Qn	Solution		
1	Let x, y and z be the amount of Francs, Pounds & Euro Mr Subash has left respectively. $1.35x+1.80y+1.55z=1151.50$ $1.40x+1.85y+1.65z=1208.25$ $1.45x+1.75y+1.60z=1189.25$ Using GC, x = 250, y = 125, z = 380. He has 250 francs, 125 pounds and 380 euros left.		
2(a)	By Factor Formula, $\sin(2\theta)\cos(3\theta) = \frac{1}{2}[\sin(5\theta)+\sin(-\theta)]$ $= \frac{1}{2}[\sin(5\theta)-\sin(\theta)]$		
	$\int \sin(2\theta)\cos(3\theta)\,d\theta = \int \frac{1}{2}[\sin(5\theta)-\sin(\theta)]\,d\theta$ $= \frac{1}{2}\cos\theta - \frac{1}{10}\cos(5\theta)+c$		
2(b)	$\theta=\sqrt{\pi} \Rightarrow \sqrt{x}=\sqrt{\pi} \Rightarrow x=\pi$ $\theta=\sqrt{\frac{\pi}{2}} \Rightarrow \sqrt{x}=\sqrt{\frac{\pi}{2}} \Rightarrow x=\frac{\pi}{2}$		
	$\theta=\sqrt{x} \Rightarrow \frac{d\theta}{dx}=\frac{1}{2\sqrt{x}}$.		
	$\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} \theta^3\cos(\theta^2)\,d\theta$ $=\int_{\frac{\pi}{2}}^{\pi} x\sqrt{x}(\cos x)\left(\frac{1}{2\sqrt{x}}\right)dx$ $=\frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} x\cos x\,dx$ $u=x$, $\frac{du}{dx}=1$ $\frac{dv}{dx}=\cos x$, $v=\sin x$ $=\frac{1}{2}\left[[x\sin x]_{\frac{\pi}{2}}^{\pi}-\int_{\frac{\pi}{2}}^{\pi}1(\sin x)\,dx\right]$ $=\frac{1}{2}\left(0-\frac{\pi}{2}+[\cos x]_{\frac{\pi}{2}}^{\pi}\right)$ $=\frac{1}{2}\left[-\frac{\pi}{2}+(-1-0)\right]$ $=-\frac{1}{2}-\frac{\pi}{4}$		

3(i)	$\sin\left[(2r+1)\theta\right]-\sin\left[(2r-1)\theta\right]$ $\equiv 2\cos\frac{(2r+1)\theta+(2r-1)\theta}{2}\sin\frac{(2r+1)\theta-(2r-1)\theta}{2}$ $\equiv 2\cos(2r\theta)\sin\theta$ [Shown]		
(ii)	From (i), $\sin\left[(2r+1)\theta\right]-\sin\left[(2r-1)\theta\right]\equiv 2\cos(2r\theta)\sin\theta$ $\Rightarrow \cos(2r\theta)=\frac{\sin\left[(2r+1)\theta\right]-\sin\left[(2r-1)\theta\right]}{2\sin\theta}$ $\therefore \sum_{r=1}^{n}\cos(2r\theta) = \sum_{r=1}^{n}\frac{\sin\left[(2r+1)\theta\right]-\sin\left[(2r-1)\theta\right]}{2\sin\theta}$ $=\frac{1}{2\sin\theta}\begin{bmatrix} \sin 3\theta-\sin\theta \\ +\ \sin 5\theta-\sin 3\theta \\ +\ \sin 7\theta-\sin 5\theta \\ +\ \cdots \\ +\ \sin(2n-1)\theta-\sin(2n-3)\theta \\ +\ \sin(2n+1)\theta-\sin(2n-1)\theta \end{bmatrix}$ $=\frac{\sin\left[(2n+1)\theta\right]-\sin\theta}{2\sin\theta}$ [Shown]		
(iii)	$\sum_{r=1}^{n}\cos^2\left(\frac{r\pi}{5}\right) = \sum_{r=1}^{n}\frac{\cos\left(\frac{2r\pi}{5}\right)+1}{2}$ $=\frac{1}{2}\sum_{r=1}^{n}\cos\left(\frac{2r\pi}{5}\right)+\sum_{r=1}^{n}\frac{1}{2}$ $\left(\text{Let } \theta=\frac{\pi}{5}\right)$ $=\frac{1}{2}\left[\frac{\sin\frac{(2n+1)\pi}{5}-\sin\frac{\pi}{5}}{2\sin\frac{\pi}{5}}\right]+\frac{1}{2}n$ $=\frac{\sin\frac{(2n+1)\pi}{5}}{4\sin\frac{\pi}{5}}-\frac{1}{4}+\frac{1}{2}n$ As $n\to\infty$, $-\frac{1}{4}+\frac{1}{2}n\to\infty$ and $\left\lvert\sin\frac{(2n+1)\pi}{5}\right\rvert\le 1$, $\therefore \sum_{r=1}^{n}\cos^2\left(\frac{r\pi}{5}\right)\to\infty.$ $\therefore$ the series $\cos^2\left(\frac{\pi}{5}\right)+\cos^2\left(\frac{2\pi}{5}\right)+\cos^2\left(\frac{3\pi}{5}\right)+\ldots$ is divergent.		

