2017 Prelim Paper 2 Solution

1	**<u>Method 1</u>** $z - w\text{i} = 3$ $\Rightarrow \quad w = \dfrac{z-3}{\text{i}} = 3\text{i} - z\text{i}$ --- (1) Substitute (1) into $z^2 - w + 6 + 3\text{i} = 0$ $z^2 - (3\text{i} - z\text{i}) + 6 + 3\text{i} = 0$ $\Rightarrow \quad z^2 + z\text{i} + 6 = 0$ $\Rightarrow \quad z = \dfrac{-\text{i} \pm \sqrt{(\text{i})^2 - 4(1)(6)}}{2} = \dfrac{-\text{i} \pm \sqrt{-1-24}}{2}$ $= \dfrac{-\text{i} \pm 5\text{i}}{2}$ $\therefore \quad z = 2\text{i}$ or $z = -3\text{i}$ $\Rightarrow w = 3\text{i} - (2\text{i})\text{i}$ $\qquad w = 3\text{i} - (-3\text{i})\text{i}$ $= 2 + 3i$ $\qquad = -3 + 3\text{i}$
	<u>Method 2</u> $z - w\text{i} = 3$ $\Rightarrow \quad z = 3 + w\text{i}$ …. (1) Substitute (1) into $z^2 - w + 6 + 3\text{i} = 0$ $(3 + w\text{i})^2 - w + 6 + 3\text{i} = 0$ $\Rightarrow \quad 9 + 6w\text{i} - w^2 - w + 6 + 3\text{i} = 0$ $\Rightarrow \quad -w^2 - (1 - 6\text{i})w + 15 + 3\text{i} = 0$ $\Rightarrow \quad -w^2 - (1 - 6\text{i})w + 15 + 3\text{i} = 0$ $\Rightarrow \quad w^2 + (1 - 6\text{i})w - 15 - 3\text{i} = 0$ $\Rightarrow \quad w = \dfrac{-(1-6\text{i}) \pm \sqrt{(1-6\text{i})^2 - 4(1)(-15-3\text{i})}}{2}$

$$= \frac{-1+6\mathrm{i} \pm \sqrt{1-12\mathrm{i}-36+60+12\mathrm{i}}}{2}$$

$$= \frac{-1+6\mathrm{i} \pm \sqrt{25}}{2}$$

$\therefore \; w = 2+3\mathrm{i}$ or $w = -3+3\mathrm{i}$

$\Rightarrow \; z = 3+(2+3\mathrm{i})\mathrm{i} = 2\mathrm{i}$ $\quad z = 3+(-3+3\mathrm{i})\mathrm{i} = -3\mathrm{i}$

