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TEMASEK JUNIOR COLLEGE, SINGAPORE
JC 2
Preliminary Examination 2017
Higher 2

TEMASEK JUNIOR COLLEGE

MATHEMATICS 9758/01

Paper 1 **29 Aug 2017**

Additional Materials: Answer paper
List of Formulae (MF26)

3 hours

READ THESE INSTRUCTIONS FIRST

Write your Civics group and name on all the work that you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

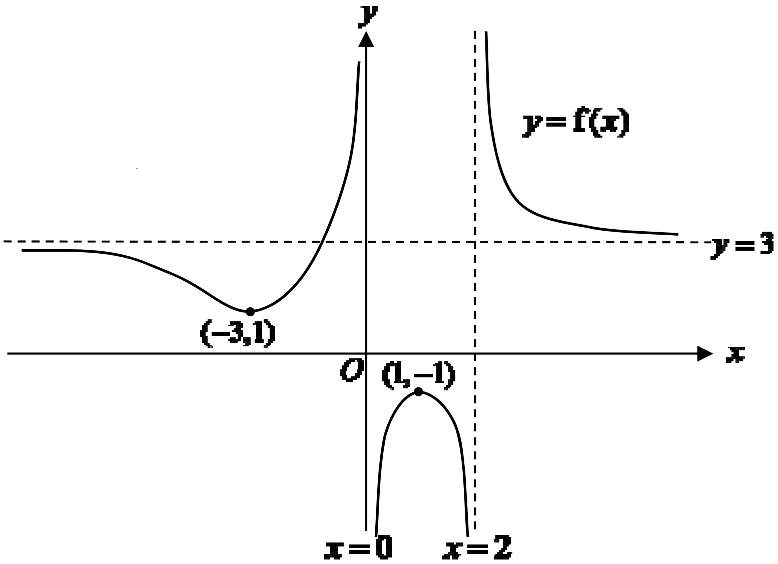
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

At the end of the examination, fasten all your work securely together.

This document consists of 7 printed pages and 1 blank page.

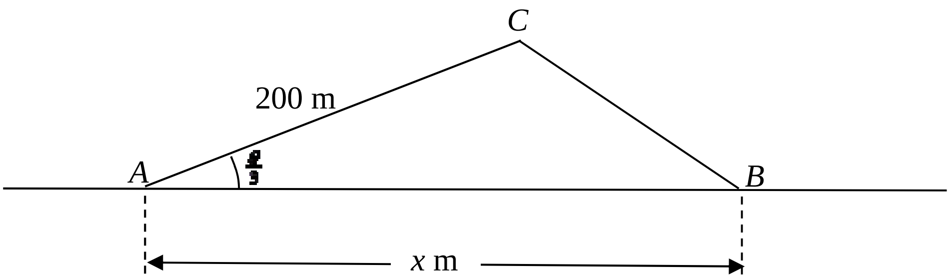
1 The graph of $y = \mathrm{f}(x)$ is shown below.



(a) The graph of $y = \mathrm{f}(2-x)$ is obtained when the graph of $y = \mathrm{f}(x)$ undergoes a sequence of transformations. Describe the sequence of transformations. [2]

(b) Sketch the graph of $y = \mathrm{f}'(x)$, stating the equations of any asymptotes and the coordinates of any points of intersection with the axes. [3]

2



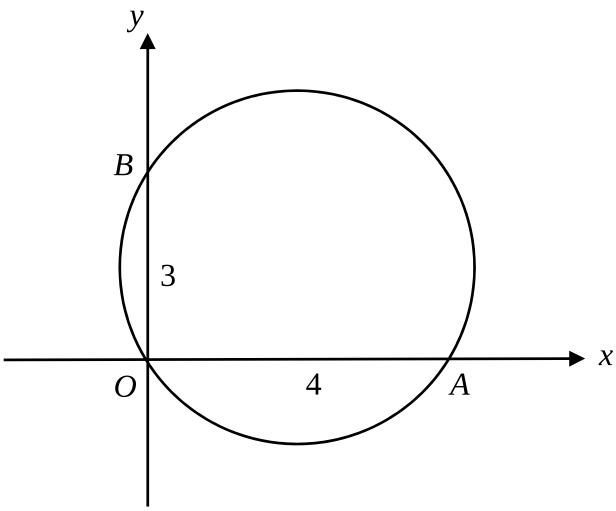
The diagram shows two points at ground level, A and B. The distance in metres between A and B is denoted by x. The angle of elevation of C from B is twice the angle of elevation of C from A. The distance AC is 200 m and $\angle BAC = \dfrac{\theta}{3}$ radians. Show that

$$x = \frac{200\sin\theta}{\sin\left(\frac{2}{3}\theta\right)}.$$ [2]

It is given that θ is a small angle such that θ^4 and higher powers of θ are negligible. By using appropriate expansions from the List of Formulae (MF26), show that

$$x \approx \frac{2700 - 250\theta^2}{9}.$$ [4]

3



The diagram above shows a circle C which passes through the origin O and the points A and B.

