HWA CHONG INSTITUTION
2017 JC2 PRELIMINARY EXAMINATION

MATHEMATICS
Higher 2

9758/01

Paper 1

Tuesday **12 September 2017** **3 hours**

Additional materials: Answer paper
List of Formula (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and CT class on all the work you hand in, including the Cover Page which is found on Page 2.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Do not write anything on the List of Formula (MF26).

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question. At the end of the examination, place the completed cover page on top of your answer scripts and fasten all your work securely together with the string provided.

This question paper consists of 8 printed pages.

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MATHEMATICS

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Name: ________ CT: 1 6

For Examiner's Use			
Question No.	Marks Obtained	Total Marks	Remarks
1		5	
2		6	
3		6	
4		7	
5		8	
6		8	
7		10	
8		12	
9		13	
10		12	
11		13	
TOTAL		100	

COVER PAGE

1. Write your name, CT group and calculator model(s) in the spaces provided.

2. Arrange your answers in numerical order.

3. Detach this cover page and place it on top of your answer paper and fasten them securely together with the string provided.

Graphing Calculator Model:

Scientific Calculator Model:

1 The *floor function*, denoted by $\lfloor x \rfloor$, is the greatest integer less than or equal to x. For example, $\lfloor -2.1 \rfloor = -3$ and $\lfloor 3.5 \rfloor = 3$.

The function f is defined by

$$\mathrm{f}(x) = \begin{cases} \lfloor x \rfloor & \text{for } x \in \mathbb{R}, \quad -1 \le x < 2, \\ 0 & \text{for } x \in \mathbb{R}, \quad 2 \le x < 3, \end{cases}$$

where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

It is given that $\mathrm{f}(x) = \mathrm{f}(x+4)$.

(i) Find the values of $\mathrm{f}(-1.2)$ and $\mathrm{f}(3.6)$. [2]

(ii) Sketch the graph of $y = \mathrm{f}(x)$ for $-2 \le x < 4$. [2]

(iii) Hence evaluate $\int_{-2}^{4} \mathrm{f}(x)\,\mathrm{d}x$. [1]

2 By writing $\sec^3 x = \sec x \sec^2 x$, find $\int \sec^3 x\,\mathrm{d}x$.

Hence find the exact value of $\int_{0}^{\tan^{-1} 2} \sec^3 x\,\mathrm{d}x$. [6]

3 **(i)** By first expressing $3x - x^2 - 4$ in completed square form, show that $3x - x^2 - 4$ is always negative for all real values of x. [2]

(ii) Hence, or otherwise, without the use of a calculator, solve the inequality

$$\frac{\left(3x - x^2 - 4\right)\left(x - 1\right)^2}{x^2 - 2x - 5} \le 0,$$

leaving your answer in exact form. [4]

4 The complex number z is given by $z = r\,\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $0 \le \theta \le \pi$. It is given that the complex number $w = \left(-\sqrt{3} - \mathrm{i}\right) z$.

(i) Find $|w|$ in terms of r, and $\arg w$ in terms of θ. [2]

(ii) Given that $\dfrac{z^8}{w^*}$ is purely imaginary, find the three smallest values of θ in terms of π. [5]

5 **(a)** It is given that three non-zero vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ satisfy the equation $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}+\mathbf{c})=\mathbf{b}\times\mathbf{c}$, where $\mathbf{b}\neq\mathbf{c}$. Find a linear relationship between $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$. [3]

(b) A point A with position vector $\overrightarrow{OA}=\alpha\,\mathbf{i}+\beta\,\mathbf{j}+\gamma\,\mathbf{k}$, where α, β and γ are real constants, has direction cosines $\cos\theta$, $\cos\phi$ and $\cos\omega$, where θ, ϕ and ω are the angles $\overrightarrow{OA}$ make with the positive x, y and z-axes respectively.

(i) Express the direction cosines $\cos\theta$, $\cos\phi$ and $\cos\omega$ in terms of α, β and γ. Hence find the value of $\cos^2\theta+\cos^2\phi+\cos^2\omega$. [3]

(ii) Hence show that $\cos 2\theta+\cos 2\phi+\cos 2\omega=-1$. [2]

6 A particle moving along a path at time t, where $0<t<\frac{\pi}{3}$, is defined parametrically by

$$x=\cot 3t \quad \text{and} \quad y=2\operatorname{cosec} 3t+1.$$

(a) The tangent to the path at the point $P(\cot 3p,\ 2\operatorname{cosec}3p+1)$ meets the y-axis at the point Q. Show that the coordinates of Q is $(0,\ 2\sin 3p+1)$. [4]

(b) The distance of the particle from the point $R(0,1)$ is denoted by s, where $s^2=x^2+(y-1)^2$. Find the exact rate of change of the particle's distance from R at time $t=\frac{\pi}{4}$. [4]

7 **(i)** It is given that $\ln y=2\sin x$. Show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-y\ln y+\dfrac{1}{y}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$. [2]

(ii) Find the first four terms of the Maclaurin series for y in ascending powers of x. [4]

(iii) Using appropriate expansions from the List of Formulae (MF26), verify the expansion found in part **(ii)**. [2]

(iv) Given that x is sufficiently small for x^4 and higher powers of x to be neglected, deduce an approximation for $\mathrm{e}^{(2\sin x)-\ln(\sec x)}$ in ascending powers of x. [2]

