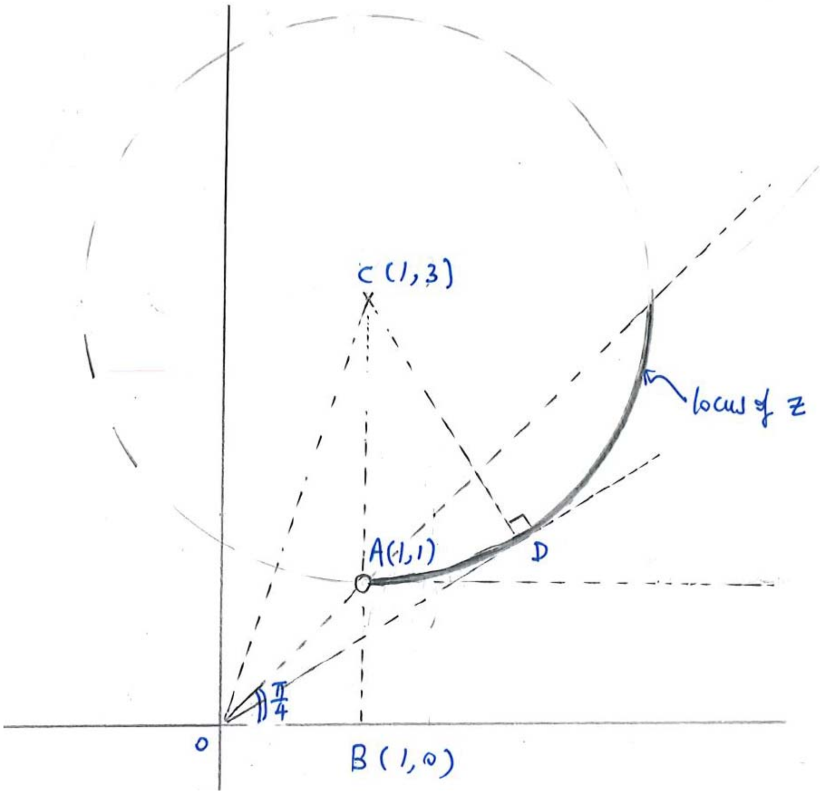
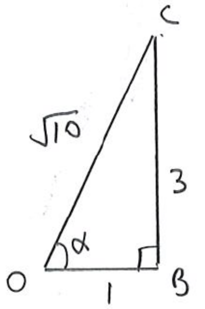
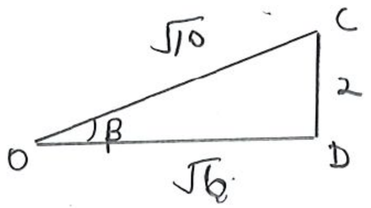
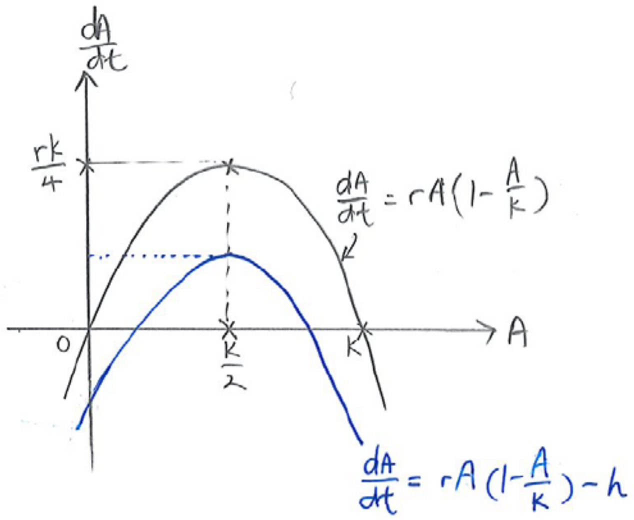
1	(i) $w_1 = 3, w_2 = 3+2\times 2 = 7$ (ii) Consider codewords of length $n-1$. If the $(n-1)^{th}$ letter is A, then there are 2 choices; A ,C for the n^{th} letter. Similarly if the $(n-1)^{th}$ letter is J, then there are 2 choices; J ,C for the n^{th} letter. If the $(n-1)^{th}$ letter is C, make it such that there are 2 choices; for the n^{th} letter say A, C. There are $2w_{n-1}$ such codewords. The case that is missing is codewords of length $n-2$ followed by C and then J, ie w_{n-2} . $\therefore \ \mathrm{w}_n = 2w_{n-1} + w_{n-2}\ , \quad n = 3,4,5...$ <u>Alternatively,</u> If the first letter of the codeword is *C*, then the number of possible codewords of length *n* is w_{n-1} . If the first letter of the codeword is A, then the number of possible codewords of length *n* is w_{n-1}- no of codewords of length n-1 starting with *J* If the first letter of the codeword is J, then the number of possible codewords of length *n* is w_{n-1}- no of codewords of length n-1 starting with *A* $w_n = w_{n-1} + \left(w_{n-1} - \text{no of codewords on length n-1 starting with J}\right)$ $\quad + \left(w_{n-1} - \text{no of codewords of length n-1 starting with A}\right)$ $= w_{n-1} + \left(2w_{n-1} - \text{no of words on length n-1 starting with A or J}\right)$ $= 3w_{n-1} - \left(\text{no of codewords on length n-1 starting with A or J}\right)$ $= 3w_{n-1} - \left(\mathrm{w}_{n-1} - \text{no of codewords on length n-2 starting with C}\right)$ $= 3w_{n-1} - \left(\mathrm{w}_{n-1} - \mathrm{w}_{n-2}\right)$ $= 2w_{n-1} + w_{n-2} \qquad , n = 3,4,5....$ (iii) Characteristic equation: $x^2 - 2x - 1 = 0$ $\Rightarrow \quad x = \dfrac{2 \pm \sqrt{4-4(1)(-1)}}{2(1)} = \dfrac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$ $\therefore$ Solution has the form $\mathrm{w}_n = c_1\left(1+\sqrt{2}\right)^n + c_2\left(1-\sqrt{2}\right)^n$ Since $u_1 = 3,\quad 3 = c_1\left(1+\sqrt{2}\right) + c_2\left(1-\sqrt{2}\right)$ ------ (1) Since $u_2 = 7,\quad 7 = c_1\left(1+\sqrt{2}\right)^2 + c_2\left(1-\sqrt{2}\right)^2$ $\Rightarrow \ 7 = c_1\left(3+2\sqrt{2}\right) + c_2\left(3-2\sqrt{2}\right)$ ------ (2) (1)x 2: $6 = 2c_1\left(1+\sqrt{2}\right) + 2c_2\left(1-\sqrt{2}\right)$ --------(3) (2)-(3): $c_1 + c_2 = 1$ From (1), $3 = c_1\left(1+\sqrt{2}\right) + \left(1-c_1\right)\left(1-\sqrt{2}\right)$ $3 = 1 - \sqrt{2} + 2c_1\sqrt{2}$

