Name:______________ Class:__________

Tutor:______________

JURONG JUNIOR COLLEGE

Preliminary Examinations

MATHEMATICS
Higher 2

9758 /02

13 September 2017

Paper 2

3 hours

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together, with the cover page in front.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

[Turn over

Section A: Pure Mathematics [40 marks]

1 It is given that $y = \ln(1+\sin x)$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. Show that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\mathrm{e}^{-y}$. [4]

(ii) Express $\frac{\mathrm{d}^4 y}{\mathrm{d}x^4}$ in terms of $\frac{\mathrm{d}y}{\mathrm{d}x}$ and e^{-y}. [3]

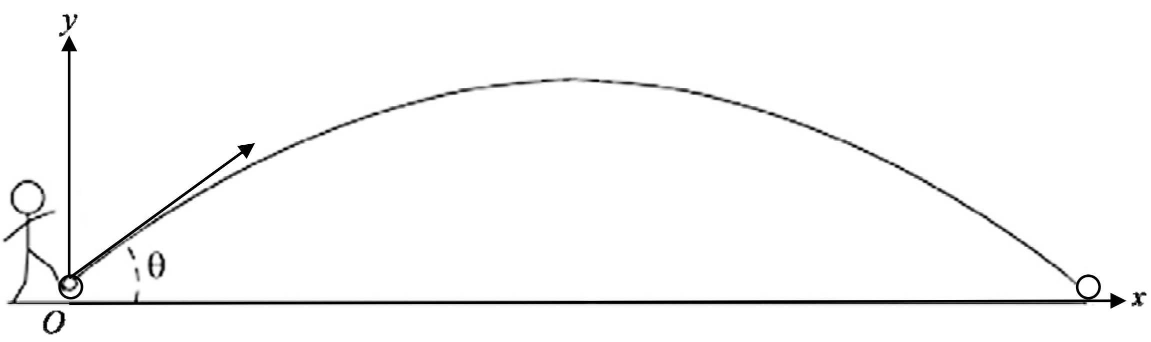
(iii) Hence, find the first four non-zero terms in the Maclaurin series for $\ln(1+\sin x)$. [3]

2 John kicked a ball at an acute angle θ made with the horizontal, and it moved in a projectile motion, as shown in the diagram. The initial velocity of the ball is u m s^{-1}. Taking John's position where he kicked the ball as the origin O, the ball's displacement curve is given by the parametric equations:

horizontal displacement, $x = ut\cos\theta,$

vertical displacement, $y = ut\sin\theta - 5t^2,$

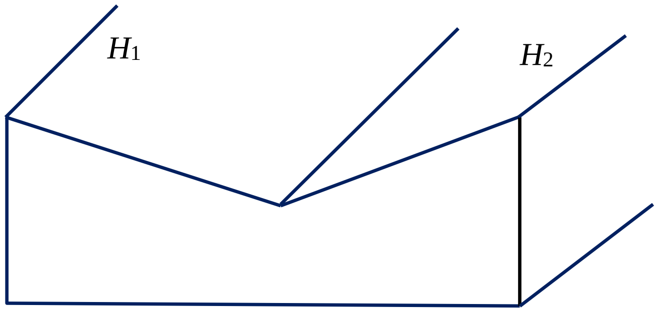
where u and θ are constants and t is the time in seconds after the ball is kicked.



(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \tan\theta - \frac{10}{u} t\sec\theta.$ [2]

(ii) If the initial velocity of the ball is 30 m s^{-1}, find the equation of the tangent to the displacement curve at the point where $t = \frac{1}{2}$, giving your answer in the form $y = (a\tan\theta + b\sec\theta)x + c$, where a, b and c are constants to be determined. [3]

3 Peter is using equations of planes to model two hillsides that meet along a river. The river is modelled by the line where the two planes meet.



One of the hillsides, H_1, contains the points A, B and C with coordinates (3, 0, 2), (1, 0, 3) and $(2, -3, 5)$ respectively. The point A is on the river. The other hillside H_2 has equation $2x - y + kz = 14$, where k is a constant.

(i) Find a vector equation of H_1 in scalar product form. [4]

(ii) Show that $k = 4$ and deduce that point B is also on the river. [3]

(iii) Write down a cartesian equation of the river. [1]

(iv) Show that B is the point on the river that is nearest to C. Hence find the exact distance from C to the river. [3]

(v) Find the acute angle between BC and H_2. [2]

4 To determine whether the amount of preservatives in a particular brand of bread meets the safety limit of preservatives present, the Food Regulatory Authority (FRA) conducted a test to examine the growth of fungus on a piece of bread over time after its expiry date.

The piece of bread has a surface area of 100 cm^2. The staff from FRA estimate the amount of fungus grown and the rate at which it is growing by finding the area of the piece of bread the fungus covers over time. They believe that the area, A cm^2, of fungus present t days after the expiry date is such that the rate at which the area is increasing is proportional to the product of the area of the piece of bread covered by the fungus and the area of the bread not covered by the fungus. It is known that the initial area of fungus is 20 cm^2 and that the area of fungus is 40 cm^2 five days after the expiry date.