<u>Method 3</u>

$$w\mathrm{i} = z-3$$

$$\Rightarrow w = -\mathrm{i}z+3\mathrm{i}$$

$$\therefore \; z^2 - (-\mathrm{i}z+3\mathrm{i}) + 6 + 3\mathrm{i} = 0$$

$$\Rightarrow \; z^2 + \mathrm{i}z + 6 = 0$$

Let $z = a+b\mathrm{i}$ where $a, b \in \mathbb{R}$

$$(a+b\mathrm{i})^2 + \mathrm{i}(a+b\mathrm{i}) + 6 = 0$$

$$\Rightarrow \; a^2 - b^2 + 2ab\mathrm{i} + a\mathrm{i} - b + 6 = 0$$

$$\Rightarrow \; a^2 - b^2 - b + 6 + (2ab+a)\mathrm{i} = 0$$

By comparing the real and imaginary parts,

$a^2 - b^2 - b + 6 = 0 \quad \ldots (1)$

$2ab + a = 0 \quad \ldots (2)$

From (2), $a = 0$ or $b = -\frac{1}{2}$

When $a = 0$, $b^2 + b - 6 = 0$

$$(b-2)(b+3) = 0$$

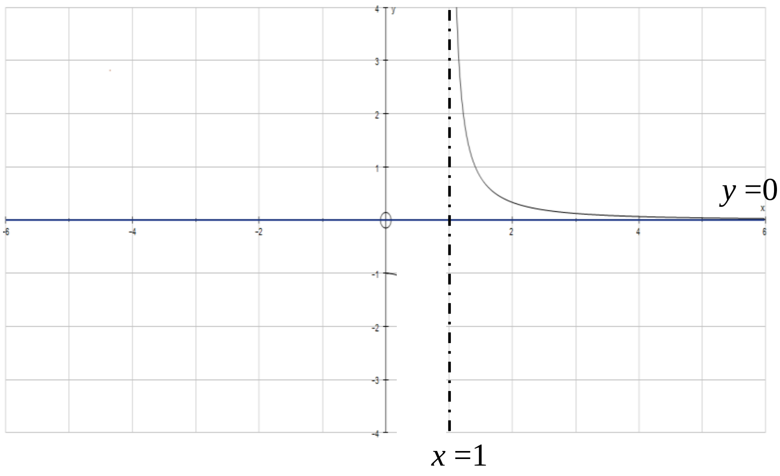
$$b = 2 \text{ or } b = -3$$

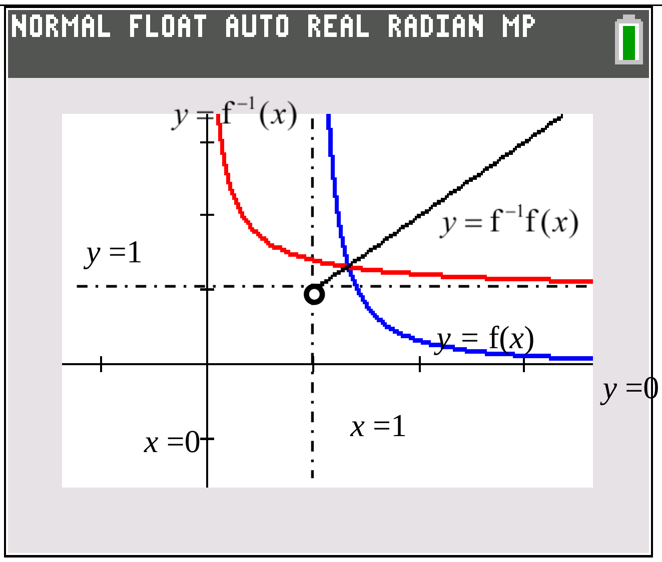
Hence $z = 2\mathrm{i}, w = -\mathrm{i}(2\mathrm{i}) + 3\mathrm{i} = 2+3\mathrm{i}$

or $\quad z = -3\mathrm{i}, w = -\mathrm{i}(-3\mathrm{i}) + 3\mathrm{i} = -3+3\mathrm{i}$

When $b = -\frac{1}{2}$, $a^2 = \frac{1}{4} - \frac{1}{2} - 6 = -\frac{25}{4}$

	There is no real solution for a.
2(i)	$\dfrac{2}{n-1}-\dfrac{3}{n}+\dfrac{1}{n+1}$ $=\dfrac{2(n)(n+1)-3(n-1)(n+1)+(n-1)(n)}{(n-1)(n)(n+1)}$ $=\dfrac{\left(2n^2+2n\right)-\left(3n^2-3\right)+\left(n^2-n\right)}{n^3-n}$ $=\dfrac{n+3}{n^3-n}$
(ii)	$\displaystyle\sum_{r=2}^{n}\frac{2r+6}{r^3-r}$ $\displaystyle=2\sum_{r=2}^{n}\frac{r+3}{r^3-r}$ $\displaystyle=2\sum_{r=2}^{n}\left(\frac{2}{r-1}-\frac{3}{r}+\frac{1}{r+1}\right)$ $=2\left[\begin{array}{l} \frac{2}{1}-\frac{3}{2}+\cancel{\frac{1}{3}} \\ +\frac{2}{2}-\cancel{\frac{3}{3}}+\cancel{\frac{1}{4}} \\ +\cancel{\frac{2}{3}}-\cancel{\frac{3}{4}}+\cancel{\frac{1}{5}} \\ +\ldots \\ +\cancel{\frac{2}{n-2}}-\cancel{\frac{3}{n-1}}+\frac{1}{n} \\ +\cancel{\frac{2}{n-1}}-\frac{3}{n}+\frac{1}{n+1} \end{array}\right]$ $=2\left(\dfrac{2}{1}-\dfrac{3}{2}+\dfrac{2}{2}+\dfrac{1}{n}-\dfrac{3}{n}+\dfrac{1}{n+1}\right)$ $=2\left(\dfrac{3}{2}-\dfrac{2}{n}+\dfrac{1}{n+1}\right)$ $=3-\dfrac{4}{n}+\dfrac{2}{n+1}$
(iii)	$\displaystyle\sum_{r=2}^{n}\frac{2r+10}{(r+1)(r+2)(r+3)}$ Let $r+2=p\Rightarrow r=p-2$