	$c_1 = \frac{2+\sqrt{2}}{2\sqrt{2}} = \frac{1+\sqrt{2}}{2}$ $c_2 = \frac{1-\sqrt{2}}{2}$ Therefore, the solution is $w_n = \left(\frac{1+\sqrt{2}}{2}\right)\left(1+\sqrt{2}\right)^n + \left(\frac{1-\sqrt{2}}{2}\right)\left(1-\sqrt{2}\right)^n$ $= \frac{1}{2}\left[\left(1+\sqrt{2}\right)^{n+1} + \left(1-\sqrt{2}\right)^{n+1}\right]$ for $n \geq 1,\ n \in Z$.
2	 For arg(z) to be minimum,  $\alpha = \tan^{-1} 3,\ \beta = \sin^{-1}\left(\frac{2}{\sqrt{10}}\right)$ $\arg(z) = \alpha - \beta = \tan^{-1} 3 - \sin^{-1}\left(\frac{2}{\sqrt{10}}\right)$ and $\lvert z \rvert = \sqrt{6}$ By trigonometric addition formula, $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

	$$=\frac{1}{\sqrt{10}}\frac{\sqrt{6}}{\sqrt{10}}+\frac{3}{\sqrt{10}}\frac{2}{\sqrt{10}}=\frac{\sqrt{6}+6}{10}$$ $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$ $$=\frac{3}{\sqrt{10}}\frac{\sqrt{6}}{\sqrt{10}}-\frac{1}{\sqrt{10}}\frac{2}{\sqrt{10}}=\frac{3\sqrt{6}-2}{10}$$ $z=\sqrt{6}\left[\cos(\alpha-\beta)+\mathrm{i}\sin(\alpha-\beta)\right]$ $$=\sqrt{6}\left[\frac{\sqrt{6}+6}{10}+\mathrm{i}\frac{3\sqrt{6}-2}{10}\right]=\frac{3+3\sqrt{6}}{5}+\mathrm{i}\frac{9-\sqrt{6}}{5}$$
3	Consider $\frac{\mathrm{d}A}{\mathrm{d}t}=rA\left(1-\frac{A}{k}\right)$  For sustainable yield, the maximum point of the graph above must be above the horizontal axis. So, $\frac{rk}{4}-h\geq 0\Rightarrow h\leq\frac{rk}{4}$ Therefore maximum sustainable yield is $h_{\max}=\frac{rk}{4}$ $$\frac{\mathrm{d}A}{\mathrm{d}t}=rA\left(1-\frac{A}{k}\right)-qA$$ $$=\frac{rA(k-A)-qkA}{k}$$ $$=\frac{A(rk-rA-qk)}{k}$$ $$=\frac{A\big(k(r-q)-rA\big)}{k}$$ $$=\frac{rA}{k}\left(\frac{k(r-q)}{r}-A\right)$$

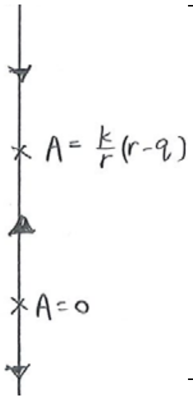
When $\frac{dA}{dt}=0$, $A=\frac{k(r-q)}{r}$ or $A=0$

For sustainability in the long run,

$$\frac{k(r-q)}{r}>0$$

$$k(r-q)>0$$

$$0<q<r$$



4 (i) Substituting the equation of the tangent into the line l,

$$\frac{x^2}{a^2}+\frac{(mx+k)^2}{b^2}=1$$

$$\Rightarrow b^2x^2+a^2\left(m^2x^2+2cmx+k^2\right)=a^2b^2$$

$$\Rightarrow \left(b^2+a^2m^2\right)x^2+2a^2kmx+a^2\left(k^2-b^2\right)=0$$

Since $y=mx+k$ is tangent to the circle, the roots of this quadratic equation are real and equal,

Discriminant = 0

$$\Rightarrow 4a^4k^2m^2=4a^2\left(b^2+a^2m^2\right)\left(k^2-b^2\right)$$

$$\Rightarrow a^2k^2m^2=b^2k^2-b^4+a^2k^2m^2-a^2b^2m^2$$

$$\Rightarrow b^2+a^2m^2=k^2$$

(ii) The equation of the line l is $mx-y+k=0$. The perpendicular distance from the focal point S $(c,0)$ to the line l is given by

$$QS=\left|\frac{mc-0+k}{\sqrt{1^2+m^2}}\right|$$

$$=\frac{|mae+k|}{\sqrt{1+m^2}} \quad \text{since } c=ae$$

Alternatively use similar Triangles.