4	(table below)		

Yr	Amount at the beginning of yr	Amount at the end of yr
1	6000	$6000(1.03)$
2	$6000(1.03)-k$	$\left[6000(1.03)-k\right](1.03)$ $=6000(1.03)^2-k(1.03)$
3	$6000(1.03)^2-k(1.03)-k$ $=6000(1.03)^2-k(1.03)-k$	$\left[6000(1.03)^2-k(1.03)-k\right](1.03)$ $=6000(1.03)^3-k(1.03)^2-k(1.03)$

By inspection, amount in the fund at the end of nth year

$$= 6000(1.03)^n - k(1.03)^{n-1} - k(1.03)^{n-2} - \ldots - k(1.03)$$

Amount in the fund at the beginning of $(n+1)$th year

$$= 6000(1.03)^n - k(1.03)^{n-1} - k(1.03)^{n-2} - \ldots - k(1.03) - k$$

$$= 6000(1.03)^n - k\left[1+1.03+(1.03)^2+\cdots+(1.03)^{n-1}\right]$$

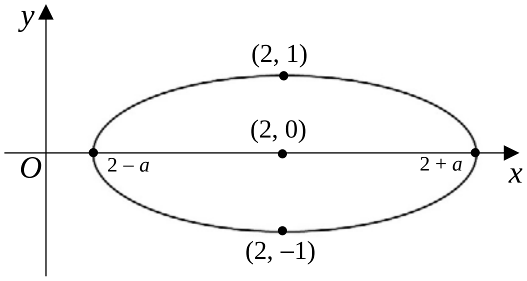
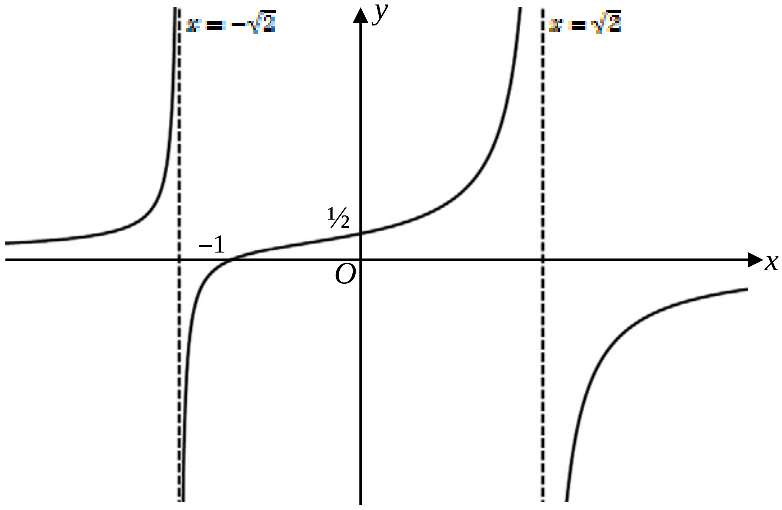
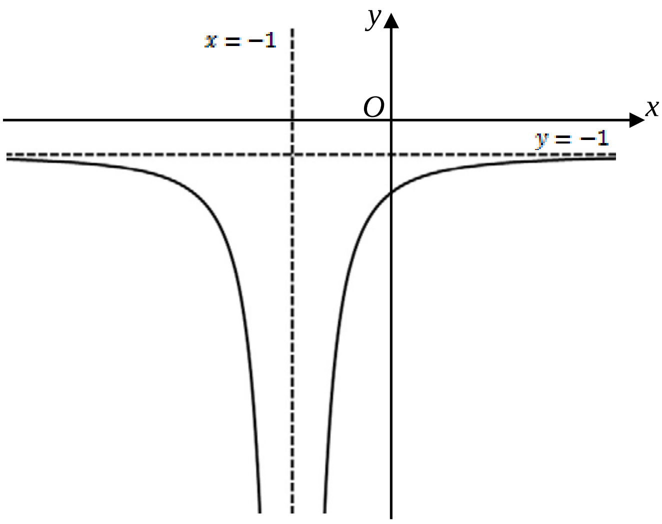
$$= 6000(1.03)^n - k\left\{\frac{1\left[1-(1.03)^n\right]}{1-1.03}\right\}$$

