Anderson Junior College
Preliminary Examination 2017
H2 Mathematics Paper 1 (9758/01) solutions with comments

1	Let x, y and z be the amounts Mr Tan invested in structured deposit account, bonds and an estate fund respectively. x+y+z = 25000 --- (1) y = z + 7000 --- (2) $\left[(1.02x)\times 1.02\right]+\left[(1.03y)\times 1.03\right]+\left[(1.045z)\times 1.045\right]=26300$ --- (3) Solving the 3 simultaneous equations : x = 13937.6 = 13938 (nearest dollars), y = 9031.2 ≈9031, z = 2031.2 ≈ 2031
2	Let $y=u\sqrt{1-3x^2}$ $\Rightarrow \quad \frac{dy}{dx}=\frac{du}{dx}\sqrt{1-3x^2}+u\left(\frac{1}{2}\right)\frac{-6x}{\sqrt{1-3x^2}}$ DE : $\quad \frac{\mathrm{d}y}{\mathrm{d}x}+\frac{3xy}{1-3x^2}-x+1=0$ $\Rightarrow \quad \frac{du}{dx}\sqrt{1-3x^2}-\frac{-3xu}{\sqrt{1-3x^2}}+\frac{3x}{1-3x^2}\left(u\sqrt{1-3x^2}\right)-x+1=0$ $\Rightarrow \quad \frac{du}{dx}\sqrt{1-3x^2}-\frac{3xu}{\sqrt{1-3x^2}}+\frac{3xu}{\sqrt{1-3x^2}}=x-1$ $\Rightarrow \quad \frac{du}{dx}\sqrt{1-3x^2}=x-1$ $\Rightarrow \quad \frac{du}{dx}=\frac{x}{\sqrt{1-3x^2}}-\frac{1}{\sqrt{1-3x^2}}$ $\Rightarrow \quad u=-\frac{1}{6}\int\frac{-6x}{\sqrt{1-3x^2}}dx-\int\frac{1}{\sqrt{1-3x^2}}dx$ $\Rightarrow \quad \frac{y}{\sqrt{1-3x^2}}=-\frac{1}{6}\left[2\sqrt{1-3x^2}\right]-\frac{\sin^{-1}\left(\sqrt{3}x\right)}{\sqrt{3}}+C$ $\Rightarrow \quad y=-\frac{1}{3}\left(1-3x^2\right)-\frac{\sqrt{1-3x^2}}{\sqrt{3}}\sin^{-1}\left(\sqrt{3}x\right)+C\sqrt{1-3x^2}$
3	Consider triangle *ABC*, $AC^2=4+2-2(2)\sqrt{2}\cos\left(\frac{\pi}{4}-\theta\right)$ $=6-4\sqrt{2}\left(\cos\frac{\pi}{4}\cos\theta+\sin\frac{\pi}{4}\sin\theta\right)=6-4\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right)$ $AC=\sqrt{6-4\cos\theta-4\sin\theta}$ (shown) Consider triangle *ACD*, $\cos\theta=\frac{AD}{AC}$ $AD=\cos\theta\sqrt{6-4\cos\theta-4\sin\theta}$

Since θ is small, $\sin\theta \approx \theta,\ \cos\theta \approx 1-\dfrac{\theta^2}{2}$,

$$AD \approx \left(1-\frac{\theta^2}{2}\right)\sqrt{6-4\left(1-\frac{\theta^2}{2}\right)-4\theta}$$

$$=\left(1-\frac{\theta^2}{2}\right)\left(2+2\theta^2-4\theta\right)^{\frac{1}{2}}$$

$$=\left(1-\frac{\theta^2}{2}\right)\sqrt{2}\left(1+\theta^2-2\theta\right)^{\frac{1}{2}}$$

$$=\sqrt{2}\left(1-\frac{\theta^2}{2}\right)\left(1+\frac{1}{2}(\theta^2-2\theta)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2}(\theta^2-2\theta)^2+\dots\right)$$