(i) Write down a differential equation expressing the relation between A and t. [1]

(ii) Find the value of t at which 50% of the piece of bread is covered by fungus, giving your answer correct to 2 decimal places. [6]

(iii) Given that this particular brand of bread just meets the safety limit of the amount of preservatives present when the test is concluded 2 weeks after the expiry date, find the range of values of A for any piece of bread of this brand to be deemed safe for human consumption in terms of the amount of preservatives present, giving your answer correct to 2 decimal places. [2]

(iv) Write the solution of the differential equation in the form $A = \mathrm{f}(t)$ and sketch this curve. [3]

Section B: Probability and Statistics [60 marks]

5 The probability distribution of a discrete random variable, X, is shown below.

x	1	2
$\mathrm{P}(X=x)$	a	b

Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ in terms of a. [5]

6 **(i)** Find the number of 3-digit numbers that can be formed using the digits 1, 2 and 3 when

(a) no repetitions are allowed, [1]

(b) any repetitions are allowed, [1]

(c) each digit may be used at most twice. [2]

(ii) Find the number of 4-digit numbers that can be formed using the digits 1, 2 and 3 when each digit may be used at most twice. [5]

7 At a canning factory, cans are filled with potato puree. The machine which fills the cans is set so that the volume of potato puree in a can has mean 420 millilitres. After the machine is recalibrated, a quality control officer wishes to check whether the mean volume has changed. A random sample of 30 cans of potato puree is selected and the volume of the puree in each can is recorded. The sample mean volume is $\overline{x}$ millilitres and the sample variance is 12 millilitres^2.

(i) Given that $\overline{x}=418.55$, carry out a test at the 1% level of significance to investigate whether the mean volume has changed. State, giving a reason, whether it is necessary for the volumes to have a normal distribution for the test to be valid. [6]

(ii) Use an algebraic method to calculate the range of values of $\overline{x}$, giving your answer correct to 2 decimal places, for which the result of the test at the 1% level of significance would be to reject the null hypothesis. [3]

8 In this question you should state clearly the values of the parameters of any normal distribution you use.

The mass of a tomato of variety A has normal distribution with mean 80 g and standard deviation 11 g.

(i) Two tomatoes of variety A are randomly chosen. Find the probability that one of the tomatoes has mass more than 90 g and the other has mass less than 90 g. [3]

The mass of a tomato of variety B has normal distribution with mean 70 g. These tomatoes are packed in sixes using packaging that weighs 15 g.

(ii) The probability that a randomly chosen pack of 6 tomatoes of variety B including packaging, weighs less than 450 g is 0.8463. Show that the standard deviation of the mass of a tomato of variety B is 6 g, correct to the nearest gram. [4]

(iii) Tomatoes of variety A are packed in fives using packaging that weighs 25 g. Find the probability that the total mass of a randomly chosen pack of variety A is greater than the total mass of a randomly chosen pack of variety B, using 6 g as the standard deviation of the mass of a tomato of variety B. [5]

9 A jar contains 5 identical balls numbered 1 to 5. A fixed number, n, of balls are selected and the number of balls with an even score is denoted by X.

(i) Explain how the balls should be selected in order for X to be well modelled by a binomial distribution. [2]

Assume now that X has the distribution $\mathrm{B}\left(n, \frac{2}{5}\right)$.

(ii) Given that $n = 10$, find $\mathrm{P}(X \geq 4)$. [2]

(iii) Given that the mean of X is 4.8, find n. [2]

(iv) Given that $\mathrm{P}(X = 0 \text{ or } 1) < 0.01$, write down an inequality for n and find the least value of n. [3]

Shawn and Arvind take turns to draw one ball from the jar at random. The first person who draws a ball with an even score wins the game. Shawn draws first.

(v) Show that the probability that Shawn wins the game is $\frac{3}{5}$ if the selection of balls is done without replacement. [2]

(vi) Find the probability that Shawn wins the game if the selection of balls is done with replacement. [2]

10 **(a)** Traffic engineers are studying the correlation between traffic flow on a busy main road and air pollution at a nearby air quality monitoring station. Traffic flow, x, is recorded automatically by sensors and reported each hour as the average flow in vehicles per hour for the preceding hour. The air quality monitoring station provides, each hour, an overall pollution reading, y, in a suitable unit (higher readings indicate more pollution). Data for a random sample of 8 hours are as follows.

Traffic flow, x	1796	1918	2120	2315	2368	2420	2588	2850
Pollution reading, y	1.0	2.2	3.5	4.2	4.3	4.5	4.9	5.5

(i) Draw the scatter diagram for these values, labelling the axes. [2]

It is thought that the pollution y can be modelled by one of the formulae

$$y = a + bx \qquad\qquad y^2 = c + dx$$

where a, b, c and d are constants.

(ii) Find the value of the product moment correlation coefficient between

(a) x and y,

(b) x and y^2. [2]

(iii) Use your answers to parts **(i)** and **(ii)** to explain which of $y = a + bx$ or $y^2 = c + dx$ is the better model. [2]

(iv) It is required to estimate the value of y for which x = 2000. Find the equation of a suitable regression line, and use it to find the required estimate. [2]

(v) The local newspaper carries a headline “Heavy traffic causes air pollution”. Comment briefly on the validity of this headline in the light of your results. [1]

(b) The diagram below shows an old research paper that has been partially destroyed. The surviving part of the paper contains incomplete information about some bivariate data from an experiment. Calculate the missing constant at the end of the equation of the second regression line. [3]