r cm and height h cm. As r and h vary, its curved surface area, $\pi r\sqrt{\left(r^2+h^2\right)}$ cm^2, remains constant.

It is given that when $r=\sqrt{2}$ cm, the magnitude of the rate of change of h is 10 times the magnitude of the rate of change of r. Given also that $h>r$, find the height of the cone at this instant. [4]

3

(a) Find $\displaystyle\int \frac{x+2}{\sqrt{\left(1-8x-4x^2\right)}}\,\mathrm{d}x$. [4]

(b) Use the substitution $x=2\sec\theta$ to find the exact value of $\displaystyle\int_2^4 \frac{1}{x}\sqrt{\left(x^2-4\right)}\,\mathrm{d}x$. [4]

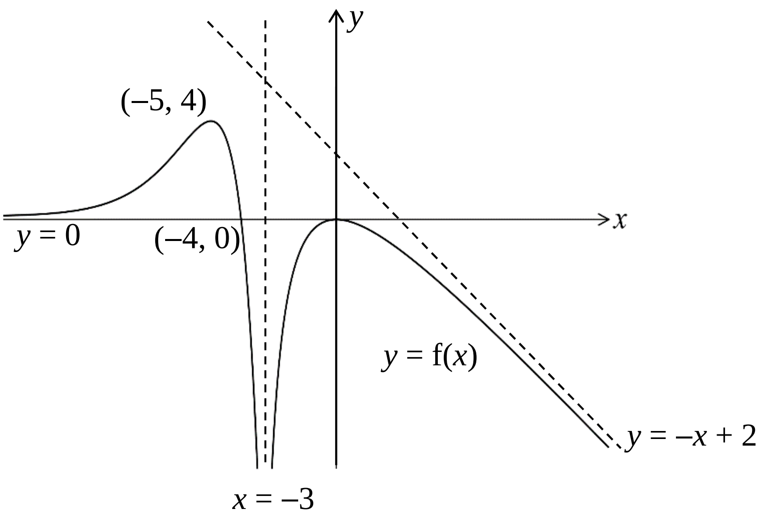
4 A curve C has equation $y=\mathrm{f}\left(x\right)$, where

$$\mathrm{f}\left(x\right)=\frac{a}{\left(x+b\right)^2}+cx,$$

and a, b and c are constants. It is given that C has a vertical asymptote $x=-1$ and a minimum point at $\left(0,\ 1\right)$.

(i) Find the values of a, b and c. [4]

(ii) Sketch the graph of $y=\mathrm{f}\left(|x|\right)$, stating the coordinates of any point(s) of intersection with the axes and the equation(s) of any asymptote(s). [3]

	(iii) Hence, solve the inequality $\mathrm{f}\left(\lvert x\rvert\right)-4>0$. [2]
5	 The diagram shows the curve $y=\mathrm{f}(x)$. The curve has maximum points at $(-5,\ 4)$ and the origin, and crosses the x-axis at $(-4,\ 0)$. The lines $y = 0$, $x = -3$ and $y = -x + 2$ are the horizontal, vertical and oblique asymptotes to the curve respectively. On separate diagrams, draw sketches of the graphs of **(a)** $y=\dfrac{1}{\mathrm{f}\left(x\right)}$, [3] **(b)** $y=\mathrm{f}'\left(x\right)$, [3] **(c)** $y=\mathrm{f}\left(\dfrac{x+1}{2}\right)$, [3] labelling clearly the equation(s) of any asymptote(s), coordinates of any axial intercept(s) and turning point(s) where applicable.
6	**(i)** Given that $y=\ln\left(1+\sin 2x\right)$, show that $\mathrm{e}^{y}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\mathrm{e}^{y}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2=-4\sin 2x$. Find the first three non-zero terms in the Maclaurin's series for y. [5] **(ii)** It is given that the three terms found in part **(i)** are equal to the first three terms in the series expansion of $ax(1+bx)^n$ for small x. Find the exact values of the constants a, b and n and use these values to find the coefficient of x^4 in the expansion of $ax(1+bx)^n$, giving your answer as a simplified rational number. [5]