It is given that $OA = 4$ units and $OB = 3$ units.

(i) Show that the coordinates of the centre of C is $\left(2, \frac{3}{2}\right)$. Hence write down the equation of C in the form $(x-2)^2 + \left(y - \frac{3}{2}\right)^2 = r^2$, where r is a constant to be determined. [2]

(ii) By adding a suitable line to the diagram above, find the range of values of m for which the equation $mx - \frac{3}{2} = \sqrt{\frac{25}{4} - (x-2)^2}$ has a solution. [4]

4 The curve C has equation $y = \sin 2x + 2\cos x$, $0 \le x \le 2\pi$.

(i) Using an algebraic method, find the exact x-coordinates of the stationary points. [You do not need to determine the nature of the stationary points.] [3]

(ii) Sketch the curve C, indicating clearly the coordinates of the turning points and the intersection with the axes. [1]

(iii) Find the area bounded by the curve C and the line $y = \frac{1}{\pi}x$. [3]

5 The curve C has equation $y = kx^3$. The tangent at the point P on C meets the curve again at point Q. The tangent at point Q meets the curve again at point R. If the x coordinates of P, Q and R are p, q, and r respectively where $p \neq 0$.

(i) Show that p and q satisfy the equation $\left(\dfrac{q}{p}\right)^2 + \left(\dfrac{q}{p}\right) - 2 = 0$. [4]

(ii) Show that p, q and r are three consecutive terms of a geometric progression. Hence determine if this geometric series is convergent. [4]

[You may use the identity $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$ for $a, b \in \mathbb{R}$.]

6 **(a)** The vectors **a** and **b** are the position vector of points A and B respectively. It is given that $OA = 2\sqrt{7}$, $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{a} \cdot \mathbf{b} = -14$.

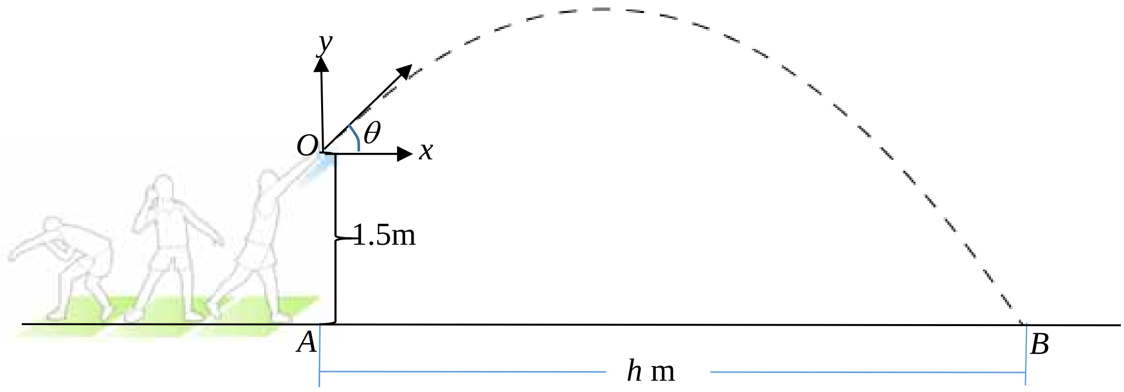
(i) Find angle AOB. [2]

(ii) State the geometrical meaning of $|\hat{\mathbf{a}} \cdot \mathbf{b}|$, where $\hat{\mathbf{a}}$ is the unit vector of **a**. [1]

(iii) Hence or otherwise, find the position vector of the foot of perpendicular from B to line OA in terms of **a**. [2]

(b) The non-zero vectors **p** and **q** are such that $|\mathbf{p} \times \mathbf{q}| = 2$. Given that **p** is a unit vector and $\mathbf{q} \cdot \mathbf{q} = 4$, show that **p** and **q** are perpendicular to each other. [3]

7



The diagram shows a shot put being projected with a velocity v ms^{-1} from the point O at an angle θ made with the horizontal. The point O is 1.5m above the point A on the ground. The x-y plane is taken to be the plane that contains the trajectory of this projectile motion with x-axis parallel to the horizontal and O being the origin. The equation of the trajectory of this projectile motion is known to be

$$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta},$$

where g ms^{-2} is the acceleration due to gravity.