	$= \sum_{p-2=2}^{p-2=n} \frac{2p+6}{(p-1)(p)(p+1)}$ $= \sum_{p=4}^{n+2} \frac{2p+6}{p^3-p}$ $= \sum_{p=2}^{n+2} \frac{2p+6}{p^3-p} - \sum_{p=2}^{3} \frac{2p+6}{p^3-p}$ $= \left(3 - \frac{4}{n+2} + \frac{2}{n+3}\right) - \left(3 - \frac{4}{3} + \frac{2}{4}\right)$ $= \frac{5}{6} - \frac{4}{n+2} + \frac{2}{n+3}$
3(i)	$\mathrm{f} : x \mapsto \frac{1}{x^2-1}$ Let $y = \frac{1}{x^2-1}$ $x^2 = \frac{1}{y} + 1$ $x = \pm\sqrt{1+\frac{1}{y}}$ Since $x > 1$, $x = \sqrt{1+\frac{1}{y}}$ $\mathrm{f}^{-1}(x) = \sqrt{1+\frac{1}{x}} = \sqrt{\frac{1+x}{x}}$.  From graph of f, $\mathrm{R_f} = (0,\infty)$ $\therefore \mathrm{D}_{\mathrm{f}^{-1}}(x) = (0,\infty)$.

(ii)	
(iii)	Since $ff^{-1}(x) = f^{-1}f(x) = x$ have the same rule, we investigate the domain $D_{f^{-1}f} = (1, \infty)$ $D_{ff^{-1}} = (0, \infty)$ Taking the intersection of these domains, Range of values is $x > 1$.
4 (i)	Equation of plane is $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}, \quad \lambda, \mu \in \mathbb{R}$ A normal vector to plane is $\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ -4 \end{pmatrix}$ Hence vector equation of the plane is

$$\mathbf{r}\cdot\begin{pmatrix}2\\1\\4\end{pmatrix}=\begin{pmatrix}1\\3\\-2\end{pmatrix}\cdot\begin{pmatrix}2\\1\\4\end{pmatrix}$$

$$\mathbf{r}\cdot\begin{pmatrix}2\\1\\4\end{pmatrix}=-3$$

(ii)

$$l_{AC}:\quad \mathbf{r}=\begin{pmatrix}-5\\2\\2\end{pmatrix}+s\begin{pmatrix}2\\1\\4\end{pmatrix},\ s\in\mathbb{R}$$

Thus $\overrightarrow{OC}=\begin{pmatrix}-5\\2\\2\end{pmatrix}+s\begin{pmatrix}2\\1\\4\end{pmatrix}$ for some $s\in\mathbb{R}$.

Since C lies on the plane:

$$\left[\begin{pmatrix}-5\\2\\2\end{pmatrix}+s\begin{pmatrix}2\\1\\4\end{pmatrix}\right]\cdot\begin{pmatrix}2\\1\\4\end{pmatrix}=-3$$

$$2(-5+2s)+(2+s)+4(2+4s)=-3$$

$$s=-\frac{3}{21}$$

Thus $\overrightarrow{OC}=\begin{pmatrix}2\left(-\frac{3}{21}\right)-5\\ \left(-\frac{3}{21}\right)+2\\ 4\left(-\frac{3}{21}\right)+2\end{pmatrix}=\frac{1}{7}\begin{pmatrix}-37\\13\\10\end{pmatrix}$

(iii)

