Suggested Solutions for NJC H2 Mathematic Prelim Exam Paper 2 2017

Q1	Suggested Solutions
	Let α, β and μ be the original amount charged per 5 min, 10 min and 15 min block for each ride by α- bike, β- bike and μ- bike respectively. $5\alpha+3\beta+\mu=5.7$ ---------- (1) $4\alpha+\beta+3(0.95\mu)=5.72$ ---------- (2) $8\alpha+4\beta+3(0.95^2\mu)=9.71$ ---------- (3) Solving the above 3 equations simultaneously by GC, $\alpha=\$0.4079329609, \beta=\$0.8402234637, \mu=\$1.139664804$ $\alpha=\$0.41, \beta=\$0.84, \mu=\$1.14$. Original pricing per 40-min block: **Using calculator values** α- bike: $\$0.4079329609\times 8=\$3.26$ β- bike: $\$0.84\times 4=\$3.36$ μ- bike: $\$1.14\times 3=\$3.42$ Thus, α- bike offers the cheapest rate for a 40-min ride.

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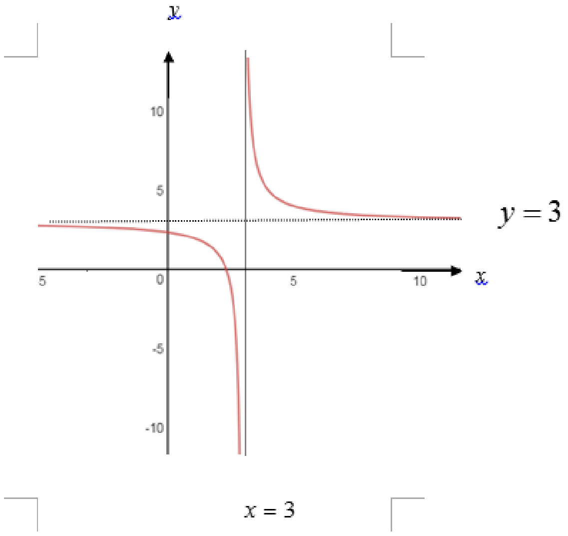
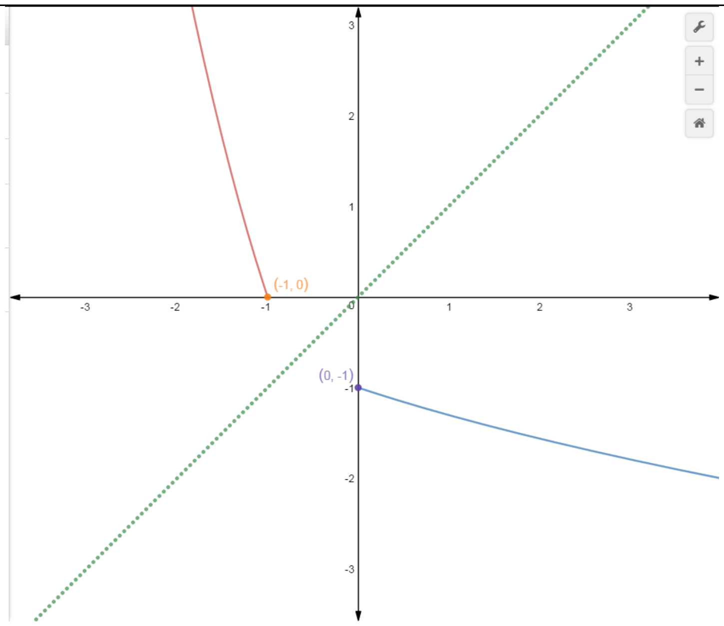
Q2	**Suggested Solutions**
(i)	 Since any horizontal line $\underline{y=a, a\in\mathbb{R}}$, intersects the graph of $y=\mathrm{f}(x)$ $\underline{\text{at most once}}$, the function f is one-one. It follows that f^{-1} exists. OR Since any horizontal line $\underline{y=a, a\in \mathrm{R}_f}$, intersects the graph of $y=\mathrm{f}(x)$ $\underline{\text{exactly once}}$, the function f is one-one. It follows that f^{-1} exists.
(i)	Let $y=\dfrac{7-3x}{3-x}$ $y(3-x)=7-x$ $x=\dfrac{7-3y}{3-y}$ Since $\mathrm{f}^{-1}(x)=\dfrac{7-3x}{3-x},\ x\in\mathbb{R}, x\neq 3,$ $\therefore \mathrm{f}^{-1}=\mathrm{f}$. (shown) $D_{\mathrm{f}^{-1}}=R_{\mathrm{f}}=(-\infty,3)\cup(3,\infty)=D_{\mathrm{f}}$
(i)	Note that $\mathrm{f}^{-1}\mathrm{f}(x)=x.$ Therefore, $\mathrm{f}^{2003}(5)=\underbrace{\mathrm{fff...f}}_{2003\text{times}}(5)=\mathrm{f}\left(\underbrace{\mathrm{f}^{-1}\mathrm{f......f}^{-1}\mathrm{f}}_{1000\text{ times of }\mathrm{f}^{-1}\mathrm{f}}(5)\right)=\mathrm{f}(5)=4.$
(ii)	$\lvert(2-x)(1+x)\rvert=\begin{cases}(2-x)(1+x), & -1\le x\le 2,\\ -(2-x)(1+x), & x<-1 \text{ or } x>2.\end{cases}$ For $x\in(-\infty,-1], y=-(2-x)(1+x)$ **Method 1** $x^2-x-2-y=0$ $x=\dfrac{-(-1)\pm\sqrt{(-1)^2-4(1)(-2-y)}}{2(1)}$ $x=\dfrac{1\pm\sqrt{9+4y}}{2}$

