ANDERSON JUNIOR COLLEGE
2017 Preliminary Examination
H2 Mathematics Paper 1 (9758/01)

Duration: 3 hours

1 Mr Tan invested a total of \$25,000 in a structured deposit account, bonds and an estate fund. He invested \$7,000 more in bonds than in estate fund. The projected annual interest rates for structured deposit account, bonds and estate fund are 2%, 3% and 4.5% respectively. Money that is not drawn out at the end of the year will be re-invested for the following year.

Mr Tan plans to draw out his money from all investments at the end of the second year and estimates that he will receive a total of \$26,300. Find the amount of money Mr Tan invested in each investment, giving your answer to the nearest dollar. [5]

2 Show that the differential equation

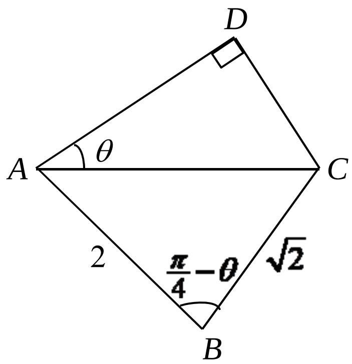
$$\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{3xy}{1-3x^2}-x+1=0$$

may be reduced by means of the substitution $y=u\sqrt{1-3x^2}$ to

$$\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{x-1}{\sqrt{1-3x^2}}.$$

Hence find the general solution for y in terms of x. [5]

3



The diagram above shows a quadrilateral $ABCD$, where AB = 2, $BC=\sqrt{2}$, angle $ABC=\dfrac{\pi}{4}-\theta$ radians and angle $CAD=\theta$ radians.

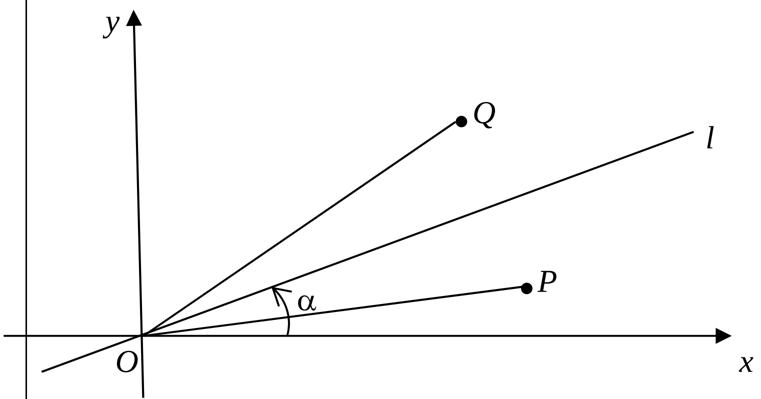
Show that

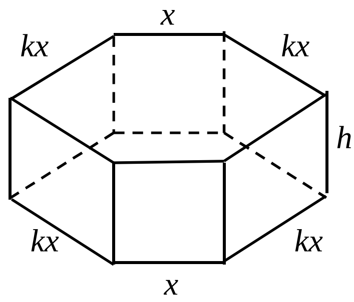
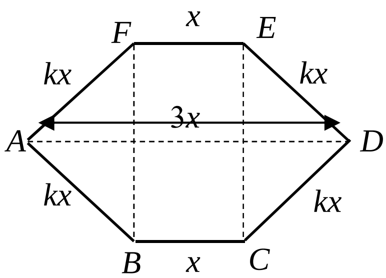
$$AC=\sqrt{6-4\cos\theta-4\sin\theta}.$$ [2]

Given that θ is small enough for θ^3 and higher powers of θ to be neglected, show that

$$AD\approx a+b\theta+c\theta^2,$$

where a, b and c are constants to be determined. [5]