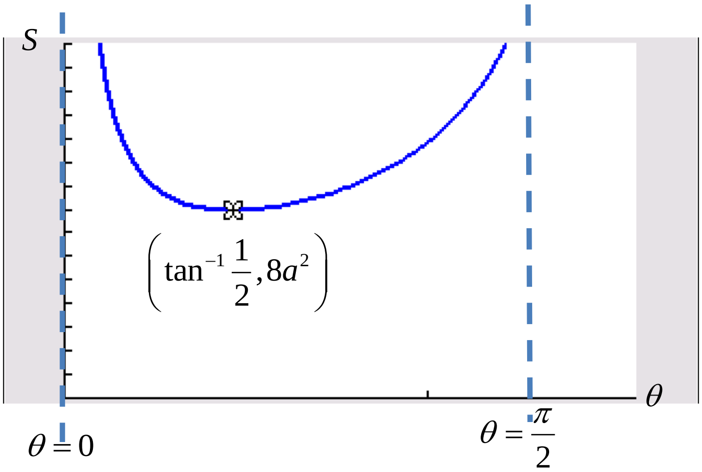
Using mid-point theorem

$$\overrightarrow{OA'}=2\overrightarrow{OC}-\overrightarrow{OA}$$

$$=\frac{2}{7}\begin{pmatrix}-37\\13\\10\end{pmatrix}-\begin{pmatrix}-5\\2\\2\end{pmatrix}=\frac{1}{7}\begin{pmatrix}-39\\12\\6\end{pmatrix}$$

B is the point of intersection of l_1 and π .

	$\overrightarrow{BA'} = \overrightarrow{OA'} - \overrightarrow{OB}$ $= \frac{1}{7}\begin{pmatrix}-39\\12\\6\end{pmatrix} - \begin{pmatrix}1\\3\\-2\end{pmatrix}$ $= \frac{1}{7}\begin{pmatrix}-46\\-9\\20\end{pmatrix}$ $l_2: \quad \mathbf{r} = \frac{1}{7}\begin{pmatrix}-39\\12\\6\end{pmatrix} + t\begin{pmatrix}-46\\-9\\20\end{pmatrix}, \ t \in \mathbb{R}$ or $l_2: \quad \mathbf{r} = \begin{pmatrix}1\\3\\-2\end{pmatrix} + t\begin{pmatrix}-46\\-9\\20\end{pmatrix}, \ t \in \mathbb{R}$
5(i)	The height of triangle ADG is $\frac{a}{\tan\theta} = \frac{a}{t}$. Hence $AH = 2a + \frac{a}{t} = a\left(2 + \frac{1}{t}\right)$.
	$BH = BE + EH = 2a\tan\theta + a = a(2t+1)$ Area $S = \frac{1}{2}(AH)(BC)$ $S = \frac{a}{2}\left(2 + \frac{1}{t}\right)\left(2a(2t+1)\right)$ $S = a^2\left(2 + \frac{1}{t}\right)(2t+1)$ $S = a^2\left(4 + 4t + \frac{1}{t}\right)$
(ii)	$\frac{dS}{dt} = a^2\left(4 - \frac{1}{t^2}\right)$ When $\frac{dS}{dt} = 0$, $t^2 = \frac{1}{4}$

	$\Rightarrow \quad t = \pm\frac{1}{2}$ Reject $t = \tan\theta = -\frac{1}{2}$ as θ is acute $\frac{\mathrm{d}^2 S}{\mathrm{d}t^2} = a^2\left(\frac{2}{t^3}\right)$ When $t = \frac{1}{2}$, $\frac{\mathrm{d}^2 S}{\mathrm{d}t^2} = a^2\left(\frac{2}{\left(\frac{1}{2}\right)^3}\right) = 16a^2 > 0$. Hence the minimum value of S occurs when $t = \frac{1}{2}$. Minimum $S = a^2(4+2+2) = 8a^2$.
(iii)	To sketch the graph of $S = a^2\left(4 + 4\tan\theta + \frac{1}{\tan\theta}\right)$ 
6 (a)	Since adjacent balls do not sum up to two, balls numbered '1' needs be separated. Number of ways of arranging the other balls with no restriction = 6!