	Method 2 $y = x^2 - x - 2 = (x-0.5)^2 - 2.25$ $x = 0.5 \pm \sqrt{y+2.25}$ $x = \dfrac{1+\sqrt{9+4y}}{2}$ (rejectd $\because x \le -1$) or $\dfrac{1-\sqrt{9+4y}}{2}$ $\therefore \mathrm{g}^{-1}(x) = \dfrac{1}{2} - \sqrt{x + \dfrac{9}{4}}$
(iii)	
	For $\mathrm{g}\,\mathrm{g}^{-1}(x) = x$, $D_{\mathrm{g}\,\mathrm{g}^{-1}} = D_{\mathrm{g}^{-1}}$. $\therefore x \in [0,\infty)$ or $x \ge 0$
(iv)	Since $R_{\mathrm{g}^{-1}} = (-\infty,-1]$ and $\mathrm{D_f} = (-\infty,\infty)\setminus\{3\}$ $R_{g^{-1}} \subseteq D_f$. $\therefore \mathrm{fg}^{-1}$ exists.
	Using the graph of $y = \mathrm{g}^{-1}(x)$ in part **(ii)**, $R_{\mathrm{g}^{-1}} = (-\infty,-1]$. From graph of $y = \mathrm{f}(x)$ in **(i)** in $(-\infty,-1]$. $\therefore R_{\mathrm{f}\,\mathrm{g}^{-1}} = [2.5,3)$