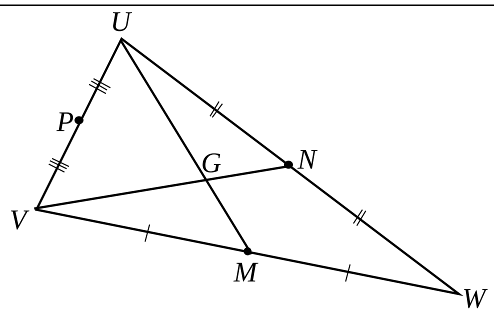
$$= 6000(1.03)^n + \frac{100}{3}k\left[1-(1.03)^n\right]$$

$$= \frac{100}{3}\left[180(1.03)^n + k - k(1.03)^n\right]$$

$$= \frac{100}{3}\left[(180-k)(1.03)^n + k\right] \quad \text{[Shown]}$$

4(i)	Given k = 400, $\frac{100}{3}\left[(180-400)(1.03)^n+400\right] < 1000$ $-220(1.03)^n+400 < 30$ $(1.03)^n > \frac{37}{22}$ (or 1.6818) $n\ln 1.03 > \ln\frac{37}{22}$ $n > \frac{\ln\frac{37}{22}}{\ln 1.03}$ = 17.6 (3 sf) Least n = 18 Or: use GC, table of values gives least n = 18 $n+1 = 19$ Therefore, at the beginning of 19th year, the amount in the fund will be less than $1000 for the first time		
4(ii)	When $n+1=16 \Rightarrow n = 15$, $\frac{100}{3}\left[(180-k)(1.03)^{15}+k\right] \leq 0$ $(180-k)(1.03)^{15}+k \leq 0$ $180(1.03)^{15}+k\left[1-(1.03)^{15}\right] \leq 0$ $k\left[1-(1.03)^{15}\right] \leq -180(1.03)^{15}$ $k\left[(1.03)^{15}-1\right] \geq 180(1.03)^{15}$ $k \geq \frac{180(1.03)^{15}}{(1.03)^{15}-1}$ $k \geq 502.6$ Least k = 503 (nearest integer) Or: from GC (plot graph or table of values), least k = 503 (nearest integer)		

5(a)	$(x-2)^2 = a^2(1-y^2)$ $\Rightarrow \dfrac{(x-2)^2}{a^2} + y^2 = 1$ $\Rightarrow \dfrac{(x-2)^2}{a^2} + \dfrac{(y-0)^2}{1^2} = 1,$ $1 < a < 2$ 		
5(b) (i)	$y = \dfrac{1}{\mathrm{f}(x)}$ 		
5(b) (ii)	$y = \mathrm{f}'(x)$ 		