Slotting in the balls numbered '1', permutation is done as balls are of different colour = ${}^{7}C_3 \times 3!$

No of ways

$= 6! \times {}^{7}C_3 \times 3!$
$= 151200$

(b) **Method 1**

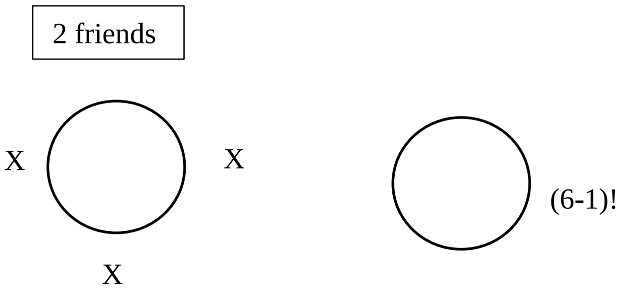


Table of 5 Table of 6

Case 1 – 2 friends are seated together at table of 5

No. of ways to select 3 other friends and arrange them at the table of 5 = ${}^{9}C_3 \times (4-1)!$

No. of ways to arrange the 2 friends = 2!

No. of ways to sit the remaining friends at the table of 6

= (6-1)! = 5! = 120

Total no. of ways = ${}^{9}C_3 \times (4-1)! \times 2! \times 5! = 120960$

Case 2 – 2 friends are seated together at table of 6

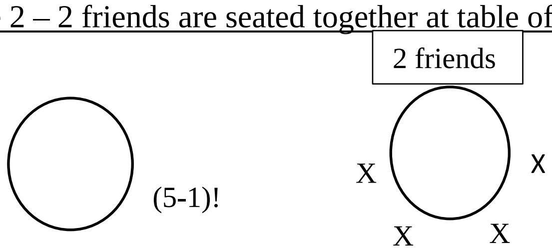
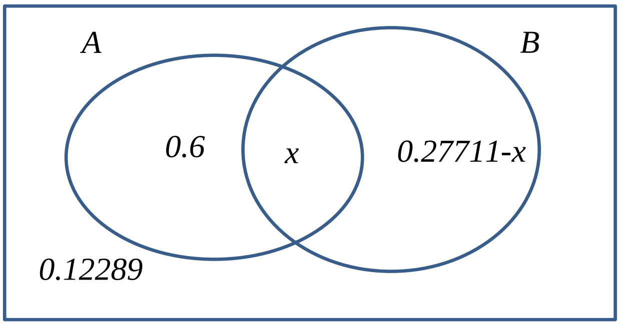


Table of 5 Table of 6

No. of ways to select 4 other friends and arrange them at the table of 6 = ${}^{9}C_4 \times (5-1)! = 3024$

No. of ways to sit the 2 friends at the table of 6 = 2!

	No. of ways to sit the remaining friends at the table of 5 = (5-1)! = 4! = 24 Total no. of ways = ${}^{9}C_4 \times (5-1)! \times 2! \times 4! = 145152$ No of ways to arrange 11 friends without restrictions = ${}^{11}C_5 \times (5-1)! \times (6-1)!$ = 1330560 Total no. of ways of arranging 11 people such that 2 particular friends are not seated together = 1330560 – 120960 – 145152 = 1064448
	<u>Method 2</u> **<u>Alternative Method</u>** Case 1: Two particular friends seated at table of 5 No of ways $= {}^{9}\mathrm{C}_3 \times 2! \times 3 \times 2 \times 5!$ $= 120960$ ${}^{9}\mathrm{C}_3$: Selection of friends to be seated at table of 5. This automatically selects friends to be seated at table of 6. (3-1)!: Arranging the 3 other friends in table of 5. ${}^{3}P_2$: Slotting in the 2 particular friends 5!: Arranging the 6 other friends in table of 6. Case 2: Two particular friends seated at table of 6 No of ways $= {}^{9}\mathrm{C}_4 \times 4! \times 3! \times 4 \times 3$ $= 217728$ ${}^{9}\mathrm{C}_4$: Selection of friends to be seated at table of 5. This automatically selects friends to be seated at table of 6. (5-1)!: Arranging the 5 friends in table of 5. 4!: Arranging the 5 friends in table of 6. ${}^{4}P_2$: Slotting in the 2 particular friends