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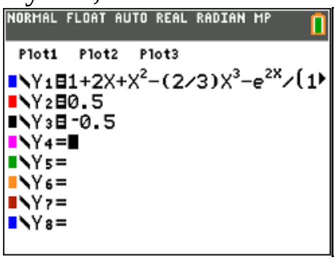
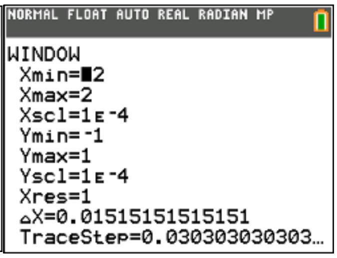
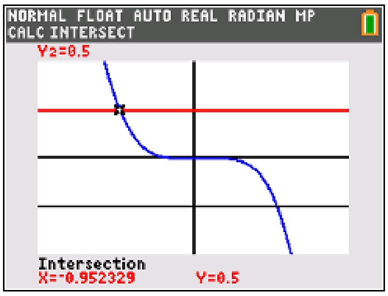
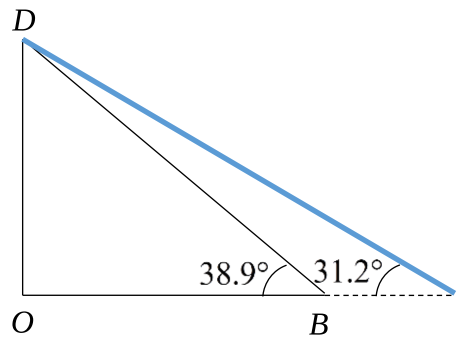
Q3	Suggested Solution
	$y=\dfrac{e^{2x}}{1+x^2}$ $\left(1+x^2\right)y=e^{2x}$ $\left(1+x^2\right)y'+2xy=2e^{2x}$ $\left(1+x^2\right)y''+2xy'+2xy'+2y=4e^{2x}$ $\Rightarrow\left(1+x^2\right)y''+4xy'+2y=4e^{2x}$ $\left(1+x^2\right)y'''+2xy''+4y'+4xy''+2y'=8e^{2x}$ $\Rightarrow\left(1+x^2\right)y'''+6xy''+6y'=8e^{2x}$ When $x=0,\ y=1, y'=2, y''=2, y'''=-4$ $y=\dfrac{e^{2x}}{1+x^2}$ $=1+2x+2\left(\dfrac{x^2}{2!}\right)-4\left(\dfrac{x^3}{3!}\right)+\cdots$ $\approx 1+2x+x^2-\dfrac{2x^3}{3}$ $a=2,\quad b=-\dfrac{2}{3}$
(a)	For $-2\le x\le 2$, $\left\|f(x)-h(x)\right\|\le 0.5$ $-0.5\le 1+2x+x^2-\dfrac{2x^3}{3}-\dfrac{e^{2x}}{1+x^2}\le 0.5$ By GC,    From the diagram above, $-0.95233\le x\le 1.072619$ $\therefore -0.952\le x\le 1.07$

Suggested Solutions for NJC H2 Mathematic Prelim Exam Paper 2 2017

Q4	Suggested Solutions
(i)	$\overrightarrow{OD}=\begin{pmatrix}0\\0\\8\end{pmatrix}$, $\overrightarrow{OE}=\begin{pmatrix}7\\0\\5\end{pmatrix}$, $\overrightarrow{OF}=\begin{pmatrix}0\\7\\5\end{pmatrix}$. Hence, $\overrightarrow{DE}=\begin{pmatrix}7\\0\\5\end{pmatrix}-\begin{pmatrix}0\\0\\8\end{pmatrix}=\begin{pmatrix}7\\0\\-3\end{pmatrix}$, $\overrightarrow{DF}=\begin{pmatrix}0\\7\\5\end{pmatrix}-\begin{pmatrix}0\\0\\8\end{pmatrix}=\begin{pmatrix}0\\7\\-3\end{pmatrix}$ and $\overrightarrow{EF}=\begin{pmatrix}0\\7\\5\end{pmatrix}-\begin{pmatrix}7\\0\\5\end{pmatrix}=\begin{pmatrix}-7\\7\\0\end{pmatrix}$ A vector perpendicular to the plane is $=\overrightarrow{DE}\times\overrightarrow{DF}$ $=\begin{pmatrix}7\\0\\-3\end{pmatrix}\times\begin{pmatrix}0\\7\\-3\end{pmatrix}$ $=\begin{pmatrix}21\\21\\49\end{pmatrix}=7\begin{pmatrix}3\\3\\7\end{pmatrix}$ Cartesian equation of the plane is $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}3\\3\\7\end{pmatrix}=\begin{pmatrix}0\\0\\8\end{pmatrix}\cdot\begin{pmatrix}3\\3\\7\end{pmatrix}\Rightarrow \mathbf{r}\cdot\begin{pmatrix}3\\3\\7\end{pmatrix}=56$ $\therefore 3x+3y+7z=56$
(ii)	Let the required angle be θ $\cos\theta=\dfrac{\left\|\begin{pmatrix}3\\3\\7\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}\right\|}{\sqrt{3^2+3^2+7^2}\sqrt{1}}=\dfrac{7}{\sqrt{67}}$ $\theta\approx 31.2^\circ$ (1 dec place) (or 0.545 rad)
(iii)	Method 1 $\left\|\overrightarrow{OB}\right\|=\sqrt{7^2+7^2}=\sqrt{98}$ $\left\|\overrightarrow{OD}\right\|=9$ Angle between DB and the ground $=\angle OBD$ $=\tan^{-1}\left(\dfrac{8}{\sqrt{7^2+7^2}}\right)$ $\approx 38.9^\circ$



From the diagram, the canvas will cover *B*.

