2017 H2 Prelim P2 Solutions

Qns	Solutions
1	$z^3-4(1+\mathrm{i})z^2+(-2+9\mathrm{i})z+5-\mathrm{i}=0$ $\left(z-(1+\mathrm{i})\right)\left(Az^2+Bz+C\right)=0$ By comparing coefficients, $z^3: A=1$ $z^0: -(1+\mathrm{i})C=5-\mathrm{i}$ $\Rightarrow C=\dfrac{5-\mathrm{i}}{-(1+\mathrm{i})}=-2+3\mathrm{i}$ $z^2: B-(1+\mathrm{i})=-4(1+\mathrm{i})$ $\Rightarrow B=-3(1+\mathrm{i})$ $\Rightarrow \left(z-(1+\mathrm{i})\right)\left(z^2-3(1+\mathrm{i})z-2+3\mathrm{i}\right)=0$ Solving $\left(z^2-3(1+\mathrm{i})z-2+3\mathrm{i}\right)=0$: $z=\dfrac{-\left(-3(1+\mathrm{i})\right)\pm\sqrt{\left(-3(1+\mathrm{i})\right)^2-4(1)\left(-2+3\mathrm{i}\right)}}{2(1)}$ $=\dfrac{3+3\mathrm{i}\pm\sqrt{8+6\mathrm{i}}}{2}$ $=\dfrac{3+3\mathrm{i}\pm(3+\mathrm{i})}{2}=3+2\mathrm{i}$ or i $\therefore$ other 2 roots are $z=3+2\mathrm{i}$ or $z=\mathrm{i}$
2(i)	$x=\cos t$ $y=\dfrac{1}{2}\sin 2t$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}=\dfrac{\cos 2t}{-\sin t}$ $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right\|_{t=p}=\dfrac{\cos 2p}{-\sin p}\Rightarrow$ gradient of normal $=\dfrac{\sin p}{\cos 2p}$ $\Rightarrow$ equation of normal at $\left(\cos p,\dfrac{1}{2}\sin 2p\right)$: $y-\dfrac{1}{2}\sin 2p=\dfrac{\sin p}{\cos 2p}\left(x-\cos p\right)$ $y=\dfrac{\sin p}{\cos 2p}x+\dfrac{1}{2}\left(\sin 2p-\tan 2p\right)$ $\Rightarrow$ equation of normal at $t=\dfrac{2\pi}{3}$: $y=\dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}x+\dfrac{1}{2}\left(-\dfrac{\sqrt{3}}{2}-\sqrt{3}\right)\Rightarrow y=-\sqrt{3}x-\dfrac{1}{4}\left(3\sqrt{3}\right)$....(1) To find point of intersection of normal and C (when the normal cuts C again),

Substitute $x=\cos t$ and $y=\dfrac{1}{2}\sin 2t$ into (1):

$$\frac{1}{2}\sin 2t=-\sqrt{3}\left(\cos t\right)-\frac{1}{4}\left(3\sqrt{3}\right)$$

$$\frac{1}{2}\sin 2t+\sqrt{3}\left(\cos t\right)+\frac{1}{4}\left(3\sqrt{3}\right)=0$$

From GC,

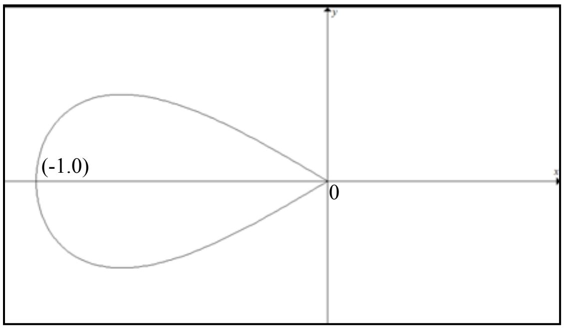
$t=2.094395$ (corresponds to $t=\dfrac{2\pi}{3}$)

or $t=3.495928$

$\Rightarrow$ point normal meets C again:

$$\left(\cos\left(3.495928\right),\frac{1}{2}\sin\left(2\left(3.495928\right)\right)\right)=\left(-0.938,0.325\right)$$