$$=\sqrt{2}\left(1-\frac{\theta^2}{2}\right)\left(1+\frac{1}{2}\theta^2-\theta-\frac{1}{2}\theta^2+\dots\right)$$

$$=\sqrt{2}\left(1-\frac{\theta^2}{2}\right)(1-\theta+\dots)$$

$$=\sqrt{2}\left(1-\theta-\frac{\theta^2}{2}+\dots\right)$$

$$\approx \sqrt{2}-\sqrt{2}\theta-\frac{\sqrt{2}}{2}\theta^2$$

4(a)

$$\sum_{n=1}^{2N}\frac{1}{4(n+1)^2-1}=\sum_{n=2}^{2N+1}\frac{1}{4n^2-1}$$

$$=\sum_{n=1}^{2N+1}\frac{1}{4n^2-1}-\frac{1}{3}$$

$$=\frac{1}{2}-\frac{1}{2\left[2(2N+1)+1\right]}-\frac{1}{3}$$

$$=\frac{1}{6}-\frac{1}{2(4N+3)}$$

$\dfrac{1}{(2n+3)^2}=\dfrac{1}{4n^2+12n+9}$ and $\dfrac{1}{4(n+1)^2-1}=\dfrac{1}{4n^2+8n+3}$

$\therefore\quad \dfrac{1}{(2n+3)^2}<\dfrac{1}{4(n+1)^2-1}$

Alternative:

$$\frac{1}{(2n+3)^2}<\frac{1}{(2n+1)(2n+3)}=\frac{1}{4(n+1)^2-1}$$

Hence

$$\sum_{n=1}^{2N}\frac{1}{(2n+3)(2n+3)}<\sum_{n=1}^{2N}\frac{1}{4(n+1)^2-1}$$

$$\sum_{n=1}^{2N}\frac{1}{(2n+3)(2n+3)}<\frac{1}{6}-\frac{1}{2(4N+3)}$$ [since N>0 & $\dfrac{1}{2(4N+3)}>0$]

$$<\frac{1}{6}$$

4b	$T_n = S_n - S_{n-1} = n\ln 2 - \frac{n^2-1}{e} - \left[(n-1)\ln 2 - \frac{(n-1)^2-1}{e}\right]$ $= \left[n-(n-1)\right]\ln 2 - \frac{1}{e}\left[(n^2-1)-(n-1)^2+1\right]$ $= \ln 2 - \frac{1}{e}\left[n^2-1-n^2+2n-1+1\right]$ $= \ln 2 - \frac{1}{e}\left[2n-1\right]$ $T_n - T_{n-1} = \ln 2 - \frac{1}{e}(2n-1) - \left[\ln 2 - \frac{1}{e}(2(n-1)-1)\right]$ $= -\frac{2}{e}$ Since $-\frac{2}{e}$ is a constant, the terms follow an AP.
5	Curve C : $y = f(x)$ Tangent to C at x = 0 is $2x - ay = 3 \Rightarrow y = -\frac{3}{a} + \frac{2}{a}x$ Since the tangent to C at x = 0 is $y = f(0) + f'(0)x$, $\therefore f(0) = -\frac{3}{a}$ and $f'(0) = \frac{2}{a}$ The 3rd term of the series for $f(x)$ is $\frac{1}{3}x^2$ $\Rightarrow \frac{f''(0)}{2!}x^2 = \frac{1}{3}x^2$ $\Rightarrow f''(0) = \frac{2}{3}$ From $(1+2x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, When x = 0, we have $\frac{2}{3} + \left(-\frac{3}{a}\right)\left(\frac{2}{a}\right) = 0$ $\Rightarrow a^2 = 9$ $\Rightarrow a = 3$ (since $a > 0$)

$$(1+2x)\frac{d^2y}{dx^2}+y\frac{dy}{dx}=0$$

Differentiate w.r.t. x :