Method 2

$\overrightarrow{OB} = \begin{pmatrix} 7 \\ 7 \\ 0 \end{pmatrix}$

Equation of perpendicular line passing through *B*, *l*:

$\mathbf{r} = \begin{pmatrix} 7 \\ 7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}$

Using normal of plane to be $\begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$ i.e. all entries are positive:

solve the equation of plane *DEF* and *l*:

$\left[\begin{pmatrix} 7 \\ 7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right] \cdot \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix} = 56$

$\lambda = 2$

Since $\lambda = 2 > 0$, *l* and plane *DEF* intersect above the horizontal ground. So the canvas covers the point *B*.

Method 3

$\begin{pmatrix} 7 \\ 7 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix} = 42 < 56$

Distance from *O* to plane parallel to DEF and passing through *B* is smaller than the distance between *O* and plane *DEF*. Hence *B* is beneath the canvas.

(iv)

Normal vector of the vertical wall is $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $(d, 0, 0)$ lies on the vertical wall.

	$\begin{pmatrix} d \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = d \Rightarrow \mathbf{r} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = d$ Hence the equation of the vertical wall is $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = d$. Direction vector of the line of intersection is $\begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \\ -3 \end{pmatrix}$ Let $(x, \text{y}, 0)$ be the common point on lying on the two planes. $\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix} = 56 \Rightarrow 3x + 3y = 56$ $\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = d \Rightarrow x = d$ Solving the above equations simultaneously $3d + 3y = 56 \Rightarrow y = \dfrac{56 - 3d}{3}$ $\therefore \mathbf{r} = \begin{pmatrix} d \\ \dfrac{56-3d}{3} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 7 \\ -3 \end{pmatrix}$, where $\lambda \in \mathbb{R}$.
(iv)	For P to shine the brightest at point B, P must be as near as possible to B. Thus P is the foot of perpendicular from B to the roof. Equation of the line passes through B and P: $\mathbf{r} = \begin{pmatrix} 7 \\ 7 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}, \alpha \in \mathbb{R}$ Thus $\overrightarrow{OP} = \begin{pmatrix} 7+3\alpha \\ 7+3\alpha \\ 7\alpha \end{pmatrix}$ for some α. Since P lies on the roof,

	$\begin{pmatrix} 7+3\alpha \\ 7+3\alpha \\ 7\alpha \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix} = 56 \Rightarrow 42+67\alpha = 56$ $\therefore \alpha = \dfrac{14}{67}$ Substitute $\alpha = \dfrac{14}{67}$ into $\overrightarrow{OP} = \begin{pmatrix} 7+3\alpha \\ 7+3\alpha \\ 7\alpha \end{pmatrix}$. $\overrightarrow{OP} = \begin{pmatrix} 7+3\left(\frac{14}{67}\right) \\ 7+3\left(\frac{14}{67}\right) \\ 7\left(\frac{14}{67}\right) \end{pmatrix} = \begin{pmatrix} \frac{511}{67} \\ \frac{511}{67} \\ \frac{98}{67} \end{pmatrix}$ **Alternatively, use projection vector:** $\overrightarrow{BD} = \begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix} - \begin{pmatrix} 7 \\ 7 \\ 0 \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \\ 8 \end{pmatrix}$ To check for the direction of normal vector of *DEF* **n**= $\begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$ $\begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix} . \overrightarrow{BD} = \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix} . \begin{pmatrix} -7 \\ -7 \\ 8 \end{pmatrix} = -21-21+56>0$ Hence, angle between $\overrightarrow{BD}$ and **n** is acute. $\overrightarrow{BP} = \left(\overrightarrow{BD}.\mathbf{n}\right)\mathbf{n}$ $= \left(\begin{pmatrix} -7 \\ -7 \\ 8 \end{pmatrix} . \dfrac{\begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}}{\sqrt{9+9+49}} \right) \dfrac{\begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}}{\sqrt{9+9+49}}$ $= \dfrac{14}{67}\begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$