2(ii)



2(iii)

Method 1:

$x=\cos t\Rightarrow x^2=\cos^2 t$

$y=\dfrac{1}{2}\sin 2t\Rightarrow y=\sin t\cos t$

$\Rightarrow y^2=\sin^2 t\cos^2 t=\left(1-\cos^2 t\right)\cos^2 t=\left(1-x^2\right)x^2$

$\therefore$ Cartesian equation: $y^2=\left(1-x^2\right)x^2$

Method 2:

$x=\cos t\Rightarrow \cos t=\dfrac{x}{1},\sin t=\dfrac{\pm\sqrt{1-x^2}}{1}\quad\left(\because \dfrac{\pi}{2}\le t\le\dfrac{3\pi}{2}\right)$

$y=\dfrac{1}{2}\sin 2t\Rightarrow y=\sin t\cos t=\pm\sqrt{1-x^2}\left(x\right)$

$\therefore$ Cartesian equation: $y=\pm x\sqrt{1-x^2}$

Method 3:

$x=\cos t\Rightarrow x^2=\cos^2 t\Rightarrow \cos 2t=2\cos^2 t-1=2x^2-1$

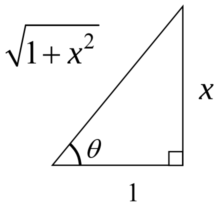
$y=\dfrac{1}{2}\sin 2t\Rightarrow \sin 2t=2y$

Using $\sin^2 2t+\cos^2 2t=1$,

$\left(2y\right)^2+\left(2x^2-1\right)^2=1$

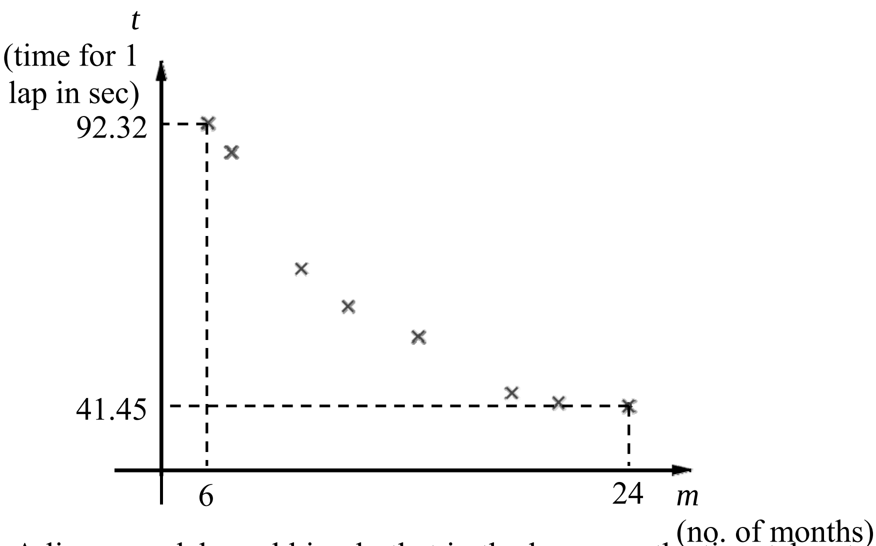
$\therefore$ Cartesian equation: $4y^2+\left(2x^2-1\right)^2=1$

