HWA CHONG INSTITUTION
2017 JC2 PRELIMINARY EXAMINATION

MATHEMATICS
Higher 2

9758/02

Paper 2

Tuesday **19 September 2017** **3 hours**

Additional materials: Answer paper
List of Formula (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and CT class on all the work you hand in, including the Cover Page.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Do not write anything on the List of Formula (MF26).

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question. At the end of the examination, place the completed cover page on top of your answer scripts and fasten all your work securely together with the string provided.

This question paper consists of 8 printed pages.

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MATHEMATICS

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Name: ____________________ **CT:** | 1 | 6 | | | |

COVER PAGE

1. Write your name, CT group and calculator model(s) in the spaces provided.
2. Arrange your answers in numerical order.
3. Detach this cover page and place it on top of your answer paper and fasten them securely together with the string provided.

For Examiner's Use			
Question No.	**Marks Obtained**	**Total Marks**	**Remarks**
1		6	
2		7	
3		13	
4		14	
5		5	
6		6	
7		7	
8		8	
9		12	
10		10	
11		12	
TOTAL		**100**	

Graphing Calculator Model: ____________________

Scientific Calculator Model: ____________________

Section A: Pure Mathematics [40 marks]

1 The sum, S_n, of the first n terms of a sequence u_1, u_2, u_3, … is given by

$$S_n = b - \frac{3a}{(n+1)!},$$

where a and b are constants.

(i) It is given that $u_1 = k$ and $u_2 = \frac{2}{3}k$, where k is a constant. Find a and b in terms of k. [3]

(ii) Find a formula for u_n in terms of k, giving your answer in its simplest form. [2]

(iii) Determine, with a reason, if the series $\sum_{r=1}^{\infty} u_r$ converges. [1]

2 The complex numbers z and w satisfy the following equations

$$2z + 3w = 20,$$

$$w - zw^* = 6 + 22\mathrm{i}.$$

(i) Find z and w in the form $a + b\mathrm{i}$, where a and b are real, $a \neq 0$. [5]

(ii) Show z and w on a single Argand diagram, indicating clearly their modulus. State the relationship between z and w with reference to the origin O. [2]

3 The function f is defined by

$$\mathrm{f} : x \mapsto \sqrt{3}\sin x + \cos x, \qquad x \in \mathbb{R}, \ -\pi < x < \tfrac{\pi}{6}.$$

(i) Express f in the form $R\sin(x+\alpha)$, where R and α are exact constants to be determined, $R > 0$, $0 \le \alpha \le \frac{\pi}{2}$. [2]

(ii) Sketch f, giving the exact coordinates of the turning point and the end-points. Deduce the exact range of f. [4]

(iii) The function g is defined by

$$\mathrm{g} : x \mapsto \frac{1}{2} - |x-1|, \qquad x \in \mathbb{R}, \ -\frac{5}{2} \le x \le \frac{1}{2}.$$

Explain why the composite function fg exists. Find the range of fg. [3]

(iv) The domain of f is restricted such that the function f^{-1} exists. Find the largest domain of f for which f^{-1} exists. Define f^{-1} in a similar form. [4]

4 Referred to the origin O, the position vector of a point A is $-\mathbf{i}+2\mathbf{j}+4\mathbf{k}$. A plane p contains A and is parallel to the vectors $4\mathbf{i}-2\mathbf{j}+\mathbf{k}$ and $2\mathbf{i}+\mathbf{k}$.

(i) Find a cartesian equation of p. [2]

(ii) A plane q has equation $x-2y+z=2$. Find a vector equation of the line l where p and q meet. [1]

A point B lies on l such that AB is perpendicular to l.

(iii) Find the position vector of B. [3]

(iv) Find the length of projection of AB on q. [2]

(v) A point C lies on q such that AC is perpendicular to q. Find the position vector of C. Hence find a cartesian equation of the line of reflection of AB in q. [6]

Section B: Statistics [60 marks]

5 The independent random variables X and Y are normally distributed with the same mean 7 but different variances $\mathrm{Var}(X)$ and $\mathrm{Var}(Y)$, respectively. It is given that $\mathrm{P}(X<10)=\mathrm{P}(Y>6)$.