	$\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{BP}$ $= \begin{pmatrix} 7 \\ 7 \\ 0 \end{pmatrix} + \frac{14}{67}\begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$ $= \frac{1}{67}\begin{pmatrix} 511 \\ 511 \\ 98 \end{pmatrix}$

Q5	Suggested Solutions
(i)	see below

	5 Badminton Players	$(m-2)$ Floorball Players	6 Swimmers Players
Case 1	1	1	2
Case 2	1	2	1
Case 3	2	1	1

Case 1: Number of selections is $\binom{5}{1}\binom{m-2}{1}\binom{6}{2}$

Case 2: Number of selections is $\binom{5}{1}\binom{m-2}{2}\binom{6}{1}$

Case 3: Number of selections is $\binom{5}{2}\binom{m-2}{1}\binom{6}{1}$

Total number of selections

$$=\binom{5}{1}\binom{m-2}{1}\binom{6}{2}+\binom{5}{1}\binom{m-2}{2}\binom{6}{1}+\binom{5}{2}\binom{m-2}{1}\binom{6}{1}$$

$$=75(m-2)+30\frac{(m-2)(m-3)}{2!}+60(m-2)$$

$$=135(m-2)+15(m-2)(m-3)$$

$$=15(m-2)(9+m-3)$$

$$=15(m-2)(m+6)$$

$\therefore k=15$

<u>Alternative method:</u>

$$\binom{5}{1}\binom{m-2}{1}\binom{6}{1}\binom{m-2+5+6-3}{1}/\,2!$$

(ii) Number of ways to select exactly one of the twins

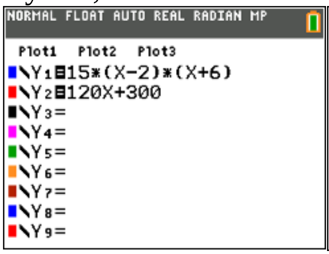
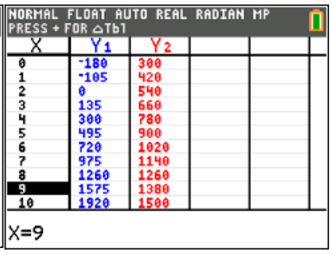
$$=\binom{5}{1}\binom{2}{1}\binom{6}{2}+\binom{5}{1}\binom{m-2}{1}\binom{2}{1}\binom{6}{1}+\binom{5}{2}\binom{2}{1}\binom{6}{1}$$

$$=150+60(m-2)+120$$

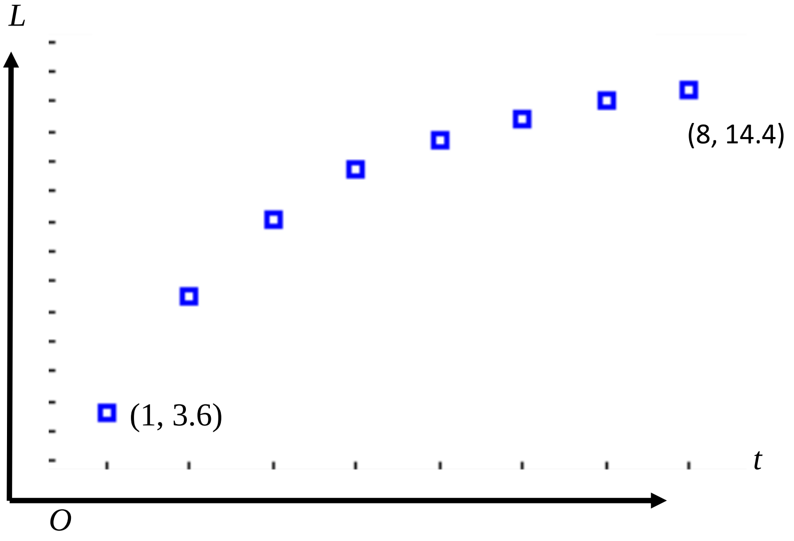
$$=60m+150$$

Number of ways that the twins are not selected > 2 times the number of ways that exactly one of the twins is selected.

$$15(m-2)(m+6)>2(60m+150)$$

	By GC,   least value of m is 9.
Last Part	Step 1: Arrange 3 units at the round table = 3!/3 Step 2: Arrange the twins among themselves = 2! Step 3: Slot in the teachers = $\binom{3}{2}\times 2!$ Number of ways for the twins to be seated together and teachers are separated $=\frac{3!}{3}\times 2!\times\binom{3}{2}\times 2!\times 6=144$