$$(1+2x)\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)=0$$

When $x=0$, $y=-\frac{3}{3}=-1$, $\frac{dy}{dx}=\frac{2}{3}$, $\frac{d^2y}{dx^2}=\frac{6}{9}=\frac{2}{3}$,

$$\frac{d^3y}{dx^3}+(2-1)\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)^2=0$$

$$\frac{d^3y}{dx^3}=-\frac{2}{3}-\frac{4}{9}=-\frac{10}{9}$$

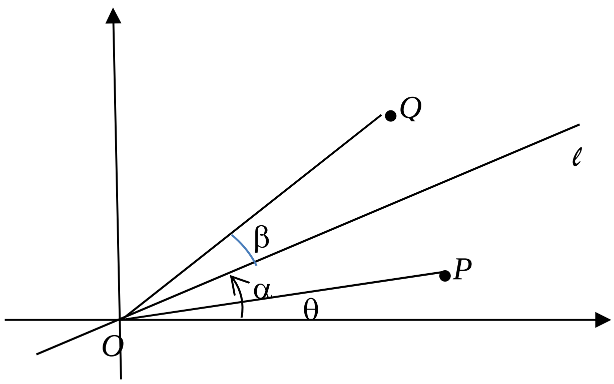
$$\therefore\ y=-1+\frac{2}{3}x+\frac{1}{3}x^2-\frac{10}{9(3!)}x^3+\ldots$$

$$=-1+\frac{2}{3}x+\frac{1}{3}x^2-\frac{5}{27}x^3+\ldots$$

6

P ≡ $z_1=re^{i\theta}$,

$|z_1|=r$ & arg (z_1) = θ



Let β be angle between lines OQ & l,

β = (α - θ) since line l **bisects** ∠POQ

arg z_1 + arg z_2

= θ+ (α + β)

= θ+ α + (α - θ)

= 2α

$|z_1z_2|=|z_1||z_2|=r^2$ AND arg (z_1 z_2) = arg z_1 + arg z_2 = 2α

Hence $z_1z_2=r^2(\cos 2\alpha+i\sin 2\alpha)$.

$\alpha=\frac{\pi}{4}$ $\Rightarrow$ $z_1z_2=r^2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)=r^2i$ (Purely imaginary).

Cartesian equation of the locus of R is $x=0, y>0$

7

$FB=EC=2\sqrt{(kx)^2-x^2}=2x\sqrt{k^2-1}$

Area of cross-section of prism

= Area of $ABCD$ + Area of $AFED$

= 2(Area of trapezium $ABCD$)

$=2\left[\frac{1}{2}(x+3x)\sqrt{(kx)^2-x^2}\right]$

$=4x^2\sqrt{k^2-1}$

Surface area of prism, $S=2\left(4x^2\sqrt{k^2-1}\right)+2xh+4kxh$

Hence $S=8x^2\sqrt{k^2-1}+2xh(1+2k)$ (shown) --- (2)

