Name:______________ Class:__________

Tutor:______________

JURONG JUNIOR COLLEGE

Preliminary Examinations

MATHEMATICS
Higher 2

9758 /01

28 August 2017

Paper 1

3 hours

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together, with the cover page in front.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

1 Mr Subash returned to Singapore after his tour in Europe and wishes to convert his foreign currencies back to Singapore Dollars (S$). Three money changers offer the following exchange rates:

Money Changer	1 Swiss Franc	1 British Pound	1 Euro	Total amount of S$ Mr Subash would receive after currency conversion
A	S$1.35	S$1.80	S$1.55	S$1151.50
B	S$1.40	S$1.85	S$1.65	S$1208.25
C	S$1.45	S$1.75	S$1.60	S$1189.25

How much of each currency has Mr Subash left after his tour? [4]

2 **(a)** Find $\int \sin(2\theta)\cos(3\theta)\,\mathrm{d}\theta$. [2]

(b) Use the substitution $\theta = \sqrt{x}$ to find the exact value of $\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} \theta^3 \cos\left(\theta^2\right)\mathrm{d}\theta$. [5]

3 **(i)** Using the formula for $\sin P - \sin Q$, show that

$$\sin\left[(2r+1)\theta\right] - \sin\left[(2r-1)\theta\right] \equiv 2\cos(2r\theta)\sin\theta.$$ [1]

(ii) Given that $\sin\theta \neq 0$, using the method of differences, show that

$$\sum_{r=1}^{n}\cos(2r\theta) = \frac{\sin\left[(2n+1)\theta\right] - \sin\theta}{2\sin\theta}.$$ [2]

(iii) Hence find $\sum_{r=1}^{n}\cos^2\left(\frac{r\pi}{5}\right)$ in terms of *n*.

Explain why the infinite series

$$\cos^2\left(\frac{\pi}{5}\right) + \cos^2\left(\frac{2\pi}{5}\right) + \cos^2\left(\frac{3\pi}{5}\right) + \dots$$

is divergent. [3]

4 A fund is started at $6000 and compound interest of 3% is added to the fund at the end of each year. If withdrawals of $*k* are made at the beginning of each of the subsequent years, show that the amount in the fund at the beginning of the (n +1)th year is

$$\$\frac{100}{3}\left[(180-k)(1.03)^n + k\right].$$ [5]

(i) It is given that $k = 400$. At the beginning of which year, for the first time, will the amount in the fund be less than $1000? [2]

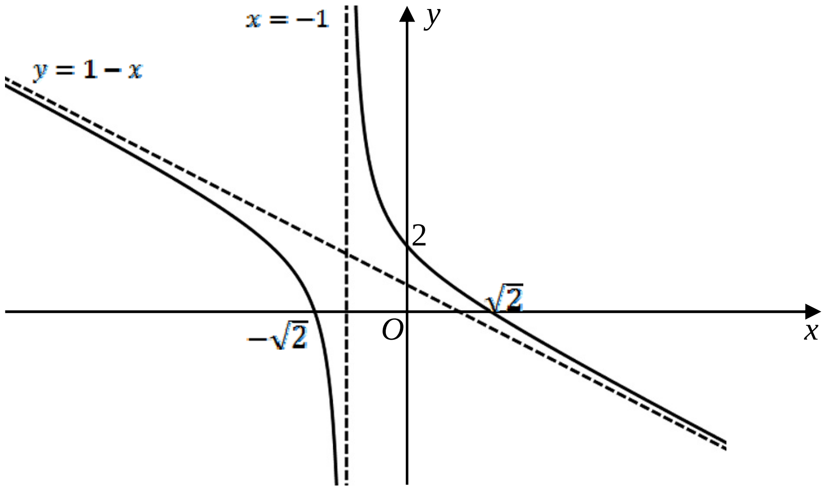
(ii) If the fund is fully withdrawn at the beginning of sixteenth year, find the least value of *k* to the nearest integer. [2]

5 **(a)** The curve C has the equation

$$(x-2)^2 = a^2\left(1-y^2\right), \qquad 1<a<2.$$

Sketch C, showing clearly any intercepts and key features. [2]

(b) The diagram shows the graph of $y=\mathrm{f}(x)$, which has an oblique asymptote $y=1-x$, a vertical asymptote $x=-1$, x-intercepts at $\left(\sqrt{2},0\right)$ and $\left(-\sqrt{2},0\right)$, and y-intercept at $(0,2)$.



Sketch, on separate diagrams, the graphs of

(i) $y=\dfrac{1}{\mathrm{f}(x)}$, [3]

(ii) $y=\mathrm{f}'(x)$, [3]

showing clearly all relevant asymptotes and intercepts, where possible.

6 With respect to the origin O, the position vectors of the points U, V and W are $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ respectively. The mid-points of the sides VW, WU and UV of the triangle UVW are M, N and P respectively.

