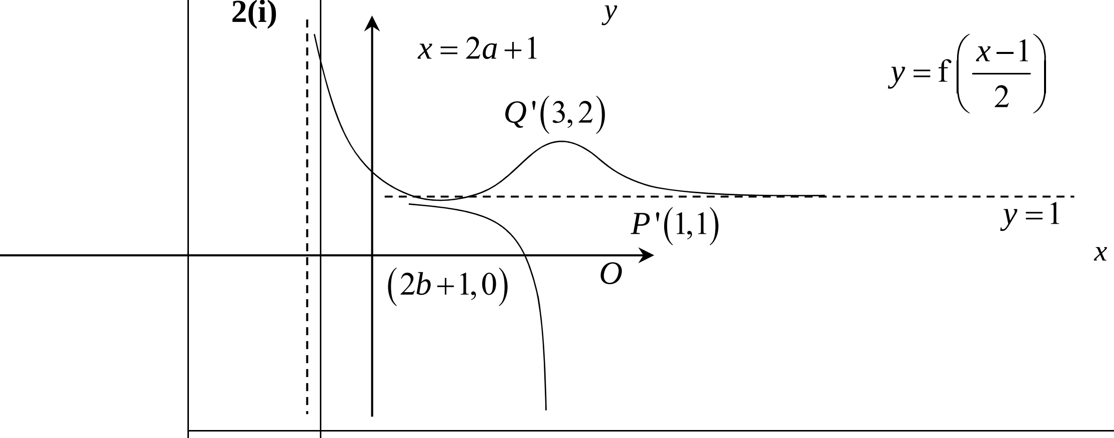
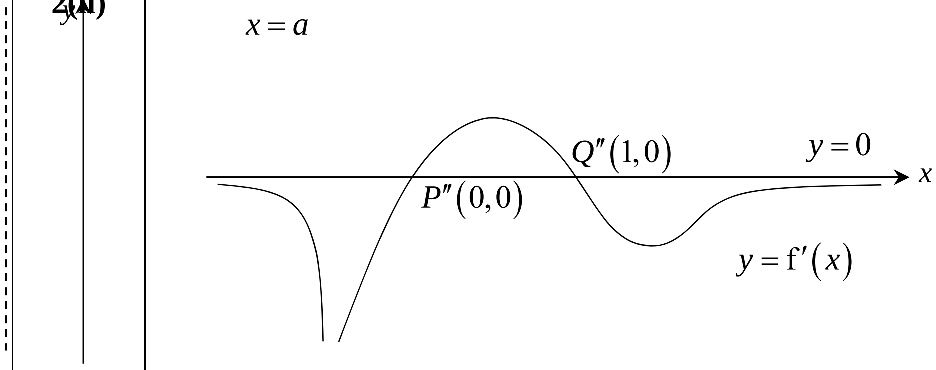
2017 H2 Prelim P1 Solutions

Qns	Solutions
1	Passes through $(-1,1)$: $1=\dfrac{-2}{a-b+c}\ \Rightarrow\ a-b+c=-2$ ……………………..(1) Turning point at $(-1,1)$: $\left.\dfrac{dy}{dx}\right\|_{x=-1}=0$ now $\dfrac{dy}{dx}=\dfrac{(ax^2+bx+c)-(x-1)(2ax+b)}{(ax^2+bx+c)^2}$ Hence $\dfrac{(a-b+c)-(-2)(-2a+b)}{(a-b+c)^2}=0$ $\Rightarrow\ (a-b+c)-(-2)(-2a+b)=0$ $\Rightarrow\ -3a+b+c=0$ …………………………………(2) When $x=-\frac{1}{3}$, $ax^2+bx+c=0$: Hence $\dfrac{a}{9}-\dfrac{b}{3}+c=0$ ………………………………...(3) Solving (1), (2) and (3) simultaneously, we get $a=3$, $b=7$ and $c=2$.
2(i)	
2(ii)	