	Method 1: $\int_{-1}^{0} \pi y^2 \mathrm{d}x$ $= \pi \int_{-1}^{0} \left(1 - x^2\right) x^2 \mathrm{d}x$ $= \pi \int_{-1}^{0} x^2 - x^4 \mathrm{d}x = \pi \left[\frac{x^3}{3} - \frac{x^5}{5} \right]_{-1}^{0} = \frac{2}{15}\pi$ units^3 Method 2 (not advised): $x = \cos t \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = -\sin t$ when $x = 0,\ t = \frac{\pi}{2}, \frac{3\pi}{2}$ (can use either) when $x = -1,\ t = \pi$ $\int_{-1}^{0} \pi y^2 \mathrm{d}x$ $= \pi \int_{\pi}^{\frac{3\pi}{2}} \left(\frac{1}{2} \sin 2t \right)^2 \left(-\sin t\right)\, \mathrm{d}t$ $= -\pi \int_{\pi}^{\frac{3\pi}{2}} \left(\sin t \cos t\right)^2 \left(\sin t\right)\, \mathrm{d}t$ $= -\pi \int_{\pi}^{\frac{3\pi}{2}} \sin^2 t \cos^2 t \left(\sin t\right)\, \mathrm{d}t$ $= -\pi \int_{\pi}^{\frac{3\pi}{2}} \left(1 - \cos^2 t\right) \cos^2 t \left(\sin t\right)\, \mathrm{d}t$ $= -\pi \int_{\pi}^{\frac{3\pi}{2}} \left(\cos^2 t - \cos^4 t\right)\left(\sin t\right)\, \mathrm{d}t$ $= -\pi \left(-\int_{\pi}^{\frac{3\pi}{2}} \left(\cos t\right)^2 \left(-\sin t\right)\, \mathrm{d}t + \int_{\pi}^{\frac{3\pi}{2}} \left(\cos t\right)^4 \left(-\sin t\right)\, \mathrm{d}t \right)$ $= -\pi \left(-\left[\frac{\left(\cos t\right)^3}{3} \right]_{\pi}^{\frac{3\pi}{2}} + \left[\frac{\left(\cos t\right)^5}{5} \right]_{\pi}^{\frac{3\pi}{2}} \right)$ $= -\pi \left(-0 - \frac{1}{3} + 0 + \frac{1}{5} \right) = \frac{2}{15}\pi$ units^3
3(i)	$\int \frac{x}{(1+x^2)^2}\, \mathrm{d}x = \frac{1}{2} \int \frac{2x}{(1+x^2)^2}\, \mathrm{d}x = -\frac{1}{2(1+x^2)} + c$

