

RI H2 Mathematics 2017 Prelim Exam Paper 2 Question

1	<p>Referred to the origin O, the points A, B and C have position vectors \mathbf{a}, \mathbf{b} and \mathbf{c} respectively such that</p> $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}, \quad \mathbf{b} = 5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \mathbf{c} = 4\mathbf{i} + \mathbf{j} - 2\mathbf{k}.$ <p>(i) Given that M is the mid-point of AC, use a vector product to find the exact area of triangle ABM. [4]</p> <p>(ii) Find the position vector of the point N on the line AB such that \overrightarrow{MN} is perpendicular to \overrightarrow{AB}. [4]</p>
2	<p>(a) (i) Show that $\frac{1}{r-1} - \frac{2}{r} + \frac{1}{r+1} = \frac{2}{r(r-1)(r+1)}$. [1]</p> <p>(ii) Hence find $\sum_{r=3}^n \frac{4}{r(r-1)(r+1)}$. (There is no need to express your answer as a single algebraic fraction). [4]</p> <p>(b) Amy and her brother Ben are saving money together for their family trip. In the first week of 2017, Amy saves \$25 and Ben saves \$2. In each subsequent week, Amy saves \$4 more than the amount she saved in the previous week, and Ben saves 22% more than the amount he saved in the previous week.</p> <p>(i) Which is the first week in which Ben saves more than Amy in that week? [2]</p> <p>(ii) They need a combined total of \$2400 for the trip. How many complete weeks do Amy and Ben need to save before they can achieve their targeted amount? [2]</p>
3	<p>The function f is defined as follows.</p> $f : x \mapsto \sqrt{3} \sin x + \cos x, \quad x \in \mathbb{R}, \quad 0 < x < \pi.$ <p>(i) Write $f(x)$ as $R \sin(x + \alpha)$, where R and α are constants with exact values to be found. [2]</p> <p>(ii) Sketch the graph of $y = f(x)$, stating the axial intercepts, and find the range of f. [3]</p> <p>(iii) Hence, solve $f(x) \leq 1$ exactly. [2]</p> <p>The function g is defined as follows:</p> $g : x \mapsto 2 \cos \left(x + \frac{\pi}{6} \right), \quad x \in \mathbb{R}, \quad -\frac{\pi}{6} \leq x \leq b.$ <p>(iv) Write down the largest exact value of b, for g^{-1} to exist. [1]</p> <p>(v) Taking the value of b found in part (iv), show that the composite function $g^{-1}f$ exists and solve $g^{-1}f(x) = x$ exactly. [3]</p>

4	<p>The line l_1 has equation $\frac{x}{-3} = \frac{y}{12} = \frac{z-1}{4}$ and the line l_2 has equation $\frac{x-1}{-3} = y-4 = \frac{z-1}{4}$.</p> <p>(i) Show that l_1 and l_2 are skew lines. [3]</p> <p>(ii) Find a cartesian equation of the plane p which is parallel to l_1 and contains l_2. [3]</p> <p>(iii) The point $A(0, a, 1)$ is equidistant from p and l_1. Calculate the possible values of a exactly. [6]</p>
5	<p>For events X and Y, it is given that $P(X Y) = \frac{1}{2}$, $P(Y X) = \frac{2}{3}$ and $P(X \cup Y) = \frac{5}{6}$. Find</p> <p>(i) $P(X)$, [3]</p> <p>(ii) $P(X \cup Y')$. [2]</p>
6	<p>The power consumption of a randomly chosen Effixion laptop has a normal distribution. The salesman at Elf Superstore claims that the average power consumption of an Effixion laptop is 100 watts. The power consumption, w watts, is measured for a random sample of 50 Effixion laptops. The results are summarised as follows.</p> $\sum (w-100) = 26 \quad \sum (w-100)^2 = 273$ <p>Test whether this data provides evidence at the 3% level of significance, that the salesman has made an understatement. [6]</p> <p>The power consumption of another random sample of 50 Effixion laptops is measured. It is found that the sample variance is 6.25. Using this sample only, find the set of values of \bar{w}, correct to 2 decimal places, for which the test would result in the rejection of the null hypothesis in favour of the alternative hypothesis at the 1% level of significance. [4]</p>
7	<p>An unbiased cubical die has the number 1 on one face, the number 2 on two faces and the number 3 on three faces. Adrian invites Benny to play a game. In each round, Benny rolls the die twice. Adrian pays Benny \$$a$ if the total score is 2 and \$3 if the total score is 3. However, if the total score is 4, Benny pays Adrian \$2. No payment is made otherwise.</p> <p>(i) Find the probability that Adrian pays Benny at least 5 times in 20 rounds. [4]</p> <p>The random variable X represents Benny's winnings in each round.</p> <p>(ii) Given that $a = 6$, find the probability distribution of X. Hence, help Benny decide if he should accept Adrian's invitation to play the game. Justify your answer. [5]</p> <p>(iii) Determine the value of a for the game to be fair. [1]</p>
8	<p>(a) In Country S, each household's monthly income per capita is calculated by taking the gross household income divided by the total number of members in the household. It is assumed</p>

that this amount for a randomly chosen household consisting of 3 members follows a normal distribution with mean \$2601 and standard deviation \$768.

(i) The Ministry of Education offers financial aid to students from households consisting of 3 members each and with a household monthly income per capita lower than \$1800. Find the probability that a randomly chosen household with 3 members does not qualify for financial aid. [1]

(ii) It is found that there is a 50% chance that a randomly chosen household with 3 members has a gross household income between \$5000 and \$ a , where $a > 5000$. Find the value of a , correct to the nearest dollar. [3]

(b) Mr Tan is self-employed and his monthly income follows a normal distribution with mean \$6000 and standard deviation \$1000 whereas Mrs Tan works part-time and earns a fixed amount of \$1500 a month. Their family's monthly expenditure follows a normal distribution with mean μ dollars and standard deviation 650 dollars.

(i) It was found that 10% of the time they spend more than \$5900 in a month. Find the value of μ , correct to the nearest dollar. [2]

(ii) Mr and Mrs Tan save the remaining amount of their income after deducting their expenditure every month. Find the probability that their monthly savings in August and in September differ by more than \$1000. [4]

(iii) State an assumption needed for your calculation in part (b)(ii). [1]

9 (i) Sketch a scatter diagram that might be expected when x and y are related approximately as given in each of the cases (A) and (B) below. In each case, your diagram should include 6 points, approximately equally spaced with respect to x , and with all x - and y -values positive. The letters a, b, c and d represent constants.

(A) $y = a + bx^2$, where a is positive and b is negative,

(B) $y = c + d \ln x$, where c is positive and d is negative. [2]

The following table shows the Gross Domestic Product (GDP) per capita, x , and infant mortality rate, y , for a sample of 9 countries.

x (\$)	1375	2502	10569	2966	11539	2036	4260	1433	7427
y	115	69	18	65	17	83	44	112	27

(ii) Draw a scatter diagram for these values, labelling the axes clearly. [2]

(iii) Calculate the product moment correlation coefficient, and explain why its value does not necessarily mean that a linear model is the best model for the relationship between x and y . [2]

8	Forward	10 May	19	Midfielder	5 February
9	Forward	1 July	20	Midfielder	1 March
10	Midfielder	1 April	21	Forward	27 March
11	Midfielder	29 October	22	Goalkeeper	31 October

(iii) Find the probability that the team formed by country N contains no players with the same date of birth. [4]