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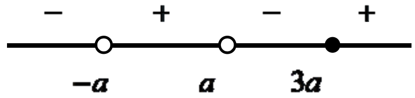
$$\frac{1}{x+a} \le \frac{2a}{x^2-a^2} \Rightarrow \frac{1}{x+a} - \frac{2a}{x^2-a^2} \le 0$$

$$\Rightarrow \frac{x-3a}{(x+a)(x-a)} \le 0$$



$$\therefore x < -a \text{ or } a < x \le 3a$$

$$\int \frac{1}{x+a} - \frac{2a}{x^2-a^2}\,\mathrm{d}x = \ln(x+a) - \ln\left(\frac{x-a}{x+a}\right) = \ln\frac{(x+a)^2}{x-a}$$

OR

$$\int \frac{x-3a}{(x+a)(x-a)}\,\mathrm{d}x = \int \frac{2}{x+a} - \frac{1}{x-a}\,\mathrm{d}x = \ln\frac{(x+a)^2}{x-a}$$

$$\int_{2a}^{4a} \left| \frac{1}{x+a} - \frac{2a}{x^2-a^2} \right| \mathrm{d}\,x$$

$$= \int_{2a}^{3a} -\left(\frac{1}{x+a} - \frac{2a}{x^2-a^2}\right) \mathrm{d}\,x + \int_{3a}^{4a} \frac{1}{x+a} - \frac{2a}{x^2-a^2}\,\mathrm{d}\,x$$

$$\int_{2a}^{3a} -\left(\frac{1}{x+a} - \frac{2a}{x^2-a^2}\right) \mathrm{d}\,x + \int_{3a}^{4a} \frac{1}{x+a} - \frac{2a}{x^2-a^2}\,\mathrm{d}\,x$$

$$= -\left[\ln\frac{(x+a)^2}{x-a}\right]_{2a}^{3a} + \left[\ln\frac{(x+a)^2}{x-a}\right]_{3a}^{4a}$$

$$= -\left(\ln\frac{16a^2}{2a} - \ln\frac{9a^2}{a}\right) + \left(\ln\frac{25a^2}{3a} - \ln\frac{16a^2}{2a}\right)$$

$$= -\ln\frac{8}{9} + \ln\frac{25}{24} = \ln\left(\frac{25}{24}\times\frac{9}{8}\right) = \ln\frac{75}{64}$$

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<td>4(i)</td>
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$$(k+x)^n = k^n\left(1+\frac{x}{k}\right)^n$$

$$= k^n\left(1 + n\left(\frac{x}{k}\right) + \frac{(n)(n-1)}{2!}\left(\frac{x}{k}\right)^2 + \ldots\right)$$

$$= k^n\left(1 + \frac{n}{k}x + \frac{(n)(n-1)}{2k^2}x^2 + \ldots\right)$$

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<td>4(ii)</td>
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$$\left|\frac{x}{k}\right| < 1 \Rightarrow |x| < |k|$$

