

1

(i)

$$\text{Let } y = \frac{3}{\sqrt{(2x-7)^3}} = 3(2x-7)^{-\frac{3}{2}}$$

$$\frac{dy}{dx} = 3(2x-7)^{-\frac{5}{2}} \cdot \left(\frac{-3}{2}\right) \cdot (2)$$

$$= -9(2x-7)^{-\frac{5}{2}} \quad \left(\text{or } -\frac{9}{(2x-7)^{\frac{5}{2}}} \right)$$

(ii)

$$\begin{aligned} & \int \frac{1}{e^{4t-3}} + \frac{1}{2t-1} dt \\ &= \int e^{-4t+3} + \frac{1}{2t-1} dt \\ &= -\frac{1}{4}e^{-4t+3} + \frac{1}{2}\ln(2t-1) + c \end{aligned}$$

2

(i)

$$3 - 10e^{2x} - 8e^{4x} = 0$$

Let $u = e^{2x}$. Then

$$\Rightarrow 3 - 10u - 8u^2 = 0$$

$$\Rightarrow (2u+3)(1-4u) = 0$$

$$\Rightarrow u = -\frac{3}{2} \quad \text{or} \quad u = \frac{1}{4}$$

$$\Rightarrow e^{2x} = -\frac{3}{2} \quad \text{or} \quad e^{2x} = \frac{1}{4}$$

(rej $\because e^{2x} > 0$)

$$\therefore 2x = \ln \frac{1}{4} \Rightarrow x = \frac{1}{2} \ln \frac{1}{4} = -\ln 2$$

$$a = -1 \text{ and } b = 2$$

(ii)

For the inequality $3 - 10e^{2x} - 8e^{4x} \leq 0$,

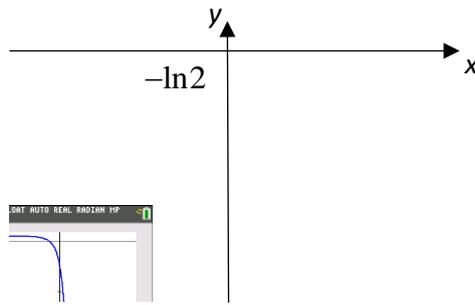
$$\begin{array}{c} \text{---} \quad \text{---} \quad \text{---} \\ | \quad \quad | \quad \quad | \\ \text{---} \quad \text{---} \quad \text{---} \\ -\frac{3}{2} \quad \quad \frac{1}{4} \end{array}$$

$$e^{2x} \leq -\frac{3}{2} \quad \text{or} \quad e^{2x} \geq \frac{1}{4}$$

$$\text{(rej } \because e^{2x} > 0)$$

$$\therefore x \geq -\ln 2$$

Or use graphical method



Therefore,
 $\therefore x \geq -\ln 2$

3

(i)
 Let $A \text{ cm}^2$ be the area of the window and $l \text{ cm}$ be the length of the rectangle.

$$\pi x + 2l + 2x = 300$$

$$\Rightarrow l = 150 - x - \frac{\pi x}{2}$$

$$A = \frac{1}{2} \pi x^2 + 2xl$$

$$= \frac{1}{2} \pi x^2 + 2x \left(150 - x - \frac{\pi x}{2} \right)$$

$$= \frac{1}{2} \pi x^2 + 300x - 2x^2 - \pi x^2$$

$$A = 300x - 2x^2 - \frac{1}{2} \pi x^2$$

(ii)

For maximum area, $\frac{dA}{dx} = 0$

$$300 - 4x - \pi x = 0$$

$$x = \frac{300}{4 + \pi} \text{ or } 42.007$$

Using 1st derivative test,

x	$\left(\frac{300}{4+\pi}\right)^-$	$\frac{300}{4+\pi}$	$\left(\frac{300}{4+\pi}\right)^+$
$\frac{dA}{dx}$	+ve	0	-ve

A is maximum when $x = \frac{300}{4+\pi}$ or 42.007.

