2017 NYJC JC2 Prelim Exam 9649/1 Solution

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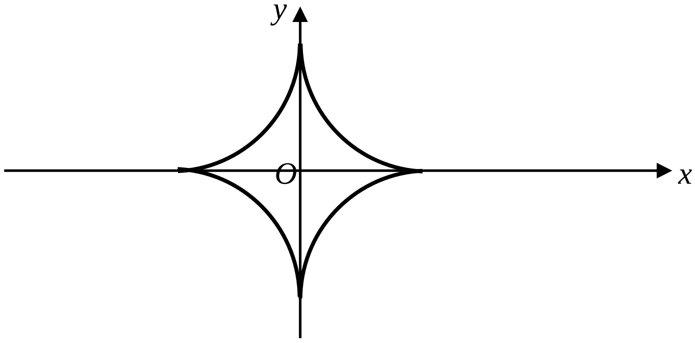
1	(i) 
	(ii) $\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2=\left(-3a\cos^2\theta\sin\theta\right)^2+\left(3a\sin^2\theta\cos\theta\right)^2$
	$=9a^2\sin^2\theta\cos^2\theta\left(\cos^2\theta+\sin^2\theta\right)$
	$=9a^2\sin^2\theta\cos^2\theta$
	(iii) Area of curved surface
	$=2\times2\pi\int_0^{\frac{\pi}{2}}y\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\ \mathrm{d}\theta$
	$=4\pi\int_0^{\frac{\pi}{2}}a\sin^3\theta\sqrt{9a^2\sin^2\theta\cos^2\theta}\ \mathrm{d}\theta$
	$=12\pi a^2\int_0^{\frac{\pi}{2}}\cos\theta\sin^4\theta\ \mathrm{d}\theta$ since $\sin\theta\geq0,\cos\theta\geq0$ for $0\leq\theta\leq\frac{\pi}{2}$
	$=12\pi a^2\left[\frac{1}{5}\sin^5\theta\right]_0^{\frac{\pi}{2}}$
	$=\frac{12\pi a^2}{5}$

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2	Define $\mathrm{f}(x)=\ln x-\frac{(x-1)^2}{x^2}=\ln x-\left(\frac{x^2-2x+1}{x^2}\right)=\ln x-\left(1-2x^{-1}+x^{-2}\right)$. Then $\mathrm{f}'(x)=\frac{1}{x}-\left(\frac{2}{x^2}-\frac{2}{x^3}\right)=\frac{x^2-2x+2}{x^3}=\frac{(x-1)^2+1}{x^3}>0$ for $x>1$. So f is strictly increasing on $(1,\infty)$. Moreover, since $\mathrm{f}(1)=\ln 1-\frac{(1-1)^2}{x^2}=0$, we deduce that for all $x>1$, $\mathrm{f}(x)>0 \Rightarrow \ln x>\frac{(x-1)^2}{x^2}$. From the previous part, C_1 lies above C_2 for $x>1$. By the shell method, Required volume = $2\pi\int_2^4 x(y_1-y_2)\,\mathrm{d}x$ $=2\pi\int_2^4 x\left[\ln x-\frac{(x-1)^2}{x^2}\right]\mathrm{d}x$ $=2\pi\int_2^4\left[x\ln x-\frac{(x-1)^2}{x}\right]\mathrm{d}x$ $=2\pi\left\{\left[\frac{1}{2}x^2\ln x\right]_2^4-\int_2^4\frac{1}{2}x\,\mathrm{d}x-\int_2^4\left(x-2+\frac{1}{x}\right)\right\}$ $=2\pi\left\{8\ln 4-2\ln 2-\frac{1}{4}\left[x^2\right]_2^4-\left[\frac{1}{2}x^2-2x+\ln x\right]_2^4\right\}$ $=2\pi\left\{16\ln 2-2\ln 2-3-(\ln 2+2)\right\}$ $=2\pi(13\ln 2-5)$
3(i)	$f_0=0$, $f_1=a$, $f_2=a$, $f_3=2a$, $f_4=3a$ $u_2=(f_1)^2-f_0f_2=a^2$ $u_3=(f_2)^2-f_1f_3=a^2-2a^2=-a^2$ $u_4=(f_3)^2-f_2f_4=4a^2-3a^2=a^2$
3(ii)	Hence, $u_n=(-1)^n a^2$
3(iii)	Let P_n be the proposition $u_n=(-1)^n a^2$, for $n\in\mathbf{Z}^+, n\geq 2$. When $n=2$, L.H.S = $u_2=(f_1)^2-f_0f_2=a^2=(-1)^2a^2$ = R.H.S $\therefore$ P_2 is true.