Let C be the point (0, k) and D be the point where the tangent cuts the x-axis ie $\left(-\frac{k}{m},0\right)$

Then $\Delta DQS \cong \Delta DOC$

$$\left|\frac{QS}{OC}\right|=\left|\frac{DS}{DC}\right| \Rightarrow \left|\frac{QS}{k}\right|=\left|\frac{-\frac{k}{m}-ae}{\sqrt{k^2+\left(-\frac{k}{m}\right)^2}}\right|$$

$$\Rightarrow \left|\frac{QS}{k}\right|=\frac{|-k-mae|}{|k|\sqrt{m^2+1}}$$

$$\Rightarrow QS=\frac{|mae+k|}{\sqrt{1+m^2}}$$

(iii) $QS \times Q'S' = \dfrac{|mae+k|}{\sqrt{1+m^2}} \times \dfrac{|mae-k|}{\sqrt{1+m^2}}$

$= \dfrac{|m^2a^2e^2 - k^2|}{1+m^2}$

$= \dfrac{|m^2a^2e^2 - (a^2m^2 + b^2)|}{1+m^2}$

$= \dfrac{|m^2a^2(e^2-1) - b^2|}{1+m^2}$

$= \dfrac{\left|m^2a^2\left(-\dfrac{b^2}{a^2}\right) - b^2\right|}{1+m^2}$ since $e = \sqrt{1-\dfrac{b^2}{a^2}}$

$= \dfrac{|-b^2m^2 - b^2|}{1+m^2}$

$= \dfrac{|b^2(m^2+1)|}{1+m^2} = b^2$

(iv) By reflective property of ellipse, $\angle S'PQ' = \angle SPQ$

Using vertically opposite angle, $\angle QPR = \angle S'PQ'$

$\therefore \angle SPQ = \angle QPR$

$\therefore \Delta SPQ \cong \Delta RPQ \quad (similar - AAA)$

But PQ is a common side ie $\therefore \Delta SPQ \equiv \Delta RPQ \quad (congruent)$

$SQ = RQ.$

(v) Since $PR = PS$ as $\Delta SPQ \equiv \Delta RPQ$, $S'R = S'P + PR = S'P + PS = 2a$

(vi) Joining O to Q, since $SO : SS' = 1:2$ and $SQ : SR = 1:2$,

$\therefore \Delta SOQ \cong \Delta SS'R$

$\therefore \dfrac{OQ}{S'R} = \dfrac{1}{2} \Rightarrow OQ = \dfrac{1}{2}(2a) = a$

Therefore Q lies on the auxiliary circle $x^2 + y^2 = a^2$.

Given, $OQ' = a$

Denote $SQ = u, S'Q' = v$

Using cosine rule, $\cos\angle OSQ = \dfrac{c^2 + u^2 - a^2}{2cu} = \dfrac{u^2 - b^2}{2cu}$ since $c^2 = a^2 - b^2$

$\cos\angle OS'Q' = \dfrac{c^2 + v^2 - a^2}{2cv} = \dfrac{v^2 - b^2}{2cv}$

Since $\cos\angle OSQ = -\cos\angle OS'Q'$ (corresponding angles),

We have $\dfrac{u^2 - b^2}{2cu} = -\left(\dfrac{v^2 - b^2}{2cv}\right)$

$vu^2 - b^2v = -uv^2 + b^2u$

$vu^2 + uv^2 = b^2v + b^2u$

$vu(u+v) = b^2(v+u)$

$\therefore uv = b^2$ i.e. $QS \times Q'S' = b^2$.

5

Take $\mathbf{u}\ ,\mathbf{v}\ \in P_3$, where $\mathbf{u}=u_0+u_1x+u_2x^2$ and $\mathbf{v}=v_0+v_1x+v_2x^2$, $u_i,v_i\in\mathbb{R}$, $i=1,2,3$

$$T(\mathbf{u}+\mathbf{v})=T\left((u_0+v_0)+(u_1+v_1)x+(u_2+v_2)x^2\right)$$

$$\begin{aligned}
&=\left((u_0+v_0)+3(u_1+v_1)+2(u_2+v_2)\right)\\
&\qquad+\left((u_0+v_0)-(u_1+v_1)-(u_2+v_2)\right)x\\
&\qquad\qquad+\left(2(u_0+v_0)+2(u_1+v_1)+\lambda(u_2+v_2)\right)x^2\\
&=(u_0+3u_1+2u_2)+(u_0-u_1-u_2)x+(2u_0+2u_1+\lambda u_2)x^2\\
&\qquad\qquad+(v_0+3v_1+2v_2)+(v_0-v_1-v_2)x+(2v_0+2v_1+\lambda v_2)x^2\\
&=T\mathbf{u}+T\mathbf{v}
\end{aligned}$$