$$\therefore -|k| < x < |k|$$

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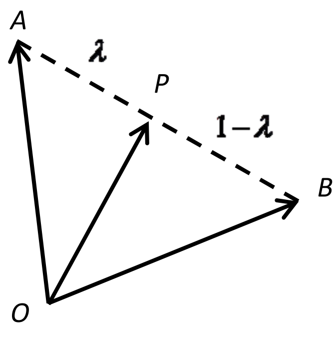
4(iii)	Let $x = y + 3y^2$ and $n = -3$: $\left(k + y + 3y^2\right)^{-3}$ $= k^{-3}\left(1 + \frac{(-3)}{k}\left(y + 3y^2\right) + \frac{(-3)(-4)}{2k^2}\left(y + 3y^2\right)^2 + \ldots\right)$ $= k^{-3}\left(1 - \frac{3}{k}y - \frac{9}{k}y^2 + \frac{6}{k^2}y^2 + \ldots\right)$ $\Rightarrow k^{-3}\left(-\frac{9}{k} + \frac{6}{k^2}\right) = 2 \Rightarrow 2k^5 + 9k - 6 = 0$ $\therefore k = 0.642$ (to 3 sf)
5	$\frac{\overrightarrow{OC}\cdot\overrightarrow{OA}}{\left\|\overrightarrow{OC}\right\|\left\|\overrightarrow{OA}\right\|} = \frac{\overrightarrow{OC}\cdot\overrightarrow{OB}}{\left\|\overrightarrow{OC}\right\|\left\|\overrightarrow{OB}\right\|}$ $\frac{\mathbf{c}\cdot\mathbf{a}}{\|\mathbf{a}\|} = \frac{\mathbf{c}\cdot\mathbf{b}}{\|\mathbf{b}\|} \Rightarrow \frac{\mathbf{c}\cdot\mathbf{a}}{\|\mathbf{a}\|} - \frac{\mathbf{c}\cdot\mathbf{b}}{\|\mathbf{b}\|} = 0 \Rightarrow \mathbf{c}\cdot\left(\frac{\mathbf{a}}{\|\mathbf{a}\|} - \frac{\mathbf{b}}{\|\mathbf{b}\|}\right) = 0$ Alternatively $\left(\frac{\mathbf{a}}{\|\mathbf{a}\|} - \frac{\mathbf{b}}{\|\mathbf{b}\|}\right)\cdot\mathbf{c} = \frac{\mathbf{a}\cdot\mathbf{c}}{\|\mathbf{a}\|} - \frac{\mathbf{b}\cdot\mathbf{c}}{\|\mathbf{b}\|}$ $= \frac{\|\mathbf{a}\|\|\mathbf{c}\|\cos\theta}{\|\mathbf{a}\|} - \frac{\|\mathbf{b}\|\|\mathbf{c}\|\cos\theta}{\|\mathbf{b}\|} = 0$
	$\left(\frac{\mathbf{a}}{\|\mathbf{a}\|} + \frac{\mathbf{b}}{\|\mathbf{b}\|}\right)\cdot\left(\frac{\mathbf{a}}{\|\mathbf{a}\|} - \frac{\mathbf{b}}{\|\mathbf{b}\|}\right) = \left(\frac{\mathbf{a}\cdot\mathbf{a}}{\|\mathbf{a}\|^2} - \frac{\mathbf{b}\cdot\mathbf{b}}{\|\mathbf{b}\|^2}\right)$ $= \left(\frac{\|\mathbf{a}\|^2}{\|\mathbf{a}\|^2} - \frac{\|\mathbf{b}\|^2}{\|\mathbf{b}\|^2}\right) = 1 - 1 = 0$
	P is on $l_{AB} \Rightarrow \overrightarrow{OP} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) = \lambda\mathbf{b} + (1-\lambda)\mathbf{a}$ P is on $l_{OC} \Rightarrow \overrightarrow{OP} = \mu\overrightarrow{OC} = \mu\left(\frac{\mathbf{a}}{\|\mathbf{a}\|} + \frac{\mathbf{b}}{\|\mathbf{b}\|}\right)$  Equating $\lambda\mathbf{b} + (1-\lambda)\mathbf{a} = \mu\left(\frac{\mathbf{a}}{\|\mathbf{a}\|} + \frac{\mathbf{b}}{\|\mathbf{b}\|}\right)$ Comparing coefficients of **a** and **b** $\lambda = \frac{\mu}{\|\mathbf{b}\|}$ and $1 - \lambda = \frac{\mu}{\|\mathbf{a}\|}$ Note that $AP : PB = \lambda : 1 - \lambda$, therefore $AP : PB = \frac{\mu}{\|\mathbf{b}\|} : \frac{\mu}{\|\mathbf{a}\|} = \|\mathbf{a}\| : \|\mathbf{b}\|$.

