NANYANG JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

Higher 2

MATHEMATICS

9758/02

Paper 2 **15th September 2017**

3 Hours

Additional Materials: Answer Paper

Graph Paper

List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

NANYANG JUNIOR COLLEGE
Internal Examinations

Section A: Pure Mathematics [40 marks]

1 The position vectors of points A and B with respect to the origin O are $\mathbf{a}$ and $\mathbf{b}$ respectively where $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors. Point C lies on OA produced such that $4OA = AC$ and point D lies on OB produced such that $OB = BD$. The lines BC and AD meet at the point M.

(i) Giving a necessary condition for $\mathbf{a}$ and $\mathbf{b}$, find the position vector of M in terms of $\mathbf{a}$ and $\mathbf{b}$. [5]

(ii) If $|\mathbf{a}| = 1$ and $|\mathbf{b}| = 2$, find the shortest distance of M from the line OC giving your answer in the form $k|\mathbf{a} \times \mathbf{b}|$ where k is a constant to be determined. [2]

2 **(a)** Find the set of values of θ lying in the interval $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$ such that the sum to infinity of the geometric series $1 + \tan\theta + \tan^2\theta + ...$ is greater than 2. [5]

(b) The sum of the first n terms of a positive arithmetic sequence $\{u_n\}$ is given by the formula $S_n = 4n^2 - 2n$. Three terms of this sequence, u_2, u_m and u_{32}, are consecutive terms in a geometric sequence. Find m. [4]

3 It is given that $y = \ln(\cos ax - \sin ax)$, where a is a non-zero constant.

(i) Show that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + a^2 = 0.$ [3]

(ii) By further differentiation of the result in **(i)**, find, in terms of a, the Maclaurin series for y, up to and including the term in x^3. [3]

(iii) Hence show that when x is small enough for powers of x higher than 2 to be neglected and $a = 2$, then $\cos 2x - \sin 2x \approx 1 + kx + kx^2$ where k is a constant to be determined. [4]

(iv) Using appropriate expansions from the List of Formulae (**MF26**), verify the correctness of your answer in **(iii)**. [2]

4 The growth of an organism in a controlled environment is monitored and the growth rate of the organism is proportional to $(N-x)x$, where x is the population (in thousands) of the organism at time t and N is a constant such that $x<N$. The initial population of the organism is $\frac{1}{3}N$.

(i) Find x in terms of t and determine the population of the organism in the long run, giving your answer in terms of N. [6]

Another model is proposed for the growth of the organism, which assumes the growth rate is purely a function of time and is modelled by the differential equation $\frac{\mathrm{d}^2 x}{\mathrm{d}t^2}=\frac{-9t}{\left(4+9t^2\right)^2}$. It predicts that the population of the organism will also eventually stabilise.

(ii) Show that under this model, $x=\frac{1}{12}\tan^{-1}\left(\frac{3t}{2}\right)+\frac{N}{3}$.

Hence state the population of the organism in the long run, giving your answer in terms of N. [6]

Section B: Probability and Statistics [60 marks]

5 From past records, the number of days of hospitalization for an individual with minor ailment can be modelled by a discrete random variable with probability density function given by

$$P(X=x)=\begin{cases}\dfrac{6-x}{15}, & \text{for } x=1,2,3,4,5,\\ 0, & \text{otherwise.}\end{cases}$$

An insurance policy pays \$100 per day for up to 3 days of hospitalization and \$25 per day of hospitalization thereafter.

(i) Calculate the expected payment for hospitalization for an individual under this policy. [4]

(ii) The insurance company will incur a loss if the total payout for 100 hospitalisation claims under this policy exceed \$24000. Using a suitable approximation, estimate the probability that the insurance company will incur a loss for 100 such claims. [4]

6 A teacher wants to randomly form two teams of 5 students from a group of 5 girls and 5 boys for a sports activity. Two of the girls, Ann and Alice, are selected as team leaders. Find the probability that one team has exactly 3 girls. [2]

The ten students are seated at a round table of 10. Find the probability that

(i) Ann and Alice are not seated together, [2]

(ii) no two of the remaining 3 girls are next to each other given that Ann and Alice are not seated together. [4]

7 In a large company, a small sample of n employees is obtained to find out their mode of transport to work. The number of employees who ride the train to work is denoted by R. Assume that R has the distribution $\mathrm{B}(n, p)$.