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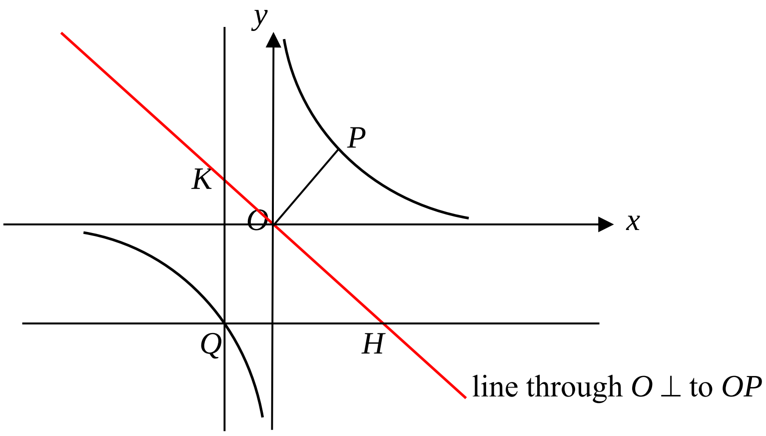
	Assume that P_k is true for some $k \in \mathbf{Z}^+, k \geq 2$, i.e. $u_k = \left(f_{k-1}\right)^2 - f_{k-2} f_k = (-1)^k a^2$. Consider P_{k+1}. $u_{k+1} = \left(f_k\right)^2 - f_{k-1} f_{k+1}$ $= \left(f_k\right)^2 - f_{k-1}\left(f_k + f_{k-1}\right)$ $= \left(f_k\right)^2 - f_{k-1} f_k - \left(f_{k-1}\right)^2$ $= \left(f_k\right)^2 - f_{k-1} f_k - f_{k-2} f_k - (-1)^k a^2$ $= \left(f_k\right)^2 - f_k\left(f_{k-1} + f_{k-2}\right) + (-1)^{k+1} a^2$ $= \left(f_k\right)^2 - f_k\left(f_k\right) + (-1)^{k+1} a^2 = (-1)^{k+1} a^2$ $\therefore$ P_k is true $\Rightarrow$ P_{k+1} is true. Since P_2 is true, by mathematical induction, P_n is true for every integer $n \geq 2$.
4	 Let $P \equiv \left(p, \dfrac{c^2}{p}\right), Q \equiv \left(q, \dfrac{c^2}{q}\right)$. Gradient of line $KH = -\dfrac{1}{\text{Gradient of } OP} = -\dfrac{1}{\frac{c^2}{p} \big/ p} = -\dfrac{p^2}{c^2}$. So equation of line KH is $y = -\dfrac{p^2}{c^2} x$ ------ (1) At H, $y = \dfrac{c^2}{q}$. Sub. into (1) gives $\dfrac{c^2}{q} = -\dfrac{p^2}{c^2} x \Rightarrow x = -\dfrac{c^4}{p^2 q}$. Thus $H \equiv \left(-\dfrac{c^4}{p^2 q}, \dfrac{c^2}{q}\right)$. At K, $x = q$. Sub. into (1) gives $y = -\dfrac{p^2 q}{c^2}$. Thus $K \equiv \left(q, -\dfrac{p^2 q}{c^2}\right)$.

