YISHUN JUNIOR COLLEGE
2017 JC2 Preliminary Examination

MATHEMATICS **9740/01**
HIGHER 2

28 AUGUST 2017
MONDAY 0800h – 1100h

Additional materials :
Answer paper
List of Formulae (MF15)

TIME 3 hours

READ THESE INSTRUCTIONS FIRST

Write your CTG and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.

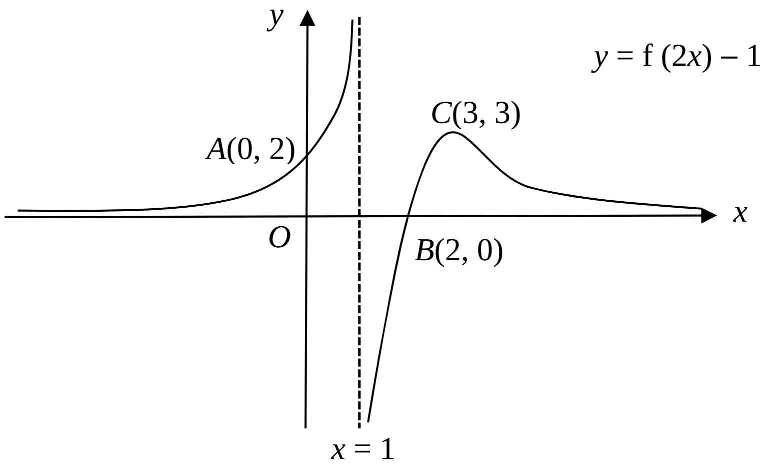
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, write down the question number of the questions attempted, model of calculator used on the spaces provided on the cover page. Tie your cover page on top of the answer scripts before submission.
The number of marks is given in brackets [ ] at the end of each question or part question.

This question paper consists of 6 printed pages.

1



The diagram shows the curve $y = \mathrm{f}(2x) - 1$ with a maximum point at $C(3, 3)$. The curve crosses the axes at the points $A(0, 2)$ and $B(2, 0)$. The line $x = 1$ and the x-axis are the asymptotes of the curve.

On separate diagrams, sketch the graphs of

(i) $y = \mathrm{f}(x)$, [2]

(ii) $y = \mathrm{f}'(x)$, [2]

stating clearly the equations of the asymptotes and the coordinates of the points corresponding to A, B and C where appropriate.

2 **(i)** Without using a calculator, solve the inequality $\dfrac{x}{x^2 - 5} \le 0$, giving your answer in exact form. [2]

(ii) Hence, find the set of values of x for which $\dfrac{\sqrt{x}}{x - 5} \le 0$. [2]

3 Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point P on OA is such that $OP : PA = 2 : 3$, and the point Q on OB is such that $OQ : QB = 1 : 2$. Given that M is the mid-point of PQ, state the position vector of M in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]
Show that the area of triangle OMP can be written as $k\left|\mathbf{a} \times \mathbf{b}\right|$, where k is a constant to be determined. [4]

4 Find

(a) $\int \cos(\ln x)\ \mathrm{d}x$, [3]

(b) $\int \dfrac{1 - 2x}{2x^2 + 1}\ \mathrm{d}x$. [3]

5 It is given that $z=\sqrt{3}+\mathrm{i}$ and $w=-1+\mathrm{i}$.

(i) Without using a calculator, find an exact expression for $\dfrac{z^2}{w^*}$. Give your answer in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$. [4]

(ii) Find the exact value of the real number q such that $\arg\left(1-\dfrac{q}{z}\right)=\dfrac{\pi}{12}$. [3]

6 It is given that $y=\ln\left(3+\mathrm{e}^x\right)$.

(i) Show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(ii) By differentiating the above result, find the first four non-zero terms of the Maclaurin series for y. Give the coefficients in exact form. [3]

(iii) Hence find the Maclaurin series for $\dfrac{\mathrm{e}^{-2x}}{3+\mathrm{e}^{-2x}}$, up to and including the term in x^2. [2]

7 The curve C has equation $y=\dfrac{ax^2+bx-8}{x-2}$, where a and b are constants. It is given that C has asymptote $y = 3 - 2x$.