8 **(a)** A curve is defined parametrically by the equations

$$x = \sin t \quad \text{and} \quad y = \cos^3 t\,, \qquad -\pi \le t \le \pi\,.$$

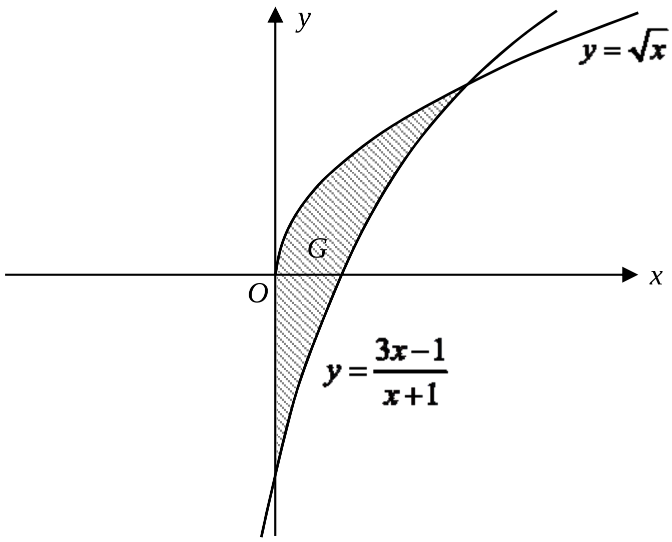
(i) Show that the area enclosed by the curve is given by

$$k\int_0^{\frac{\pi}{2}} \cos^4 t \;\mathrm{d}t\,,$$

where k is a constant to be determined. [3]

(ii) Hence find the exact area enclosed by the curve. [3]

(b) In the diagram, the region G is bounded by the curves $y = \dfrac{3x-1}{x+1}$, $y = \sqrt{x}$ and the y-axis.



Find the exact volume of the solid generated when G is rotated about the y-axis through 2π radians. [6]

9 A curve C_1 has equation $y = \dfrac{ax^2 - bx}{x^2 - c}$, where a, b and c are constants. It is given that C_1 passes through the point $\left(3, \frac{9}{5}\right)$ and two of its asymptotes are $y = 2$ and $x = -2$.

(i) Find the values of a, b and c. [3]

In the rest of the question, take the values of a, b and c as found in part **(i)**.

(ii) Using an algebraic method, find the exact set of values of y that C_1 cannot take. [3]

(iii) Sketch C_1, showing clearly the equations of asymptotes and the coordinates of the turning points. [3]

(iv) It is given that the equation $e^y = x - r$, where $r \in \mathbb{R}^+$, has exactly one real root. State the range of values of r. [1]

(v) The curve C_2 has equation $y = 2 + \dfrac{3x + 5}{x^2 - 2x - 3}$. State a sequence of transformations which transforms C_1 to C_2. [3]

10 Food energy taken in by a man goes partly to maintain the healthy functioning of his body and partly to increase body mass. The total food energy intake of the man per day is assumed to be a constant denoted by I (in joules). The food energy required to maintain the healthy functioning of his body is proportional to his body mass M (in kg). The increase of M with respect to time t (in days) is proportional to the energy not used by his body. If the man does not eat for one day, his body mass will be reduced by 1%.

(i) Show that I, M and t are related by the following differential equation:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{I - aM}{100a}, \text{ where } a \text{ is a constant.}$$

State an assumption for this model to be valid. [3]

(ii) Find the total food energy intake per day, I, of the man in terms of a and M if he wants to maintain a constant body mass. [1]

It is given that the man's initial mass is 100kg.

(iii) Solve the differential equation in part **(i)**, giving M in terms of I, a and t. [3]

(iv) Sketch the graph of M against t for the case where $I > 100a$. Interpret the shape of the graph with regard to the man's food energy intake. [3]

(v) If the man's total food energy intake per day is $50a$, find the time taken in days for the man to reduce his body mass from 100kg to 90kg. [2]

11 A manual hoist is a mechanical device used primarily for raising and lowering heavy loads, with the motive power supplied manually by hand. Three hoists, A, B and C are used to lift a load vertically.

(i) For hoist A, the first pull will raise the load by a vertical distance of 45 cm. On each subsequent pull, the load will raise 1.6 cm lesser than the vertical distance covered by the previous pull. Determine the number of pulls needed for the load to achieve maximum total height. Hence find this maximum total height. [4]

(ii) For hoist B, the first pull will raise the load by a vertical distance of 45 cm. On each subsequent pull, the vertical distance raised will be 95% of the distance covered by the previous pull. Find the theoretical maximum total height that the load can reach. [2]

(iii) For hoist C, every pull will raise the load by a constant vertical distance of 45 cm. However, after each pull, the load will slip and drop by 2% of the total vertical height the load has reached. Show that just before the 4^{th} pull, the load would have reached a total vertical height of 130 cm, correct to 3 significant figures.

Hence show that before the $(n+1)^{\text{th}}$ pull, the load would have reached a total vertical height of $X+Y(0.98)^{n+1}$, where X and Y are integers to be determined. [5]

(iv) Explain clearly if hoist C can lift the load up a building of height 25 metres. [2]

End of Paper