3(ii)	$x=\tan\theta \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}\theta}=\sec^2\theta$ $\displaystyle\int \frac{1}{(1+x^2)^2}\,\mathrm{d}x=\int \frac{1}{\left(1+\tan^2\theta\right)^2}\sec^2\theta\,\mathrm{d}\theta$ $\displaystyle=\int \frac{1}{\sec^2\theta}\,\mathrm{d}\theta=\int \cos^2\theta\,\mathrm{d}\theta$ $\displaystyle=\int \frac{\cos 2\theta+1}{2}\,\mathrm{d}\theta=\frac{1}{2}\left(\frac{\sin 2\theta}{2}+\theta\right)$ $=\frac{1}{2}\left(\sin\theta\cos\theta+\theta\right)+c$ $=\frac{1}{2}\left(\frac{x}{1+x^2}+\tan^{-1}x\right)+c$ $x=\tan\theta$ $\sin\theta=\frac{x}{\sqrt{1+x^2}}$ $\cos\theta=\frac{1}{\sqrt{1+x^2}}$ 
3(iii)	$\displaystyle\int \frac{x^2}{(1+x^2)^2}\,\mathrm{d}x=\int \frac{x^2+1-1}{(1+x^2)^2}\,\mathrm{d}x=\int \frac{1}{1+x^2}-\frac{1}{\left(1+x^2\right)^2}\,\mathrm{d}x$ $=\tan^{-1}x-\frac{1}{2}\left(\frac{x}{1+x^2}+\tan^{-1}x\right)+c$ $=\frac{1}{2}\left(\tan^{-1}x-\frac{x}{1+x^2}\right)+c$
3(iv)	$\displaystyle\int \frac{x^2+2x+5}{(1+x^2)^2}\,\mathrm{d}x=\int \frac{x^2}{(1+x^2)^2}+\frac{2x}{(1+x^2)^2}+\frac{5}{(1+x^2)^2}\,\mathrm{d}x$ $=\frac{1}{2}\left(\tan^{-1}x-\frac{x}{1+x^2}\right)+2\left(-\frac{1}{2(1+x^2)}\right)+5\left(\frac{1}{2}\left(\frac{x}{1+x^2}+\tan^{-1}x\right)\right)+c$ $=3\tan^{-1}x+\frac{2x-1}{1+x^2}+c$
4(a) (i)	$\mathbf{d}=\cos 60^\circ\mathbf{i}+\cos 60^\circ\mathbf{j}+\cos\gamma\mathbf{k}$ $\cos^2 60^\circ+\cos^2 60^\circ+\cos^2\gamma=1$ $\Rightarrow \cos^2\gamma=1-\frac{1}{4}-\frac{1}{4}=\frac{1}{2}$ $\Rightarrow \cos\gamma=\frac{1}{\sqrt{2}}$ $(\because \gamma$ is acute$)$ $\mathbf{d}=\frac{1}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}+\frac{1}{\sqrt{2}}\mathbf{k} \mathbin{/\!/} \mathbf{i}+\mathbf{j}+\sqrt{2}\mathbf{k}$
(a)(ii)	$m:\mathbf{r}=\begin{pmatrix}2\\-1\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}$ $\left(\begin{pmatrix}2\\-1\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}-\begin{pmatrix}3\\2\\0\end{pmatrix}\right)\cdot\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}=0\Rightarrow\left(\begin{pmatrix}-1\\-3\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}\right)\cdot\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}=0$ $\therefore (-1-3)+\lambda(1^2+1^2+\sqrt{2}^2)=0\Rightarrow\lambda=1$ Therefore position vector of point is $\begin{pmatrix}2\\-1\\0\end{pmatrix}+\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}=\begin{pmatrix}3\\0\\\sqrt{2}\end{pmatrix}$

	Coordinates = $\left(3,0,\sqrt{2}\right)$ <u>OR</u> $\overrightarrow{AN}=\left(\overrightarrow{AP}\cdot\hat{\mathbf{d}}\right)\hat{\mathbf{d}}=\frac{\left(\begin{pmatrix}3\\2\\0\end{pmatrix}-\begin{pmatrix}2\\-1\\0\end{pmatrix}\right)\cdot\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}}{\sqrt{1+1+2}}\frac{\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}}{\sqrt{1+1+2}}=\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}$ $\therefore\overrightarrow{ON}=\overrightarrow{OA}+\overrightarrow{AN}=\begin{pmatrix}2\\-1\\0\end{pmatrix}+\begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}=\begin{pmatrix}3\\0\\\sqrt{2}\end{pmatrix}$
4(b)	$l:\frac{x-a}{2}=\frac{y-1}{b}=-\frac{z}{2}\Rightarrow l:\mathbf{r}=\begin{pmatrix}a\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\b\\-2\end{pmatrix}$ $\begin{pmatrix}2\\b\\-2\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}=0\Rightarrow 2+2b-4=0\Rightarrow b=1$ $\begin{pmatrix}a\\1\\0\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}=5\Rightarrow a+2=5\Rightarrow a=3$
(b)(i)	p_2 perpendicular to $p_1\ \Rightarrow\mathbf{n}_1\ //\ p_2$ $p_2:\mathbf{r}=\begin{pmatrix}3\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}+\mu\begin{pmatrix}1\\2\\2\end{pmatrix}$ $\begin{pmatrix}2\\1\\-2\end{pmatrix}\times\begin{pmatrix}1\\2\\2\end{pmatrix}=\begin{pmatrix}6\\-6\\3\end{pmatrix}//\begin{pmatrix}2\\-2\\1\end{pmatrix}$ $p_2:\mathbf{r}\cdot\begin{pmatrix}2\\-2\\1\end{pmatrix}=\begin{pmatrix}3\\1\\0\end{pmatrix}\begin{pmatrix}2\\-2\\1\end{pmatrix}=4$
(b)(ii)	$\frac{1}{\sqrt{9}}\left\|\begin{pmatrix}x-3\\y-1\\z\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}\right\|=\frac{1}{\sqrt{9}}\left\|\begin{pmatrix}x-3\\y-1\\z\end{pmatrix}\cdot\begin{pmatrix}2\\-2\\1\end{pmatrix}\right\|$ $\|x-3+2(y-1)+2z\|=\|2(x-3)-2(y-1)+z\|$ $\Rightarrow x+2y+2z-5=2x-2y+z-4$ $\Rightarrow x-4y-z=-1$ *or* $\Rightarrow x+2y+2z-5=-(2x-2y+z-4)$ $\Rightarrow 3x+3z=9\Rightarrow x+z=3$
5 (i)	Number of ways = (3 – 1)!·5!·4!·3! = 34 560
5 (ii)	Number of ways = N(5 bowlers together) + N(4 canoeists together) – N(5 bowlers together & 4 canoeists together) = 8!·5! + 9!·4! – 5!·5!·4! = 4 838 400 + 8 709 120 – 345 600 = 13 201 920