(i) Find the value of a and show that $b = 7$. [3]

(ii) Sketch C, stating clearly the equations of any asymptotes and the coordinates of any stationary points and any points of intersection with the axes. [3]

(iii) By drawing another suitable curve on the same diagram, deduce the number of real roots of the equation $\left(-2x^2+7x-8\right)^2-25\left(x-2\right)^3=0$. [3]

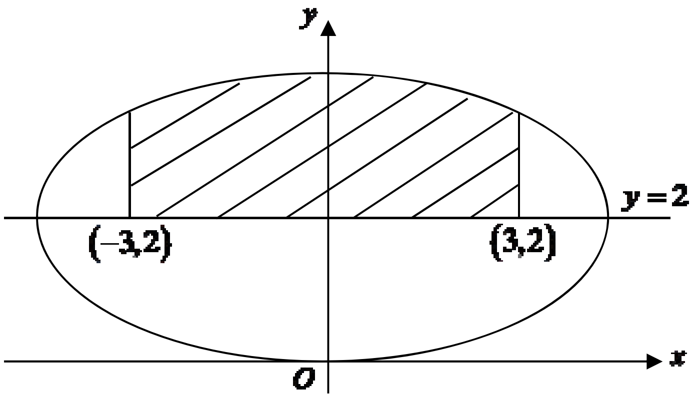
8 Emily has 1016 toy bricks.

(i) Emily wishes to build a brick structure with one brick in the first row, two bricks in the second row, three bricks in the third row and so on. What is the maximum number of rows that she can build and how many bricks will be left unused? [4]

(ii) Emily keeps all her 1016 bricks in $\left(2k-1\right)$ bags of different sizes. She packs m bricks into the smallest bag. For each subsequent bag, she packs double the number of bricks she packs in the previous bag. Given that she has 64 bricks in the kth bag, find the value of m and the number of bags. [5]

9 **(a)** By using the substitution $x = 3\sec\theta$, evaluate $\displaystyle\int_{3\sqrt{2}}^{6} \frac{3x+1}{\sqrt{x^2-9}}\,\mathrm{d}x$ exactly. [5]

(b)



The diagram shows an ellipse with equation $\dfrac{x^2}{16} + \dfrac{(y-2)^2}{4} = 1$.

(i) Find the area of the shaded region, giving your answer correct to 3 decimal places. [2]

(ii) Find the exact volume of the solid generated when the shaded region is rotated 180° about the y-axis. [4]

10 **(a)** By using the substitution $z = x - y$, solve the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x-y-1}{x-y+1}$.

Find the particular solution for which $y = 1$ when $x = 1$. [4]

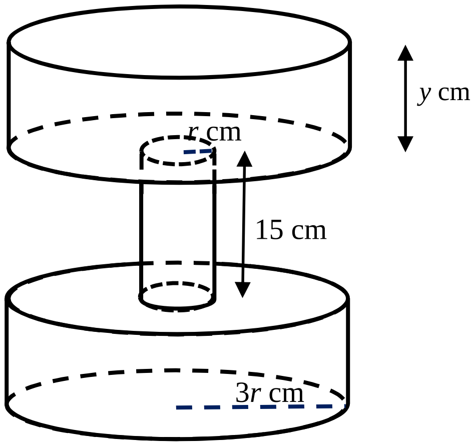
(b) A sky diver jumped out of an aeroplane over a certain mountainous valley with zero speed and t seconds later, the speed of his descent was v metres per second. He experienced gravitational force and air resistance which affect v. Gravity would increase his speed by a constant 10 metres per second^2 and the air resistance would decrease his speed at a rate proportional to the square of his speed. It is given that when his speed reaches 50 metres per second, the rate of change of his speed is 7.5 metres per second^2.