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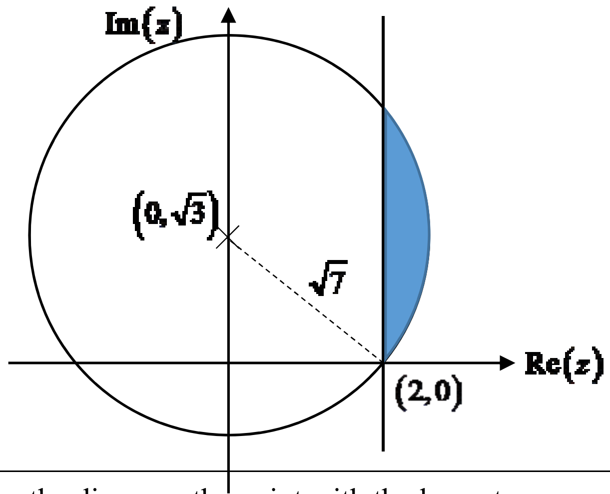
	(Gradient of PH) × (Gradient of PK) $=\left(\dfrac{\frac{c^2}{q}-\frac{c^2}{p}}{-\frac{c^4}{p^2q}-p}\right)\times\left(\dfrac{-\frac{p^2q}{c^2}-\frac{c^2}{p}}{q-p}\right)$ $=\left(-\dfrac{c^2\left(p-q\right)}{pq}\cdot\dfrac{p^2q}{c^4+p^3q}\right)\times\left(-\dfrac{p^3q+c^4}{pc^2}\cdot\dfrac{1}{q-p}\right)$ $=-1$ Hence $PH\perp PK$ and so P lies on the circle with HK as a diameter.
5(i)	
5(ii)	From the diagram, the point with the largest argument is $\left(2,2\sqrt{3}\right)$. $0\le\arg z\le\dfrac{\pi}{3}$
5(iii)	Area $=$ area of sector – area of triangle $=\dfrac{1}{2}\left(\sqrt{7}\right)^2\left(2\tan^{-1}\dfrac{\sqrt{3}}{2}\right)-\dfrac{1}{2}\left(\sqrt{7}\right)^2\sin\left(2\tan^{-1}\dfrac{\sqrt{3}}{2}\right)$ $=4.9960-3.4641$ ≈ 1.5319 $=1.53$ (3sf)
5(iv)	$z-2=\mathrm{e}^{\frac{2k\pi}{5}\mathrm{i}}$ $z=2+\mathrm{e}^{\frac{2k\pi}{5}\mathrm{i}},\ k=-2,-1,0,1,2$ Using the diagram, $z=2+\mathrm{e}^{\frac{2\pi}{5}\mathrm{i}}=2+\cos\left(\dfrac{2\pi}{5}\right)+\mathrm{i}\sin\left(\dfrac{2\pi}{5}\right)$

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Let $\mathrm{f}(x)=2x^3+x-1$ and let *r* denotes a root of the equation $\mathrm{f}(x)=0$.

Since $\mathrm{f}(0)=-1<0$ and $\mathrm{f}(1)=2>0$, $r\in(0,1)$.

Since $\mathrm{f}(0.5)=2(0.5)^3+0.5-1=-0.25<0$, $r\in(0.5,1)$.

(i) Since $\mathrm{f}(0.75)=2(0.75)^3+0.75-1=0.59375>0$, $r\in(0.5,0.75)$.

Since $\mathrm{f}(0.625)=2(0.625)^3+0.625-1=0.11328125>0$, $r\in(0.5,0.625)$.

Since $\mathrm{f}(0.5625)=2(0.5625)^3+0.5625-1=-0.0815429688<0$, $r\in(0.5625,0.625)$.

Hence $r\approx 0.6$ correct to one decimal place.

Let $a_1,a_2,a_3,\ldots$ be a sequence of approximations to *r* in the intervals $(0,1),(0.5,1),(0.5,0.75),\ldots$ respectively.

That is, the lengths of the intervals to which $a_1,a_2,a_3,\ldots$ belong are $1,\frac{1}{2},\frac{1}{2^2},\ldots$ respectively.

In general, a_n belongs to the interval of length $\frac{1}{2^{n-1}}$, $n=1,2,3,\ldots$.

We want least *n* such that $|a_n-r|<0.001\Rightarrow a_n-0.001<r<a_n+0.001$.

That is the length of the interval to which a_n belongs is less than $a_n+0.001-(a_n-0.001)=0.002$.

Therefore $\frac{1}{2^{n-1}}<0.002\Rightarrow 2^{n-1}>\frac{1}{0.002}\Rightarrow n>9.97$.

So least value of *n* is 10.

