ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

MATHEMATICS 9758/02

Paper 2 **21 August 2017**

3 hours

Additional Materials: Cover Sheet
Answer Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT
JC2 Preliminary Examination 2017

MATHEMATICS 9758
Higher 2
Paper 2

/ 100

Index No: [ ][ ][ ][ ] Form Class: ____________
Name: ______________________________

Calculator model: ____________________

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

Question No.	Marks
1	/4
2	/10
3	/12
4(a)	/6
4(b)	/8
5	/6
6	/7
7	/11
8	/10
9	/13
10(a)	/3
10(b), 10(c)	/10

Summary of Areas for Improvement			
Knowledge (K)	Careless Mistakes (C)	Read/Interpret Qn wrongly (R)	Presentation (P)

Section A: Pure Mathematics [40 marks]

1 Given that $1+\mathrm{i}$ is a root of the equation $z^3-4(1+\mathrm{i})z^2+(-2+9\mathrm{i})z+5-\mathrm{i}=0$, find the other roots of the equation. [4]

2 A curve C has parametric equations

$$x=\cos t$$
$$y=\frac{1}{2}\sin 2t$$

where $\frac{\pi}{2}\le t\le\frac{3\pi}{2}$.

(i) Find the equation of the normal to C at the point P with parameter p. [2]

The normal to C at the point when $t=\frac{2\pi}{3}$ cuts the curve again. Find the coordinates of the point of intersection. [2]

(ii) Sketch C, clearly labelling the coordinates of the points where the curve crosses the x- and y- axes. [1]

(iii) Find the cartesian equation of C. [2]

The region bounded by C is rotated through π radians about the x-axis. Find the exact volume of the solid formed. [3]

3 **(i)** Find $\displaystyle\int \frac{x}{\left(1+x^2\right)^2}\,\mathrm{d}x$. [2]

(ii) By using the substitution $x=\tan\theta$, show that

$$\int \frac{1}{\left(1+x^2\right)^2}\,\mathrm{d}x = k\left(\frac{x}{1+x^2}+\tan^{-1}x\right)+c,$$

where c is an arbitrary constant, and k is a constant to be determined. [5]

(iii) Hence find $\displaystyle\int \frac{x^2}{\left(1+x^2\right)^2}\,\mathrm{d}x$. [3]

(iv) Using all of the above, find $\displaystyle\int \frac{x^2+2x+5}{\left(1+x^2\right)^2}\,\mathrm{d}x$, simplifying your answer. [2]

4 **(a)** **(i)** The unit vector $\mathbf{d}$ makes angles of $60°$ with both the x- and y-axes, and θ with the z-axis, where $0° \le \theta \le 90°$. Show that $\mathbf{d}$ is parallel to $\mathbf{i}+\mathbf{j}+\sqrt{2}\mathbf{k}$. [3]

(ii) The line m is parallel to $\mathbf{d}$ and passes through the point with coordinates $(2,-1,0)$. Find the coordinates of the point on m that is closest to the point with coordinates $(3,2,0)$. [3]

(b) The plane p_1 has equation $\mathbf{r}\cdot(\mathbf{i}+2\mathbf{j}+2\mathbf{k})=5$, and the line l has equation $\dfrac{x-a}{2}=\dfrac{y-1}{b}=-\dfrac{z}{2}$, where a and b are constants.

Given that l lies on p_1, show that $b=1$ and find the value of a. [2]

(i) The plane p_2 contains l and is perpendicular to p_1. Find the equation of p_2 in the form $\mathbf{r}\cdot\mathbf{n}=c$, where c is a constant to be determined. [3]

(ii) The variable point $P(x,y,z)$ is equidistant from p_1 and p_2. Find the cartesian equation(s) of the locus of P. [3]

Section B: Statistics [60 marks]

5 A group of 12 students consists of 5 bowlers, 4 canoeists and 3 footballers.

(i) The group sits at a round table with 12 seats. In how many different ways can they sit so that all the players of the same sport sit together? [2]

(ii) The group stands in a line. In how many different ways can they stand so that *either* the bowlers are all next to one another *or* the canoeists are all next to one another *or* both? [2]

(iii) Find the number of ways in which a delegation of 8 can be selected from this group if it must include at least 1 student from each of the 3 sports. [2]

6 Alex and his friend stand randomly in a queue with 3 other people. The random variable X is the number of people standing between Alex and his friend.

(i) Show that $\mathrm{P}(X = 2) = 0.2$. [2]

(ii) Tabulate the probability distribution of X. [2]

(iii) Find $\mathrm{E}(X)$ and $\mathrm{E}(X-1)^2$. Hence find $\mathrm{Var}(X)$. [3]

7 It has been suggested that the optimal pH value for shampoo should be 5.5, to match the pH level of healthy scalp. Any pH value that is too low or too high may have undesirable effects on the user's hair and scalp. A shampoo manufacturer wants to investigate if the pH level of his shampoo is at the optimal value, by carrying out a hypothesis test at the 10% significance level. He measures the pH value, x, of n randomly chosen bottles of shampoo, where n is large.