6(i)	$e^y = (1+\sin x)^2$ Differentiating w.r.t. x, $e^y \frac{dy}{dx} = 2(1+\sin x)\cos x$ $e^y \frac{dy}{dx} = 2\cos x + \sin 2x$ Differentiating w.r.t. x again, $e^y \frac{d^2y}{dx^2} + \frac{dy}{dx} e^y \frac{dy}{dx} = -2\sin x + 2\cos 2x$ $e^y \left[\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] = 2(\cos 2x - \sin x)$ (shown) Differentiating w.r.t. x: $e^y \left[\frac{d^3y}{dx^3} + 2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}\right] + \left[\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] e^y \frac{dy}{dx} = 2(-2\sin 2x - \cos x)$ Substituting $x = 0$, $y = 0;\quad \frac{dy}{dx} = 2;\quad \frac{d^2y}{dx^2} = -2;\quad \frac{d^3y}{dx^3} = 2$ $\Rightarrow y = 0 + 2x + \frac{-2}{2!}x^2 + \frac{2}{3!}x^3 + \ldots$ $\therefore y = 2x - x^2 + \frac{1}{3}x^3 + \ldots$
6(ii)	**<u>Method 1</u>**: $e^y = (1+\sin x)^2$ $\Rightarrow y = \ln(1+\sin x)^2$ $= 2\ln(1+\sin x)$ $= 2\ln\left(1 + \left(x - \frac{x^3}{3!}\right) + \ldots\right)$ $= 2\left(\left(x - \frac{x^3}{3!}\right) - \frac{\left(x - \frac{x^3}{3!}\right)^2}{2} + \frac{\left(x - \frac{x^3}{3!}\right)^3}{3} + \ldots\right)$ $= 2\left(x - \frac{x^3}{6} - \frac{x^2}{2} + \frac{x^3}{3} + \ldots\right)$ $= 2x - x^2 + \frac{1}{3}x^3 + \ldots$ which is same as the expansion for y found in (i), up to and including the term in $x^3 \Rightarrow$ verified.

Method 2:

$$\text{RHS} = (1+\sin x)^2$$

$$= \left(1 + x - \frac{x^3}{3!}\right)^2$$

$$= 1 + x - \frac{x^3}{6} + x + x^2 - \frac{x^3}{6} + \ldots$$

$$= 1 + 2x + x^2 - \frac{x^3}{3} + \ldots$$

$$\text{LHS} = \mathrm{e}^y$$

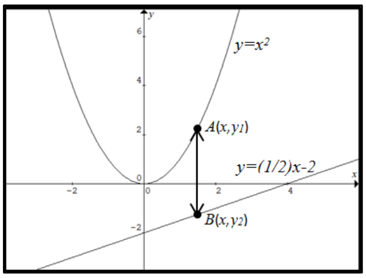
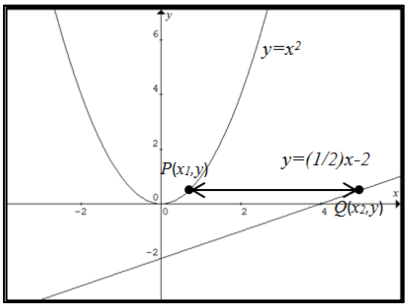
$$= \mathrm{e}^{\left(2x - x^2 + \frac{1}{3}x^3 + \ldots\right)} \qquad \text{(using expansion for } y \text{ in (i))}$$

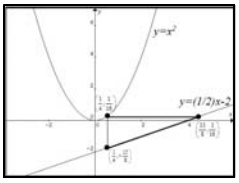
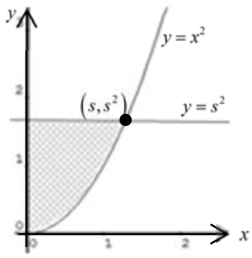
$$= 1 + \left(2x - x^2 + \frac{1}{3}x^3\right) + \frac{\left(2x - x^2 + \frac{1}{3}x^3\right)^2}{2!} + \frac{\left(2x - x^2 + \frac{1}{3}x^3\right)^3}{3!} + \ldots \quad \text{LHS} = \text{RHS} \Rightarrow \text{verified.}$$