6 (i)	 By Ratio Theorem, $\overrightarrow{UM} = \dfrac{\overrightarrow{UW}+\overrightarrow{UV}}{2}$ $= \dfrac{\mathbf{w}-\mathbf{u}+\mathbf{v}-\mathbf{u}}{2}$ $= \dfrac{1}{2}(\mathbf{v}+\mathbf{w}-2\mathbf{u})$ (Shown)		
(ii)	Vector equation of line *UM* is $\mathbf{r} = \mathbf{u} + \lambda(\mathbf{w}+\mathbf{v}-2\mathbf{u}),\ \lambda \in \mathbb{R}$ $\overrightarrow{VN} = \dfrac{\overrightarrow{VW}+\overrightarrow{VU}}{2}$ $= \dfrac{\mathbf{w}-\mathbf{v}+\mathbf{u}-\mathbf{v}}{2} = \dfrac{1}{2}(\mathbf{w}+\mathbf{u}-2\mathbf{v})$ Vector equation of line *VN* is $\mathbf{r} = \mathbf{v} + \mu(\mathbf{w}+\mathbf{u}-2\mathbf{v}),\ \mu \in \mathbb{R}$ At point of intersection *G*, $\mathbf{u}+\lambda(\mathbf{w}+\mathbf{v}-2\mathbf{u}) = \mathbf{v}+\mu(\mathbf{w}+\mathbf{u}-2\mathbf{v})$ For **u**: $1-2\lambda = \mu$ For **w**: $\lambda = \mu$ Solving, $\lambda = \dfrac{1}{3} = \mu$ $\overrightarrow{OG} = \mathbf{u}+\dfrac{1}{3}(\mathbf{w}+\mathbf{v}-2\mathbf{u})$ $= \dfrac{1}{3}(\mathbf{u}+\mathbf{v}+\mathbf{w})$ (Shown)		
(iii)	$\mathbf{u} = \begin{pmatrix}1\\0\\0\end{pmatrix}, \mathbf{v} = \begin{pmatrix}0\\1\\0\end{pmatrix}, \mathbf{w} = \begin{pmatrix}0\\0\\1\end{pmatrix}$ $\overrightarrow{OG} = \dfrac{1}{3}\left[\begin{pmatrix}1\\0\\0\end{pmatrix}+\begin{pmatrix}0\\1\\0\end{pmatrix}+\begin{pmatrix}0\\0\\1\end{pmatrix}\right] = \begin{pmatrix}\frac{1}{3}\\\frac{1}{3}\\\frac{1}{3}\end{pmatrix}$ $\left\|\overrightarrow{OG}\right\| = \sqrt{3\left(\dfrac{1}{3^2}\right)} = \sqrt{\dfrac{1}{3}}$ Direction cosines of $\overrightarrow{OG}$ are $\dfrac{\frac{1}{3}}{\sqrt{\frac{1}{3}}}, \dfrac{\frac{1}{3}}{\sqrt{\frac{1}{3}}}, \dfrac{\frac{1}{3}}{\sqrt{\frac{1}{3}}}$, i.e., $\sqrt{\dfrac{1}{3}}, \sqrt{\dfrac{1}{3}}, \sqrt{\dfrac{1}{3}}$		

7 (a)	$u=2-\mathrm{i}\sin^2\theta,\ v=2\cos^2\theta+\mathrm{i}\sin^2\theta$ $u-v = 2-\mathrm{i}\sin^2\theta-2\cos^2\theta-\mathrm{i}\sin^2\theta$ $= 2-2\cos^2\theta-2\mathrm{i}\sin^2\theta$ $= 2\left(1-\cos^2\theta\right)-2\mathrm{i}\sin^2\theta$ $= \underline{\underline{2\sin^2\theta-2\mathrm{i}\sin^2\theta}}$ or $\underline{\underline{2\left(\sin^2\theta\right)(1-\mathrm{i})}}$ $\lvert u-v\rvert = 2\lvert\sin^2\theta-\mathrm{i}\sin^2\theta\rvert$ $= 2\sqrt{\sin^4\theta+\sin^4\theta}$ $= 2\sqrt{2\sin^4\theta}$ $= \underline{\underline{2\sqrt{2}\sin^2\theta}}$ or $2\lvert\sin^2\theta\rvert\lvert 1-\mathrm{i}\rvert$ $= 2\left(\sin^2\theta\right)\sqrt{1+1}$ $= \underline{\underline{2\sqrt{2}\sin^2\theta}}$ Note that $u-v$ lies in the 4th quadrant. $\arg(u-v) = -\tan^{-1}\dfrac{2\sin^2\theta}{2\sin^2\theta}$ $= -\tan^{-1}1 = \underline{\underline{-\dfrac{\pi}{4}}}$ Or: $\arg(u-v) = \arg\left(2\sin^2\theta-2\mathrm{i}\sin^2\theta\right)=\arg\left[2\left(\sin^2\theta\right)(1-\mathrm{i})\right]$ $= \arg\left(2\sin^2\theta\right)+\arg(1-\mathrm{i})$ $= 0+\left(-\dfrac{\pi}{4}\right) = \underline{\underline{-\dfrac{\pi}{4}}}$		
7 (b)	<u>Method 1 Solve α first then factorise quadratic expression or use sum of roots</u> $x^2+(\mathrm{i}-3)x+2(1-\mathrm{i}) = 0$ Sub. $x=\alpha\in\mathbb{R}$, $\alpha^2+(\mathrm{i}-3)\alpha+2(1-\mathrm{i}) = 0$ $\left(\alpha^2-3\alpha+2\right)+\mathrm{i}(\alpha-2) = 0$ Comparing imaginary parts, $\alpha-2 = 0$ $\underline{\underline{\alpha = 2}}$ $x^2+(\mathrm{i}-3)x+2(1-\mathrm{i}) = (x-2)(x-\beta)$ Comparing constants, $2(1-\mathrm{i}) = 2\beta$ $\therefore \underline{\underline{\beta = 1-\mathrm{i}}}$ Or: Sum of roots, $\alpha+\beta = -(\mathrm{i}-3)$ $2+\beta = 3-\mathrm{i}$ $\therefore \underline{\underline{\beta = 1-\mathrm{i}}}$		