5 (iii)	Number of ways = N(Total) – N(0 bowlers) – N(0 canoeists) – N(0 footballers) $= {}^{12}C_8 - 0 - {}^{8}C_8 - {}^{9}C_8 = 485$
6 (i)	$\mathrm{P}(X=2)=\mathrm{P}(A{*}{*}F{*},{*}A{*}{*}F)=2\left(\dfrac{2\times 3!}{5!}\right)=\dfrac{1}{5}=0.2$ (shown)
6 (ii)	$\mathrm{P}(X=0)=\mathrm{P}(AF{*}{*}{*},{*}AF{*}{*},{*}{*}AF{*},{*}{*}{*}AF)=4\left(\dfrac{2\times 3!}{5!}\right)=\dfrac{2}{5}$ $=0.4$ $\mathrm{P}(X=1)=\mathrm{P}(A{*}F{*}{*},{*}A{*}F{*},{*}{*}A{*}F)=3\left(\dfrac{2\times 3!}{5!}\right)=\dfrac{3}{10}=0.3$ $\mathrm{P}(X=3)=\mathrm{P}(A{*}{*}{*}F)=\left(\dfrac{2\times 3!}{5!}\right)=\dfrac{1}{10}=0.1$ <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P(X = x)</td><td>0.4</td><td>0.3</td><td>0.2</td><td>0.1</td></tr></table>
6 (iii)	$\mathrm{E}(X)=\sum_{all\ x} x\mathrm{P}(X=x)=0(0.4)+1(0.3)+2(0.2)+3(0.1)=1$ $\mathrm{E}(X-1)^2=\sum_{all\ x}(x-1)^2\mathrm{P}(X=x)=1(0.4)+0(0.3)+1(0.2)+4(0.1)=1$ $\mathrm{Var}(X)=\mathrm{E}(X-\mu)^2=\mathrm{E}(X-1)^2=1$
7(a) (i)	Let X be the random variable "pH value of a randomly chosen bottle of shampoo". Unbiased estimate of population mean $=\bar{x}$ $=\dfrac{178.2}{30}$ $=\underline{5.94}$ Unbiased estimate of population variance $=s^2$ $=\dfrac{1}{29}\left(1238.622-\dfrac{178.2^2}{30}\right)$ $=6.21083$ $=\underline{6.21}$ (3 s.f.) To test $\mathrm{H}_0: \mu=5.5$ against $\mathrm{H}_1: \mu\neq 5.5$ at 10% significance level Under H_0, since $n=30>20$ is large, $\bar{X}\sim \mathrm{N}\left(5.5,\dfrac{6.21083}{30}\right)$ approx. by Central Limit Theorem Test statistic $Z=\dfrac{\bar{X}-5.5}{\sqrt{\dfrac{6.21083}{30}}}\sim \mathrm{N}(0,1)$ approx. Value of test statistic $z=\dfrac{5.94-5.5}{\sqrt{\dfrac{6.21083}{30}}}=0.967$ (3 s.f.) **Either** Since $-1.64<0.967<1.64$, z lies <u>outside</u> the critical region $\Rightarrow$ Do <u>not</u> reject H_0 **Or** p-value$=0.334>0.1 \Rightarrow$ Do <u>not</u> reject H_0 $\therefore$ There is insufficient evidence at 10% significance level to conclude that the mean pH