7a	Volume of prism $= 400 = \left(4x^2\sqrt{k^2-1}\right)h$ $h = \dfrac{100}{x^2\sqrt{k^2-1}}$ --- (1) (1) in (2): $S = 8x^2\sqrt{k^2-1} + 2(1+2k)\left(\dfrac{100}{x\sqrt{k^2-1}}\right)$ $\dfrac{\mathrm{d}S}{\mathrm{d}x} = 16x\sqrt{k^2-1} - \dfrac{200(1+2k)}{x^2\sqrt{k^2-1}}$ When $\dfrac{\mathrm{d}S}{\mathrm{d}x} = 0,\quad x^3 = \dfrac{200(1+2k)}{16(k^2-1)} \quad \Rightarrow\ x = \sqrt[3]{\dfrac{25(1+2k)}{2(k^2-1)}}$
7b	When $k = 2$, $S = 8x^2\sqrt{k^2-1} + 2xh(1+2k) = 8\sqrt{3}x^2 + 10xh$ and $V = \left(4x^2\sqrt{k^2-1}\right)h = 4\sqrt{3}x^2h$ Given that $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\mathrm{d}h}{\mathrm{d}t}$ $\Rightarrow \dfrac{\mathrm{d}h}{\mathrm{d}x} = \dfrac{\mathrm{d}h}{\mathrm{d}t} \times \dfrac{\mathrm{d}t}{\mathrm{d}x} = 1$ <u>Method 1</u> $\dfrac{\mathrm{d}S}{\mathrm{d}x} = 8\sqrt{3}(2x) + 10h + 10x\dfrac{\mathrm{d}h}{\mathrm{d}x} = 16\sqrt{3}x + 10h + 10x$ --- (1) $\dfrac{\mathrm{d}V}{\mathrm{d}x} = 4\sqrt{3}\left(h.2x + x^2\dfrac{\mathrm{d}h}{\mathrm{d}x}\right) = 4\sqrt{3}\left(2xh + x^2\right)$ --- (2) When $x = 3,\ h = 8,\quad \dfrac{\mathrm{d}S}{\mathrm{d}t} = 0.5,$ $\dfrac{\mathrm{d}S}{\mathrm{d}x} = 16\sqrt{3}(3) + 10(8+3) = 48\sqrt{3} + 110$ $\dfrac{\mathrm{d}V}{\mathrm{d}x} = 4\sqrt{3}\left(2.3.8 + 3^2\right) = 228\sqrt{3}$ $\dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}x} \times \dfrac{\mathrm{d}x}{\mathrm{d}S} \times \dfrac{\mathrm{d}S}{\mathrm{d}t}$ $= 228\sqrt{3} \times \dfrac{1}{48\sqrt{3}+110} \times 0.5$ $= 1.02$ (to 3 s.f.) <u>Method 2</u> $\dfrac{\mathrm{d}S}{\mathrm{d}t} = 8\sqrt{3}\left(2x\dfrac{\mathrm{d}x}{\mathrm{d}t}\right) + 10\left(h\dfrac{\mathrm{d}x}{\mathrm{d}t} + x\dfrac{\mathrm{d}h}{\mathrm{d}t}\right) = \left(16\sqrt{3}x + 10h + 10x\right)\dfrac{\mathrm{d}x}{\mathrm{d}t}$ --- (1) And $\dfrac{\mathrm{d}V}{\mathrm{d}t} = 4\sqrt{3}\left(h.2x\dfrac{\mathrm{d}x}{\mathrm{d}t} + x^2\dfrac{\mathrm{d}h}{\mathrm{d}t}\right) = 4\sqrt{3}\left(2xh + x^2\right)\dfrac{\mathrm{d}x}{\mathrm{d}t}$ --- (2)

	When $x=3,\ h=8,\ \frac{\mathrm{d}S}{\mathrm{d}t}=0.5$, using eqn (1) to find $\frac{\mathrm{d}x}{\mathrm{d}t}$ $0.5=\left(16\sqrt{3}(3)+10(8)+10(3)\right)\frac{\mathrm{d}x}{\mathrm{d}t}$ $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{0.5}{48\sqrt{3}+110}\approx 0.0025888$ Sub into (2) to get $\frac{\mathrm{d}V}{\mathrm{d}t}$ $\frac{\mathrm{d}V}{\mathrm{d}t}=4\sqrt{3}\left(2.3.8+3^2\right)(0.0025888)=1.022343317\approx 1.02$ (to 3 sf)
8i)	$y=\frac{4x^2-kx+2}{x-2}$ By long division, $y=4x+8-k+\frac{18-2k}{x-2}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x-2)(8x-k)-(4x^2-kx+2)(1)}{(x-2)^2}$ $=\frac{4x^2-16x+2k-2}{(x-2)^2}$ Let $\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow 4x^2-16x+2k-2=0$ $\Rightarrow 2x^2-8x+k-1=0$ $\Rightarrow x=\frac{8\pm\sqrt{64-4(2)(k-1)}}{4}=2\pm\sqrt{\frac{9-k}{2}}$ C has stationary point when $k\le 9$ However, when k = 9, the value x=2 is undefined on the curve. In fact, the curve C is a straight line, $y = 4x - 1$. Hence C has stationary point when $k < 9$. <u>Alternative Presentation 1:</u> Let $\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow 4x^2-16x+2k-2=0$ $\Rightarrow 2x^2-8x+k-1=0$ For $\frac{dy}{dx}$=0 to have real roots, "$b^2-4ac\ge 0$" $\Rightarrow 8^2-4(2)(k-1)\ge 0$ $\Rightarrow 64-8k+8\ge 0$ $\Rightarrow 8k\le 72$ $\Rightarrow k\le 9$ <u>Alternative Presentation 2:</u> $\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow 4x^2-16x+2k-2=0$ $\Rightarrow 2x^2-8x+k-1=0$ $\Rightarrow 2\left(x-2\right)^2+k-9=0$ $\Rightarrow 2(x-2)^2=9-k$ For $\frac{dy}{dx}=0$ to have roots x, $9-k\ge 0 \Rightarrow k\le 9$ However, when k = 9, the value x=2 is undefined on the curve. In fact, the curve C is a straight line, $y = 4x - 1$. Hence C has stationary point when $k < 9$.