Take $\mathbf{w}\in\mathbb{R}^3$ and $\alpha\in\mathbb{R}$, where $\mathbf{w}=w_0+w_1x+w_2x^2$, $w_i\in\mathbb{R}$, $i=1,2,3$

$$T(\alpha\mathbf{w})=T\left(\alpha\left(w_0+w_1x+w_2x^2\right)\right)$$

$$\begin{aligned}
&=T\left(\alpha w_0+\alpha w_1x+\alpha w_2x^2\right)\\
&=(\alpha w_0+3\alpha w_1+2\alpha w_2)+(\alpha w_0-\alpha w_1-\alpha w_2)x+(2\alpha w_0+2\alpha w_1+\lambda\alpha w_2)x^2\\
&=\alpha\left(w_0+3w_1+2w_2\right)+\alpha\left(w_0-w_1-\alpha_2\right)x+\alpha\left(2w_0+2w_1+\lambda\alpha w_2\right)x^2\\
&=\alpha\left((w_0+3w_1+2w_2)+(w_0-w_1-\alpha_2)x+(2w_0+2w_1+\lambda\alpha w_2)x^2\right)\\
&=\alpha T\left(w_0+w_1x+w_2x^2\right)
\end{aligned}$$

Therefore, T is a linear transformation. (shown)

$T\left(a_0+a_1x+a_2x^2\right)=(a_0+3a_1+2a_2)+(a_0-a_1-a_2)x+(2a_0+2a_1+\lambda a_2)x^2$ may be expressed as

$T\begin{pmatrix}a_0\\a_1\\a_2\end{pmatrix}=\begin{pmatrix}a_0+3a_1+2a_2\\a_0-a_1-a_2\\2a_0+2a_1+\lambda a_2\end{pmatrix}$ and T may be represented by the matrix $\mathbf{A}=\begin{pmatrix}1&3&2\\1&-1&-1\\2&2&\lambda\end{pmatrix}$

$$\begin{pmatrix}1&3&2\\1&-1&-1\\2&2&\lambda\end{pmatrix}\rightarrow\begin{pmatrix}1&3&2\\0&4&3\\0&0&1-\lambda\end{pmatrix}\rightarrow\begin{pmatrix}1&3&2\\0&1&\frac{3}{4}\\0&0&1-\lambda\end{pmatrix}$$

For $\lambda=1$, rank R = 2

For $\lambda\neq1$, rank R = 3

When $\lambda=1$,

$$\begin{pmatrix}1&3&2\\1&-1&-1\\2&2&1\end{pmatrix}\rightarrow\begin{pmatrix}1&3&2\\0&1&\frac{3}{4}\\0&0&0\end{pmatrix}$$

Basis for $R=\left\{1+x+2x^2,3-x+2x^2\right\}$

Let $x_3=\beta$,

$$x_2=-\frac{3}{4}\beta$$

$$x_1=-3\left(-\frac{3}{4}\beta\right)-2\beta=\frac{1}{4}\beta$$

Basis for $N=\{1-3x+4x^2\}$

Take $1-3x+4x^2 \in N, 1+x+2x^2 \in R$,

we have $(1-3x+4x^2)+(1+x+2x^2)=2-2x+6x^2$

Equating $2-2x+6x^2=\phi(1-3x+4x^2)$ gives no solution. So, $2-2x+6x^2 \notin N$

Equating $2-2x+6x^2=\varphi(1+x+2x^2)+\gamma(3-x+2x^2)$

i.e. $2-2x+6x^2=(\varphi+3\gamma)+(\varphi-\gamma)x+(2\varphi+2\gamma)x^2$, which gives no solution.

So, $2-2x+6x^2 \notin R$

Therefore, $2-2x+6x^2 \notin R\cup N$ and $R\cup N$ is not closed under vector addition, hence it cannot be a vector space.

