| Name | ( ) | Class | |
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RIVER VALLEY HIGH SCHOOL
2017 Year 6 Preliminary Examination II
**Higher 2**

# MATHEMATICS **9758/01**

Paper 1 **14 September 2017**

**3 hours**

Additional Materials: Answer Paper
Graph Paper
List of Formulae (MF26)
Cover Page

**READ THESE INSTRUCTIONS FIRST**

**Do not open this booklet until you are told to do so.**
Write your name, class and index number in the space at the top of this page.

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphic calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages and **2** blank pages.

**1.** **(i)** Describe a sequence of transformations that transform the graph of $y = \ln x$ onto the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \ln(x+a) + b$ and that $a$ and $b$ are constants such that $a > 1$ and $b > 1$. [2]

**(ii)** By sketching the graph of $y = \mathrm{f}(x)$ or otherwise, sketch the graph of $y = \dfrac{1}{\mathrm{f}(x)}$.

State, in terms of $a$ and $b$, the coordinates of any points where $y = \dfrac{1}{\mathrm{f}(x)}$ crosses the axes and the equations of any asymptotes. [3]

**2.** A curve $C$ has equation $y = \dfrac{2x^2+3}{x-1}$, $x \in \mathbb{R}$, $x \neq 1$.

**(i)** Sketch $C$, stating the equations of the asymptotes, axial intercepts and the coordinates of the turning points, if any. [3]

**(ii)** Using part **(i)**, solve the inequality $2x + 2 \leq \mathrm{e}^x - \dfrac{5}{x-1}$. [2]

**(iii)** Hence, solve the inequality $2x + 4 \leq \mathrm{e}^{x+1} - \dfrac{5}{x}$. [2]

**3.** **(i)** By using the substitution $t = 3\sec\theta$, find $\displaystyle\int \frac{\sqrt{t^2-9}}{t}\,\mathrm{d}t$. [4]

**(ii)** The curve $C$ is defined by the parametric equations

$$x = \ln t, \quad y = \sqrt{t^2-9}, \text{ where } t \geq 3.$$

Find the exact value of the area of the region bounded by $C$, the line $x = \ln 6$ and the $x$-axis. [4]

**4.** Henry and Isaac take part in a marathon race. In their first training session, they run a distance of 2.4 km each.

**(a)** Henry increases the distance he runs in each subsequent training session by 400 m.

**(i)** Find the distance he runs in the $20^{\text{th}}$ session. [2]

**(ii)** Find the minimum number of sessions he needs to attend in order to run a total distance of 99 km. [3]

**(b)** **(i)** Isaac increases the distance he runs in each subsequent session by $x$ %. Find $x$ if Isaac runs a total distance of 200 km at the end of 20 sessions. [3]

**(ii)** Isaac feels that the training is too tough after the first session. He decides to decrease the distance he runs in each subsequent session by 5% and increase the numbers of sessions. Will he be able to run a total distance of 200 km? Justify your answer. [2]

**5.** With reference to the origin $O$, the positon vectors of three points $A$, $B$ and $C$ are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. Given that $|\mathbf{a}| = 4$, $|\mathbf{b}| = 3$, $\mathbf{c}$ is a unit vector and the angle $AOC$ is $\frac{\pi}{3}$ radians.

**(i)** Find the value of $\mathbf{a} \cdot \mathbf{c}$ and give the geometrical interpretation of this value. [2]

**(ii)** Given $\mathbf{a} - \mathbf{c} = k\mathbf{b}$ where $k \in \mathbb{R}$, $k \neq 0$. By considering $(\mathbf{a} - \mathbf{c}) \cdot (\mathbf{a} - \mathbf{c})$, find the exact values of $k$. [3]

The point $M$ divides $OC$ in the ratio $OM : OC = 2 : 3$.