4	**(a)** Given that $\sum_{n=1}^{N} \frac{1}{4n^2-1} = \frac{1}{2} - \frac{1}{2(2N+1)}$, find $\sum_{n=1}^{2N} \frac{1}{4(n+1)^2-1}$. Deduce that $\sum_{n=1}^{2N} \frac{1}{(2n+3)^2}$ is less than $\frac{1}{6}$. [5] **(b)** The sum to n terms of a series is given by $S_n = n\ln 2 - \frac{n^2-1}{\mathrm{e}}$. Find an expression for the n^{th} term of the series, in terms of n. Show that the terms of the series follow an arithmetic progression. [4]
5	A curve C has equation $y = \mathrm{f}(x)$. The equation of the tangent to the curve C at the point where $x = 0$ is given by $2x - ay = 3$ where a is a positive constant. It is also given that $y = \mathrm{f}(x)$ satisfies the equation $(1+2x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and that the third term in the Maclaurin's expansion of f(x) is $\frac{1}{3}x^2$. Find the value of a. Hence, find the Maclaurin's series for f(x) in ascending powers of x, up to and including the term in x^3. [7]
6	The diagram below shows the line l that passes through the origin and makes an angle α with the positive real axis, where $0 < \alpha < \frac{\pi}{2}$. Point P represents the complex number z_1 where $0 < \arg z_1 < \alpha$ and length of OP is r units. Point P is reflected in line l to produce point Q, which represents the complex number z_2.  Prove that $\arg z_1 + \arg z_2 = 2\alpha$. [2] Deduce that $z_1 z_2 = r^2(\cos 2\alpha + i\sin 2\alpha)$. [1] Let R be the point that represents the complex number $z_1 z_2$. Given that $\alpha = \frac{\pi}{4}$, write down the cartesian equation of the locus of R as z_1 varies. [2]

7	 Fig 1  Fig 2 Figure 1 shows a solid metal hexagonal prism of height h cm. Figure 2 shows the hexagonal cross-section $ABCDEF$ of the prism where $AD = 3x$ cm, $BC = FE = x$ cm and the remaining 4 sides are of length kx cm each, where k is a constant. Show that $$S = 8x^2\sqrt{k^2-1} + 2xh(1+2k),$$ [3] where S is the surface area of this solid hexagonal prism. **(a)** If the volume of the prism is fixed at 400 cm^3, use differentiation to find, in terms of k, the exact value of x that gives a stationary value of S. [3] Let $k = 2$. **(b)** The prism is heated and it expands in such a way that, at time t seconds, the rate of increase of x is the same as the rate of increase of its height h. At the instant when $x = 3$, the prism's height is 8 cm and its surface area is increasing at a constant rate of 0.5 cm^2/s. Find the rate of change of the volume of the prism at this instant. [6]
8	The curve C has equation $y = \dfrac{4x^2 - kx + 2}{x-2}$, where k is a constant. **(i)** Show that curve C has stationary points when $k < 9$. [3] **(ii)** Sketch the graph of C for the case where $6 < k < 9$, clearly indicating any asymptotes and points of intersection with the axes. [4] **(iii)** Describe a sequence of transformations which transforms the graph of $y = 2x + \dfrac{1}{x}$ to the graph of $y = \dfrac{4x^2 - 8x + 2}{x-2}$. [3] **(iv)** By drawing a suitable graph on the same diagram as the graph of C, solve the inequality $$\frac{4x^2-8x+2}{x-2} > \frac{1}{x^2}.$$ [3]