7 (b)

Method 2 Factorise the quadratic expression first

$$x^2+(\mathrm{i}-3)x+2(1-\mathrm{i}) = (x-\alpha)(x-\beta)$$

Comparing coefficients of x,

$$\mathrm{i}-3 = -(\alpha+\beta)$$

$$\alpha+\beta = 3-\mathrm{i} \qquad (1)$$

Comparing constants,

$$\alpha\beta = 2-2\mathrm{i} \qquad (2)$$

From (1), $\beta = 3-\mathrm{i}-\alpha \qquad (3)$

Sub. (3) into (2), $\alpha(3-\mathrm{i}-\alpha) = 2-2\mathrm{i}$

$$3\alpha-\alpha^2-\alpha\mathrm{i} = 2-2\mathrm{i}$$

Comparing imaginary parts, $\underline{\underline{\alpha = 2}}$

Sub. into (3), $\beta = 3-\mathrm{i}-2$

$$\therefore \underline{\underline{\beta = 1-\mathrm{i}}}$$

Or:

Let $\beta = a+b\mathrm{i}$, where $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $b \neq 0$

$$x^2+(\mathrm{i}-3)x+2(1-\mathrm{i}) = (x-\alpha)\left[x-(a+b\mathrm{i})\right]$$

Comparing coefficients of x,

$$\mathrm{i}-3 = -a-b\mathrm{i}-\alpha$$

$b = -1$ (Comparing imaginary parts)

$a+\alpha = 3 \quad (1)$ (Comparing real parts)

Comparing constants,

$$2-2\mathrm{i} = \alpha(a+b\mathrm{i})$$

$$= \alpha(a-\mathrm{i}) = \alpha a-\alpha\mathrm{i}$$

$\underline{\underline{\alpha = 2}}$ (Comparing imaginary parts)

Sub. into (1), $a = 3-\alpha = 3-2 = 1$

$$\therefore \underline{\underline{\beta = 1-\mathrm{i}}}$$

Method 3 Solve x first using quadratic formula

$$x^2+(\mathrm{i}-3)x+2(1-\mathrm{i}) = 0$$

$$x = \frac{-(\mathrm{i}-3)\pm\sqrt{(\mathrm{i}-3)^2-4(1)\left[2(1-\mathrm{i})\right]}}{2}$$

$$= \frac{3-\mathrm{i}\pm\sqrt{\mathrm{i}^2-6\mathrm{i}+9-8+8\mathrm{i}}}{2} = \frac{3-\mathrm{i}\pm\sqrt{2\mathrm{i}}}{2}$$

$$= \frac{3-\mathrm{i}\pm(1+\mathrm{i})}{2}$$ (use GC to find $\sqrt{2\mathrm{i}}$)

$$= 2 \text{ or } 1-\mathrm{i}$$

$$\therefore \underline{\underline{\alpha = 2}} \text{ and } \underline{\underline{\beta = 1-\mathrm{i}}}$$

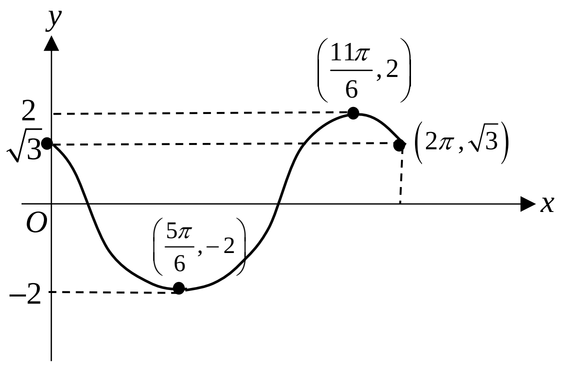
<table>
<tr><td></td><td>For comparison purpose:
If GC is not used to find $\sqrt{2\mathrm{i}}$, then the algebraic works will look as follows:

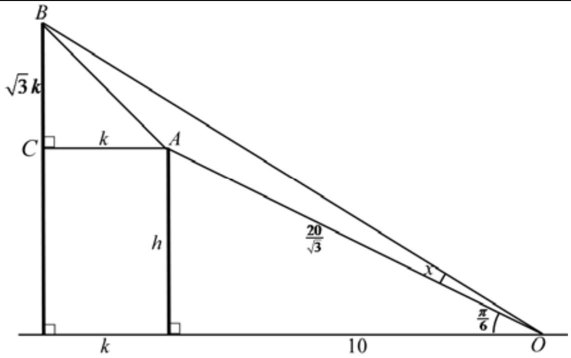
Let $\sqrt{2\mathrm{i}} = a+b\mathrm{i}$, where $a \in \mathbb{R}$, $b \in \mathbb{R}$
$2\mathrm{i} = a^2 - b^2 + 2ab\mathrm{i}$
Compring real parts, $a^2 - b^2 = 0$
$a^2 = b^2$
$a = \pm b$ (1)
Compring imaginary parts, $ab = 1$ (2)
When $a = b$,
Sub. into (2), $a^2 = 1$
$a = \pm 1$
When a = 1, b = 1. When $a = -1$, $b = -1$
$\pm\sqrt{2\mathrm{i}} = \pm(1+\mathrm{i})$
When $a = -b$
Sub. into (2), $-b^2 = 1$ (NA $\because b \in \mathbb{R}$)
$\therefore\ x = \dfrac{3-\mathrm{i}\pm(1+\mathrm{i})}{2}$ = 2 or $1-\mathrm{i}$
$\therefore\ \underline{\underline{\alpha = 2}}$ and $\underline{\underline{\beta = 1-\mathrm{i}}}$</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>

8(a)(i)	Let A cm^2 be area of the circular patch. $A = \pi r^2$ $\frac{dA}{dr} = 2\pi r$ Given $\frac{dA}{dt} = 6\pi$ cm^2/s, a **constant** This means that, in 1 s, A increases by 6π cm^2 **constantly**. When $t = 0$, $A = 0$ When $t = 24$, $A = 24 \times 6\pi = 144\pi$ $\pi r^2 = 144\pi$ $r = 12$ (reject $r = -12$ since $r > 0$) $\frac{dA}{dr} = 2\pi(12) = 24\pi$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $6\pi = 24\pi \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{1}{4}$ $\therefore$ rate of change of the radius is $\frac{1}{4}$ cm/ s.		
(ii)	$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $6\pi = 2\pi r \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{6\pi}{2\pi r} = \frac{3}{r}$ <u>Method 1</u> As r increases, $\frac{dr}{dt} = \frac{3}{r}$ decreases, $\therefore \frac{dr}{dt}$ will <u>decrease</u> as time passes. <u>Method 2</u> $\frac{d\left(\frac{dr}{dt}\right)}{dt} = \frac{d\left(\frac{3}{r}\right)}{dr} \times \frac{dr}{dt}$ $= \frac{-3}{r^2}\left(\frac{3}{r}\right) = \frac{-9}{r^3} < 0$ $\therefore \frac{dr}{dt}$ will <u>decrease</u> as time passes.		

<table>
<tr><td>8(b)
(i)</td><td>$V = \pi r^2 h$
$355 = \pi r^2 h$
$\pi r h = \dfrac{355}{r}$

$C = K(2\pi r h) + 2K(4r^2)$
$= K\left[2\left(\dfrac{355}{r}\right) + 8r^2\right]$
$= K\left(\dfrac{710}{r} + 8r^2\right)$ (Shown)</td><td></td><td></td></tr>
<tr><td>(ii)</td><td>$\dfrac{dC}{dr} = \left(-\dfrac{710}{r^2} + 16r\right)K$
For C to be a minimum, $\dfrac{dC}{dr} = 0$.
$-\dfrac{710}{r^2} + 16r = 0$
$-710 + 16r^3 = 0$
$r^3 = \dfrac{355}{8}$
$r = \sqrt[3]{\dfrac{355}{8}} = 3.54$ (3 sf)</td><td></td><td></td></tr>
<tr><td></td><td>$\dfrac{d^2C}{dr^2} = \left(\dfrac{1420}{r^3} + 16\right)K = \left(\dfrac{1420}{\frac{355}{8}} + 16\right)K = 48K > 0$
Or
<table><tr><td>r</td><td>3.5</td><td>$\sqrt[3]{\dfrac{355}{8}} \approx 3.54$</td><td>3.6</td></tr><tr><td>$\dfrac{dC}{dr}$</td><td>$-1.96K < 0$</td><td>0</td><td>$2.82K > 0$</td></tr></table>So, $r = \sqrt[3]{\dfrac{355}{8}}$ does give the minimum cost.</td><td></td><td></td></tr>
<tr><td></td><td>Recall $355 = \pi r^2 h$
$h = \dfrac{355}{\pi r^2}$
$\therefore \dfrac{h}{r} = \dfrac{355}{\pi r^3} = \dfrac{355}{\pi\left(\frac{355}{8}\right)}$
$= \dfrac{8}{\pi}$ (Shown)</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>