By setting up and solving a differential equation, show that

$$v = \frac{100\left(1-\mathrm{e}^{-mt}\right)}{1+\mathrm{e}^{-mt}},$$ where m is a constant to be found. [7]

Describe briefly what his speed would be after he had descended for a long time and just before he deployed his parachute. [1]

11



A plastic water dumbbell consists of a cylinder as a handle and two cylinders as the weights. The handle has a radius r cm and height 15 cm. Each weight has radius $3r$ cm and height y cm. The dumbbell is made of plastic of negligible thickness and the volume of the dumbbell is a fixed value k cm^3.

(i) Given that $r = r_1$ is the value of r which gives the minimum external surface area, show that r_1 satisfies the equation $102\pi r^3 + 30\pi r^2 - k = 0$. [6]

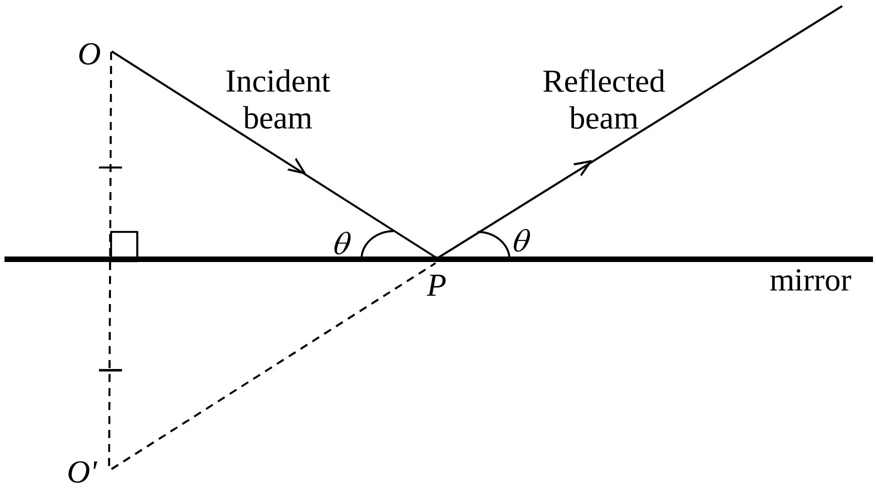
(ii) Find the value of r_1 if $k = 450$. [1]

(iii) It is given instead that $r = 2$ and $y = 7$. Water is pumped into an empty dumbbell through an opening from the top at a rate of 15 cm^{3}s^{-1}. Find the exact rate at which the depth of the water is increasing after 1 minute. [5]

[Question 12 is printed on the next page.]

12 Laser (Light Amplification by Stimulated Emission of Radiation) has many applications including medicine, data storage, military and industrial uses. It has the property of spatial coherence, which allows the laser beam to stay narrow over long distances. When a laser beam is projected onto a mirror at an angle, it reflects off the mirror at the same angle.

An engineer is designing a device that does industrial cutting using a laser beam. To make the device compact, the device has a mirror to reflect the beam before it leaves the device. The laser beam source is located at the origin *O*. It projects an incident beam with direction vector $\mathbf{i}+2\mathbf{j}+\mathbf{k}$. The beam hits the mirror at the point *P* with angle θ. The mirror has an equation $-x+2y+3z=12$.



(i) Find the acute angle θ that the beam makes with the mirror. [2]

(ii) By finding *O'*, the image of *O* in the mirror, find a vector equation of the line that the reflected beam is on. [7]

(iii) The engineer plans to install a sensor at (3, 1, 0) to monitor the heat produced by the laser. For the sensor to work properly, the sensor must be less than 2 units away from either the incident or the reflected beam. Determine if the sensor will work properly. [4]

~ ~ End of paper ~ ~