	value of the shampoo is not 5.5. *[Note: No B1 mark if students write* $Z=\dfrac{5.94-5.5}{s/\sqrt{n}}\sim \mathrm{N}(0,1)$ *]*
(a)(ii)	It is not necessary to assume X is normally distributed. As the sample size is large, by Central Limit Theorem, $\bar{X}$ is approximately normally distributed.
(b)(i)	Critical region of the test is $z<-1.64485$ or $z>1.64485$ $\Rightarrow$ $\underline{z<-1.64 \text{ or } z>1.64}$ (3 s.f.)
(b)(ii)	Value of test statistic $z=\dfrac{5.94-5.5}{\sqrt{\dfrac{6.5}{n}}}=\dfrac{0.44\sqrt{n}}{\sqrt{6.5}}$ For a favourable outcome at 10% significance level, do not reject H_0 $\Rightarrow$ z lies <u>outside</u> the critical region $\Rightarrow -1.64485<\dfrac{0.44\sqrt{n}}{\sqrt{6.5}}<1.64485$ $\Rightarrow \dfrac{-1.64485\sqrt{6.5}}{0.44}<\sqrt{n}<\dfrac{1.64485\sqrt{6.5}}{0.44}$ $\Rightarrow n<\left(\dfrac{1.64485\sqrt{6.5}}{0.44}\right)^2$ $\Rightarrow n<90.837$ Hence largest $n=\underline{90}$
8(i)	 A linear model would imply that in the long run, the time taken to swim a lap would be negative, which is unrealistic. *(Note: Extrapolation is not accepted as a reason, as the question isn't looking for a reason based on the data obtained.)*
8(ii)	Using GC, for $C=37$, $r=\underline{-0.992555}$

