SAINT ANDREW'S JUNIOR COLLEGE

Preliminary Examination

MATHEMATICS

Higher 2 **9758/02**

Wednesday **13 September 2017** **3 hours**

Additional materials : Answer paper
List of Formulae (MF 26)
Cover Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Answer **all** the questions. Total marks : **100**

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

At the end of the examination, fasten all your work securely together.

Section A: Pure Mathematics [40 marks]

1 Without the use of a calculator, find the complex numbers z and w which satisfy the simultaneous equations

$$z - w\mathrm{i} = 3$$
$$z^2 - w + 6 + 3\mathrm{i} = 0$$ **[6]**

2 **(i)** Show that $\dfrac{2}{n-1} - \dfrac{3}{n} + \dfrac{1}{n+1} = \dfrac{An+B}{n^3-n}$, where A and B are constants to be found. **[2]**

(ii) Hence find $\displaystyle\sum_{r=2}^{n} \frac{2r+6}{r^3-r}$. **[3]**

(iii) Use your answer to part **(ii)** to find $\displaystyle\sum_{r=2}^{n} \frac{2r+10}{(r+1)(r+2)(r+3)}$. **[3]**

3 The function f is defined by $\mathrm{f} : x \mapsto \dfrac{1}{x^2-1}, \quad x \in \mathbb{R}, \quad x > 1$.

(i) Find $\mathrm{f}^{-1}(x)$ and write down the domain of f^{-1}. **[3]**

(ii) On the same diagram, sketch the graphs of $y = \mathrm{f}(x)$, $y = \mathrm{f}^{-1}(x)$ and $y = \mathrm{f}^{-1}\mathrm{f}(x)$ stating the equations of any asymptotes and showing the relationships between the graphs clearly. **[4]**

(iii) State the set of values of x such that $\mathrm{ff}^{-1}(x) = \mathrm{f}^{-1}\mathrm{f}(x)$. **[1]**

4 Referred to the origin O, the point A has position vector $-5\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$. The plane π has equation:

$$\mathbf{r} = (1 + \lambda - 2\mu)\mathbf{i} + (3 - 2\lambda)\mathbf{j} + (\mu - 2)\mathbf{k} \quad \text{where } \lambda, \mu \in \mathbb{R}$$

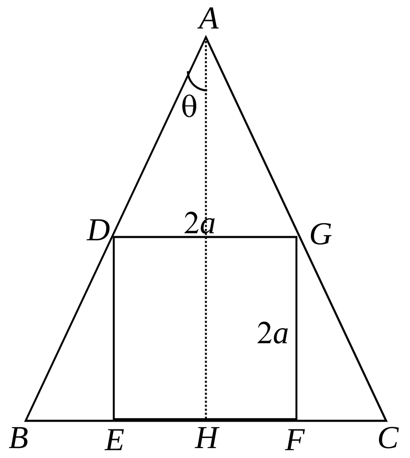
(i) Find the vector equation of plane π in scalar product form. **[2]**

(ii) Find the position vector of the foot of perpendicular, C, from A to π. **[3]**

The line l_1 passes through the points A and B.

(iii) The line l_2 is the reflection of the line l_1 about the plane π. Find a vector equation of l_2. **[3]**

5



It is given that *DEFG* is a square with fixed side $2a$ cm and it is inscribed in the isosceles triangle *ABC* with height *AH* , where $AB = AC$ and angle $BAH = \theta$.

(i) Taking $t = \tan\theta$, show that the area of the triangle *ABC* is given by $S = a^2\left(4 + 4t + \dfrac{1}{t}\right)$. **[3]**

(ii) Find the minimum area of S in terms of a when t varies. **[4]**

(iii) Hence sketch the graph showing the area of the triangle *ABC* as θ varies. **[3]**

Section B: Statistics [60 marks]

6 **(a)** There are three yellow balls, three red balls and three blue balls. Balls of each colour are numbered 1, 2, and 3. Find the number of ways of arranging the balls in a row such that adjacent balls do not sum up to two. **[2]**

(b) In a restaurant, there were two round tables available, a table for five and a table for six. Find the number of ways eleven friends can be seated if two particular friends are not seated next to each other. **[4]**

7 For the events A and B, it is given that

$$\mathrm{P}(A \cap B') = 0.6, \quad \mathrm{P}(A \cup B') = 0.83 \quad \text{and} \quad \mathrm{P}(A \mid B') = 0.83$$

Find,

(i) $\mathrm{P}(B)$ **[2]**

(ii) $\mathrm{P}(A \cap B)$ **[2]**

(iii) $\mathrm{P}(B \mid A')$ **[2]**

Hence determine whether A and B are independent. **[1]**

8 A fairground game involves trying to hit a moving target with a gunshot. A round consists of a **maximum** of 3 shots. Ten points are scored if a player hits the target. The **round** ends **immediately** if the player misses a shot. The probability that Linda hits the target in a single shot is 0.6. All shots taken are independent of one another.