(ii)

$$y=\frac{4x^2-kx+2}{x-2}=4x+(8-k)+\frac{18-2k}{x-2}$$

Asymptotes of C are $y = 4x + 8 - k$ and $x = 2$

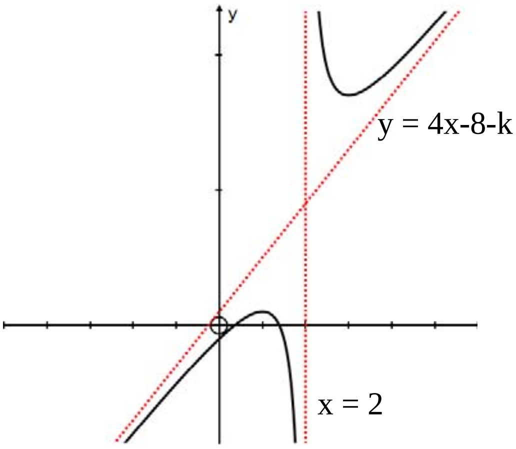
When $x=0$, $y==-1$.

When $y=0$, $4x^2-kx+2=0$

$$\Rightarrow x=\frac{k\pm\sqrt{k^2-32}}{8}$$

The axial intercepts are $(0,-1)$, $\left(\frac{k-\sqrt{k^2-32}}{8},0\right)$ and $\left(\frac{k+\sqrt{k^2-32}}{8},0\right)$.

ii)



(iii)

When k = 8, $y=4x+(8-8)+\frac{18-2(8)}{x-2}=4x+\frac{2}{x-2}$

$$y=2x+\frac{1}{x}\xrightarrow{A}y=2\left(2x+\frac{1}{x}\right)=4x+\frac{2}{x}$$

$$y=4x+\frac{2}{x}\xrightarrow{B}y=4(x-2)+\frac{2}{(x-2)}=y=4x-8+\frac{2}{(x-2)}$$

$$y=4x-8+\frac{2}{(x-2)}\xrightarrow{C}y=\left(4x-8+\frac{2}{(x-2)}\right)+8=4x+\frac{2}{(x-2)}$$

A – Scaling, parallel to the y-axis by a scale factor of 2.
B - Translate the graph by 2 units in the direction of x-axis
C - Translate the graph by 8 units in the direction of y-axis