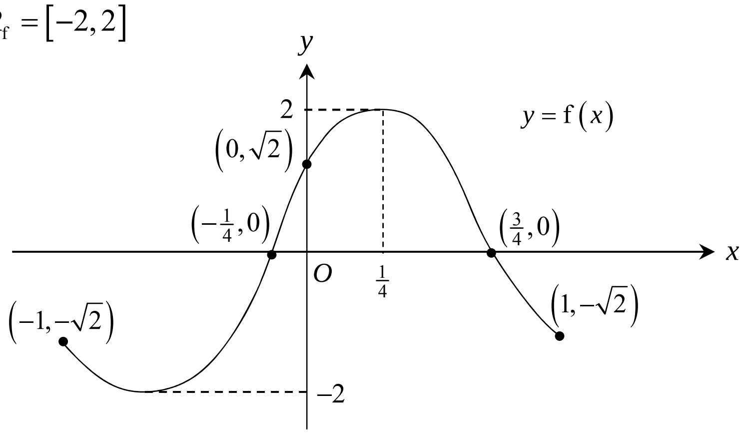
$$= 1 + 2x - x^2 + \frac{1}{3}x^3 + \frac{4x^2 - 2x^3 - 2x^3}{2} + \frac{8x^3}{6} + \ldots$$

$$= 1 + 2x + x^2 - \frac{1}{3}x^3 + \ldots$$

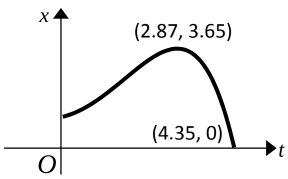
7(a)	$2z+1=\|w\|$ ....(1) $2w-z=4+8\text{i}$.......(2) $2z+1=$ a positive real number $\Rightarrow$ Let $z=x$ and $w=a+b\text{i}$ From (2): $2(a+bi)-x=4+8\text{i}$ $\Rightarrow$ Comparing Re and Im parts, $2a-x=4$ $2b=8\Rightarrow b=4$ From (1): $2x+1=\sqrt{a^2+b^2}$....(3) Substitute $b=4$ and $x=2a-4$ into (3): $2(2a-4)+1=\sqrt{a^2+16}\Rightarrow(4a-7)^2=a^2+16$ $16a^2-56a+49=a^2+16\Rightarrow 15a^2-56a+33=0$ $\Rightarrow a=\frac{11}{15}$ or $a=3$ $\Rightarrow x=-\frac{98}{15}$ or $x=2$ but $2z+1=$ a positive real number $\Rightarrow$ when $x=-\frac{98}{15}$, $2z+1=2\left(-\frac{98}{15}\right)+1<0$ $\Rightarrow$ reject $x=-\frac{98}{15}$ and $a=\frac{11}{15}$ $\Rightarrow x=2,\ a=3, b=4$ $\Rightarrow z=2,\ w=3+4\text{i}$
7(b)	$2\text{e}^{-\left(\frac{3+x+\text{i}y}{\text{i}}\right)}=1-\text{i}$ $2\text{e}^{3\text{i}+x\text{i}-y}=\sqrt{2}\text{e}^{\text{i}\left(-\frac{\pi}{4}\right)}$ $2\text{e}^{-y}\text{e}^{\text{i}(3+x)}=\sqrt{2}\text{e}^{\text{i}\left(-\frac{\pi}{4}\right)}$ $\Rightarrow$ By comparing modulus and args: $2\text{e}^{-y}=\sqrt{2}$ and $\qquad 3+x=-\frac{\pi}{4}$ $-y=\ln\left(\frac{\sqrt{2}}{2}\right) \qquad \Rightarrow x=-\frac{\pi}{4}-3$ $\Rightarrow y=-\ln\left(\frac{\sqrt{2}}{2}\right)$ (or $\ln\sqrt{2}$ or $\frac{1}{2}\ln 2$)