(ii) $\mathrm{f}(x)=2x^3+x-1\Rightarrow \mathrm{f}'(x)=6x^2+1$. By Newton-Raphson method,

$$x_{n+1}=x_n-\frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}=x_n-\frac{2x_n^{\,3}+x_n-1}{6x_n^{\,2}+1}.$$

$$x_2=x_1-\frac{2x_1^{\,3}+x_1-1}{6x_1^{\,2}+1}=0.8-\frac{2(0.8)^3+0.8-1}{6(0.8)^2+1}\approx 0.63$$

(iii) $\mathrm{F}(x)=1-2x^3\Rightarrow \mathrm{F}'(x)=-6x^2$.

Since $|\mathrm{F}'(0.6)|=\left|-6(0.6)^2\right|=2.16>1$, using the approximate root 0.6 obtained in (i), the condition for convergence for the fixed-point iteration method is not satisfied. Therefore this method will fail in this case.

For the second attempt, $\mathrm{F}(x)=\left(\frac{1-x}{2}\right)^{\frac{1}{3}}\Rightarrow \mathrm{F}'(x)=-\frac{1}{6}\left(\frac{1-x}{2}\right)^{-\frac{2}{3}}$.

The iteration is $x_{n+1}=\mathrm{F}(x_n)=\left(\frac{1-x_n}{2}\right)^{\frac{1}{3}}$, $n=1,2,3,\ldots$, $x_1=1$.

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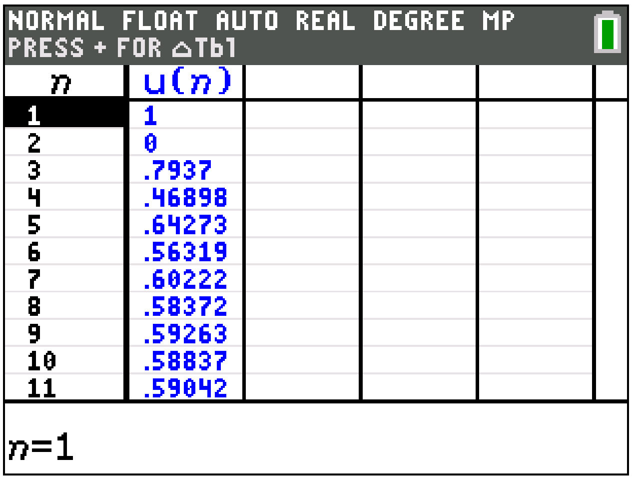
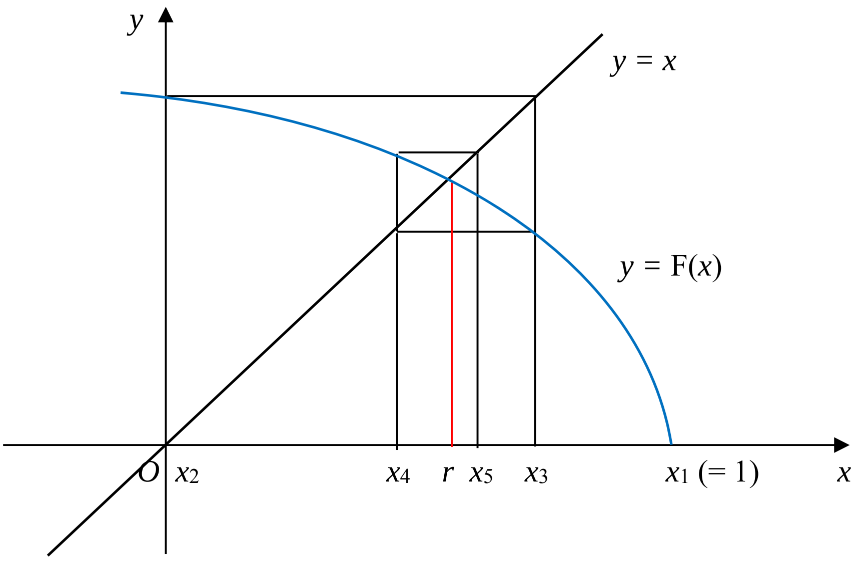
	Since $\left\|\mathrm{F}'(0.6)\right\| = \left\| -\frac{1}{6}\left(\frac{1-0.6}{2}\right)^{-\frac{2}{3}} \right\| = 0.49 < 1$ and $x_2 = \mathrm{F}(x_1) = \mathrm{F}(1) = 0$ such that
	$\left\|\mathrm{F}'(x_2)\right\| = \left\|\mathrm{F}'(0)\right\| = \left\| -\frac{1}{6}\left(\frac{1-0}{2}\right)^{-\frac{2}{3}} \right\| = 0.26 < 1$, the iteration converges to the root r.
	By GC, $x_{10} \approx 0.5884$ (4 dp)





