Candidate Name: ______________________

Class	Adm No

H

millennia institute

2017 Preliminary Examination II
Pre-University 3

MATHEMATICS **9740/02**

Paper 2 **14 September 2017**

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your admission number, name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, arrange your answers in NUMERICAL ORDER and fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 The curve C has the equation $4(x-1)^2+9y^2=36$.

(i) Sketch, for $y \geq 0$, the curve C, stating the coordinates of the end points and the turning point. [3]

(ii) By adding a suitable graph to your sketch in part **(i)**, solve the inequality

$$2\sqrt{\left[1-\frac{(x-1)^2}{9}\right]}+2-(x-1)^2 \geq 0. \qquad [2]$$

(iii) Hence, solve the inequality $2\sqrt{\left[1-\frac{(\mathrm{e}^x-1)^2}{9}\right]} \geq \left(\mathrm{e}^x-1\right)^2-2$. [2]

2 Two loci in the Argand diagram are given by the equations

$$|z-2+2\mathrm{i}|=1 \quad \text{and} \quad \arg z=-\frac{\pi}{6}.$$

The complex numbers z_1 and z_2, where $|z_1|<|z_2|$, correspond to the points of intersection of these loci.

(i) Draw an Argand diagram to show both loci, and mark the points represented by z_1 and z_2. [3]

(ii) Find the two values of z which represent points on $|z-2+2\mathrm{i}|=1$ such that $|z-z_1|=|z-z_2|$. [4]

(iii) Given that the complex number w satisfies $|w-2+2\mathrm{i}| \leq 1$ and $\arg w \leq -\frac{\pi}{6}$, find the range of values of $\arg(w+3\mathrm{i})$. [3]

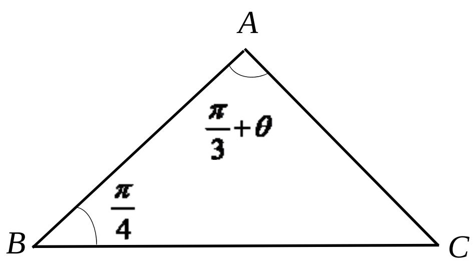
3 **(a)** It is given that $\tan^{-1} y = \ln(1+x)$.

(i) Show that $(1+x)\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1+y^2$. [1]

(ii) By successively differentiating this result, find the Maclaurin series for $\tan[\ln(1+x)]$, up to and including the term in x^3. [3]

(iii) It is given that $\mathrm{f}(x) = \mathrm{e}^x \tan[\ln(1+x)]$. Using your answer to part **(a)(ii)**, estimate the value of $\mathrm{f}'\left(\dfrac{1}{2}\right)$. [3]

(b) The diagram shows triangle *ABC*, where $AC = k$ cm, $BC = h$ cm, $\angle BAC = \dfrac{\pi}{3} + \theta$ and $\angle ABC = \dfrac{\pi}{4}$.



Given that θ is a sufficiently small angle, show that

$$\frac{h}{k} \approx \frac{\sqrt{2}}{4}\left[2\sqrt{3} + 2\theta - \left(\sqrt{3}\right)\theta^2\right].$$ [3]

[Turn Over

4 The plane π_1 contains the line $l_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, where $\lambda \in \mathbb{R}$, and is parallel to the line $l_2 : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, where $\mu \in \mathbb{R}$.

(i) Find the vector equation of π_1 in scalar product form. [2]

(ii) Find the position vector of the foot of the perpendicular from the point $A(1, 0, 1)$ to the plane π_1. [3]

(iii) Find the position vector of the point A', which is the reflection of A about π_1.[2]

(iv) Given that the angle between $l_3 : \mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, where $\alpha \in \mathbb{R}$, and the plane $\pi_2 : ax + 2y - z = 3$, where $a \in \mathbb{R}$, is $\frac{\pi}{4}$, find the value of *a*. [2]

(v) Find the line of intersection between the planes π_1 and π_2. [1]

(vi) π_3 has equation $bx + y + z = c$, where $b, c \in \mathbb{R}$. Given that π_1, π_2 and π_3 have no points in common, describe the geometrical relationship between the three planes. What can be said about the values of *b* and *c*? [3]

Section B: Statistics [60 marks]

5 Resilience Primary School has 500 students who are either Chinese, Indian or Malay, as seen in the table below.

	Chinese	Indian	Malay
Boys	114	8	93
Girls	122	77	86

The National Eye Centre wishes to conduct a survey at Resilience Primary School to find out the number of hours students spend on electronic devices each week, using a sample of 50 students.

(i) Explain how stratified sampling can be carried out in this context. [2]

(ii) Give two reasons why systematic sampling may not be appropriate. [2]

6 In another survey conducted by the National Eye Centre, it was found that p% are boys and the remaining are girls. The probability that a randomly chosen boy wears spectacles is 0.3 and the probability that a randomly chosen girl wears spectacles is 0.24.

