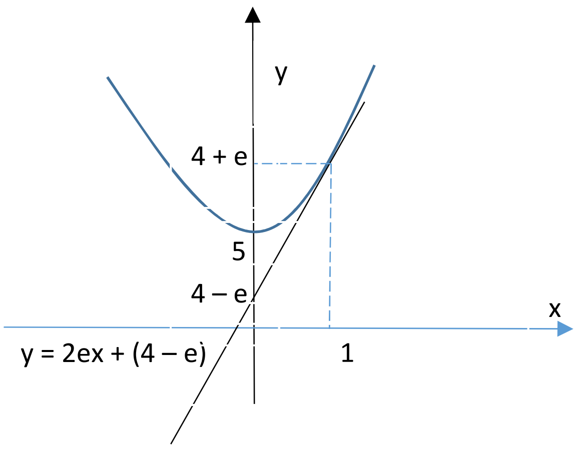
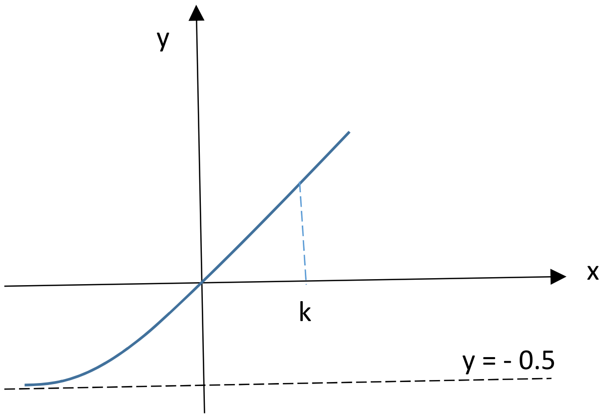
Solutions to 2017 Y6 H1 Maths Preliminary Exam II

1	Working	
	$y = 4 + e^{x^2} \Rightarrow \frac{dy}{dx} = 2xe^{x^2}$ when $x = 1, y = 4 + e, \frac{dy}{dx} = 2e$ $y - (4+e) = 2e(x-1)$ $y = 2ex + (4-e)$  $\int_0^1 4 + e^{x^2} - (2ex + 4 - e)\, dx = 1.46$	

2	Working	
i)	$\frac{d\ln(x^2+1)}{dx} = \frac{1}{x^2+1}.2x = \frac{2x}{x^2+1}$	
ii)	$\frac{(x+1)^2}{x^2+1} = \frac{x^2+2x+1}{x^2+1}$ $= \frac{x^2+1}{x^2+1} + \frac{2x}{x^2+1}$ $= 1 + \frac{2x}{x^2+1}$	
iii)	$\int \frac{(x+1)^2}{x^2+1}\, \mathrm{d}x = \int 1 + \frac{2x}{x^2+1}\, \mathrm{d}x = x + \ln(x^2+1) + c$	

3	**Working**	
i)	Let $x = a$ $\frac{1}{2}\left(e^{a}-1\right)=a$ $e^{a}-1=2a \Rightarrow e^{a}=1+2a$	
ii)	 $\int_{0}^{k}\frac{1}{2}\left(e^{x}-1\right)dx=\frac{1}{2}\left[e^{x}-x\right]_{0}^{k}$ $=\frac{1}{2}\left(e^{k}-k-1\right)$ $\frac{1}{2}\left(e^{k}-k-1\right)=\frac{a}{2}$ $e^{k}-k=1+a=e^{a}-a$ (from (i)) $\therefore k=a$	
iii)	Using GC, the points of intersection between y = x and the curve is $x=0$ and 1.256 The range of values of x: $x<0$ or $x>1.26$	

4	**Working**	

Let a, b and c be the interest rates of each fund in per cent.

$$\frac{a}{100}(2000)+\frac{b}{100}(1500)+\frac{c}{100}(1000)=309$$

$$20a+15b+10c=309$$

$$2.\frac{a}{100}(2000)=\frac{b}{100}(1500)+\frac{c}{100}(1000)$$

$$40a-15b-10c=0$$

$$\frac{a}{100}(2500)+\frac{b}{100}(2000)=320$$

$$25a+20b=320$$

Solving the 3 equations, a = 5.15, b = 9.56 and c = 6.26

Fund B gave the highest rate = 9.56%

5	**Working**	

i)

$$\frac{dS}{dt}=3t^2-20t+28=0$$

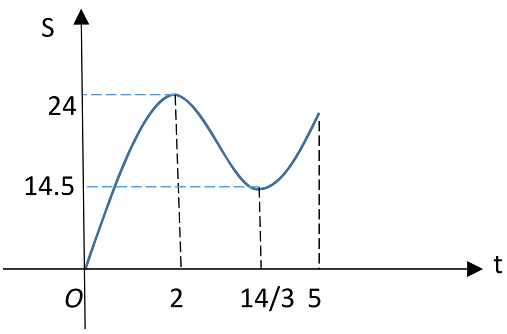
$$t=\frac{20\pm\sqrt{64}}{6}=2 \text{ or } \frac{14}{3}$$

