VICTORIA JUNIOR COLLEGE

JC 2 PRELIMINARY EXAMINATION 2017

H2 MATHEMATICS **9758/02**

Paper 2 **3 hours**

Additional Materials: Answer Paper

Graph Paper

List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and CT group on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages

[Turn over

Section A: Pure Mathematics [40 marks]

1. A curve C is defined by the parametric equations

$$x=\frac{t}{1+t},\qquad y=\frac{t^2}{1+t},$$

where t takes all real values except -1.

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, leaving your answer in terms of t. [3]

(i) Show that the equation of the tangent to C at the point $\left(\dfrac{p}{1+p},\ \dfrac{p^2}{1+p}\right)$ is

$$y=p(p+2)x-p^2.$$ [2]

(ii) Find the acute angle between the two tangents to C which pass through the point $(2,5)$. [3]

2. Referred to the origin O, the points A, B and D are such that $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OB}=\mathbf{b}$ and $\overrightarrow{OD}=\mathbf{d}$. The point C is such that $OACB$ is a parallelogram and angle OAC is $\dfrac{2\pi}{3}$ radians.

(i) Given that $\mathbf{a}$ is a unit vector and $|\mathbf{b}|=4$, find the length of projection of $\overrightarrow{OC}$ onto $\overrightarrow{OA}$. [3]

(ii) Given that $\lambda\mathbf{a}+\mu\mathbf{b}+\mathbf{d}=\mathbf{0}$ and $\lambda+\mu+1=0$, show that A, B and D are collinear. [3]

If $\mu=4$, find the area of triangle OBD, leaving your answer in the form $k|\mathbf{a}\times\mathbf{b}|$, where k is a constant to be determined. [3]

3. A geometric series has common ratio r, and an arithmetic series has first term a and common difference d, where a and d are non-zero and $a>0$. The first three terms of the geometric series are equal to the first, eighth and thirteenth terms respectively of the arithmetic series.

(i) Show that $7r^2-12r+5=0$. [2]

(ii) Deduce that the geometric series is convergent. [2]

(iii) The sum of the first n terms of the geometric series is denoted by S_n. Find the smallest value of n for S_n to be within 0.1% of the sum to infinity of the geometric series. [4]

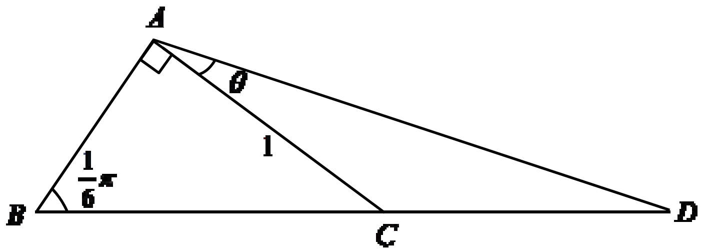
(iv) Find exactly the sum of the first 2017 terms of the arithmetic series, leaving your answer in terms of a. [3]

4. **(a)** It is given that $y = \mathrm{f}(x)$ is such that $my^2 \dfrac{\mathrm{d}y}{\mathrm{d}x} - y^3 = -\mathrm{e}^x \sin x$ and that the Maclaurin series for $\mathrm{f}(x)$ is given by $1 + \dfrac{1}{3}x + nx^2 + \ldots$, where m and n are some real constants.

(i) State the values of $\mathrm{f}(0)$ and $\mathrm{f}'(0)$. [2]

(ii) Find the values of m and n. [3]

(b) In the triangle ABC, $AC = 1$, angle $BAC = \dfrac{\pi}{2}$ radians and angle $ABC = \dfrac{\pi}{6}$ radians. D is a point on BC produced such that angle $CAD = \theta$ radians (see diagram).



(i) Show that $AD = \dfrac{\sqrt{3}}{\sqrt{3}\cos\theta - \sin\theta}$. [4]

(ii) Given that θ is a sufficiently small angle, show that

$$AD \approx 1 + a\theta + b\theta^2 \ ,$$

for constants a and b to be determined exactly. [3]

Section B: Statistics [60 marks]

5. John and Peter play a game of chess. It is equally likely for either player to make the first move. If John makes the first move, the probability of him winning the game is 0.3 while the probability of Peter winning the game is 0.2. If Peter makes the first move, the probability of him winning the game is 0.5 while the probability of John winning the game is 0.4. If there is no winner, then the game ends in a draw.

(i) Find the probability that Peter made the first move given that he won the game. [3]

(ii) John and Peter played a total of three games. Assuming that the results of the three games are independent, find the probability that John wins exactly one game. [3]

6. An experiment to determine the effect of a fertilizer on crop yield was carried out. A field was divided into eight plots of equal area and eight different amounts of fertilizer, one for each plot, were used. The table below shows the amount of fertilizer, x grams, and the crop yield, y grams, for each plot.