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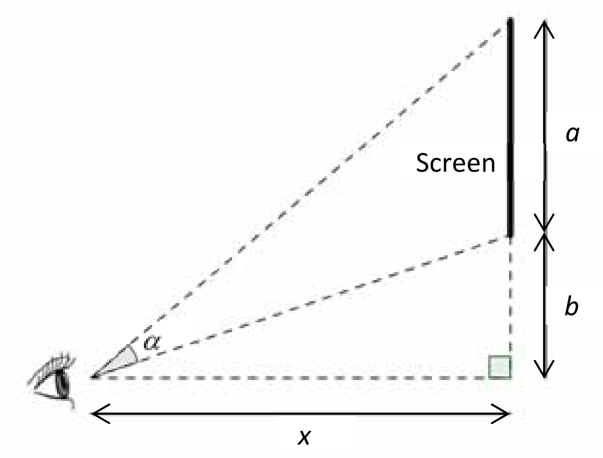


Fig. 1

Mr Tan is planning to set up a home theatre in his spacious rectangular living room. A projector screen with height a metres is to be positioned against one of the walls b metres above the eye level (see Fig. 1). He is trying to decide on the horizontal distance between the sofa and the screen so that the viewing angle α of the projection screen is as large as possible.

(i) Show that $\alpha = \tan^{-1}\frac{a+b}{x} - \tan^{-1}\frac{b}{x}$, where x is the horizontal distance between the sofa and the screen in metres. [1]

(ii) Use differentiation to show that the value of x which gives the maximum value of α satisfies the equation

$$\frac{a+b}{x^2+(a+b)^2} = \frac{b}{x^2+b^2}.$$

Solve for x and leave your answer in terms of a and b. [4]
[It is not necessary to verify the nature of the maximum point in this part.]

Mrs Tan proposed an alternative way of arrangement. She proposed to place the sofa against the wall opposite the screen, which is c metres away, and to vary the vertical position of the screen placed y metres above the eye level in order to maximise the angle α (see Fig. 2).

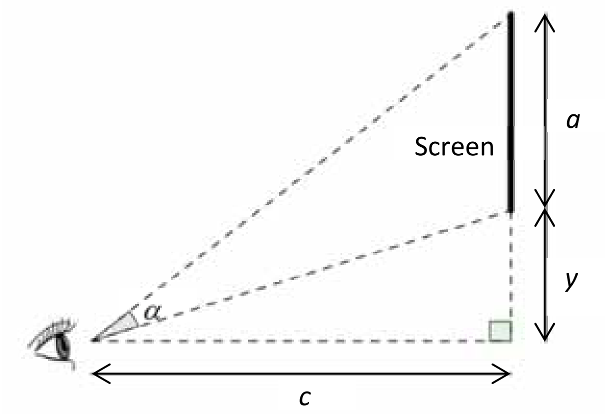


Fig. 2

(iii) Use differentiation to find the value of y which gives the maximum value of α, leaving your answer in terms of a. Interpret the answer in this context. [5]

8 A curve C has parametric equations

$$x = \sin^2 t, \quad y = 2\cos t, \quad \text{for } 0 \le t \le \frac{\pi}{2}.$$

(i) Find a cartesian equation of C. [2]

The tangent to the curve at the point P where $t = \frac{\pi}{3}$ is denoted by l.

(ii) Find an equation of l. [3]

(iii) On the same diagram, sketch C and l, stating the coordinates of the axial intercepts and the point of intersection. [3]

The region R is bounded by the curve C, the line l and the y-axis.

(iv) Find the exact value of the volume of revolution formed when R is rotated completely about the x-axis. [3]

9 **Do not use a calculator in answering this question.**

(a) One root of the equation $z^4 + 2z^3 + az^2 + bz + 50 = 0$, where a and b are real, is $z = 1+$

(i) Show that $a = 7$ and $b = 30$ and find the other roots of the equation. [5]