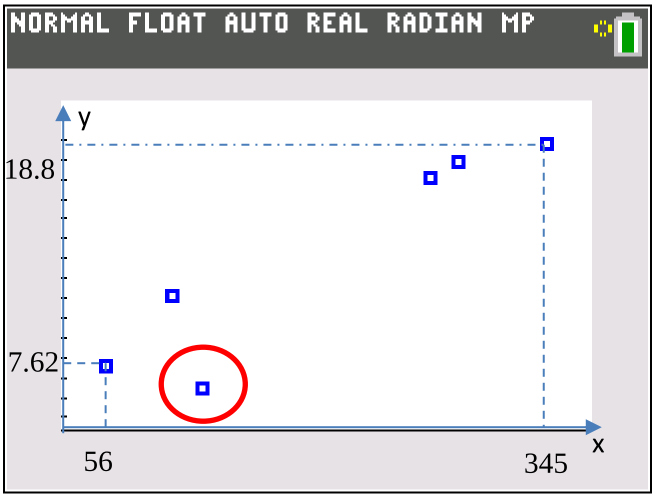
	Case 3: Two particular friends seated at separate tables No of ways $= {}^{9}\text{C}_{4} \times 4! \times 5! \times 2$ $= 725760$ ${}^{9}\text{C}_{4}$: Selection of friends to be seated at table of 5. This automatically selects friends to be seated at table of 6. (5-1)!: Arranging the 5 friends in table of 5. (6-1)!: Arranging the 6 friends in table of 6. x2: The 2 particular friends can switch tables Total no. of ways $= 120960 + 217728 + 725760$ $= 1064448$
7(i)	Given $\text{P}(A \mid B') = 0.83$ $\Rightarrow \dfrac{\text{P}(A \cap B')}{\text{P}(B')} = 0.83$ $\Rightarrow \dfrac{0.6}{1 - \text{P}(B)} = 0.83$ $\Rightarrow P(B) = 1 - 0.72289 = 0.27711 = 0.277$
(ii)	Let P($A \cap B$) = x  $\text{P}(A \cup B) = P(A \cap B') + P(B)$ $= 0.6 + x + 0.27711 - x$ $= 0.87711$ $\text{P}(A \cup B)' = 1 - 0.87711 = 0.12289$ Since $\text{P}(A \cup B') = 0.83$ $\therefore 0.6 + x + 0.12289 = 0.83$ $\Rightarrow x = 0.10711$

<table>
<tr><td></td><td>$\therefore \mathrm{P}(A \cap B) = 0.107$.</td></tr>
<tr><td>(iii)</td><td>$$\mathrm{P}(B \mid A') = \frac{\mathrm{P}(B \cap A')}{\mathrm{P}(A')}$$
$$= \frac{0.27711 - 0.10711}{1-(0.6+0.10711)}$$
$$= \frac{0.17}{0.29289}$$
$$= 0.58042$$
$$= 0.580$$
Since $\mathrm{P}(B \mid A') \neq \mathrm{P}(B) \Rightarrow B$ is not independent of A'

$\therefore$ A and B are not independent.</td></tr>
<tr><td>8 (i)</td><td>P(Linda scores 30 points) = P({hit, hit, hit})
$= 0.6^3$
$= \frac{27}{125}$ (0.216)</td></tr>
<tr><td>(ii)</td><td>Let X be the number of points scored by Linda in a round.
<table>
<tr><td>X</td><td>0</td><td>10</td><td>20</td><td>30</td></tr>
<tr><td>P(X=x)</td><td>0.4</td><td>0.6×0.4
$=0.24$</td><td>$0.6^2\times0.4$
$=0.144$</td><td>0.216</td></tr>
</table></td></tr>
<tr><td>(iii)</td><td>$\mathrm{E}(X) = 0\times0.4 + 10\times0.24 + 20\times0.144 + 30\times0.216$
$=11.76$

$\mathrm{E}(X^2) = 0^2\times0.4 + 10^2\times0.24 + 20^2\times0.144 + 30^2\times0.216$
$= 276$

$\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$
$= 276 - 11.76^2 = 137.7024$</td></tr>
<tr><td>(iv)</td><td>Let X_1 be the number of points scored by Linda in Round 1 and let X_2 be the number of points scored by Linda in Round 2.</td></tr>
</table>