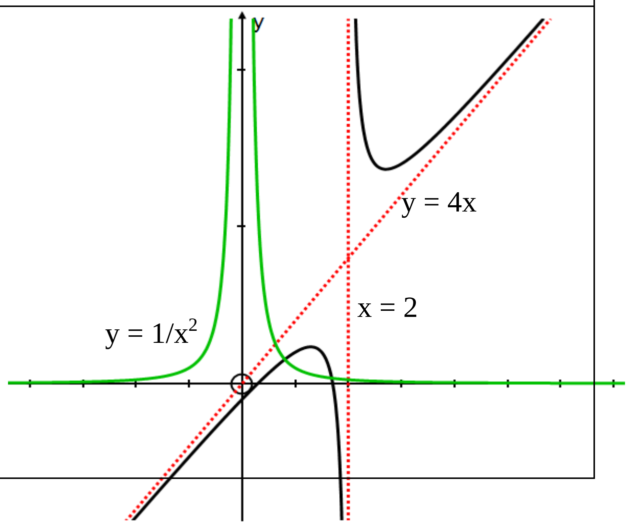
<u>Alternate Sequence of Transformations:</u>
A – Translate the graph by 2 units in the direction of x-axis
B - Translate the graph by 4 units in the direction of y-axis
C - Scaling, parallel to the y-axis by a scale factor of 2.

iv)

When $k=8$, $\frac{4x^2-kx+2}{x-2}>\frac{1}{x^2}$

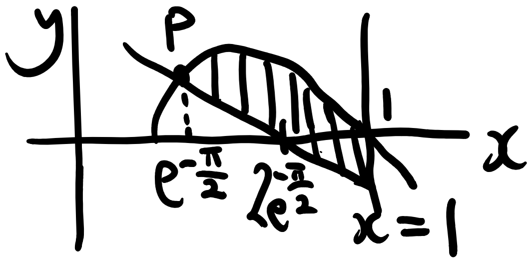
$$\Rightarrow \frac{4x^2-8x+2}{x-2}>\frac{1}{x^2}$$

From G.C,
$0.805<x<1.69$ or $x>2$.



9(i)	$(\mathbf{a}+\mathbf{b}).(\mathbf{a}+\mathbf{b})=\mathbf{a.a}+2\mathbf{a.b}+\mathbf{b.b}$ Since $(\mathbf{a}+\mathbf{b}).(\mathbf{a}+\mathbf{b})=\lvert\mathbf{a}+\mathbf{b}\rvert^2$ and given $\lvert\mathbf{a}+\mathbf{b}\rvert^2=\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2$ $\therefore\ \mathbf{a.a}+2\mathbf{a.b}+\mathbf{b.b}=\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2$ $\lvert\mathbf{a}\rvert^2+2\mathbf{a.b}+\lvert\mathbf{b}\rvert^2=\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2$ $2\mathbf{a.b}=0$ $\mathbf{a.b}=0$ $\therefore\ \mathbf{a}\perp\mathbf{b}$
ii)	Using ratio theorem, $\overrightarrow{OC}=\dfrac{4\mathbf{b}+\mathbf{a}}{5}=\dfrac{1}{5}\mathbf{a}+\dfrac{4}{5}\mathbf{b}$. **Length of projection of $\vec{\mathbf{OC}}$ onto $\vec{\mathbf{OA}}$** $=\dfrac{\lvert\overrightarrow{OC}\cdot\overrightarrow{OA}\rvert}{\lvert\overrightarrow{OA}\rvert}$ $=\dfrac{\left\lvert\left(\frac{1}{5}\mathbf{a}+\frac{4}{5}\mathbf{b}\right)\cdot\mathbf{a}\right\rvert}{\lvert\mathbf{a}\rvert}=\dfrac{\left\lvert\frac{1}{5}\mathbf{a}\cdot\mathbf{a}+\frac{4}{5}\mathbf{b}\cdot\mathbf{a}\right\rvert}{\lvert\mathbf{a}\rvert}$ $=\dfrac{\left\lvert\frac{1}{5}\lvert\mathbf{a}\rvert^2+\frac{4}{5}\mathbf{b}\cdot\mathbf{a}\right\rvert}{\lvert\mathbf{a}\rvert}\quad=\dfrac{1}{5}\lvert\mathbf{a}\rvert$ **(since $\mathbf{a}\perp\mathbf{b}$)**
iii)	$\lvert\mathbf{c}\times\mathbf{f}\rvert$ denotes twice the area of the triangle COF.
(iv)	$\overrightarrow{OX}\cdot\overrightarrow{OY}=\begin{pmatrix}\cos 3t\\ \sin 3t\\ \frac{1}{2}\end{pmatrix}\cdot\begin{pmatrix}\sin t\\ \cos t\\ -2\end{pmatrix}=\cos 3t\sin t+\sin 3t\cos t-1=\sin(4t)-1$ $\cos\angle XOY=\dfrac{\overrightarrow{OX}\cdot\overrightarrow{OY}}{\lvert\overrightarrow{OX}\rvert\lvert\overrightarrow{OY}\rvert}=\dfrac{\sin 4t-1}{\sqrt{\cos^2 3t+\sin^2 3t+\frac{1}{4}}\sqrt{\sin^2 t+\cos^2 t+4}}$ $=\dfrac{\sin 4t-1}{\sqrt{\frac{5}{4}}\sqrt{5}}$ $=\dfrac{2}{5}(\sin 4t-1)$ Maximum $\angle XOY$ occurs when cos (∠XOY) is most negative. i.e. when $\sin 4t=-1$. At that value of t, $\cos\angle XOY=\dfrac{2}{5}(-1-1)=-\dfrac{4}{5}$ $\therefore\angle XOY=\cos^{-1}\left(-\dfrac{4}{5}\right)=143.1°$
10	$x+(x+2)+(x+2(2))\ +\ \dots\ +(x+\ 23(2))\le 2000$ This is an AP with first term= x, common difference = 2, number of terms = 24 $\dfrac{24}{2}\left[2x+23(2)\right]\le 2000$ $0<x\le\dfrac{181}{3}$