(i) Find the value of p, given that the probability that a randomly chosen child wears spectacles is 0.267. [2]

(ii) For a general value of p, the probability that a randomly chosen child that wears spectacles is a girl is denoted by $\mathrm{f}(p)$. Show that $\mathrm{f}(p) = \dfrac{4(100-p)}{(400+p)}$. Prove by differentiation that f is a decreasing function for $0 \le p \le 100$, and explain what this statement means in the context of the question. [5]

[Turn Over

7 In this question you should state clearly all distributions that you use, together with the values of the appropriate parameters.

The mass, in grams, of broccoli and carrots are normally distributed with means and standard deviations as shown in the table below.

	Mean (g)	**Standard deviation (g)**
Broccoli	μ	σ
Carrot	180	15

(i) Given that the probability that the mass of a randomly chosen broccoli does not exceed 250g is 0.788 and the probability that the mass of a randomly chosen broccoli exceeds 236g is 0.625, find the values of μ and σ. [3]

(ii) Find the probability that the mass of a randomly chosen broccoli lies within 5 grams of a randomly chosen carrot. [2]

(iii) 120 broccoli are randomly chosen. Using a suitable approximation, find the probability that there are fewer than 90 broccoli with a mass not exceeding 250g.[3]

(iv) Determine, with explanation, whether the mass of a vegetable chosen randomly from a basket containing an equal number of broccoli and carrots follows a normal distribution. [1]

8 The table gives the values of eight observations of bivariate data, x and y.

x	1	2	3	4	5	6	7	8
y	5	1	18	23	28	31	33	34

(i) Draw a scatter diagram for these values, labelling the axes clearly. Determine the outlier by labelling it as P in your scatter diagram. [2]

(ii) By omitting P, explain if $y = ax^2 + b$ or $y = a\ln x + b$ is the better model for the data. [2]

(iii) Using the more appropriate model found in part **(ii)**, calculate the equation of the least-squares regression line. [1]

(iv) Interpret, in the context of the question, the least squares estimates of a and b. [2]

(v) Use the regression line found in part **(iii)** to predict the value of y when x = 4.5. Comment on the reliability of your answer. [2]

9 Based on past records, the mean number of rainy days per year in Singapore was reported as 178. The authorities suspect that due to global warming, the number of rainy days has changed. A random sample of 12 years is taken and the number of rainy days per year, X, is summarised by

$$\sum(x-8) = 2017.7, \qquad \sum x^2 = 372\,500.$$

(i) Calculate the unbiased estimates of the mean and variance of X. [2]

(ii) Test, at the 5% level of significance, whether the mean number of rainy days per year has changed. State any assumptions used in your calculations. [4]

(iii) Explain, in the context of the question, the meaning of the p-value. [1]

(iv) The population variance is found to be 9 and the assumption used in part **(ii)** holds true. A test at the 5% level of significance whether the mean number of rainy days per year has changed was conducted. Find the range of values of $\bar{x}$ such that the null hypothesis is not rejected. [3]

[Turn Over

10 **(a)** Find the number of ways in which the letters of the word MILLENNIUM can be arranged if

(i) there are no restrictions, [1]

(ii) the first and last letters are the same, and the letters E and U must be separated. [2]

Four letters are randomly selected from the letters of the word MILLENNIUM to form a code word. Find the number of possible code words that can be formed. [2]

(b) Mr See (together with his wife and daughter) and Mrs Saw (together with her husband and two sons) came to visit their former teacher Mdm Rain during Millennium Institute's Homecoming Day. Find the number of ways Mr See and his family, Mrs Saw and her family, and Mdm Rain can be arranged if

(i) they are around a table with ten indistinguishable chairs, such that the children are seated together. [2]

(ii) the two empty chairs are removed and Mr See's daughter is seated between her parents and the See family are to be seated directly opposite Mdm Rain. [3]

11 In this question you should state clearly all distributions that you use, together with the values of the appropriate parameters.

The number of people queuing to buy coffee at CoffeeVille in a period of 1 minute during lunch hour (12pm to 2pm) is a random variable with an average number of 2.9.

State, in context, a condition under which a Poisson distribution would be a suitable probability model. [1]

Assume that the number of people queuing to buy coffee at CoffeeVille in a period of 1 minute during the lunch hour follows the distribution $\text{Po}(2.9)$.

(i) State the most probable number of people queuing in 1 minute. [1]

(ii) Find the probability that in a period of 3 minutes, there are at most 5 people queuing to buy coffee. [2]

(iii) N periods of 3 minutes are taken. Given that the probability that at least 7 periods of 3 minutes have at most 5 people queuing to buy coffee is more than 0.99, find the least value of N. [3]

(iv) A random sample of 120 periods of 3 minutes is taken. Using a suitable approximation, find the probability that more than 12 periods of 3 minutes have exactly 4 people queuing. [3]

(v) Explain why the Poisson model would probably not be valid if applied to the operating hours of CoffeeVille from 11am to 10pm. [1]

End of Paper