Maximum A

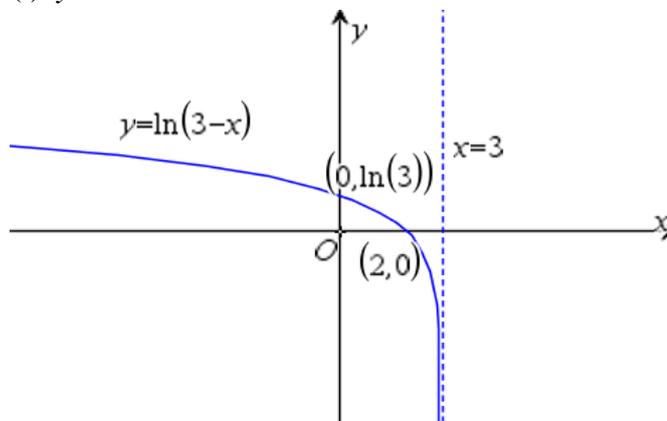
$$= 300\left(\frac{300}{4+\pi}\right) - 2\left(\frac{300}{4+\pi}\right)^2 - \frac{1}{2}\pi\left(\frac{300}{4+\pi}\right)^2$$

$$= 6301.115$$

$$= 6301 \text{ cm}^2 \text{ (nearest whole number)}$$

4

(i) $y = \ln(3-x)$



(ii)

$$\frac{dy}{dx} = -\frac{1}{3-x}$$

At point P , $x=1$; $\frac{dy}{dx} = -\frac{1}{2}$; $y = \ln(2)$

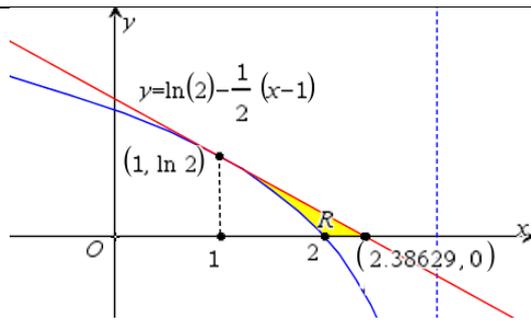
Eqn of tangent at P : $y - \ln(2) = -\frac{1}{2}(x-1)$

$$y = \ln(2) - \frac{1}{2}(x-1)$$

$$y = -\frac{1}{2}x + \ln 2 + \frac{1}{2}$$

(iii)

$$y = -\frac{1}{2}x + \ln 2 + \frac{1}{2} \quad y = \ln 2 - \frac{1}{2}(x-1)$$



The area of R
 = Area of triangle – area under the curve
 = $\frac{1}{2}(2.38629 - 1)(\ln 2) - \int_1^2 \ln(3 - x) dx$
 = 0.0941586528
 $\approx 0.0942 \text{ units}^2$

5

(i)

$$\frac{dy}{dx} = 4x^3 - 12x^2 + 9x - 2$$

Let $\frac{dy}{dx} = 0$

$$\Rightarrow 4x^3 - 12x^2 + 9x - 2 = 0$$

Using GC, $x = 2$ or $\frac{1}{2}$

When $x = 2$, $y = (2)^4 - 4(2)^3 + 4.5(4) - 2(2) + 2$
 $= 0$

When $x = \frac{1}{2}$, $y = \left(\frac{1}{2}\right)^4 - 4\left(\frac{1}{2}\right)^3 + 4.5\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 2$
 $= \frac{27}{16}$ or 1.6875

The coordinates of the stationary points are $(2, 0)$ and $\left(\frac{1}{2}, \frac{27}{16}\right)$.