(i) Show that $\overrightarrow{UM}=\dfrac{1}{2}(\mathbf{v}+\mathbf{w}-2\mathbf{u})$. [2]

(ii) Find the vector equations of the lines UM and VN. Hence show that the position vector of the point of intersection, G, of UM and VN is $\dfrac{1}{3}(\mathbf{u}+\mathbf{v}+\mathbf{w})$. [5]

(iii) It is now given that $\mathbf{u}=\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\mathbf{v}=\begin{pmatrix}0\\1\\0\end{pmatrix}$, $\mathbf{w}=\begin{pmatrix}0\\0\\1\end{pmatrix}$. Find the direction cosines of $\overrightarrow{OG}$. [2]

[Turn over

7 **(a)** If $u = 2 - \mathrm{i}\sin^2\theta$ and $v = 2\cos^2\theta + \mathrm{i}\sin^2\theta$ where $-\pi < \theta \le \pi$, find $u - v$ in terms of $\sin^2\theta$, and hence determine the exact expression for $|u-v|$ and the exact value of $\arg(u-v)$. [6]

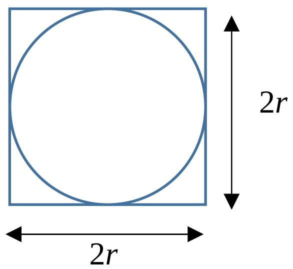
(b) The roots of the equation $x^2 + (\mathrm{i}-3)x + 2(1-\mathrm{i}) = 0$ are α and β, where α is a real number and β is not a real number. Find α and β. [4]

8 **(a)** When a liquid is poured onto a flat surface, a circular patch is formed. The area of the circular patch is expanding at a constant rate of 6π cm^2/s.

(i) Find the rate of change of the radius 24 seconds after the liquid is being poured. [3]

(ii) Explain whether the rate of change of the radius will increase or decrease as time passes. [1]

(b) A cylindrical can of volume $355\,\text{cm}^3$ with height h cm and base radius r cm is made from 3 pieces of metal. The curved surface of the can is formed by bending a rectangular sheet of metal, assuming that no metal is wasted in creating this surface. The top and bottom surfaces of the can are cut from square sheets of metal with length $2r$ cm, as shown below. The cost of the metal sheets is $\$K$ per cm^2.



(i) Show that the total cost of metal used, denoted by $\$C$, is given by

$$C = K\left(\frac{710}{r} + 8r^2\right).$$ [3]

(ii) Use differentiation to show that, when the cost of metal used is a minimum, then $\dfrac{h}{r} = \dfrac{8}{\pi}$. [5]

9 **(i)** Express $\sqrt{3}\cos x - \sin x$ in the form $R\cos(x+\alpha)$ where R and α are exact positive constants to be found. [2]

(ii) State a sequence of transformations which transform the graph of $y = \cos x$ to the graph of $y = \sqrt{3}\cos x - \sin x$. [2]

The function f is defined by $\mathrm{f} : x \mapsto \sqrt{3}\cos x - \sin x,\ 0 \le x \le 2\pi$.

(iii) Sketch the graph of $y = \mathrm{f}(x)$ and state the range of f. [3]

The function g is defined by $\mathrm{g} : x \mapsto \mathrm{f}(x),\ 0 \le x \le k$.

(iv) Given that g^{-1} exists, state the largest exact value of k and find $\mathrm{g}^{-1}(x)$. [3]

The function h is defined by $\mathrm{h} : x \mapsto x - 2,\ x \ge 0$.

(v) Explain why the composite function fh does not exist. [1]

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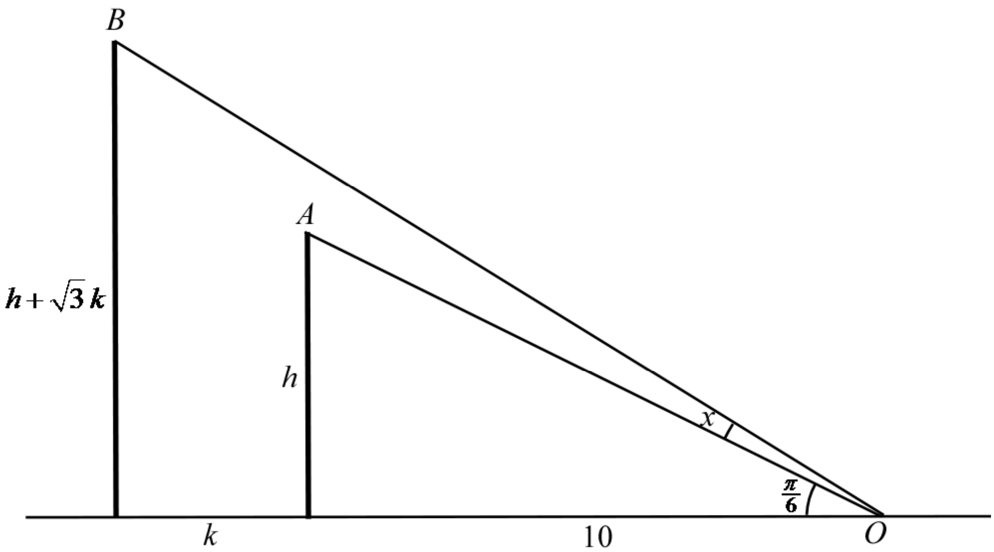