10(i)	<table><tr><td>n</td><td>Amount paid at T_n</td></tr><tr><td>2</td><td>$60(0.05)$</td></tr><tr><td>3</td><td>$60(0.05)+2(0.05)(0.98)$</td></tr><tr><td>4</td><td>$60(0.05)+2(0.05)(0.98)+2(0.05)(0.98)^2$</td></tr><tr><td></td><td>. .</td></tr><tr><td>n</td><td>$60(0.05)+2(0.05)(0.98)+2(0.05)(0.98)^2+...+2(0.05)(0.98)^{n-2}$</td></tr></table> Amount of fees at T_n $=3+\frac{0.098(1-0.98^{n-2})}{1-0.98}$ $=3+4.9(1-0.98^{n-2})$ $=7.9-4.9(0.98^{n-2})$
ii	$\sum_{r=2}^{n}\left[7.9-4.9(0.98^{r-2})\right]$ $=\sum_{r=2}^{n}7.9-4.9\sum_{r=2}^{n}(0.98^{r-2})$ $=7.9(n-1)-4.9\left[\frac{1(1-0.98^{n-1})}{1-0.98}\right]$ $=7.9(n-1)-245(1-0.98^{n-1})$ $=7.9n+245(0.98^{n-1})-252.9$
iii	Let $\mathrm{f}(n)=7.9n+245(0.98^{n-1})-252.9$. Note that $\mathrm{f}(n)$ is increasing in n Consider $7.9n+245(0.98^{n-1})-252.9>200$ <table><tr><td>44</td><td>197.47</td></tr><tr><td>45</td><td>203.32</td></tr><tr><td>46</td><td>209.21</td></tr></table> Using GC, $n\geq 45$ He will not have sufficient money at the 45th toll station.
11i	$\int e^{-2x}\sin x\ \mathrm{d}x$ $=(-\cos x)(e^{-2x})-\int(-\cos x)(-2e^{-2x})\ \mathrm{d}x$ $=-e^{-2x}\cos x-2\left[(\sin x)(e^{-2x})-\int \sin x(-2e^{-2x})\ \mathrm{d}x\right]$ $=-e^{-2x}\cos x-2e^{-2x}\sin x-4\int e^{-2x}\sin x\ \mathrm{d}x$ $5\int e^{-2x}\sin x\ \mathrm{d}x=-e^{-2x}\cos x-2e^{-2x}\sin x+C$ $\int e^{-2x}\sin x\ \mathrm{d}x=-\frac{2}{5}e^{-2x}\sin x-\frac{1}{5}e^{-2x}\cos x+A$
11ii	$\frac{\mathrm{d}x}{\mathrm{d}t}=-e^{-t}$ $\quad\frac{\mathrm{d}y}{\mathrm{d}t}=-e^{-t}\sin t+e^{-t}\cos t$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-e^{-t}\sin t+e^{-t}\cos t}{-e^{-t}}=\sin t-\cos t$