(i) Given that $n = 10$, find the value of p if the probability that 6 employees ride the train to work is twice the probability that 4 employees ride the train to work. [3]

(ii) Given that $p = 0.25$, find the largest value of n such that the probability that fewer than 2 employees who ride the train to work is more than 0.15. [3]

(iii) Given that $n = 11$ and $p = 0.7$, find the probability that at least 5 employees ride the train to work if at least 3 employees do not ride the train to work. [4]

8 **(a)** Comment briefly on the following statements:

(i) Flowers in a garden are watered and the product moment correlation coefficient between petal size and the amount of water given is 0.073, so it follows that there is no relation between petal size and quantity of water given to the flower. [1]

(ii) The product moment correlation coefficient between the risk of heart disease and amount of red wine intake is found to be approximately -1. Therefore we conclude that red wine intake causes the risk of heart disease to decrease. [1]

(b) The median age of residents in Singapore across the years are given in the table.

Year (x)	**1984**	**1988**	**1992**	**1996**	**2000**	**2004**	**2008**	**2012**	**2016**
Median age (y)	26.7	28.8	27.0	32.3	34.0	35.4	36.7	38.4	40.0

It is thought that the median age of residents in year x can be modelled by one of the formulae

$$y = \frac{a}{x} + b, \quad y = c\ln x + d,$$

where a, b, c and d are constants.

(i) Plot a scatter diagram on graph paper for these values, labelling the axis, using a scale of 2cm to represent 5 years on the y-axis and an appropriate scale for the x-axis. One of the values of y was quoted wrongly. Indicate this point as P on your diagram. [2]

For parts **(ii)**, **(iii)**, **(iv)** of this question, you should **exclude** the point P.

(ii) Find, correct to 5 decimal places, the value of the product moment correlation coefficient between

(A) x^{-1} and y

(B) $\ln x$ and y. [2]

(iii) Explain which model is more appropriate to predict the median age of residents in Singapore and find the equation of the least squares regression line for this model, giving your answer to 2 decimal places. [2]

(iv) Explain why neither the regression line of x^{-1} on y nor the regression line of $\ln x$ on y should be used to estimate the year when the median age is 30. [1]

(v) Give a possible reason for the rise in the median age. [1]

9 A manufacturing process produces ball bearings with diameters with known standard deviation 0.04 cm. Under normal circumstances, the manufacturing process will produce ball bearings of mean diameter 0.5 cm.

(i) During a routine quality control check, a random sample of 25 ball bearings gives a mean of 0.51 cm. Is there evidence to believe that the manufacturing process is producing ball bearings of the stated diameter? Perform an appropriate test at 5% level of significance. State a necessary assumption for the test to be valid. [4]

An enhancement on the manufacturing process will ensure that the diameters of the ball bearings produced are less variable.

(ii) Measurements of a sample of 100 ball bearings give the following summary statistics:

$$\Sigma x = 50.6\,,\ \Sigma(x-0.5)^2 = 0.08345\,.$$

Show that the unbiased estimate of the population variance is 8.07×10^{-4}.
Is there evidence at the 5% level of significance that after the enhancement, the manufacturing process is producing oversized ball bearings? [4]

(iii) Another sample of 100 ball bearings yield the same summary statistics as the previous sample in **(ii)**. Explain, with justification, whether the combined sample will give a different conclusion to **(ii)**. [4]

10 The diameters of the bolts produced by two manufacturers *A* and *B* follow a normal distribution with a standard deviation of 0.16 mm.

The mean diameter of the bolts produced by manufacturer *A* is 1.56 mm. Of the bolts produced by manufacturer *B*, 24.2% have a diameter less than 1.52 mm.

(i) Show that the mean diameter of the bolts produced by manufacturer *B* is 1.632 mm. [3]

(ii) Find the probability that the diameter of a randomly chosen bolt from manufacturer *A* differs from the diameter of a randomly chosen bolt from manufacturer *B* by less than 0.1 mm. [3]

(iii) Find the probability that the total diameter of 5 randomly chosen bolt from manufacturer *A* is more than 5 times the diameter of a randomly chosen bolt from manufacturer *B*. [3]

(iv) A trading company buys 44% of its stock of bolts from manufacturer *A* and the rest from manufacturer *B*. A bolt is chosen at random from the trading company's stock. Show that the probability that the diameter of the bolt is less than 1.52 mm is 0.312. [3]

———END OF PAPER———