A laser from a fixed point O on a flat ground projects light beams to the top of two vertical structures A and B as shown above. To project the beam to the top of A, the laser makes an angle of elevation of $\frac{\pi}{6}$ radians. To project the beam to the top of B, the laser makes an angle of elevation of $\left(\frac{\pi}{6}+x\right)$ radians. The two structures A and B are of heights h m and $\left(h+\sqrt{3}\,k\right)$ m respectively and are 10 m and (10 + k) m away from O respectively.

(i) Show that the length of the straight beam from O to A is $\frac{20}{\sqrt{3}}$ m. [1]

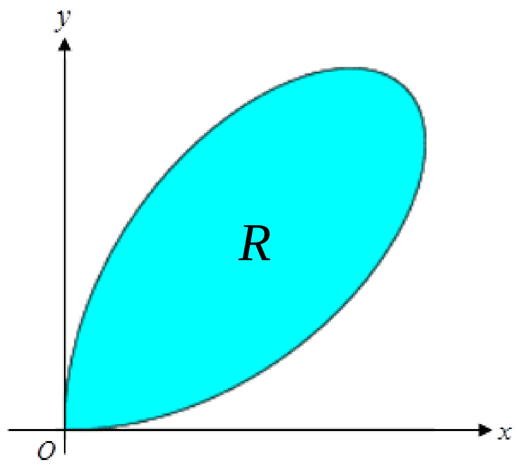
(ii) Show that the length of AB is $2k$ m and that the angle of elevation of B from A is $\frac{\pi}{3}$ radians. [3]

(iii) Hence, using the sine rule, show that $k=\dfrac{10\sin x}{\sqrt{3}\sin\left(\frac{\pi}{6}-x\right)}$. [2]

(iv) If x is sufficiently small, show that $k\approx\frac{20}{\sqrt{3}}\left(x+ax^2\right)$, where a is a constant to be determined. [6]

11 **(a)** The diagram below shows a section of *Folium of Descartes* curve which is defined parametrically by

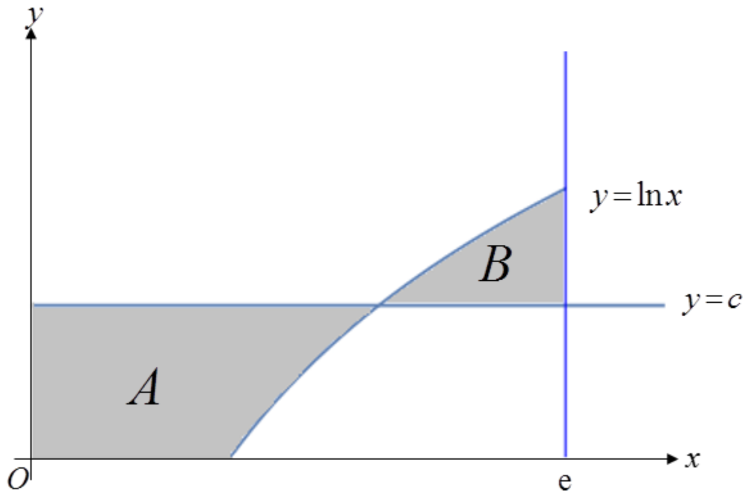
$$x = \frac{3m}{1+m^3}, \quad y = \frac{3m^2}{1+m^3}, \quad m \geq 0 .$$



(i) It is known that the curve is symmetrical about the line $y = x$. Find the values of m where the curve meets the line $y = x$. [1]

(ii) Region R is the region enclosed by the curve in the first quadrant. Show that the area of R is given by $2\left(\int_0^{\frac{3}{2}} x \, \mathrm{d}y - \frac{9}{8}\right)$, and evaluate this integral. [5]

(b) The diagram below shows a horizontal line $y = c$ intersecting the curve $y = \ln x$ at a point where the x-coordinate is such that $1 < x < \mathrm{e}$.



The region A is bounded by the curve, the line $y = c$, the x-axis and the y-axis while the region B is bounded by the curve and the lines $x = \mathrm{e}$ and $y = c$. Given that the volumes of revolution when A and B are rotated completely about the y-axis are equal, show that $c = \dfrac{\mathrm{e}^2 + 1}{2\mathrm{e}^2}$. [6]