9(i)	$\sqrt{3}\cos x - \sin x = R\cos(x+\alpha)$ $R = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = \underline{\underline{2}}$ $\alpha = \tan^{-1}\frac{1}{\sqrt{3}} = \underline{\underline{\frac{\pi}{6}}}$		
(ii)	$y = \sqrt{3}\cos x - \sin x = 2\cos\left(x + \frac{\pi}{6}\right)$ $y = \cos x \xrightarrow{A} y = \cos(x+\alpha) \xrightarrow{B} y = R\cos(x+\alpha)$ *A*: Translation by α radians in the negative *x*-direction, followed by *B*: Scaling parallel to the *y*-axis by a scale factor *R*. [can be *B* followed by *A*]		
(iii)	$f : x \mapsto \sqrt{3}\cos x - \sin x,\ 0 \le x \le 2\pi$  Range of f, $R_f = \underline{\underline{[-2, 2]}}$.		
(iv)	$g : x \mapsto f(x),\ 0 \le x \le k.$ Largest $k = \underline{\underline{\frac{5\pi}{6}}}$. Let $y = g(x)$. $y = 2\cos\left(x + \frac{\pi}{6}\right)$ $\cos\left(x + \frac{\pi}{6}\right) = \frac{y}{2}$ $\Rightarrow x = \cos^{-1}\frac{y}{2} - \frac{\pi}{6}$ $\therefore g^{-1}(x) = \underline{\underline{\cos^{-1}\frac{x}{2} - \frac{\pi}{6}}}$		
(v)	$h : x \mapsto x - 2,\ x \ge 0$ Since $R_h = [-2, +\infty)$ and $D_f = [0, 2\pi]$, $R_h \not\subset D_f$, fh does not exist.		
10 (i)			

	$\cos\frac{\pi}{6}=\frac{10}{OA}$ $\frac{\sqrt{3}}{2}=\frac{10}{OA}$ $OA=\frac{20}{\sqrt{3}}$ m (Shown)  		
(ii)	$AB = \sqrt{k^2+3k^2}=\sqrt{4k^2}=2k$ (Shown) $\angle BAC = \tan^{-1}\frac{\sqrt{3}\,k}{k}=\tan^{-1}\sqrt{3}$ $= \frac{\pi}{3}$ (Shown)		
(iii)	$\angle CBO=\frac{\pi}{2}-\left(\frac{\pi}{6}+x\right)=\frac{\pi}{3}-x$ $\angle CBA=\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$ $\angle ABO=\frac{\pi}{3}-x-\frac{\pi}{6}=\frac{\pi}{6}-x$ Or: $\angle BAO=2\pi-\frac{\pi}{2}-\frac{\pi}{3}-\frac{\pi}{3}$ ($\angle$ at a pt) $=\frac{5\pi}{6}$ $\angle ABO=\pi-x-\frac{5\pi}{6}=\frac{\pi}{6}-x$ In ΔABO, $\frac{2k}{\sin x}=\frac{\frac{20}{\sqrt{3}}}{\sin\left(\frac{\pi}{6}-x\right)}$ $k=\frac{10\sin x}{\sqrt{3}\sin\left(\frac{\pi}{6}-x\right)}$		
(iv)	$k=\frac{10\sin x}{\sqrt{3}\sin\left(\frac{\pi}{6}-x\right)}$ $=\frac{10\sin x}{\sqrt{3}\left(\sin\frac{\pi}{6}\cos x-\cos\frac{\pi}{6}\sin x\right)}$ $\approx\frac{10x}{\sqrt{3}\left[\frac{1}{2}\left(1-\frac{x^2}{2}\right)-\frac{\sqrt{3}}{2}x\right]}$ $=\frac{10x}{\frac{\sqrt{3}}{2}\left[\left(1-\frac{x^2}{2}\right)-\sqrt{3}\,x\right]}$ $=\frac{20x}{\sqrt{3}}\left[1-\left(\sqrt{3}x+\frac{x^2}{2}\right)\right]^{-1}$ $\approx\frac{20x}{\sqrt{3}}\left(1+\sqrt{3}\,x\right)$ $=\underline{\underline{\frac{20}{\sqrt{3}}\left(x+\sqrt{3}\,x^2\right)}}$		

11 (a) (i)