8(i)	Let V be the distance AB.  $V = y_1 - y_2$ $= x^2 - \left(\frac{1}{2}x - 2\right)$ $= x^2 - \frac{1}{2}x + 2$ $\frac{\mathrm{d}V}{\mathrm{d}x} = 2x - \frac{1}{2}$ when $\frac{\mathrm{d}V}{\mathrm{d}x} = 0, \quad x = \frac{1}{4}$ $\frac{\mathrm{d}^2V}{\mathrm{d}x^2} = 2 > 0 \Rightarrow$ min. value when $x = \frac{1}{4}$ when $x = \frac{1}{4}$, $y = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$ $y = \frac{1}{2}\left(\frac{1}{4}\right) - 2 = -\frac{15}{8}$ $\therefore$ coords on C (Pt A): $\left(\frac{1}{4}, \frac{1}{16}\right)$ & coords on L (Pt B): $\left(\frac{1}{4}, -\frac{15}{8}\right)$.
8(ii)	Let H be the distance PQ.  $H = x_2 - x_1 = 2(y+2) - \sqrt{y}$ $\frac{\mathrm{d}H}{\mathrm{d}y} = 2 - \frac{1}{2}y^{-\frac{1}{2}}$ when $\frac{\mathrm{d}H}{\mathrm{d}y} = 0$, $2 - \frac{1}{2}y^{-\frac{1}{2}} = 0 \Rightarrow 2 = \frac{1}{2}y^{-\frac{1}{2}}$ $\Rightarrow y = 4^{-2} = \frac{1}{16}$ $\frac{\mathrm{d}^2H}{\mathrm{d}y^2} = \frac{1}{4}y^{-\frac{3}{2}}$ $\Rightarrow$ when $y = \frac{1}{16}, \frac{\mathrm{d}^2H}{\mathrm{d}y^2} = \frac{1}{4}\left(\frac{1}{16}\right)^{-\frac{3}{2}} = 16 > 0$ $\Rightarrow$ min. value when $y = \frac{1}{16}$ when $y = \frac{1}{16}$, $x = \sqrt{\frac{1}{16}} = \frac{1}{4}$ $x = 2\left(\frac{1}{16}\right) + 2 = \frac{33}{8}$ $\therefore$ coords on C (Pt P): $\left(\frac{1}{4}, \frac{1}{16}\right)$ & coords on L (Pt Q): $\left(\frac{33}{8}, \frac{1}{16}\right)$.

8(iii)	Area of polygon = Area of triangle  Minimum distance $AB = \frac{1}{16} - \left(-\frac{15}{8}\right) = \frac{31}{16}$ Minimum distance $PQ = \frac{33}{8} - \left(\frac{1}{4}\right) = \frac{31}{8}$ $\therefore$ Area of polygon $= \frac{1}{2} \times \frac{31}{16} \times \frac{31}{8} = \frac{961}{256}$ sq units
8(iv)	 $\frac{\mathrm{d}s}{\mathrm{d}t} = 2$ Method 1: Area $= A = \int_0^{s^2} x \ \mathrm{d}y = \int_0^{s^2} \sqrt{y} \ \mathrm{d}y = \left[\frac{y^{\frac{3}{2}}}{3/2}\right]_0^{s^2} = \frac{2}{3}s^3$ $\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}s} \times \frac{\mathrm{d}s}{\mathrm{d}t} = 2s^2 \times 2 = 4s^2$ $\therefore$ when $s = \sqrt{2}$, $\frac{\mathrm{d}A}{\mathrm{d}t} = (4)\left(\sqrt{2}\right)^2 = 8$ units^2/s Method 2: Area $= A$ = Area of rectangle − Area bounded by curve, x-axis and $x = s$ $= s \times s^2 - \int_0^s y \ \mathrm{d}x = s^3 - \int_0^s x^2 \ \mathrm{d}x = s^3 - \left[\frac{x^3}{3}\right]_0^s = \frac{2}{3}s^3$ $\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}s} \times \frac{\mathrm{d}s}{\mathrm{d}t} = 2s^2 \times 2 = 4s^2$ $\therefore$ when s $= \sqrt{2}, \Rightarrow \frac{\mathrm{d}A}{\mathrm{d}t} = 4\left(\sqrt{2}\right)^2 = 8$ units^2/s
9(a)	By factor formula, $\sin\left(x+\frac{1}{4}\right)\pi - \sin\left(x-\frac{3}{4}\right)\pi = 2\cos\left[\frac{1}{2}\left(2x-\frac{1}{2}\right)\pi\right]\sin\left(\frac{1}{2}\pi\right)$ $= 2\cos\left(x-\frac{1}{4}\right)\pi.$ Hence