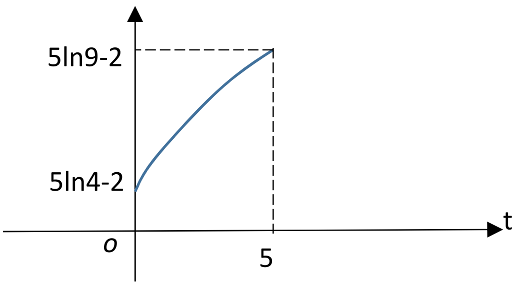
	1.995	**2**	**2.005**
gradient	+	**0**	-
shape			

	4.665	**14/3**	**4.675**
gradient	-	**0**	+
shape			

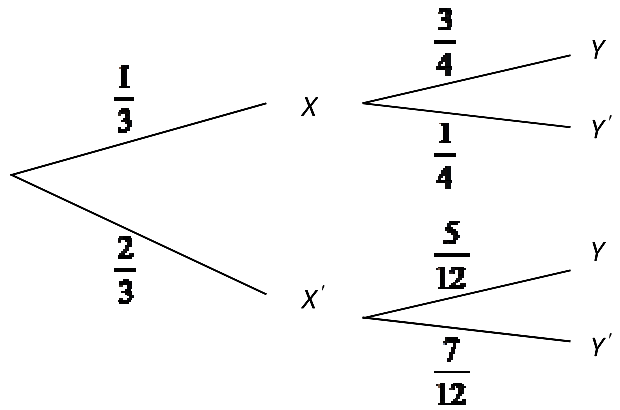
When $t=2, S=24$ which is maximum.

ii)



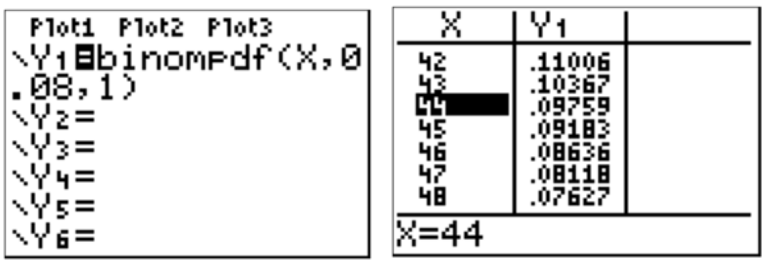
iii)	$\int_0^5 t^3 - 10t^2 + 28t\, dt \approx 89.6$ **89.6 metres** refers to the total distance covered by Johnson during 5 seconds.	
iv)	$5\ln(t+4) - 2 = 23$ $\ln(t+4) = 5$ $t + 4 = e^5$ $t = e^5 - 4$ seconds	
v)		
vi)	When t = 1, $\frac{d\sigma}{dt} = \frac{5}{1+4} = 1$ cm^3/s	

6	**Working**	
6(i)	The probability = $\frac{1! \times 7! \times 1!}{9!} = \frac{1}{8 \times 9} = \frac{1}{72}$	
6(ii)	The probability = $\frac{(4! \times 3! \times 2!) \times 3!}{9!}$ $= \frac{1728}{362880} = \frac{1}{210}$	
6(iii)	The probability = $\frac{\binom{4}{2}\binom{3}{2}\binom{2}{1} \times 5!}{\binom{9}{5} \times 5!}$ $= \frac{36}{126} = \frac{2}{7}$	

	Working	Marks
7(i)	$\mathrm{P}(X)=\frac{2}{6}=\frac{1}{3}$	
7(ii)	**The tree diagram for the events *X* and *Y*:**  	
7(iii)	$\mathrm{P}(X\|Y)=\frac{\mathrm{P}(X\cap Y)}{P(Y)}$ $=\frac{\frac{1}{3}\times\frac{3}{4}}{\frac{1}{3}\times\frac{3}{4}+\frac{2}{3}\times\frac{5}{12}}$ $=\frac{9}{19}$	
7(iv)	**Since** $\mathrm{P}(X\|Y)=\frac{9}{19}, \mathrm{P}(X)=\frac{1}{3}$ **and thus,** $\mathrm{P}(X\|Y)\neq\mathrm{P}(X)$ **we can conclude that event *A* and *B* are not independent events.** **Alternative:** **P(*Y*) =** $\frac{1}{3}\times\frac{3}{4}+\frac{2}{3}\times\frac{5}{12}=\frac{19}{36}$ **Then P(*X*∩*Y*) =** $\frac{1}{4}\neq\mathrm{P}(X)\times\mathrm{P}(Y)$ **Hence, event *A* and *B* are not independent events.**	