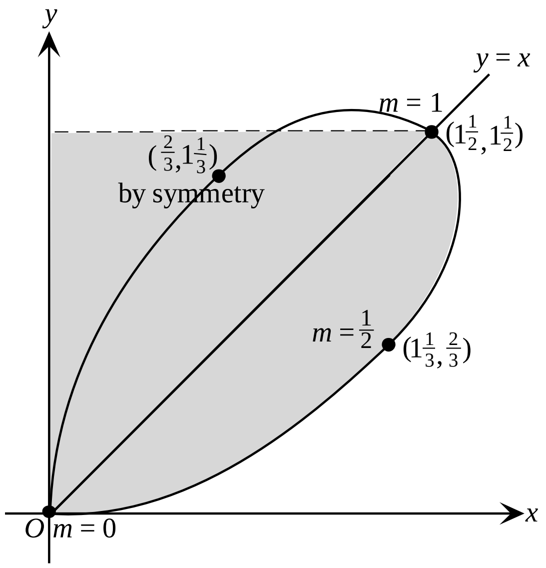
$$x=\frac{3m}{1+m^3},\quad y=\frac{3m^2}{1+m^3},\quad m\ge 0$$

$$\begin{aligned} y &= x \\ \frac{3m^2}{1+m^3} &= \frac{3m}{1+m^3} \\ m(m-1) &= 0 \\ m &= \underline{\underline{0 \text{ or } 1}} \end{aligned}$$

(ii)

When $m = 0$, $y = 0$.

When $m = 1$, $y = \frac{3}{1+1}=\frac{3}{2}$.



<u>Notes</u>:
Use GC to trace the path to see how m varies when the point moves along the path.

Area of (lower) half of the "leaf" is

$$\begin{aligned} \frac{1}{2}A &= \int_0^{\frac{3}{2}} x\,\mathrm{d}y - \text{area of } \Delta \qquad \left(\text{Note: } \int_0^{\frac{3}{2}} x\,\mathrm{d}y = \text{shaded area}\right) \\ A &= 2\left[\int_0^{\frac{3}{2}} x\,\mathrm{d}y - \frac{1}{2}\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)\right] \\ &= 2\left(\int_0^{\frac{3}{2}} x\,\mathrm{d}y - \frac{9}{8}\right) \qquad \text{(Shown)} \end{aligned}$$

$$\begin{aligned} 2\left(\int_0^{\frac{3}{2}} x\,\mathrm{d}y - \frac{9}{8}\right) &= 2\int_0^1 \frac{3m}{1+m^3}\left[\frac{6m\left(1+m^3\right)-3m^2\left(3m^2\right)}{\left(1+m^3\right)^2}\right]\mathrm{d}m-\frac{9}{4} \\ &= 2\int_0^1 \frac{3m\left(6m-3m^4\right)}{\left(1+m^3\right)^3}\mathrm{d}m-\frac{9}{4} \\ &= \frac{15}{4}-\frac{9}{4} \qquad \text{(by GC)} \\ &= \underline{\underline{\frac{3}{2}}} \end{aligned}$$

11 (b)	$y = \ln x$ $x = \mathrm{e}^{y}$		
	$V_A = \pi\int_0^c \left(\mathrm{e}^{y}\right)^2 \,\mathrm{d}y$ $= \pi\int_0^c \mathrm{e}^{2y} \,\mathrm{d}y$ $= \pi\left[\frac{1}{2}\mathrm{e}^{2y}\right]_0^c$ $= \frac{\pi}{2}\left(\mathrm{e}^{2c}-1\right)$		
	$V_B = (1-c)\pi\mathrm{e}^2 - \pi\int_c^1 \left(\mathrm{e}^{y}\right)^2 \,\mathrm{d}y$ or $\pi\int_c^1 \left[\mathrm{e}^2-\left(\mathrm{e}^{y}\right)^2\right]\mathrm{d}y$ $= \pi(1-c)\mathrm{e}^2 - \pi\left[\frac{1}{2}\mathrm{e}^{2y}\right]_c^1$ $= \pi(1-c)\mathrm{e}^2 - \frac{\pi}{2}\left(\mathrm{e}^2-\mathrm{e}^{2c}\right)$		
	$V_A = V_B$ $\frac{\pi}{2}\left(\mathrm{e}^{2c}-1\right) = \pi(1-c)\mathrm{e}^2 - \frac{\pi}{2}\left(\mathrm{e}^2-\mathrm{e}^{2c}\right)$ $\mathrm{e}^{2c}-1 = 2\mathrm{e}^2(1-c)-\mathrm{e}^2+\mathrm{e}^{2c}$ $= 2\mathrm{e}^2-2c\mathrm{e}^2-\mathrm{e}^2+\mathrm{e}^{2c}$ $2c\mathrm{e}^2 = \mathrm{e}^2+1$ $c = \frac{\mathrm{e}^2+1}{2\mathrm{e}^2}$ (Shown)		