	$\sum_{x=1}^{n} 2\cos\left(x-\tfrac{1}{4}\right)\pi$ $=\sum_{x=1}^{n}\left[\sin\left(x+\tfrac{1}{4}\right)\pi-\sin\left(x-\tfrac{3}{4}\right)\pi\right]$ $=\left[\sin\tfrac{5}{4}\pi-\sin\tfrac{1}{4}\pi\right]+\left[\sin\tfrac{9}{4}\pi-\sin\tfrac{5}{4}\pi\right]+\ldots$ $+\left[\sin\left(n-\tfrac{3}{4}\right)\pi-\sin\left(n-\tfrac{7}{4}\right)\pi\right]+\left[\sin\left(n+\tfrac{1}{4}\right)\pi-\sin\left(n-\tfrac{3}{4}\right)\pi\right]$ $=\sin\left(n+\tfrac{1}{4}\right)\pi-\sin\tfrac{1}{4}\pi$ $=\sin\left(n+\tfrac{1}{4}\right)\pi-\tfrac{1}{\sqrt{2}}$ Therefore, $\sum_{x=1}^{n}\cos\left(x-\tfrac{1}{4}\right)\pi=\tfrac{1}{2}\sin\left(n+\tfrac{1}{4}\right)\pi-\tfrac{1}{2\sqrt{2}}$.
9(b)(i)	$R_{\mathrm{f}}=[-2,2]$ 
(b)(ii)	Least value of a is $\tfrac{1}{4}$. Let $y=2\cos\left(x-\tfrac{1}{4}\right)\pi$. Then $\cos^{-1}\left(\tfrac{y}{2}\right)=\left(x-\tfrac{1}{4}\right)\pi \quad\Rightarrow\quad x=\dfrac{\cos^{-1}\left(\tfrac{y}{2}\right)}{\pi}+\dfrac{1}{4}$. $\therefore \quad \mathrm{f}^{-1}: x\mapsto \dfrac{1}{\pi}\cos^{-1}\left(\dfrac{x}{2}\right)+\dfrac{1}{4}, \qquad x\in\left[-\sqrt{2},2\right]$
(b)(iii)	fg exists $\quad\Rightarrow\quad R_{\mathrm{g}}\subseteq D_{\mathrm{f}}$ now $R_{\mathrm{g}}=\left[-\dfrac{13}{4},-2\right)$ and $D_{\mathrm{f}}=[a,1]$ since fg exists, $a\le-\dfrac{13}{4}$. Hence the greatest value of a is $-\tfrac{13}{4}$. $R_{\mathrm{fg}}=\mathrm{f}\left(R_{\mathrm{g}}\right)=\mathrm{f}\left[-\dfrac{13}{4},-2\right)=\left[-2,\sqrt{2}\right)$.
10(a)(i)	After one month, if she pays \$$x$ at the beginning of the month, she will owe the bank $\$(50000-x)\times(1.001)$ Hence $(50000-x)\times(1.001)=50000 \quad\Rightarrow\quad x=49.95$