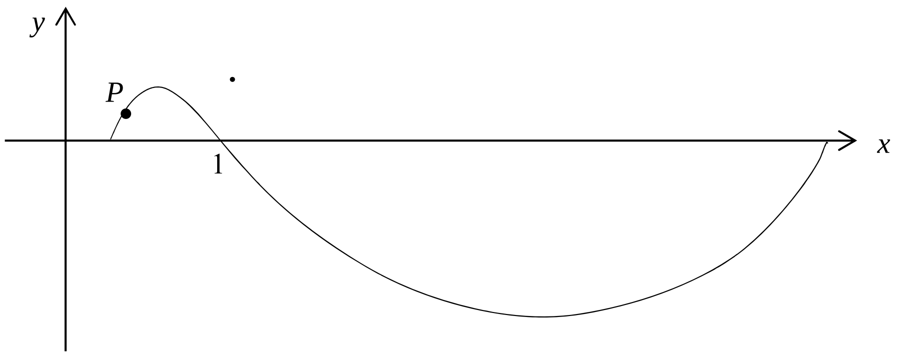
9	The position vectors of A, B and C with respect to the origin O are **a, b** and **c** respectively. It is given that $\overrightarrow{AC} = 4\overrightarrow{CB}$ and $\lvert\mathbf{a}+\mathbf{b}\rvert^2 = \lvert\mathbf{a}\rvert^2 + \lvert\mathbf{b}\rvert^2$. **(i)** By considering $(\mathbf{a}+\mathbf{b}).(\mathbf{a}+\mathbf{b})$, show that **a** and **b** are perpendicular. [2] **(ii)** Find the length of the projection of **c** on **a** in terms of $\lvert\mathbf{a}\rvert$. [3] **(iii)** Given that F is the foot of the perpendicular from C to OA and **f** denotes the position vector $\overrightarrow{OF}$, state the geometrical meaning of $\lvert\mathbf{c}\times\mathbf{f}\rvert$. [1] **(iv)** Two points X and Y move along line segments OA and AB respectively such that $\overrightarrow{OX} = (\cos 3t)\mathbf{i} + (\sin 3t)\mathbf{j} + \frac{1}{2}\mathbf{k},$ $\overrightarrow{OY} = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} - 2\mathbf{k},$ where t is a real parameter, $0 \le t \le 2\pi$. By expressing the scalar product of $\overrightarrow{OX}$ and $\overrightarrow{OY}$ in the form of $p\sin(qt)+r$ where p, q and r are real values to be determined, find the greatest value of the angle XOY. [5]
10	There are 25 toll stations, represented by T_1, T_2, T_3,……, T_{25} along a 2000 km stretch of highway. T_1 is located at the start of the highway and T_2 is located x km from T_1. Subsequently, the distance between two consecutive toll stations is 2 km more than the previous distance. Find the range of values x can take. [3] Use $x = 60$ for the rest of this question. Each toll station charges a fee based on the distance travelled from the previous toll station. The fee structure at each toll station is as follows: For the first 60 km, the fee per km will be 5 cents. For every additional 2 km, the fee per km will be 2% less than the previous fee per km. **(i)** Find, in terms of n, the amount of fees a driver will need to pay at T_n. [3] **(ii)** Find the total amount of fees a driver will need to pay, if he drives from T_1 to T_n. Leave your answer in terms of n. [4] More toll stations are built along the highway in the same manner, represented by T_{26}, T_{27}, T_{28},…….. beyond the 2000 km stretch. **(iii)** If a driver starts driving from T_1 and only has \$200, at which toll station will he not have sufficient money for the fees? [2]
11	**(i)** Show by integration that

$$\int e^{-2x}\sin x \ \mathrm{d}x = -\frac{2}{5}e^{-2x}\sin x - \frac{1}{5}e^{-2x}\cos x + A$$

where A is an arbitrary constant. [3]

The diagram below shows a sketch of curve C, with parametric equations

$$x = e^{-t}\ ,\ y = e^{-t}\sin t\ ,\ -\pi \le t \le \pi\ .$$



Point P lies on C where $t = \frac{\pi}{2}$.

(ii) Find the equation of the normal at P. [3]

(iii) Find the exact area bounded by the curve C for $0 \le t \le \pi$, the line $x = 1$ and the normal at P. [5]

(iv) The normal at P cuts the curve C again at two points where $t = q$ and $t = r$. Find the values of q and r. [3]

End of paper