Amount of fertilizer (x)	15	22	37	55	62	69	78	90
Yield (y)	101	123	137	150	150	154	158	160

(i) Draw the scatter diagram for these values, labelling the axes. [1]

It is thought that the yield of a crop, y grams, can be modelled by one of the formulae

$$y = a + bx \quad \text{or} \quad y = c + d\ln x$$

where a, b, c and d are constants.

(ii) Find the value of the product moment correlation coefficient between
(a) x and y,
(b) $\ln x$ and y. [2]

(iii) Use your answers to parts **(i)** and **(ii)** to explain which of $y = a + bx$ or $y = c + d\ln x$ is the better model. [2]

(iv) For a plot of land, the yield of the crop was 144 grams. Using a suitable regression line estimate the amount of fertilizer used, giving your answer to the nearest gram. [2]

(v) Comment on the reliability of the estimate when the model in part **(iv)** is used to estimate the value of y when $x = 110$. [1]

7. Four digits are randomly selected from the set $\{1,2,3,4,5,6,7,8,9\}$ to form a four-digit number. Repetitions are not allowed.

(i) Find the probability that none of the digits in the four-digit number are odd. [2]

The random variable X denotes the number of odd digits in the four-digit number formed.

(ii) Show that $\mathrm{P}(X = 1) = \dfrac{10}{63}$, and find the rest of the probability distribution of X, giving each probability as a fraction in its lowest terms. [3]

(iii) Find the expectation and variance of X. [3]

(iv) Two independent observations of X are denoted by X_1 and X_2.
Find $\mathrm{P}(|X_1 - X_2| < 3)$. [4]

8. In this question, you should state clearly the values of the parameters of any normal distribution you use.

A supermarket sells two types of durians, D25 and Musang Queen. The durians are sold by weight. The masses, in kilograms, of D25 and Musang Queen are modelled as having normal distributions. The means and standard deviations of these distributions, and the selling prices, in \$ per kilogram, are shown in the following table.

	Mean (kg)	Standard deviation (kg)	Selling price (\$ per kg)
D25	1.5	0.02	9
Musang Queen	1.8	0.035	18

(i) A customer buys 3 D25 durians and 2 Musang Queen durians. Find the probability that the total cost of his purchase is more than \$107. [5]

(ii) State an assumption needed for your calculations in part **(i)**. [1]

(iii) The probability that the average weight of n randomly chosen D25 durians exceeding m kg is at least 0.1. Show that n satisfies the inequality

$$(m-1.5)\sqrt{n} \leqslant 0.025631.$$

Hence find the largest possible value of n when $m = 1.51$. [4]

9. Ryde, a leading private hire car company, announced JustRyde, a new service that promises more affordable fixed fare rides and shorter waiting times. In their advertisement, Ryde claimed that the mean waiting time, in seconds, was 240. A random sample of 50 JustRyde customers is taken and their waiting times, x seconds, is recorded. The data are summarised by

$$\sum(x-240)=120, \quad \sum(x-240)^2=11200.$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Test, at the 10% significance level, whether the population mean waiting time is more than 240 seconds. [5]

(iii) State, giving a valid reason, whether any assumptions about the population are needed in order for the test to be valid. [1]

(iv) Explain, in the context of the question, the meaning of ‘at the 10% significance level’. [1]

(v) In another test, using the same data and also at the 10% significance level, the hypotheses are as follows:

Ho : the population mean waiting time is equal to k seconds.
H_1 : the population mean waiting time is not equal to k seconds.

Given that the null hypothesis is rejected in favour of the alternative hypothesis, find the set of possible values of k. [3]

10. It is a common practice for airlines to sell more plane tickets than the number of seats available. This is to maximise their profits as it is expected that some passengers will not turn up for the flight.

The plane used by Victoria Airline for her daily 10 am flight from Singapore to Hong Kong has a maximum capacity of 150 seats. For this particular flight, 154 tickets are sold every day. On average, p out of 100 customers who have purchased a plane ticket for this flight turn up. Customers who turn up after the flight is full will be turned away. The number of customers who turn up for the 10 am flight, on a randomly chosen day, is denoted by X.

(i) State, in the context of this question, two assumptions needed to model X by a binomial distribution. [2]

(ii) Explain why one of the assumptions stated in part **(i)** may not hold in this context. [1]

Assume now that these assumptions do in fact hold.

(iii) It is known that there is a 0.05 probability that at least 153 customers will turn up for the 10 am flight. Write down an equation for the value of p, and find this value numerically. [3]

It is given instead that $p = 94$.

(iv) Find the probability that, on a randomly chosen day,

(a) there are at least 141 but not more than 148 customers who turn up for the 10 am flight, [2]

(b) every customer who turns up gets a seat on the 10 am flight. [1]

(v) Find the probability that every customer who turns up gets a seat on the 10 am flight on more than 5 days in a week. [3]