(i) Find the probability that Linda scores 30 points in a round. **[1]**

The random variable X is the number of points Linda scores in a round.

(ii) Find the probability distribution of X. **[3]**

(iii) Find the mean and variance of X. **[3]**

(iv) A game consists of 2 rounds. Find the probability that Linda scores more points in round 2 than in round 1. **[3]**

9 Six cities in a certain country are linked by rail to city O. The rail company provides the information about the distance of each city to city O and the rail fare from that city to city O on its website. Charles copied the table below from the website, but he had copied one of the rail fares wrongly.

City	A	B	C	D	E	F
Distance, x km	100	270	120	56	289	347
Rail fare, \$$y$	11.1	17.1	6.44	7.62	17.9	18.8

(i) Give a sketch of the scatter diagram for the data as shown on your calculator. On your diagram, circle the point that Charles has copied wrongly. **[2]**

For parts **(ii)**, **(iii)** and **(iv)** of this question you should **exclude** the point for which Charles has copied the rail fare value wrongly.

(ii) Find, correct to 4 decimal places, the product moment correlation coefficient between
(a) $\ln x$ and y,
(b) x^2 and y. **[2]**

(iii) Using parts **(i)** and **(ii)**, explain which of the cases in part **(ii)** is more appropriate for modelling the data. **[2]**

(iv) By using the equation of a suitable regression line, estimate the rail fare when the distance is 210 km. Explain if your estimate is reliable. **[3]**

10 A factory manufactures round tables in two sizes: small and large. The radius of a small table, measured in cm, has distribution $\mathrm{N}(30, 2^2)$ and the radius of a large table, measured in cm, has distribution $\mathrm{N}(50, 5^2)$.

(i) Find the probability that the sum of the radius of 5 randomly chosen small tables is less than 160 cm. **[2]**

(ii) Find the probability that the sum of the radius of 3 randomly chosen small tables is less than twice the radius of a randomly chosen large table. **[2]**

(iii) State an assumption needed in your calculation in part **(ii)**. **[1]**

A shipment of 12 large tables is to be exported. Before shipping, a check is done and the shipment will be rejected if there are at least two tables whose radius is less than 40 cm.

(iv) Find the probability that the shipment is rejected. **[3]**

The factory decides now to manufacture medium sized tables. The radius of a medium sized table, measured in cm, has distribution $\mathrm{N}(\mu, \sigma^2)$. It is known that 20% of the medium sized tables have radius greater than 44 cm and 30% have radius of less than 40 cm.

(v) Find the values of μ and σ. **[4]**

11 The Kola Company receives a number of complaints that the volume of cola in their cans are less than the stated amount of 500 ml. A statistician decides to sample 50 cola cans to investigate the complaints. He measures the volume of cola, x ml, in each can and summarised the results as follows:

$$\sum x = 24730, \sum x^2 = 12242631.$$

(i) Find unbiased estimates of the population mean and variance correct to 2 decimal places and carry out the test at the 1% level of significance. **[6]**

(ii) One director in the company points out that the company should test whether the volume of cola in a can is 500 ml at the 1% significance level instead. Using the result of the test conducted in **(i)**, explain how the p-value of this test can be obtained from p-value in part **(i)** and state the corresponding conclusion. **[2]**

The head statistician agrees the company should test that the volume of cola in a can is 500 ml at the 1% level of significance. He intends to make a simple rule of reference for the production managers so that they will not need to keep coming back to him to conduct hypothesis tests. On his instruction sheet, he lists the following:

1. Collect a random sample of 40 cola cans and measure their volume.
2. Calculate the mean of your sample, $\bar{x}$ and the variance of your sample, s_x^2.
3. Conclude that the volume of cola differs from 500 ml if the value of $\bar{x}$ lies…..

(iii) Using the above information, complete the decision rule in step 3 in terms of s_x.

[4]

A party organiser has n cans of cola and $2n$ packets of grape juice. Assume now that the volume of a can of cola has mean 500 ml and variance 144 ml^2, and the volume of a packet of grape juice has mean 250 ml and variance 25 ml^2. She mixes all the cola and grape juice into a mocktail, which she pours into a 120-litre barrel. Assume that n is sufficiently large and that the volumes of the cans of cola and packets of grape juice are independent.

(iv) Show that if the party organiser wants to be at least 95% sure that the barrel will not overflow, n must satisfy the inequality $1000n + 22.9\sqrt{n} - 120,000 \le 0$. **[4]**

End of Paper