(ii)

When $x = 2$,

x	$x = 2^-$	$x = 2$	$x = 2^+$
$\frac{dy}{dx}$	-	0	+
slope			

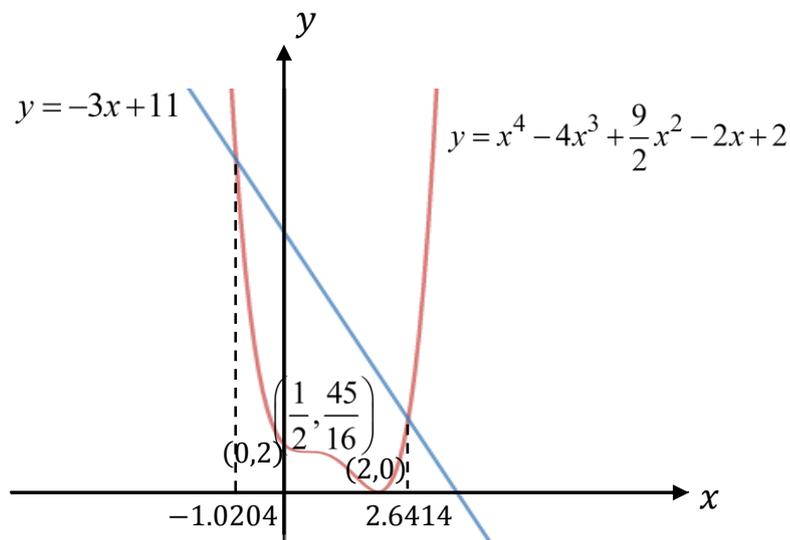
Thus $(2, 0)$ is a minimum point.

When $x = \frac{1}{2}$,

x	$x = \frac{1}{2}^-$	$x = \frac{1}{2}$	$x = \frac{1}{2}^+$
$\frac{dy}{dx}$	-	0	-
slope			

Thus $\left(\frac{1}{2}, \frac{27}{16}\right)$ is a stationary point of inflexion.

(iii)



(iv)

$$x^4 - 4x^3 + \frac{9}{2}x^2 + x - 9 < 0$$

$$x^4 - 4x^3 + \frac{9}{2}x^2 - 2x + 3x + 2 - 11 < 0$$

$$x^4 - 4x^3 + \frac{9}{2}x^2 - 2x + 2 < -3x + 11$$

Using GC, the x -coordinates of intersection are $x = -1.0204$ or 2.6414

From the graph, $-1.0204 < x < 2.6414$.

6

(i) Systematic Sampling

(ii)

One disadvantage is that not all students get an equal chance of being selected as the students who are absent from school will not get a chance to be interviewed.

(iii)

Stratified Sampling

Split the school population into **mutually exclusive subgroups/strata** based on the **four levels, Sec 1 to 4**.

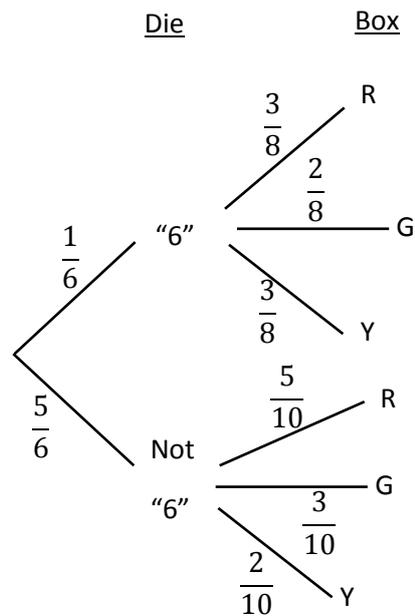
$$\frac{80}{1200} \times 100\% \approx 6.67\%$$

Choose **random samples** from each subgroup that is **proportional to the size of the subgroup/stratum (6.67%) to make up the sample of 80 students**.

or

Choose **randomly 6.67%** of students from **each stratum to form a sample of 80 students**.

7



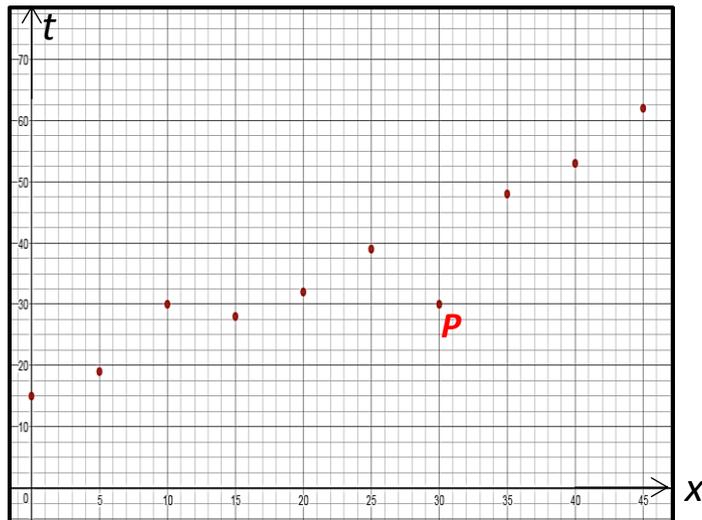
(a)(ii)