	P(Linda scores more in round 2 than in round 1) $= P(X_1 = 0 \ \& \ X_2 \geq 10)$ $+P(X_1 = 10 \ \& \ X_2 \geq 20)$ $+P(X_1 = 20 \ \& \ X_2 = 30)$ $= P(X_1 = 0)P(X_2 \geq 10)$ $+P(X_1 = 10)P(X_2 \geq 20)$ $+P(X_1 = 20)P(X_2 = 30)$ $= 0.4 \times (1 - 0.4)$ $+0.24 \times (0.144 + 0.216) + 0.144 \times 0.216$ $= 0.357504 = 0.358$ (3 s.f.)
9 (i)	
(ii) (a)	Product moment correlation coefficient , $r = 0.9996$
(b)	Product moment correlation coefficient, $r = 0.9514$
(iii)	From the scatter diagram, as x increases, the value of y increases at a decreasing rate that seems to fit model (a) better. Also, the value of $\|r\|$ for model (a) is closer to 1 as compared to model (b).
(iv)	We use the regression line y on $\ln x$ $y = 6.1619(\ln x) - 17.223 \approx 6.16 \ln x - 17.2$ When $x = 210$, $y = 6.1619(\ln 210) - 17.223 = 15.725 \approx 15.7$

	As the value of $\lvert r \rvert$ is close to 1 and $x = 210$ is within the given data range, the estimation may be reliable.
10 (i)	Let S be the random variable "radius of a small table in cm'. Let L be the random variable "radius of a large table in cm'. $S \sim \mathrm{N}(30,\ 2^2)$ $L \sim \mathrm{N}(50,\ 5^2)$ $S_1+S_2+S_3+S_4+S_5 \sim \mathrm{N}(5\times 30,\ 5\times 2^2)$ $S_1+S_2+S_3+S_4+S_5 \sim \mathrm{N}\left(150,\ 20\right)$ $\mathrm{P}(S_1+S_2+S_3+S_4+S_5 < 160) = 0.98733 \approx 0.987$
(ii)	$S_1+S_2+S_3-2L \sim \mathrm{N}(3\times 30-2\times 50,\ 3\times 2^2+2^2\times 5^2)$ $S_1+S_2+S_3-2L \sim \mathrm{N}\left(-10,\ 112\right)$ $P(S_1+S_2+S_3 < 2L) = P(S_1+S_2+S_3-2L < 0) = 0.82765 \approx 0.828$
(iii)	The radii of the large and small round tables are independent of one another.
(iv)	Let X be the random variable "number of large tables, out of 12, with radius less than 40 cm". $X \sim \mathrm{B}(12,\ \mathrm{P}(L < 40\ ))$ $X \sim \mathrm{B}(12,\ 0.022750)$ $\mathrm{P}(X \geq 2\ ) = 1-\mathrm{P}\left(X \leq 1\right)$ $= 1-0.97064$ $= 0.029357$ ≈ 0.0294
(v)	Let Y be the random variable "radius of a medium sized table in cm"

	$\text{P}(Y \geq 44\ ) = 0.20$ $\text{P}(Y < 44\ ) = 0.80$ $P\left(Z < \frac{44-\mu}{\sigma}\right) = 0.80$ $\frac{44-\mu}{\sigma} = 0.84162$ $\mu = 44 - 0.84162\sigma - - - - (1)$ $\text{P}(Y < 40\ ) = 0.30$ $P\left(Z < \frac{40-\mu}{\sigma}\right) = 0.30$ $\frac{40-\mu}{\sigma} = -0.52440$ $\mu = 40 + 0.52440\sigma - - - - (2)$ Solving (1) and (2), $44 - 0.84162\sigma = 40 + 0.5244\sigma$ $4 = 1.3660\sigma$ $\sigma = 2.9283 \approx 2.93$ $\mu = 41.535 \approx 41.5$
11 (i)	Unbiased estimate of population mean, $\bar{x} = \frac{24730}{50} = 494.60$ Unbiased estimate for population variance, $s^2 = \frac{1}{49}\left(12242631 - \frac{24730^2}{50}\right) = 228.02$ Let X be the volume of beer in one beer can in ml and μ be the population mean volume of beer of the beer cans. $H_0 : \mu = 500$ $H_1 : \mu < 500$ Under H_0 , since n = 50 is large, by the Central Limit Theorem,