(a) In the case where $n = 30$, it is found that $\sum x = 178.2$ and $\sum x^2 = 1238.622$.

(i) Find unbiased estimates of the population mean and variance, and carry out the test at the 10% significance level. [6]

(ii) Explain if it is necessary for the manufacturer to assume that the pH value of a bottle of shampoo follows a normal distribution. [1]

(b) In the case where n is unknown, assume that the sample mean is the same as that found in **(a)**.

(i) State the critical region for the test. [1]

(ii) Given that n is large and that the population variance is found to be 6.5, find the greatest value of n that will result in a favourable outcome for the manufacturer at the 10% significance level. [3]

8 A swim school takes in both male and female primary school students for competitive swimming lessons. The school assesses its students' progress each year by recording the time, t seconds, each student takes to swim a 50-metre lap in breaststroke, and the number of months, m, that he or she has been at the school. The records for 8 randomly chosen students are shown in the following table.

m	6	7	10	12	15	19	21	24
t	92.32	87.11	66.12	59.41	53.94	43.82	42.07	41.45

(i) Labelling the axes clearly, draw a scatter diagram for the data and explain, in context, why a linear model would not be suitable to predict the time taken by a student to swim a lap of breaststroke given the number of months that he or she has been at the school. [2]

It is desired to fit a model of the form $\ln(t - C) = a + bm$, where C is a suitable constant. The product moment correlation coefficient r between m and $\ln(t - C)$ for some possible values of C are shown in the table below.

C	36	37	38	39
r	–0.992114		–0.992681	–0.992192

(ii) Calculate the value of r for $C = 37$, giving your answer correct to 6 decimal places. [1]

(iii) Use the table and your answer to **(ii)** to choose the most appropriate value for C. Explain your choice. [2]

For the remainder of this question, use the value of C that you have chosen in **(iii)**.

(iv) Find the equation of the least squares regression line of $\ln(t - C)$ on m. Give an interpretation of C in the context of the question. [2]

(v) Another student who has been swimming at the school for 9 months clocked a time of 60.33 seconds for a lap of breaststroke. Using your regression line, comment on the student's swimming ability. [2]

(vi) Suggest an improvement to the data collection process so that the results could provide a fairer gauge of the expected outcome for the students in the first 2 years of lessons. [1]

9 **(i)** A procedure for accepting or rejecting a large batch of manufactured articles is such that an inspector first selects and examines a random sample of 10 articles from the batch. If the sample contains at least 2 defective articles, the batch is rejected.
It is known that the proportion of articles that are defective is 0.065. Show that the probability that a batch of articles is accepted is 0.866, correct to three significant figures. [1]
To confirm the decision, another inspector follows the same procedure with another random sample of 10 articles from the batch. If the conclusion of both inspectors are the same, the batch will be accepted or rejected as the case may be. Otherwise, one of the inspectors will select a further random sample of 10 from the same batch to examine. The batch is then rejected if there are at least 2 defective articles. Otherwise, it is accepted. Find

(a) the probability that a batch is eventually accepted, [3]

(b) the expected number of articles examined per batch. [4]

(ii) In order to cut labour cost, an alternative procedure is introduced. A random sample of 10 articles is taken from the batch and if the sample contains not more than 1 defective article then the batch is accepted. If the sample contains more than 2 defective articles, the batch is rejected. If the sample contains exactly 2 defective articles, a second sample of 10 articles is taken and if this contains no defective article then the batch is accepted. Otherwise, the batch is rejected. Given that the proportion of defective articles in the batch is p, show that the probability that the batch is accepted is A where

$$A = (1+9p)(1-p)^9 + 45p^2(1-p)^{18}.$$ [2]

If the probability that, of 100 batches inspected, more than 80 of them will be accepted is 0.98, find the value of p. [3]

10 **(a)** An examination taken by a large number of students is marked out of a total score of 100. It is found that the mean is 73 marks and that the standard deviation is 15 marks.

(i) Give a reason why the normal distribution is not a good model for the distribution of marks for the examination. [1]

(ii) The marks for a random sample of 50 students is recorded. Find the probability that the mean mark of this sample lies between 70 and 75. [2]

(b) The interquartile range of a distribution is the difference between the upper and lower quartile values for the distribution. The lower quartile value, l, of a distribution X, is such that $\mathrm{P}(X < l) = 0.25$. The upper quartile value, u, of the same distribution is such that $\mathrm{P}(X < u) = 0.75$.

The marks of another examination is known to follow a normal distribution. If a student who scores 51 marks is at the 80th percentile, and the interquartile range is found to be 10.8 marks, find the mean mark and the standard deviation of the marks scored by students who took the examination. [5]

(c) In a third examination, the marks scored by students are normally distributed with a mean of 52 marks and a standard deviation of 13 marks.

(i) If 50 is the passing mark and 289 students are expected to pass, how many candidates are there? [2]

(ii) Find the smallest integer value of m such that more than 90% of the candidates will score within m marks of the mean. [3]