X	Y1	Y2
0	-180	300
1	-105	420
2	0	540
3	135	660
4	300	780
5	495	900
6	720	1020
7	975	1140
8	1260	1260
9	1575	1380
10	1920	1500

Q6	**Suggested Solutions**
(i)	 As L <u>increases at a decreasing rate/concave downwards</u> with respect to t, the linear model $L = at + b$ should not be used.
(ii)	The r value for $L = a\sqrt{t} + b$ is 0.972. The r value for $L = c\ln t + d$ is 0.996. Since the value of $\lvert r\rvert$ for $L = c\ln t + d$, is closer to 1, $L = c\ln t + d$ is a better model. $\therefore c = 5.28248 \approx 5.28$ $\therefore d = 3.92267 \approx 3.92$
(iii)	$L = 5.28248\ln(0.5) + 3.92267$ $= 0.2611$ $= 0.261(3 \text{ sf})$ This estimate is not reliable as as the age of the Sole is <u>out of the range of the data.</u>
(iv)	Since $G = 14.8 - L$ r_1 is positive but r_2 is negative. $\therefore r_2 = -r_1$

Q7	**Suggested Solutions**
(i)	$\mathrm{P}(X=4)$ $=\frac{3}{3}\times\frac{2}{3}\times\frac{1}{3}\times\frac{3}{3}$ $=\frac{2}{9}$
(ii)	$\mathrm{E}(X)$ $=\frac{1}{3}\times 2+\frac{4}{9}\times 3+\frac{2}{9}\times 4$ $=\frac{26}{9}$ $\mathrm{E}(X^2)$ $=\frac{1}{3}\times 2^2+\frac{4}{9}\times 3^2+\frac{2}{9}\times 4^2$ $=\frac{80}{9}$ Var(X) $=\frac{80}{9}-\left(\frac{26}{9}\right)^2$ $=\frac{44}{81}$
(iii)	Since n =44 <u>is large</u>, by Central Limit Theorem, $\overline{X} \sim N\left(\frac{26}{9},\frac{44}{81}\div 44\right)$ approx. $P\left(\overline{X}>3\right)$ $=0.159$ (By GC)

Table for (i):

x	2	3	4
P(X = x)	$\frac{1}{3}$	$\frac{4}{9}$	$\frac{2}{9}$

Suggested Solutions for NJC H2 Mathematic Prelim Exam Paper 2 2017

<table>
<tr><th>Q8</th><th>Suggested Solutions</th></tr>
<tr><td>(i)</td><td>2-tail as HPB is looking for a change in either way.</td></tr>
<tr><td>(ii)</td><td>Central Limit Theorem states that the sample <u>mean</u> heart rate will follow a normal distribution approximately when the sample is large (in this case, 70 > 20).

An unbiased estimate for the unknown population variance can be found obtained from the sample.</td></tr>
<tr><td>(iii)</td><td>Unbiased estimate of population mean $\bar{h} = \frac{5411}{70} = 77.3$.
Unbiased estimate of population variance,
$s^2 = \frac{1}{69}\left(426433 - \frac{5411^2}{70}\right) = 118.3$.

Let μ denote the mean heart rate of the teenagers in the obesity group.

To test at 10% significance level:
H_0: $\mu = 75$
H_1: $\mu \neq 75$

Under H_0, since n is large, by CLT, $\overline{H} \sim N\left(75, \frac{118.3}{70}\right)$ approximately,
(AND/OR $\frac{\overline{H} - 75}{\sqrt{\frac{118.3}{70}}} \sim N(0,1)$)

By GC, p-value = 0.0768 < 0.10.
(Alternatively, CR: $|z| > 1.645$, $z = 1.769$ is in CR)

Hence we <u>reject H_0</u> at the <u>10% level of significance</u> and conclude there is <u>sufficient evidence</u> that obesity will cause change in the mean heart rate.</td></tr>
<tr><td>(iv)</td><td>An one-tail test is used instead:
H_0: $\mu = 75$
H_1: $\mu > 75$