$$\begin{aligned}
 P(\text{wins a prize}) &= \left(\frac{1}{6} \times \frac{3}{8}\right) + \left(\frac{5}{6} \times \frac{2}{10}\right) \\
 &= \frac{11}{48}
 \end{aligned}$$

	<p>(a)(iii)</p> $P(\text{from box A} \mid \text{wins the prize}) = \frac{\frac{1}{6} \times \frac{3}{8}}{\frac{11}{48}}$ $= \frac{3}{11}$ <p>(b)</p> $P(\text{wins the grand prize}) = \left(\frac{1}{6} \times \frac{3}{8} \times \frac{2}{7}\right) + \left(\frac{5}{6} \times \frac{2}{10} \times \frac{1}{9}\right)$ $= \frac{55}{1512}$
8	<p>(i) The assumption is that the event of a resident using ShareBike or not is independent of any other residents in the neighbourhood.</p> <p>(ii) The assumption may not hold as usually families may use ShareBike together as they are going for the activity together.</p> <p>(iii)</p> <p>$X \sim B(n, 0.09)$ $P(X \geq 1) < 0.99$ $P(X = 0) > 0.01$</p> $\binom{n}{0} (0.09)^0 (1 - 0.09)^n > 0.01$ $(0.91)^n > 0.01$ $n < \frac{\ln 0.01}{\ln 0.91}$ $n < 48.830$ <p>\therefore Greatest value of n is 48.</p> <p><u>Alternative method</u> Using GC, When $n = 47$, $P(X = 0) = 0.0119 (> 0.01)$ When $n = 48$, $P(X = 0) = 0.0108 (> 0.01)$ When $n = 49$, $P(X = 0) = 0.0098 (< 0.01)$ \therefore Greatest value of n is 48.</p> <p>(iv)</p> <p>$X \sim B(40, 0.09)$ $P(3 < X \leq 5) = P(X \leq 5) - P(X \leq 3)$ $= 0.344$ (3 s.f.)</p>

	<p>(v) $X \sim B(70, 0.09)$</p> <p>Since $n = 70$ is large, $np = 70 \times 0.09 = 6.3 (> 5)$ $nq = 70 \times (1 - 0.09) = 63.7 (> 5)$ $X \sim N(6.3, 5.733)$ approximately</p> <p>$P(X < 7) = P(X < 6.5)$ (continuity correction) $= 0.533$ (3 s.f.)</p>
9	<p>(i) $A \sim N(43, 8^2)$ $B \sim N(40, 6^2)$</p> <p>$P(A_1 + A_2 + A_3 < 3B) = P(A_1 + A_2 + A_3 - 3B < 0)$</p> <p>Let $S = A_1 + A_2 + A_3 - 3B$ $E(S) = 3(43) - 3(40) = 9$ $\text{Var}(S) = 3(8^2) + 3^2(6^2) = 516$ $P(S < 0) = 0.346$ (3 s.f.)</p> <p>(ii) $B \sim N(40, 6^2)$</p> <p>$E(B_1 + B_2) = 2(40) = 80$ $\text{Var}(B_1 + B_2) = 2(6^2) = 72$ $B_1 + B_2 \sim N(80, 72)$</p> <p>Required probability = $P(A > 38) \times P(B_1 + B_2 > 82)$ $= 0.299$ (3 s.f.)</p> <p>(iii) $E(A + B_1 + B_2) = 43 + 2(40) = 123$ $\text{Var}(A + B_1 + B_2) = 8^2 + 2(6^2) = 136$ $A + B_1 + B_2 \sim N(123, 136)$</p> <p>Required probability = $P(A + B_1 + B_2 > 120)$ $= 0.602$ (3 s.f.)</p> <p>(iv) Because the case in (ii) is a proper subset of the case in (iii). For eg, Part iii contains cases whereby the lifespan of component A may not exceed 38 weeks (eg. 36 weeks) but total lifespan of 2 components of B exceeds 82 weeks (eg. 84 weeks), and yet the total lifespan is more than 120 weeks.</p>