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7	(i) $(x+1)\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = \mathrm{e}^x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} + \left(\frac{2}{x+1}\right)y = \frac{\mathrm{e}^x}{x+1}$
	IF = $\mathrm{e}^{\int \frac{2}{x+1}\mathrm{d}x} = (x+1)^2$. Multiplying DE by IF gives
	$\frac{\mathrm{d}}{\mathrm{d}x}\left[(x+1)^2 y\right] = (x+1)\mathrm{e}^x$

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	$(x+1)^2 y=\int (x+1)\mathrm{e}^x \ \mathrm{d}x$ $=(x+1)\mathrm{e}^x-\int \mathrm{e}^x \mathrm{d}x$ $=x\mathrm{e}^x+A$ $\Rightarrow y=\dfrac{x\mathrm{e}^x+A}{(x+1)^2}$ (ii) $\mathrm{a}(x)\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}+\mathrm{b}(x)\dfrac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{c}(x)y$ $=\dfrac{\mathrm{d}}{\mathrm{d}x}\left\{\mathrm{f}(x)\dfrac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{g}(x)y\right\}$ $=\mathrm{f}(x)\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}+\mathrm{f}'(x)\dfrac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{g}(x)\dfrac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{g}'(x)y$ $=\mathrm{f}(x)\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}+\left[\mathrm{f}'(x)+\mathrm{g}(x)\right]\dfrac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{g}'(x)y$ Comparing, we have $\mathrm{f}(x)=\mathrm{a}(x)$ and $\begin{array}{l}\mathrm{f}'(x)+\mathrm{g}(x)=\mathrm{b}(x)\\ \Rightarrow \mathrm{g}(x)=\mathrm{b}(x)-\mathrm{f}'(x)=\mathrm{b}(x)-\mathrm{a}'(x)\end{array}$. (iii) $(x^2+x)\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}+(4x+1)\dfrac{\mathrm{d}y}{\mathrm{d}x}+2y=(x+1)\mathrm{e}^x$ ------ (1) Set $\mathrm{a}(x)=x^2+x,\ \mathrm{b}(x)=4x+1,\ \mathrm{c}(x)=2$ which satisfy $\mathrm{a}''(x)-\mathrm{b}'(x)+\mathrm{c}(x)=0$. Let $z=\mathrm{f}(x)\dfrac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{g}(x)y$. If $\dfrac{\mathrm{d}z}{\mathrm{d}x}=\dfrac{\mathrm{d}}{\mathrm{d}x}\left\{\mathrm{f}(x)\dfrac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{g}(x)y\right\}=\underbrace{(x^2+x)}_{\mathrm{a}(x)}\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}+\underbrace{(4x+1)}_{\mathrm{b}(x)}\dfrac{\mathrm{d}y}{\mathrm{d}x}+\underbrace{2}_{\mathrm{c}(x)}y$, then by (ii), $\mathrm{f}(x)=\mathrm{a}(x)=x^2+x$ and $\mathrm{g}(x)=\mathrm{b}(x)-\mathrm{a}'(x)=4x+1-(2x+1)=2x$. Thus $z=(x^2+x)\dfrac{\mathrm{d}y}{\mathrm{d}x}+2xy$. So DE (1) can be written as $\dfrac{\mathrm{d}z}{\mathrm{d}x}=(x+1)\mathrm{e}^x$. Integrating w.r.t. x, $z=\int (x+1)\mathrm{e}^x \mathrm{d}x=x\mathrm{e}^x+B$. That is, $(x^2+x)\dfrac{\mathrm{d}y}{\mathrm{d}x}+2xy=x\mathrm{e}^x+B$ which gives $(x+1)\dfrac{\mathrm{d}y}{\mathrm{d}x}+2y=\mathrm{e}^x+\dfrac{B}{x}$ ------ (2) From (i), $(x+1)\dfrac{\mathrm{d}y}{\mathrm{d}x}+2y=\mathrm{e}^x \Rightarrow y=\dfrac{x\mathrm{e}^x+A}{(x+1)^2}$ ------ (3)