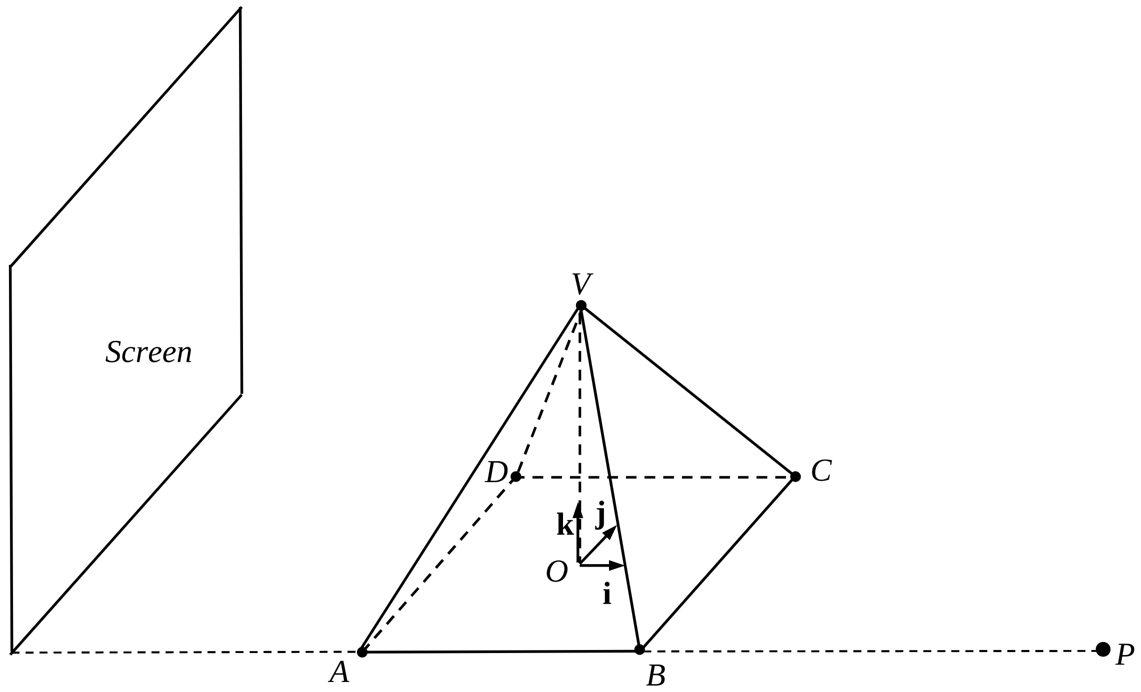
(ii) Deduce the roots of the equation $w^4 - 2\mathrm{i}w^3 - 7w^2 + 30\mathrm{i}w + 50 = 0$. [2]

(b) Given that $p^* = \dfrac{\left(-\frac{1}{\sqrt{3}} + \mathrm{i}\right)^5}{(1-\mathrm{i})^4}$, by considering the modulus and argument of p^*, find the exact expression for p, in cartesian form $x + \mathrm{i}y$. [4]

10 In a model of forest fire investigation, the proportion of the total area of the forest which has been destroyed is denoted by x. The destruction rate of the fire is defined to be the rate of change of x with respect to the time t, in hours, measured from the instant the fire is first noticed. A particular forest fire is initially noticed when 20% of the total area of the forest is destroyed.

(a) One model of forest fire investigation shows that the destruction rate is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{10}x(1-x).$$

	(i) Express the solution of the differential equation in the form $x = \mathrm{f}(t)$ and sketch the part of the curve for $t \geq 0$. [6] **(ii)** Find the exact time when the destruction rate is at its maximum. [2] **(iii)** Explain briefly why this model cannot be used to estimate how long the forest has been burning when it is first noticed. [1] **(b)** A second model for the investigation of forest fire is suggested and given by the differential equation $$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{5\pi\left[1+\left(\frac{t}{10}+\tan\frac{\pi}{10}\right)^2\right]}.$$ Determine how long the forest has been burning when the fire is first noticed. [3]
11	 A right opaque pyramid with square base *ABCD* and vertex *V* is placed at ground level for a shadow display, as shown in the diagram. *O* is the centre of the square base *ABCD*, and perpendicular unit vectors **i**, **j**, **k** are in the directions of $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{OV}$ respectively. The length of *AB* is 8 units and the length of *OV* is 2*h* units. A point light source for this shadow display is placed at the point *P*(20, −4, 0) and a screen of height 35 units is placed with its base on the ground such that the screen lies on a plane with

	vector equation $\mathbf{r}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}=\alpha$ where $\alpha<-4$ (see diagram). **(i)** Find a vector equation of the line depicting the path of the light ray from P to V in terms of h. [2] **(ii)** Find an inequality between α and h so that the shadow of the pyramid cast on the screen will not exceed the height of the screen. [3] The point light source is now replaced by a parallel light source whose light rays are perpendicular to the screen and it is also given that $h = 10$. **(iii)** Find the exact length of the shadow cast by the edge VB on the screen. [3] A mirror is placed on the plane VBC to create a special effect during the display. **(iv)** Find a vector equation of the plane VBC and hence find the angle of inclination made by the mirror with the ground. [4]