	At $t=\frac{\pi}{2}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\sin\left(\frac{\pi}{2}\right)-\cos\left(\frac{\pi}{2}\right)=1-0=1$, so gradient of normal = -1 $x=e^{-\pi/2}$, $\quad y=e^{-\pi/2}\sin\frac{\pi}{2}=e^{-\pi/2}$ Equation of normal: $y-e^{-\pi/2}=-1(x-e^{-\pi/2}) \quad \Rightarrow \quad y=-x+2e^{-\pi/2}$
11iii	 Area $=\int_{e^{-\pi/2}}^{1} y_C - y_{normal}\,\mathrm{d}x$ $=\int_{e^{-\pi/2}}^{1} y_C - \left(-x+2e^{-\pi/2}\right)\mathrm{d}x$ $=\int_{\pi/2}^{0} e^{-t}\sin t(-e^{-t})\mathrm{d}t+\int_{e^{-\pi/2}}^{1} x-2e^{-\pi/2}\ \mathrm{d}x$ $=-\int_{\pi/2}^{0} e^{-2t}\sin t\ \mathrm{d}t+\left[\frac{x^2}{2}\right]_{e^{-\pi/2}}^{1}-\left[2e^{-\pi/2}x\right]_{e^{-\pi/2}}^{1}$ $=-\left[-\frac{2}{5}e^{-2x}\sin x-\frac{1}{5}e^{-2x}\cos x\right]_{\pi/2}^{0}+\left[\frac{1}{2}-\frac{e^{-\pi}}{2}\right]-\left[2e^{-\pi/2}\left(1-e^{-\pi/2}\right)\right]$ $=\frac{2}{5}e^{0}\sin 0+\frac{1}{5}e^{0}\cos 0-\frac{2}{5}e^{-\pi}\sin\left(\frac{\pi}{2}\right)-\frac{1}{5}e^{-\pi}\cos\frac{\pi}{2}+\frac{1}{2}-\frac{e^{-\pi}}{2}-2e^{-\pi/2}+2e^{-\pi}$ $=\frac{1}{5}-\frac{2}{5}e^{-\pi}+\frac{1}{2}-\frac{e^{-\pi}}{2}-2e^{-\pi/2}+2e^{-\pi}$ $=\frac{11}{10}e^{-\pi}-2e^{-\pi/2}+\frac{7}{10}$ ***Alternative:*** Area = $\int_{e^{-\pi/2}}^{1} e^{-t}\sin t-\frac{1}{2}(e^{-\pi/2})(2e^{-\pi/2}-e^{-\pi/2})+\frac{1}{2}(1-2e^{-\pi/2})^2$ [When x =1, y = -1+ $2e^{-\pi/2}$]
11iv	For normal to meet curve again, Substitute parametric eqns into $y=-x+2e^{-\pi/2}$ $e^{-t}\sin t=-e^{-t}+2e^{-\pi/2}$ $e^{-t}(\sin t+1)-2e^{-\pi/2}=0$ Using GC, $t=-1.92148,\quad -1.0145,\quad 1.5707$ (rej, this is $\frac{\pi}{2}$) So $q=-1.92$ and $r=-1.01$ (to 3 sf)