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	Consider $\left(x+1\right)\frac{\mathrm{d}y}{\mathrm{d}x}+2y=\frac{B}{x}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}+\left(\frac{2}{x+1}\right)y=\frac{B}{x\left(x+1\right)}$. Multiplying DE by IF = $\mathrm{e}^{\int\frac{2}{x+1}\mathrm{d}x}=\left(x+1\right)^2$ gives $\frac{\mathrm{d}}{\mathrm{d}x}\left[\left(x+1\right)^2 y\right]=\frac{B\left(x+1\right)}{x}=B\left(1+\frac{1}{x}\right)$. Integrating w.r.t. x, $\left(x+1\right)^2 y=B\left(x+\ln x\right)+C$ (assume $x>0$) $y=\frac{B\left(x+\ln x\right)+C}{\left(x+1\right)^2}$ ------ (4) Hence general solution of (1) is the sum of the GS (3) and (4), i.e $y=\frac{x\mathrm{e}^x+A}{\left(x+1\right)^2}+\frac{B\left(x+\ln x\right)+C}{\left(x+1\right)^2}$ $=\frac{x\mathrm{e}^x+B\left(x+\ln x\right)+D}{\left(x+1\right)^2}$ where $A+C = D$.
8(a) (i)	Since $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ are linearly independent vectors of R^3 and the dimension of $\mathbb{R}^3$ is 3, thus $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ will form a basis for $\mathbb{R}^3$.
8(a) (ii)	$\mathbf{A}^n\mathbf{x}=\mathbf{A}^n\left(a_1\mathbf{e}_1+a_2\mathbf{e}_2+a_3\mathbf{e}_3\right)$ $=a_1\mathbf{A}^n\mathbf{e}_1+a_2\mathbf{A}^n\mathbf{e}_2+a_3\mathbf{A}^n\mathbf{e}_3$ $=a_1\lambda_1^n\mathbf{e}_1+a_2\lambda_2^n\mathbf{e}_2+a_3\lambda_3^n\mathbf{e}_3$
8(a) (iii)	Since $\|\lambda_2\|<1$ and $\|\lambda_3\|<1$, we have $\lambda_2^n\to 0$ and $\lambda_3^n\to 0$ as $n\to\infty$. Further $\lambda_1^n=1$ for all $n\in\mathbb{Z}^+$. Thus $\lim_{n\to\infty}\mathbf{A}^n\mathbf{x}=\lim_{n\to\infty}\left[a_1\lambda_1^n\mathbf{e}_1+a_2\lambda_2^n\mathbf{e}_2+a_3\lambda_3^n\mathbf{e}_3\right]$ $=a_1\mathbf{e}_1+\lim_{n\to\infty}\left[a_2\lambda_2^n\mathbf{e}_2+a_3\lambda_3^n\mathbf{e}_3\right]$ $=a_1\mathbf{e}_1$
8(b) (i)	Since $\lambda\mathbf{A}\mathbf{e}_2=\mathbf{0}$ for all $\lambda\in\mathbb{R}$, thus a basis for K is $\left\{\begin{pmatrix}-1\\1\\0\end{pmatrix}\right\}$. Since the eigenspace of 0 is of dimension 1, the dimension of K is 1.
8(b) (ii)	$\mathbf{A}=\begin{pmatrix}1&-1&0\\0&1&0\\2&0&1\end{pmatrix}\begin{pmatrix}-1&0&0\\0&0&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&-1&0\\0&1&0\\2&0&1\end{pmatrix}^{-1}$ $=\begin{pmatrix}-1&-1&0\\0&0&0\\-4&-4&1\end{pmatrix}$