	Abbie needs to pay \$49.95 (to the nearest cent) a month.
(a)(ii)	One month after graduating, she owes $(50000-k)\times(1.00375)$. n months after graduating, she will owe $1.00375^n(50000-k)-1.00375^{n-1}k-\ldots-1.00375k$ $=1.00375^n(50000)-k\left(1.00375^n+1.00375^{n-1}+\ldots+1.00375\right)$ $=1.00375^n(50000)-k\left[\dfrac{1.00375\left(1.00375^n-1\right)}{1.00375-1}\right]$ $=1.00375^n(50000)-\dfrac{803}{3}k\left(1.00375^n-1\right)$ (shown).
(a)(iii)	Sub n = 120, and k = 500: $1.00375^{120}(50000)-\dfrac{803}{3}(500)\left(1.00375^{120}-1\right)=2467.11>0$. No, she cannot. A monthly payment of \$500 is not enough. When n = 120, $1.00375^{120}(50000)-\dfrac{803}{3}k\left(1.00375^{120}-1\right)=0$ $\Rightarrow \quad k=516.26$ (nearest cent) She needs to pay \$516.26 per month.
(b)(i)	Oustanding amount upon graduation $=1.001^{36}(50000)$ $=51831.86$ Using Abbie's formula, but with a starting outstanding amount of \$51831.86, $1.00375^{120}(51831.86)-\dfrac{803}{3}k\left(1.00375^{120}-1\right)=0$ $\Rightarrow \quad k=535.17$ (nearest cent) He needs to pay \$535.17 per month.
(b)(ii)	$120\times535.17-50000=14220.43$ (to 2 d.p.) He paid \$14220.43 in interest altogether.
11(i)	$\int t^2\,\mathrm{e}^{-kt}\,\mathrm{d}t=-\dfrac{1}{k}\mathrm{e}^{-kt}(t^2)-\int -\dfrac{1}{k}\mathrm{e}^{-kt}(2t)\,\mathrm{d}t$ $=-\dfrac{1}{k}t^2\,\mathrm{e}^{-kt}+\dfrac{2}{k}\left[-\dfrac{1}{k}\mathrm{e}^{-kt}(t)-\int -\dfrac{1}{k}\mathrm{e}^{-kt}(1)\,\mathrm{d}t\right]$ $=-\dfrac{1}{k}t^2\,\mathrm{e}^{-kt}-\dfrac{2}{k^2}t\,\mathrm{e}^{-kt}-\dfrac{2}{k^3}\mathrm{e}^{-kt}+D$ $=-\mathrm{e}^{-kt}\left(\dfrac{1}{k}t^2+\dfrac{2}{k^2}t+\dfrac{2}{k^3}\right)+D$
(ii)	$\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{3}{4}x-pt^2$

(iii)	$x = u\,e^{\frac{3}{4}t} \Rightarrow \dfrac{dx}{dt} = \dfrac{3}{4}u\,e^{\frac{3}{4}t} + e^{\frac{3}{4}t}\dfrac{du}{dt}$ $\dfrac{3}{4}u\,e^{\frac{3}{4}t} + e^{\frac{3}{4}t}\dfrac{du}{dt} = \dfrac{3}{4}u\,e^{\frac{3}{4}t} - pt^2 \Rightarrow \dfrac{du}{dt} = -pt^2\,e^{-\frac{3}{4}t}$ $u = p\,e^{-\frac{3}{4}t}\left(\dfrac{1}{\frac{3}{4}}t^2 + \dfrac{2}{\left(\frac{3}{4}\right)^2}t + \dfrac{2}{\left(\frac{3}{4}\right)^3}\right) + D$ $= p\,e^{-\frac{3}{4}t}\left(\dfrac{4}{3}t^2 + \dfrac{32}{9}t + \dfrac{128}{27}\right) + D$ $\Rightarrow \dfrac{x}{e^{\frac{3}{4}t}} = p\,e^{-\frac{3}{4}t}\left(\dfrac{4}{3}t^2 + \dfrac{32}{9}t + \dfrac{128}{27}\right) + D$ $\therefore x = p\left(\dfrac{4}{3}t^2 + \dfrac{32}{9}t + \dfrac{128}{27}\right) + D\,e^{\frac{3}{4}t}$ When $t = 0, x = 1$, $1 = p\left(\dfrac{128}{27}\right) + D \Rightarrow D = 1 - \dfrac{128}{27}p$ $x = p\left(\dfrac{4}{3}t^2 + \dfrac{32}{9}t + \dfrac{128}{27}\right) + \left(1 - \dfrac{128}{27}p\right)e^{\frac{3}{4}t}$
(iv)	When $p = \frac{1}{3}$, $x = \dfrac{1}{3}\left(\dfrac{4}{3}t^2 + \dfrac{32}{9}t + \dfrac{128}{27}\right) + \left(-\dfrac{47}{81}\right)e^{\frac{3}{4}t}$  Maximum number of players on the game = 365 000. Yes, $x = 0$ when $t = 4.35$ months.
(v)	For $x = 0$ after some time, $1 - \dfrac{128}{27}p < 0 \Rightarrow p > \dfrac{27}{128} = 0.211$