The constant g is taken to be 10 and the distance between A and B is denoted by h m. Given that v = 10, show that h satisfies the equation

$$h^2 - 10h\sin 2\theta - 15\cos 2\theta - 15 = 0. \qquad [3]$$

As θ varies, h varies. Show that stationary value of h occurs when θ satisfies the following equation

$$3\tan^2 2\theta - 20\sin 2\theta \tan 2\theta - 20\cos 2\theta - 20 = 0. \qquad [5]$$

Hence find the stationary value of h. [2]

8 **(a)** In an Argand diagram, points P and Q represent the complex numbers $z_1 = 2+3\mathrm{i}$ and $z_2 = \mathrm{i}z_1$.

(i) Find the area of the triangle OPQ, where O is the origin. [2]

(ii) z_1 and z_2 are roots of the equation $\left(z^2+az+b\right)\left(z^2+cz+d\right)=0$, where $a, b, c, d \in \mathbf{R}$. Find a, b, c and d. [4]

(b) Without using the graphing calculator, find in exact form, the modulus and argument of $v^* = \left(\dfrac{\sqrt{3}+\mathrm{i}}{-1+\mathrm{i}}\right)^{14}$. Hence express v in exponential form. [5]

9 A curve C has parametric equation defined by

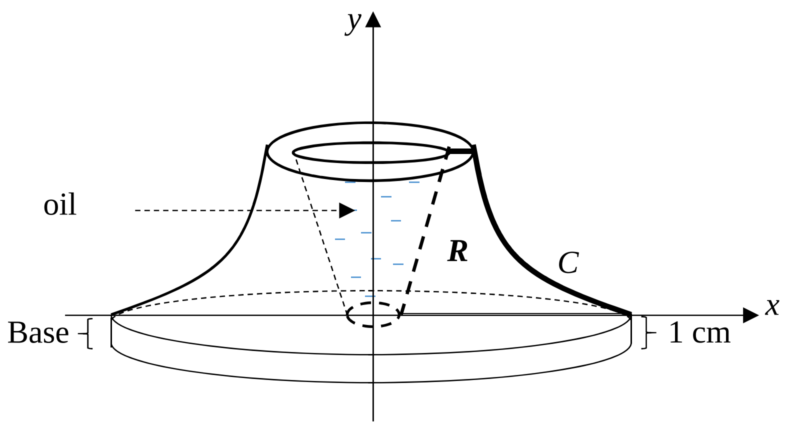
$$x = 4\sec t \text{ and } y = 8(1 - \tan t) \text{ where } -\frac{\pi}{4} \le t \le \frac{\pi}{4}.$$

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t and hence show that the equation of tangent at the point $t = -\frac{\pi}{6}$ is $y = 4x + 8(1 - \sqrt{3})$. [3]

(ii) Find the Cartesian equation of C. [2]

R is the region bounded by C, the tangent in **(i)**, the normal to C at $t = 0$ and the x axis. Part of an oil burner is formed by rotating R 2π radians about the y-axis as shown in the diagram below (not drawn to scale). The base of the burner is a solid cylinder of thickness 1 cm.

[You may assume each unit along the x and y axis to be 1 cm]



Find the volume of the material required to make the burner. [6]

10 The point A has coordinates $(3, 1, 1)$. The line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$, where λ is a parameter. P is a point on l when $\lambda = t$.

(i) Find cosine of the acute angle between AP and l in terms of t. Hence or otherwise, find the position vector of the point N on l such that N is the closest point to A. [6]

(ii) Find the coordinates of the point of reflection of A in l. [2]

The line L has equation $x = -1$, $2y = z + 2$.

(iii) Determine whether L and l are skew lines. [2]

(iv) Find the shortest distance from A to L. [3]

11 A hot air balloon rises vertically upwards from the ground as the balloon operator intermittently fires and turns off the burner. At time t minutes, the balloon ascends at a rate inversely proportional to $t + \lambda$, where λ is a positive constant. At the same time, due to atmospheric factors, the balloon descends at a rate of 2 km per minute. It is also known that initially the rate of change of the height of the balloon is 1 km per minute.

(i) Find a differential equation expressing the relation between H and t, where H km is the height of the hot air balloon above ground at time t minutes. Hence solve the differential equation and find H in terms of t and λ. [7]

Using $\lambda = 15$,

(ii) Find the maximum height of the balloon above ground in exact form. [3]

(iii) Find the total vertical distance travelled by the balloon when $t = 8$. [3]

(iv) Can we claim that the rate of change of the height of the balloon above the ground is decreasing? Explain your answer. [2]