**(iii)** Find the exact area of triangle $AMC$. [4]

6. **Do not use a calculator in answering this question.**

**(a)** Solve the simultaneous equations

$$z - 4w = 11 + 6\mathrm{i} \text{ and } 3z + 6\mathrm{i}w = 27$$

giving $z$ and $w$ in the form $x + \mathrm{i}y$ where $x$ and $y$ are real. [4]

**(b)** **(i)** The complex numbers $z$ and $w$ are given as $z = 4\left(\cos\frac{\pi}{3} - \mathrm{i}\sin\frac{\pi}{3}\right)$ and $w = 1 + \mathrm{i}\sqrt{3}$. $w^*$ denotes the conjugate of $w$. Find the modulus $r$ and the argument $\theta$ of $\frac{w^*}{z^2}$, where $r > 0$ and $-\pi < \theta \le \pi$. [3]

**(ii)** Find the set of possible values of $n$ such that $\left(\frac{w^*}{z^2}\right)^n$ is purely imaginary. [3]

7. **(a)** Show that $\int \sqrt{5 - x^2}\ \mathrm{d}x = \frac{x}{2}\sqrt{5 - x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right) + c$. [4]

**(b)** **(i)** Let $C$ be the curve $y^4 + x^2 = 5$. The $x$-coordinate of the point $P$ on $C$ is 1 and the $y$-coordinate of the point $P$ on $C$ is positive. Show that the gradient of the normal to $C$ at the point $P$ is $4\sqrt{2}$. Hence find the equation of the normal to $C$ at the point $P$ in exact form. [4]

**(ii)** The region $R$ is bounded by the curve $C$. The solid $S$ is formed by rotating the region $R$ through $\pi$ radians about the $x$-axis. Using part (**a**), find the exact volume of the solid $S$ in terms of $\pi$. [3]

**8.** **(a)** Using differentiation, find the exact dimensions of the rectangle of largest area that can be inscribed in the ellipse, $\frac{x^2}{9}+\frac{y^2}{36}=1$. Hence, find the area of this largest rectangle. [8]

**(b)** In the triangle *DEF*, angle $EDF=\frac{\pi}{3}$ and angle $DFE=\frac{\pi}{3}+\alpha$ and $EF=6$. Given that $\alpha$ is sufficiently small, show that

$$DF-DE\approx d\alpha\ ,$$

where *d* is an exact constant to be determined. [5]

**9.** The line *l* has equation $\frac{x-2}{4}=\frac{z+3}{1}$, $y=2$ and the plane $p_1$ has equation $\mathbf{r}.\begin{pmatrix}1\\1\\1\end{pmatrix}=16$.

Referred to the origin *O*, the position vector of the point *A* is $2\mathbf{i}+2\mathbf{j}-3\mathbf{k}$.

**(i)** Find the acute angle between the line *l* and the plane $p_1$. [2]

**(ii)** Find the coordinates of the foot of perpendicular, *N*, from point *A* to the plane $p_1$.

[3]

**(iii)** Find the coordinates of the point *B* which is the reflection of *A* in plane $p_1$. [2]

**(iv)** Hence, determine the equation of the line which is a reflection of line *l* in the plane $p_1$. [4]

**(v)** Another plane, $p_2$, contains the point *B* and is parallel $p_1$. Determine the exact distance between $p_1$ and $p_2$. [2]

**10.** In a farm, the growth of the population of prawns is studied.

**(a)** The population of prawns of size $n$ thousand at time $t$ months satisfies the differential equation

$$\frac{\mathrm{d}^2 n}{\mathrm{d}t^2} = \mathrm{e}^{-\frac{t}{5}}.$$

**(i)** Find the general solution of this differential equation. [2]

**(ii)** It is given that initially, the size of the population of prawns is 50 000. Sketch on a single diagram, two distinct solution curves for the differential equation to illustrate the following two cases for large values of $t$ :

I. the size of the population of prawns increases indefinitely,

II. the size of the population of prawns stabilizes at a certain positive number. [3]

(**b**) In order for the prawns to grow faster and be more resistance to diseases, a drug is administered to the prawns. The prawn's body metabolizes (breaks down) the drug at a rate proportional to the amount of drug, $x$ mg, present in the body at time $t$ hours.

**(i)** Given that the initial dosage is 0.1 mg, show that $x = \frac{1}{10}\mathrm{e}^{-kt}$, where $k > 0$. [4]

**(ii)** The half-life of a drug is defined as the time taken for half of it to be metabolized. Given that the half-life of this drug is 4 hours, find the exact value of $k$. [2]

**(iii)** If 0.1 mg of this drug is administered to the prawn every 8 hours, show that the total amount of drug present in the prawn's body at any time $t$ is always less than 0.15 mg. [3]

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