<table>
<tr><th>8</th><th>Working</th><th>Marks</th></tr>
<tr><td>8(i)</td><td>The two assumptions are
(i) The event of a particular chosen curry puff being too salty is independent of other curry puffs chosen by the MOH officials.
(ii) The probability of choosing a curry puff that is too salty is constant for all curry puffs chosen by the MOH officials.
(i) may not be true in general as curry puffs are manufactured in batches, thus if 1 curry puff is found to be too salty, curry puffs from the same batch will be salty too. (or any other similar reasoning)</td><td></td></tr>
<tr><td>8(ii)</td><td>Assume $X \sim B(n, 0.08)$,
Then we need to have
$\mathrm{P}(X = 1) < 0.01$.

Using binomial pdf in GC, we can set the necessary and obtain the following probability table for checking:



Hence, from the above table, we deduce that the least value of n should be 44.</td><td></td></tr>
<tr><td>8(iii)</td><td>Let n = 44. Then $X \sim B(44, 0.08)$,
Then we have
$\mathrm{P}(3 \le X < 10)$
$= \mathrm{P}(X \le 9) - \mathrm{P}(X \le 2)$.
$= 0.692$</td><td></td></tr>
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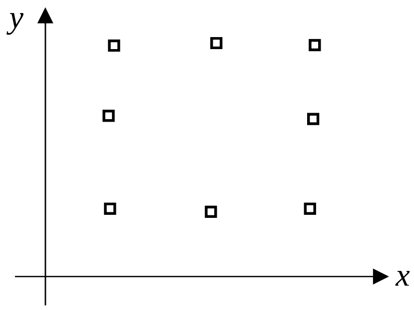
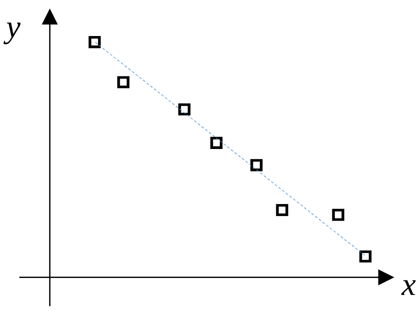
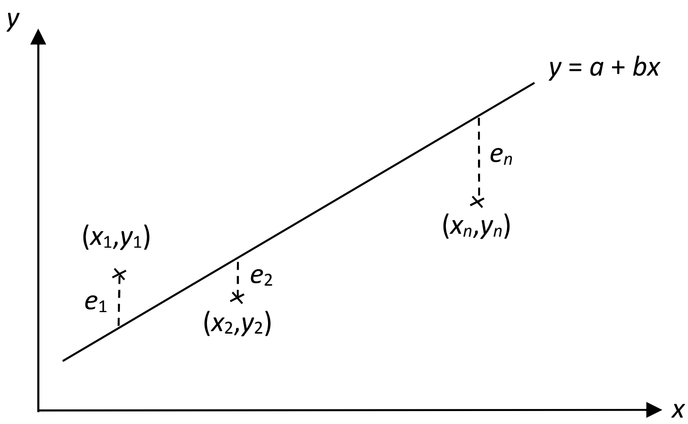
9	Working	Marks
9 (i)	The required scatter diagram are as follows: (i)  (ii) 	
9b(i)	Let the sample of bivariate data be (x_i, y_i) where $i = 1, 2, \ldots n$. Let $e_i = y_i - (a + bx_i)$ be the vertical deviation between the point (x_i, y_i) and the line $y = a + bx$.  The line $y = a + bx$ is the least square regression line for the sample of bivariate data if the sum of the squares of the vertical deviations, i.e. $\sum_{i=1}^{n} (e_i)^2$, is the minimum.	

	Using GC and the table of values, LinReg y=a+bx a=.5590909091 b=.0774125874 r²=.9835387842 r=.991735239 the regression line of *y* on *x* is $y = 0.559 + 0.0774x$	
9b(ii)	*a* is the amount of cough syrup in ml produced with no amount of the chemical compound KH_2 added in the production process. *b* is the amount of cough syrup in ml produced with the addition of 1 mg of the chemical compound KH_2 added in the production processs.	
9b(iii)	As *x* is the independent variable and *y* is the dependent variable in the data set, we will need to use the regression line of *y* on *x* to estimate the value of *x* given *y* value. Thus, when *y* = 4.0, $4.0 = 0.55909 + 0.077413x$ $\Rightarrow x = 44.448736 \approx 44.4$ mg (3 sig fig)	
9b(iv)	The estimate for *x* value in part 9b(iii) is reliable for the following reasons: i. the correlation coefficient *r* = 0.992 is very close to 1, ii. the given *y* value of 4.0 is within the data range of the *y* value iii. the correct line *y* on *x* is used as *y* is the dependent variable.	