8(iii)	The most appropriate value for C is <u>38</u>, as the magnitude of its corresponding value of r is closest to 1.
8(iv)	From GC, least squares regression line of $\ln(t-38)$ on m is $\ln(t-38)=5.01236-0.16349m$ $\Rightarrow \underline{\ln(t-38)=5.01-0.163m}$ (3 s.f.) $C=38$ is the fastest time that a student can expect to complete a lap of breaststroke after spending a long time at the swim school. (Making t the subject in the equation of the regression line gives us $t=38+e^{5.01-0.163m}$, so as $m\to\infty$, $t\to 38$.)
8(v)	When $m=9$, $t=38+e^{5.01236-0.16349(9)}$ $=72.50$ (2 d.p.) A timing of 60.33 seconds is well below the expected timing of 72.50 seconds. Therefore, we can say that the student is exceptionally strong in his/her swimming ability.
8(vi)	The 8 randomly selected students might have been of different genders and ages. To make the results fairer, data could be collected separately based on genders and age ranges.
9 (a)	Let X be the random variable 'number of defective articles in sample of 10'. $X \sim \mathrm{B}(10, 0.065)$ $\mathrm{P}(\text{accepting a batch})=\mathrm{P}(X\le 1)=0.86563=0.866$
(i)	P(batch eventually accepted) $=(0.86563)^2+2(0.86563)(1-0.86563)(0.86563)$ $=0.95069$ $=0.951$
(ii)	Let N be the number of articles examined per batch. $N=\begin{cases}20 & \text{if both findings agree}\\ 30 & \text{otherwise}\end{cases}$ $\mathrm{P}(N=20)=(0.86563)^2+(1-0.86563)^2=0.76737$ $\mathrm{P}(N=30)=1-0.76737=0.23263$ $\therefore\ \mathrm{E}(N)=20(0.76737)+30(0.23263)=22.3$
9 (b)	Let Y be the random variable 'number of defective articles in a sample of 10'. $Y \sim \mathrm{B}(10, p)$ $A=\mathrm{P}(Y\le 1)+\mathrm{P}(Y=2)\cdot\mathrm{P}(Y=0)$ $={}^{10}C_0p^0(1-p)^{10}+{}^{10}C_1p^1(1-p)^9+{}^{10}C_2p^2(1-p)^8\cdot{}^{10}C_0p^0(1-p)^{10}$ $=(1-p)^{10}+10p(1-p)^9+45p^2(1-p)^{18}$ $=(1+9p)(1-p)^9+45p^2(1-p)^{18}$ (shown)
9 (b)	Let W be the random variable 'number of acceptable batches, out of 100 inspected'. $W\sim \mathrm{B}(100, A)$ $\mathrm{P}(W>80)=0.98\Rightarrow \mathrm{P}(W\le 80)=0.02$ By GC, $A=0.876235$ $\therefore\ A=(1+9p)(1-p)^9+45p^2(1-p)^{18}=0.87624$ By GC, $p=0.08$
10 (a) (i)	Let X be the random variable 'marks of an examination'. By GC, $\mathrm{P}(X>100)=0.0359$ if $X\sim \mathrm{N}(73, 15^2)$ i.e., there are 3.59% of the students scoring more than the maximum mark of 100, which is impossible.

10 (a) (ii)	Since $n = 50 \geq 20$ is large, by Central Limit Theorem, $$\overline{X} \sim \mathrm{N}(73, \frac{15^2}{50}) \text{ approximately.}$$ $\therefore$ $\mathrm{P}(70 < \overline{X} < 75) = 0.748$
10 (b)	Let Y be the random variable 'marks of a school examination'. $Y \sim \mathrm{N}(\mu, \sigma^2)$ $\mathrm{P}(Y < 51) = 0.8$ $\mathrm{P}(Z < \frac{51-\mu}{\sigma}) = 0.8$ $\frac{51-\mu}{\sigma} = 0.84162$ $\mu + 0.84162\sigma = 51$ $\mathrm{P}(\mu - 5.4 < Y < \mu + 5.4) = 0.5$ $\mathrm{P}(\frac{-5.4}{\sigma} < Z < \frac{5.4}{\sigma}) = 0.5$ $\mathrm{P}(Z < -\frac{5.4}{\sigma}) = 0.25$ $-\frac{5.4}{\sigma} = -0.67449$ $\therefore \quad \sigma = 8.01$ $\therefore \quad \mu = 51 - 0.84162(8.0061) = 44.3$
10 (c) (i)	Let M be the random variable 'marks of another school examination'. $M \sim \mathrm{N}(52, 13^2)$ $\mathrm{P}(50 < M) = 0.56113$ Number of passes $=$ (total candidature) $\times 0.56113 = 289$ $\therefore$ total candidature $= 289 \div 0.56113 = 515$
10 (c) (ii)	$\mathrm{P}(\lvert M - 52 \rvert < m) > 0.9 \Rightarrow \mathrm{P}(52 - m < M < 52 + m) > 0.9$ where $M \sim \mathrm{N}(52, 13^2)$ $\Rightarrow \mathrm{P}(M < 52 - m) < 0.05$ $\Rightarrow \quad 52 - m < 30.6$ $\Rightarrow \quad m > 21.4$ $\therefore$ Smallest integral value of $m = 22$