CR: $z > z_{0.9} = 1.28155$

To reject H_0,
$$\frac{79.4 - 75}{\sqrt{\frac{\sigma^2}{80}}} \geq 1.28155$$
$$\sigma^2 \leq \left(\frac{4.4\sqrt{80}}{1.28155}\right)^2 = 943\ (3\text{ sf})$$</td></tr>
</table>

	The researcher should conclude that obese teenagers evidentially has a higher mean heart rate if and only if the variance is not more (less) than 943.
(v)	$\mathrm{P}\left(\mu-\sigma<X<\mu+\sigma\right)=\mathrm{P}\left(-1<Z<1\right)=0.68268\approx 0.683$.
	Since heart rates follow a normal distribution, $\mathrm{P}\left(\mu-\sigma<H<\mu+\sigma\right)\approx 0.683$ We know that from (iv), null hypothesis will be rejected whenever $\sigma\leq 30.7$. Taking $\sigma=30.7$, under H_0, $\mathrm{P}\left(75-30.7<H<75+30.7\right)\approx 0.683$ $\Rightarrow \mathrm{P}\left(44.3<H<105.7\right)\approx 0.683$ and null hypothesis will be rejected. We can say that for $\sigma\leq 30.7$ and when null hypothesis is rejected , $\mathrm{P}\left(44.3<H<105.7\right)\geq 0.683$ or $\mathrm{P}\left(H<44.3\right)+\mathrm{P}\left(H>105.7\right)<0.317$ We know that the teenager's heart rate is rarely below 44.3 or above 105.7 in a resting state, so it is likely for the researcher to reject the null hypothesis. In reality, it is unlikely for sigma to be as large as 30.7 such that the probability for H to be within one standard deviation from mean to be 0.683.

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Q9	**Suggested Solutions**
	Let D be the random variable denoting the number of days of gestation for a Dutch Belted cow. $\mathrm{P}(D<278)=0.0808$ $\mathrm{P}(Z<\frac{278-\mu}{\sigma})=0.0808$ $\frac{278-\mu}{\sigma}=-1.39971---(1)$ $\mathrm{P}(D>289)=0.212$ $\mathrm{P}(Z<\frac{289-\mu}{\sigma})=0.788$ $\frac{289-\mu}{\sigma}=0.799501---(2)$ Solving (1)&(2), $\mu=285.001064$ and $\sigma=5.0017961$ $\mu=285$ (3 s.f.) and $\sigma=5.00$ (3 s.f.) .
(i)	$\overline{D}\sim\mathrm{N}(285.001064,\frac{5.0017961^2}{32})$. $\mathrm{P}(\overline{D}>287)=0.0118629=0.0119$ (3 s.f.) The <u>number of days of gestation for a Dutch Belted cow</u> is <u>independent</u> of <u>the number of days of gestation of another Dutch Belted cow.</u>
(ii)	$J\sim\mathrm{N}(278,2.50^2)$ $D\sim\mathrm{N}(285.001064,5.0017961^2)$ Let $X=26J-\frac{1}{2}29D$ $X\sim\mathrm{N}(3095.48457\ ,\ 9485.02698)$ $\mathrm{P}(0<\ X<a)=0.35$ $\therefore a=3057.95778\approx 3058$
(iii)	$D\sim\mathrm{N}(285.001064,5.0017961^2)$ $J\sim\mathrm{N}(278,2.50^2)$ Let C_1 denote the random variable of the amount of feed cosumed by 3 pregnant Dutch belted cows. Let C_2 denote the random variable of the amount of feed cosumed by 4 pregnant Jersey cows. $C_1=29(D_1+D_2+D_3)\sim\mathrm{N}(24795.09257,63120.32374)$ $C_2=26(J_1+J_2+J_3+J_4)\sim\mathrm{N}(28912,16900)$ $C_1-C_2\sim\mathrm{N}(-4117,80020.3237)$