6

(i)

Let D = total before beer – total after 1 pint of beer

$H_0: m_D=0$

$H_1: m_D<0$

Player	A	B	C	D	E	F	G
Total before drink	101	85	140	100	100	65	85
Total after one pint of beer	141	81	180	125	101	60	100
D	-40	4	-40	-25	-1	5	-15
Rank	6.5	2	6.5	5	1	3	4

$P=$ sum of positive ranks = 5

$T=\min(P,Q)=5$

From MF26, for $n=7$, 1-tailed test at 5% significance level, critical region is $T\le 3$.

Since $T=5$ lies outside the critical region, we do not reject H_0 and conclude that there is insufficient evidence at 5% significance level that the median score has improved after drinking one pint of beer.

(ii)

The positive differences are very small and the negative differences are large, so a simple sign test based on signs and not taking the magnitude of the differences is likely to mislead.

As n = 40 is large, $T=min(P,Q)\sim N\left(\frac{n(n+1)}{4},\frac{n(n+1)(2n+1)}{24}\right)$ approximately

$$ie \quad T\sim N(410, 5535)$$

To accept H_o at 5% level of significance, we must have

$$\frac{T-410}{\sqrt{5535}}>-1.645$$

$$ie \quad T>287.62$$

But since $T=min(P,Q)$, the largest T is 410.

Therefore $287.62<T<410$

7

(i) For one packet of seeds the expected frequencies would be 12, 6 and 2 respectively. As 2 is less than 5 the χ^2 test cannot be conducted

	(ii) For all E_i to be ≥ 5, the minimum is 30:15:5, ie 50 seeds, so at least 3 packets are needed. H_0 : Ratio of red, blue and white flowers is 6:3:1 respectively. H_1 : Ratio of red, blue and white flowers is not 6:3:1 respectively. Total frequency = 5 x 20 = 100 <table><tr><td>Freq colour</td><td>Oi</td><td>Ei</td></tr><tr><td>red</td><td>51</td><td>$\frac{6}{10}\times 100 = 60$</td></tr><tr><td>blue</td><td>38</td><td>$\frac{3}{10}\times 100 = 30$</td></tr><tr><td>white</td><td>11</td><td>$\frac{1}{10}\times 100 = 10$</td></tr><tr><td></td><td>$\sum O_i = 100$</td><td>$\sum E_i = 100$</td></tr></table> Number of classes, n = 3 , $\therefore$ number of degree of freedom v = no of classes – no of restrictions = 3 - 1 = 2 At 5% level of significance, we reject H_0 if $p-value < 0.05$ $\chi^2_{calc} = \sum_{i=1}^{3}\frac{(O_i - E_i)^2}{E_i}$ Using GC, $p-value = 0.167 > 0.05$ $\therefore$ do not reject H_0 We conclude at the 5% sig level that the observed colours are consistent with the message on the packet.
8	(i) Since $n = 60$ is large, by Central Limit Theorem, $\bar{X} \sim N\left(\lambda, \frac{\lambda}{60}\right)$ $Z = \frac{\bar{X} - \lambda}{\sqrt{\frac{\lambda}{60}}} \sim N(0,1)$ $P(-1.96 < Z < 1.96) = 0.95$ $P\left(-1.96 < \frac{\bar{X} - \lambda}{\sqrt{\frac{\lambda}{60}}} < 1.96\right) = 0.95$ $P\left(-1.96\sqrt{\frac{\lambda}{60}} < \bar{X} - \lambda < 1.96\sqrt{\frac{\lambda}{60}}\right) = 0.95$ (ii) For the confidence limits,