(i) Show that $\mathrm{Var}(X)=9\mathrm{Var}(Y)$. [3]

(ii) If $\mathrm{Var}(Y)=1$, find $\mathrm{P}(X<9)$. [2]

6 A biased tetrahedral (4-sided) die has its faces numbered '-1', '0', '2' and '3'. It is thrown onto a table and the random variable X denotes the number on the face in contact with the table. The probability distribution of X is as shown.

x	-1	0	2	3
$\mathrm{P}(X=x)$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{4}$

(i) The random variable Y is defined by X_1+X_2, where X_1 and X_2 are 2 independent observations of X. Show that $\mathrm{P}(Y=2)=\frac{3}{16}$. [2]

(ii) In a game, a player pays \$2 to throw two such biased tetrahedral dice simultaneously on a table. For each die, the number on the face in contact with the table is the score of the die. The player receives \$16 if the maximum of the two scores is -1, and receives \$3 if the sum of the two scores is prime. For all other cases, the player receives nothing. Find the player's expected gain in the game. [4]

7 Mandy has 10 beads, of which 5 are spherical and 5 are cubical, each of different colours. She wishes to decorate a card by forming a circle using 8 of the 10 beads. Find the number of ways Mandy can arrange the beads if

(i) there are no restrictions, [1]

(ii) 3 particular beads are used and not all are next to one another, [3]

(iii) spherical beads and cubic beads must alternate. [3]

8 A man wishes to buy a 4-digit number lottery. He plays by randomly choosing any number from 0000 to 9999. It is assumed that each number is equally likely to be chosen. Find the probability that a randomly chosen 4-digit number has

(i) four different digits, [1]

(ii) exactly one of the first three digits is the same as the last digit, and the last digit is even, [3]

(iii) four different digits with the first digit greater than 6, given that the 4-digit number has odd and even digits that alternate. [4]

9 In a large shipment of glass stones used for the *Go* board game, a proportion p of the glass stones is chipped. The glass stones are sold in boxes of 361 pieces each. Let X denote the number of chipped glass stones in a box.

(i) Based on this context, state two assumptions in order for X to be well modelled by a binomial distribution. [2]

In the rest of the question, assume that X follows a binomial distribution.

(ii) It is known that the probability of a box containing at most 2 chipped glass stones is 0.90409. Find p. [2]

(iii) A box is deemed to be of inferior quality if it contains more than 2 chipped glass stones. Find the probability that, in a batch of 20 boxes of glass stones, there are more than 5 boxes of inferior quality in the batch. [3]

(iv) Each week, a distributor purchases 50 batches of glass stones, each batch consisting of 20 boxes of glass stones. A batch will be rejected if it contains more than 5 boxes of inferior quality. The distributor will receive a compensation of \$10 for each rejected batch in the first 20 weeks of a year, and a compensation of \$20 for each rejected batch in the remaining weeks of the year. Assuming that there are 52 weeks in a year, find the probability that the total compensation in a year is more than \$250. [5]

10 A large cohort of students sat for a mathematics examination. Based on selected data of the examination results, the following table shows y, the proportions of students who scored x marks.

x	20	30	40	50	60	70	80	90
y	0.00029	0.00174	0.00663	0.0161	0.0252	0.0252	0.0161	0.00663

(i) Draw a scatter diagram for these values, labelling the axes. [2]

(ii) Explain why, in this context, a linear model is not appropriate. [1]

It is decided to fit a model of the form $\ln y = -a(x-m)^2 + b$, where $a > 0$ and m is a suitable constant, to the data. The product moment correlation coefficient between $(x-m)^2$ and $\ln y$ is denoted by r. The table below gives values of r for some possible values of m.

m	62.5	65	67.5
r	– 0.9899292		– 0.9938968

(iii) Calculate the value of r for $m = 65$, giving your answer correct to 7 decimal places. [1]

(iv) Use the table and your answer in part **(iii)** to suggest with a reason which of 62.5, 65 or 67.5 is the most appropriate value for m. [1]

(v) Using the value of m found in part **(iv)**, calculate the values of a and b, and use them to predict the proportion of students who scored 45 marks.
Comment on the reliability of your prediction. [5]

11 Yummy Berries Farm produces blueberries and raspberries packed in boxes.

(a) Yummy Berries Farm claims that the mass, x grams, of each box of blueberries is no less than 125 grams. After receiving ~~a~~ complaints from consumers, the Consumers Association of Singapore (CASE) took a random sample of 50 boxes of blueberries from Yummy Berries Farm and the mass of each box was recorded. The data obtained are summarised in the table.

x (grams)	120	121	122	123	124	125	126	127	128	129	130
No. of boxes	3	6	6	6	3	10	3	4	6	2	1

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Test, at the 10% level of significance, whether Yummy Berries Farm has overstated its claim.

State, giving a reason, whether any assumptions about the masses of boxes of blueberries are needed in order for the test to be valid. [6]

(b) The masses of boxes of raspberries, each of y grams, are assumed to have a mean of 170 grams with standard deviation 15 grams. CASE took a random sample of n boxes of raspberries and the mean mass of boxes of raspberries from the sample is found to be 165 grams. A test is to be carried out at the 5% level of significance to determine if the mean mass of the boxes of raspberries is not 170 grams. Find the minimum number of boxes of raspberries to be taken for which the result of the test would be to reject the null hypothesis. [4]

End of Paper