10	**Working**	**Marks**
10(i)	The unbiased estimate for population mean = $\overline{x}$ $= \frac{720}{50} + 100 = 114.4$ The unbiased estimate for population variance = s^2 $= \frac{1}{50-1}\left(\sum(x-100)^2 - \frac{\left(\sum(x-100)\right)^2}{50}\right)$ $= \frac{1}{49}\left(30500 - \frac{720^2}{50}\right) = \frac{2876}{7} = 410.8571429 \approx 411$ (3 sig. fig.)	
10(ii)	Let μ be the mean of the amount of energy released in each collision. For testing of the new claim by the young physicists, Let $H_0: \mu = 108$ $H_1: \mu \neq 108$ We then perform a one tail test at 5% level of significance i.e. $\alpha = 0.05$	

	Under H_0, $\overline{X} \sim N\left(108, \frac{2876/7}{50}\right)$ Then test statistics is $Z = \frac{\overline{X} - 108}{\sqrt{\frac{2876/7}{50}}} \sim N(0, 1)$ Using GC, the p-value = 0.0255723191. Since p-value < α, we reject H_0 and conclude that there is sufficient evidence at 5% level of significance that the population mean of the energy released in each collision is not 108 MeV. As the sample size n is large (50), it is not necessary to assume that X, the amount of energy released in each collision follows a normal distribution as by the Central Limit Theorem, $\overline{X}$ can be approximated by a normal distribution for the test to be valid.	
10(iii)	For testing of the new claim by the young physicists, Let $H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$ We then need to perform a one tailed test at 3% level of significance i.e. $\alpha = 0.03$ Under H_0, $\overline{X} \sim N\left(\mu_0, \frac{2876/7}{50}\right)$ Then test statistics is $Z = \frac{\overline{X} - \mu_0}{\sqrt{\frac{2876/7}{50}}} \sim N(0, 1)$. For H_0 not to be rejected, $z_{\text{calculated}}$ must not be in the critical region, hence, $\frac{114.4 - \mu_0}{\sqrt{\frac{2876/7}{50}}} \le 1.88079361$ $\Rightarrow 114.4 - \mu_0 \le 1.88079361 \times \sqrt{\frac{2876/7}{50}}$ $\Rightarrow \mu_0 \ge 114.4 - 1.88079361 \times \sqrt{\frac{2876/7}{50}}$ $\Rightarrow \mu_0 \ge 109.0085999$ $\Rightarrow \mu_0 \ge 109$ (3 sig fig)	

11	Working	Marks
i)	Let W be the r.v. of the amount of electricity used by a household in a month. $W \sim N(500, 80^2)$ $P(W < 600) = 0.89435 \approx 0.894$	
ii)	Let H be the number of households out of 10 which use less than 960 kWh, H ~B(10, 0.89435) $P(H \le 8 \mid H \ge 4) = \dfrac{P(4 \le H \le 8)}{P(H \ge 4)} = \dfrac{P(H \le 8) - P(H \le 3)}{1 - P(H \le 3)}$ $= \dfrac{0.28583}{0.9999868} = 0.28583 \approx 0.286$	
iii)	Let X be the r.v. of number of months that use more than 600 kWh, $X \sim B(12, 0.10565)$. $P(X \ge 5) = 1 - P(X \le 4) \approx 0.00550$	
iv)	$W_1 - W_2 \sim N(0, 12800)$ $P(\lvert W_1 - W_2 \rvert < 100) = P(-100 < W_1 - W_2 < 100) = 0.62324 \approx 0.623$	
v)	Assume that the amount of electricity used by a household of the two months is independent. This assumption may not be true if the family knows that they have over-used energy in the first month, they may use less in the later months. (or any other reasonable explanations)	
vi)	Let C = 0.22W be the r.v. of the cost of electricity for a household in each month. $C \sim N(110, 309.76)$ $P(C > 150) = 0.011521 \approx 0.0115$	
vii)	$T = C_1 + C_2 + C_3 \sim N(330, 929.28)$ $P(T < m) \ge 0.96$ $m \ge 383.368 \approx 383$ (3 sig. fig)	
viii)	Let V be the r.v of the amount of electricity used by a household per month at the end of the campaign. Supposed $V \sim N(185, 80^2)$ Consider: a) $P(V < 0) = 0.0104$ Since the probability that a household uses negative amount of electricity is 0.0104 (1.04%) which is quite significant, Normal distribution is deemed as unsuitable. OR b) $P(185 - 3(80) < W < 185 + 3(80)) = 0.997$ Since probability of 0.997 of amount of electricity used is within 3 standard deviation and there is quite significant probability that the amount of electricity used is negative, Normal distribution is deemed unsuitable.	