	$\overline{X} \sim N\left(500, \frac{s^2}{50}\right)$ approximately. Use a left-tailed z-test at the 1% level of significance. Test statistic: $Z = \frac{\overline{X} - 500}{\frac{s}{\sqrt{50}}} \sim N(0,1)$. Reject H_0 if *p*-value ≤ 0.01. From the sample, $p - \text{value} = 0.0057248 = 0.00572$ Since *p*-value $= 0.00572 \leq 0.01$, we reject H_0. There is sufficient evidence at the 1% level of significance to conclude that the volume of cola in a can is less than 500 ml.
(ii)	As we are using a two tailed test instead of a one tailed test, *p*-value = 2 (0.00572) = 0.01144. Hence we do not reject H_0. There is insufficient evidence at the 1% level of significance to conclude that the volume of cola in a can is not 500 ml.
(iii)	Let *X* be the volume of cola in one can in ml and μ be the population mean volume of cola of the cans. $H_0 : \mu = 500$ $H_1 : \mu \neq 500$ Unbiased estimate of population variance, $s^2 = \frac{40}{39}\left(s_x\right)^2$ Under H_0 , since *n* = 40 is large, by the Central Limit Theorem, $\overline{X} \sim N\left(500, \frac{{s_x}^2}{39}\right)$ approximately. Use a two-tailed z-test at the 1% level of significance.

	Test statistic: $Z = \dfrac{\overline{X} - 500}{\dfrac{s_x}{\sqrt{39}}} \sim N(0,1)$ Critical values: $z_{crit(1)} = -2.5758 \quad z_{crit(2)} = 2.5758$. Reject H_0 if $z_{cal} \le -2.5758$ or $z_{cal} \ge 2.5758$. Since H_0 is rejected, $-2.5758 \le z_{cal}$ or $z_{cal} \ge 2.5758$ $-2.5758 \le \dfrac{\overline{x} - 500}{\sqrt{\dfrac{s_x{}^2}{39}}}$ or $\dfrac{\overline{x} - 500}{\sqrt{\dfrac{s_x{}^2}{39}}} \ge 2.5758$ $500 - 2.5758\sqrt{\dfrac{s_x{}^2}{39}} \le \overline{x}$ or $\overline{x} \ge 500 + 2.5758\sqrt{\dfrac{s_x{}^2}{39}}$ $500 - 0.41246 s_x \le \overline{x}$ or $\overline{x} \ge 500 + 0.41246 s_x$ $500 - 0.412 s_x \le \overline{x}$ or $\overline{x} \ge 500 + 0.412 s_x$ Hence the decision rule should read: Conclude that the volume of cola differs from 500 ml if the value of $\overline{x}$ lies within this range : $500 - 0.412 s_x \le \overline{x}$ or $\overline{x} \ge 500 + 0.412 s_x$.
(iv)	Let X be the volume of cola in one can in ml. since n is large, by the Central Limit Theorem, $X_1 + X_2 + ... + X_n \sim N\left(500n, 144n\right)$ approximately. Let Y be the volume of grape juice in one packet in ml. since $2n$ is large, by the Central Limit Theorem, $Y_1 + Y_2 + ... + Y_{2n} \sim N\left(500n, 50n\right)$ approximately. $X_1 + X_2 + ... + X_n + Y_1 + Y_2 + ... + Y_{2n} \sim N(1000n, 194n)$

	$P(X_1 + X_2 + ... + X_n + Y_1 + Y_2 + ... + Y_{2n} \leq 120,000) \geq 0.95$ $P\left(Z \leq \frac{120,000 - 1000n}{\sqrt{194n}}\right) \geq 0.95$ $\frac{120,000 - 1000n}{\sqrt{194n}} \geq 1.6449$ $120,000 - 1000n \geq 1.6449\sqrt{194n}$ $1000n + 22.9\sqrt{n} - 120,000 \leq 0$