	$-1.96\sqrt{\frac{\lambda}{60}}=\bar{X}-\lambda$ and $1.96\sqrt{\frac{\lambda}{60}}=\bar{X}-\lambda$ $\left(1.96\sqrt{\frac{\lambda}{60}}\right)^2=\left(\bar{X}-\lambda\right)^2$ $\left(1.96\sqrt{\frac{\lambda}{60}}\right)^2=\left(\bar{X}-\lambda\right)^2$ $\left(1.96\sqrt{\frac{\lambda}{60}}\right)^2=\left(20.1-\lambda\right)^2$ $0.064\lambda=404.01-40.2\lambda+\lambda^2$ $\lambda^2-40.264\lambda+404.01=0$ From GC, $\lambda=19.00$ and $\lambda=21.27$ (2dp) Therefore the confidence limits are 19.00 and 21.27 . (iii) Since $\mu=\lambda=22$ lies outside the confidence interval $\left(19.00, 21.27\right)$, we reject H_0 and conclude at 5% significance level that there is sufficient evidence that the discipline master's claim is not correct.
9	Assumptions: The samples are independent and taken from populations with equal variance Let μ_A and μ_B be the mean yield of blackcurrant due to fertiliser *A* and *B* respectively. $H_0:\mu_A=\mu_B$ $H_1:\mu_A>\mu_B$ Under H_0, the test statistic is $T=\frac{\bar{A}-\bar{B}}{S\sqrt{\frac{1}{n_A}+\frac{1}{n_B}}}\sim t\left(n_A+n_B-2\right)$ ie $t\left(18\right)$ where *s* is the pooled estimate of the common population variance. For one-tailed test at 10% significance level, we reject H_0 if $p-value<0.1$ Using GC, $\bar{a}=47.1$ $\bar{b}=42.5$ $p-value=0.279$ Since the *p*-value = 0.279 > 0.1 , we do not reject H_0 and conclude at the 10% significance level that there is insufficient evidence that fertiliser *A* produces a greater yield than fertiliser *B*
10	For $y\geq c$, $G(y)=P\left(Y\leq y\right)$ $=P\left(X\leq y\mid X\geq c\right)$

$$= \frac{P(c \le X \le y)}{P(X \ge c)}$$

$$= \frac{P(X \le y) - P(X \le c)}{1 - P(X < c)}$$

$$= \frac{F(y) - F(c)}{1 - F(c)}$$

$$g(y) = \frac{d}{dy}\left(G(y)\right)$$

$$= \frac{d}{dy}\left(\frac{F(y) - F(c)}{1 - F(c)}\right)$$

$$= \frac{f(y)}{1 - F(c)}$$

[A1]

Therefore

$$g(y) = \begin{cases} \dfrac{f(y)}{1 - F(c)} & , \quad y \ge c \\ 0 & , \quad y < c \end{cases}$$

[A1]

(i) $1 - F(1) = P(T > 1) = \int_1^\infty \frac{2}{3} t e^{-\frac{1}{3}t^2} dt = -\left[e^{-\frac{1}{3}t^2} \right]_1^\infty = e^{-\frac{1}{3}}$

(ii) $f(t) = \begin{cases} \frac{2}{3} t e^{-\frac{1}{3}t^2}, & t \ge 0 \\ 0, & t < 0 \end{cases}$

Let W be the chargeable duration per call in minutes

$$g(w) = \begin{cases} \dfrac{f(w)}{1 - F(1)} & , \quad w \ge 1 \\ 0 & , \quad w < 1 \end{cases}$$

$$g(w) = \begin{cases} \dfrac{2}{3} e^{\frac{1}{3}} w e^{-\frac{1}{3}w^2} & , \quad w \ge 1 \\ 0 & , \quad w < 1 \end{cases}$$

$$E(W) = \int_1^\infty w . \frac{2}{3} e^{\frac{1}{3}} w e^{-\frac{1}{3}w^2} dw$$

Using GC, $E(W) = 1.887355$

Therefore, expected amount being charged

$= 1.887355 \times 0.3$

$= 0.56620$

= $0.57 or 56.6 cents

(iii) From GC, $E(W^2) = \int_1^\infty w^2 . \frac{2}{3} e^{\frac{1}{3}} w e^{-\frac{1}{3}w^2} dw = 4$

$$Var(W) = E(W^2) - E^2(W)$$
$$= 4 - (1.887355)^2$$
$$= 0.43789$$

Let $S := \sum_{i=1}^{30} (0.3W)$ be r.v. denoting the call charges in \$ per month.