	$\mathrm{P}(\lvert C_1 - C_2 \rvert > 4000)$ $= \mathrm{P}(C_1 - C_2 < -4000) + \mathrm{P}(C_1 - C_2 > 4000)$ $= 0.6604182314$ $= 0.660$ (3 s.f.) Or $D \sim \mathrm{N}(285.00, 5.0018^2)$ $J \sim \mathrm{N}(278, 2.50^2)$ Let C_1 denote the random variable of the amount of feed cosumed by 3 pregnant Dutch belted cows. Let C_2 denote the random variable of the amount of feed cosumed by 4 pregnant Jersey cows. $C_1 = 29(D_1 + D_2 + D_3) \sim \mathrm{N}(24795, 63120.42217)$ $C_2 = 26(\mathrm{J}_1 + J_2 + J_3 + J_4) \sim \mathrm{N}(28912, 16900)$ $C_1 - C_2 \sim \mathrm{N}(-4117, 80020.422)$ $\mathrm{P}(\lvert C_1 - C_2 \rvert > 4000)$ $= \mathrm{P}(C_1 - C_2 < -4000) + \mathrm{P}(C_1 - C_2 > 4000)$ $= 0.66041814$ $= 0.660$ (3 s.f.)

Q10	Suggested Solutions

(i) The event of a bulb being defective may not be independent of another bulb being defective.

(ii) Let X be the random variable for the number of defective light bulbs produced by Factory A.

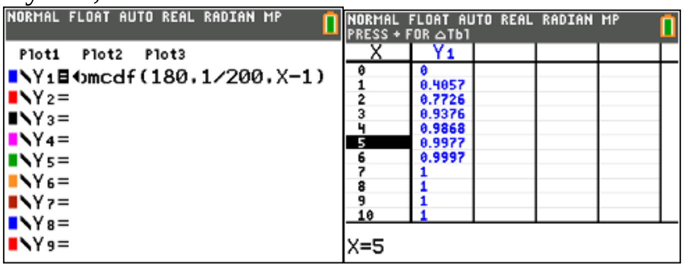
$$X \sim \mathrm{B}\left(180, \frac{1}{200}\right)$$

Given

$$\mathrm{P}(X < r) = 0.998$$

$$\Rightarrow \mathrm{P}(X \le r-1) = 0.998$$

By GC,



$$\therefore r = 5$$

(iii) Let Y be the random variable for the number of defective light bulbs produced by Factory B.

$$Y \sim \mathrm{B}(30, p)$$

P(the batch is accepted)

$$= \mathrm{P}(Y_1 = 0) + \mathrm{P}(Y_1 = 1 \text{ or } 2)\mathrm{P}(Y_2 = 0)$$

$$= \binom{30}{0} p^0 (1-p)^{30}$$

$$+ \left[\binom{30}{1} p(1-p)^{29} + \binom{30}{2} p^2 (1-p)^{28}\right]\binom{30}{0} p^0 (1-p)^{30}$$

$$= (1-p)^{30} + \left[30p(1-p)^{29} + 435p^2(1-p)^{28}\right](1-p)^{30}$$

$$= (1-p)^{30} + 30p(1-p)^{59} + 435p^2(1-p)^{58}$$

(iv) Let U be the random variable for the number of defective light bulbs produced by Factory A.

Let V be the random variable for the number of defective light bulbs produced by Factory B.

$$U \sim \mathrm{B}\left(1200, \frac{1}{200}\right)$$

$$V \sim \mathrm{B}(1200, 0.007)$$

P(1 bulb from B is defective| there is exactly one defective bulb)

	$=\dfrac{\mathrm{P}(U=0,V=1)}{\mathrm{P}(U=0,V=1)+\mathrm{P}(U=1,V=0)}$ ≈ 0.5838223 $=0.584$ (3 s.f.) Reference for $\dfrac{\mathrm{P}(U=0,V=1)}{\mathrm{P}(U=0,V=1)+\mathrm{P}(U=1,V=0)}$: $\dfrac{\left[\binom{1200}{1}0.007^1(1-0.007)^{1199}\times\binom{1200}{0}\left(\frac{1}{200}\right)^0\left(1-\frac{1}{200}\right)^{1200}\right]}{\left[\begin{array}{l}\binom{1200}{1}0.007^1(1-0.007)^{1199}\times\binom{1200}{0}\left(\frac{1}{200}\right)^0\left(1-\frac{1}{200}\right)^{1200}\\+\binom{1200}{0}0.007^0(1-0.007)^{1200}\times\binom{1200}{1}\left(\frac{1}{200}\right)^1\left(1-\frac{1}{200}\right)^{1199}\end{array}\right]}$