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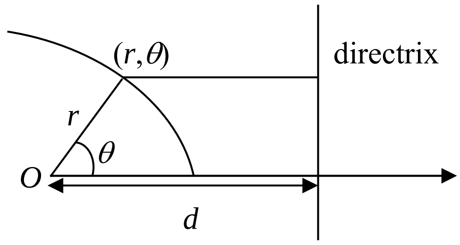
8(b) (iii)	Note that $\mathbf{e}_1$ and $\mathbf{e}_3$ forms a basis for R. Thus the range space is of the form $\mathbf{x} = \lambda\mathbf{e}_1 + \mu\mathbf{e}_3$ where $\lambda,\ \mu \in \mathbb{R}$. $\begin{pmatrix}1\\0\\2\end{pmatrix}\times\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}0\\-1\\0\end{pmatrix}$ Hence R is a plane that contains the origin with normal vector $\begin{pmatrix}0\\-1\\0\end{pmatrix}$.
9	In standard position, equation of elliptical path of planet X is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. So a = 3 and c = 3 – 1 = 2. Thus eccentricity, $e = \frac{c}{a} = \frac{2}{3}$.  By definition, $\frac{r}{d-r\cos\theta}=e \Rightarrow r = e\left(d-r\cos\theta\right)$ $\Rightarrow r = \frac{ed}{1+e\cos\theta}$ where $e=\frac{2}{3}$. Now when planet X is at the point P_0 closest to the sun, $r = OP_0 = 1$ and the distance from P_0 to the directrix is d – 1. Hence by definition, $\frac{1}{d-1}=e=\frac{2}{3}\Rightarrow d=\frac{5}{2}$. So the polar equation of the elliptical path of planet X is $r=\frac{\frac{2}{3}\left(\frac{5}{2}\right)}{1+\frac{2}{3}\cos\theta}=\frac{5}{3+2\cos\theta}$. $r^2+\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2=\left(\frac{5}{3+2\cos\theta}\right)^2+\left(\frac{10\sin\theta}{\left(3+2\cos\theta\right)^2}\right)^2$ $=\frac{25\left(3+2\cos\theta\right)^2+100\sin^2\theta}{\left(3+2\cos\theta\right)^4}$ $=\frac{25\left(13+12\cos\theta\right)}{\left(3+2\cos\theta\right)^4}$ Distance traversed by planet X in moving from P_0 to $P_{\pi/2}$ $=\int_0^{\frac{\pi}{2}}\sqrt{r^2+\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\ \mathrm{d}\theta = 5\int_0^{\frac{\pi}{2}}\frac{\sqrt{13+12\cos\theta}}{\left(3+2\cos\theta\right)^2}\ \mathrm{d}\theta \approx 2.02530413$ AU.

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	Area enclosed by elliptical path of planet X $= \pi ab$ where $a = 3,\ b = \sqrt{a^2 - c^2} = \sqrt{3^2 - 2^2} = \sqrt{5}$ from the above results. $= 3\sqrt{5}\pi$ AU^2 [Alternatively, area enclosed = $\frac{1}{2}\int_0^{2\pi}\left(\frac{5}{3+2\cos\theta}\right)^2 \, d\theta \approx 21.07444419$] Area swept out by radius vector from O to planet X in moving from P_0 to $P_{\pi/2}$ = $\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(\frac{5}{3+2\cos\theta}\right)^2 \, d\theta \approx 1.154363415$ AU^2. Therefore time taken for planet X to traverse the path $P_0P_{\pi/2}$ $= \frac{1.154363415}{3\sqrt{5}\pi} Y \approx 0.0548Y$.
10(i)	$(0.85)^4(25) = 13.1$
10(ii)	Just before the $(n+1)$-th dose, amount present in body is $(0.85)^4 u_n$. Thus $u_{n+1} = 0.85^4 u_n + 25$, i.e $u_{n+1} = 0.522 u_n + 25$.
10(iii)	C.F.: $u_n = A(0.522)^n$ P.S.: $u_n = B$. Since $B = 0.522B + 25 \Rightarrow$ B=52.3$ G.S.: $u_n = A(0.522)^n + 52.3$ Since $u_1 = 25$, thus $A = -52.3$. Thus $u_n = 52.3(1 - 0.522^n)$
10(iv)	Need $0.522u_n > 20$, thus $0.522 \times 52.3(1 - 0.522^n) > 20$ $\Rightarrow 0.522^n < 0.61759$ $\Rightarrow n > 2.0288$ $\Rightarrow n \geq 3$ Thus the interrogation can begin after the 3rd dose.
10(v)	Note that u_n is an increasing sequence and $u_n \to 52.3$ as $n \to \infty$. Thus amount of serum is body is always less than 55 regardless of the number of doses. Thus there is no maximum length of time.