$$E(S) = 30 \times 0.3 \times 1.887355 = 16.986195$$
$$Var(S) = 30 \times (0.3)^2 \times 0.43789 = 1.1823049$$

Since $n = 30$ large, by Central Limit Theorem,

$S \sim N(16.986195\,, 1.1823049)$

$P(S > 18) = 0.175571$

So, expected number of months = $\dfrac{1}{0.175571} = 5.70$ (3 sig. fig)

11

(i) $X \sim P_0(\lambda)$

$$P(Y = r) = P(X = r \cap Y = r)$$
$$+ P(X = r+1 \cap Y = r)$$
$$+ P(X = r+2 \cap Y = r)$$
$$+ \ldots\ldots.$$
$$= \sum_{s=r}^{\infty} P(X = s \cap Y = r)$$
$$= \sum_{s=r}^{\infty} P(X = s).P(Y = r \mid X = s)$$
$$= \sum_{s=r}^{\infty} \frac{e^{-\lambda}\lambda^s}{s!} \times \binom{s}{r} p^r (1-p)^{s-r}$$

(ii)

$$P(Y = r) = \sum_{s=r}^{\infty} \frac{e^{-\lambda}\lambda^s}{s!} \times \binom{s}{r} p^r (1-p)^{s-r}$$
$$= \sum_{s=r}^{\infty} \frac{e^{-\lambda}\lambda^s}{s!} \times \frac{s!}{r!(s-r)!} p^r (1-p)^{s-r}$$
$$= \frac{e^{-\lambda}}{r!} \sum_{s=r}^{\infty} \frac{\lambda^s}{(s-r)!} p^r (1-p)^{s-r}$$
$$= \frac{e^{-\lambda}}{r!} \sum_{s=r}^{\infty} \frac{\lambda^r.\lambda^{s-r}}{(s-r)!} p^r (1-p)^{s-r}$$
$$= \frac{e^{-\lambda}(\lambda p)^r}{r!} \sum_{s=r}^{\infty} \frac{(\lambda(1-p))^{s-r}}{(s-r)!}$$
$$= \frac{e^{-\lambda}(\lambda p)^r}{r!} \times e^{\lambda(1-p)}$$
$$= \frac{e^{-\lambda+\lambda(1-p)}(\lambda p)^r}{r!}$$

$$= \frac{e^{-\lambda p}(\lambda p)^r}{r!}, \quad r = 0,1,2,3.....$$

Therefore the number of customers joining the queue for corporate banking service in any one-minute interval follows a Poisson distribution with mean λp.

(ii)

$Y \sim P_0(\lambda p)$

$P(\text{no customer joins the queue for the personal banking service} \mid Y = k)$

$= P(X = k \mid Y = k)$

$$= \frac{P(X = k \cap Y = k)}{P(Y = k)}$$

$$= \frac{\dfrac{e^{-\lambda}\lambda^k}{k!} \times p^k}{\dfrac{e^{-\lambda p}(\lambda p)^k}{k!}}$$

$= e^{-\lambda + \lambda p}$

$= e^{-\lambda(1-p)}$

For t-minutes interval, $Y \sim P_0(\lambda p t)$

$F(t) = P(T \le t) = P(Y \ge 1)$

$\quad = 1 - P(Y = 0)$

$\quad = 1 - e^{-\lambda p t}$

Differentiate to obtain p.d.f. of T, $\quad f(t) = \lambda p e^{-\lambda p t}$

ie T follows an exponential distribution with parameter λp.

Let the time he can take for his toilet break be a.

$P(T > a) \ge 0.9$

$\int_a^\infty \lambda p e^{-\lambda p t}\, dt \ge 0.9$

$\left[-e^{-\lambda p t}\right]_a^\infty \ge 0.9$

$e^{-\lambda p a} \ge 0.9$

$a \le -\dfrac{\ln 0.9}{\lambda p} = 2.34$ mins (3 sig fig.)