(i)



(ii)

Acceptable reasons:

The traffic condition on the road was good (Lesser cars on the road, no traffic jam) and thus he required much shorter travelling time though he left home only at 7.30am.

It was a public holiday/school holiday/Sunday and yet Mr Lee has to work.

(iii)

$$r \approx 0.987$$

The pmcc is close to 1, indicating a strong positive linear correlation between x and t . I.e. the later Mr Lee leaves home after 7 am, the longer the travelling time would take.

NORMAL FLOAT AUTO REAL RA	
	LinReg
$y=ax+b$	
$a=.9783333333$	
$b=15.025$	
$r^2=.9749009733$	
$r=.9873707375$	

$$(iv) \quad t = 0.978x + 15.025$$

(v)



$a = 0.978$ means that for every additional minute that Mr Lee delays in leaving home after 7am, his travelling time will increase by 0.978 minutes.

(vi)

Method 1:

There are 90 minutes from 7 am to 8.30 am.

$$x + t \leq 90$$

$$x + (0.97833x + 15.025) \leq 90$$

$$1.97833x \leq 74.975$$

$$x \leq 37.898$$

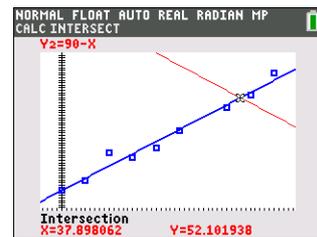
The largest possible value of x is 37 (correct to the nearest minute)

The latest time Mr Lee could leave home without being late for work is 7.37 am.

Method 2:

Sketch the line $x + t = 90$ and find x -coordinate of the point of intersection with the regression line.

If $x = 38$, Mr Lee will arrive late for work. Thus the latest time he needs to leave home is 7.37am.



Method 3:

By Trial & Error, using GC

From part (vi),

$$\text{if } x = 40, t = 54.158, x + t > 90$$

$$x = 39, t = 53.18, x + t > 90$$

$$x = 38, t = 52.202, x + t > 90$$

$$x = 37, t = 51.223, x + t < 90$$

Thus the latest time he needs to leave home is 7.37am.

X	Y1
35	49.267
36	50.245
37	51.223
38	52.202
39	53.18
40	54.158
41	55.137
42	56.115
43	57.093
44	58.072
45	59.05

X=35

11

$$(i) \sum x = 2526, \quad \sum (x - \bar{x})^2 = 544$$

Unbiased estimates of the population mean μ is

$$\bar{x} = \frac{2526}{30} = 84.2$$

$$s^2 = \frac{30}{29} \left[\frac{544}{30} \right]$$

Unbiased estimates of the population variance σ^2 is = 18.75862

$$= 18.8 \text{ (3 s.f)}$$

(ii)

$$H_0: \mu = 85$$

$$H_1: \mu < 85$$

Test at 10% significance level

Assuming that H_0 is true,

Since $n = 30$ is sufficiently large, by the Central Limit Theorem, $\bar{X} \sim N\left(85, \frac{s^2}{30}\right)$ approximately.

Test statistic: $Z = \frac{\bar{X} - 85}{\sqrt{\frac{18.75862}{30}}} \sim N(0, 1)$ approximately.

Using GC, p -value = 0.15584 = 0.156 (3 s.f)

$$\left(\text{or } z = \frac{84.2 - 85}{\sqrt{\frac{18.75862}{30}}} = -1.0117 \right)$$

Since p -value = 0.15584 > 0.1 (or $z = -1.0117 > -1.28155$), we do not reject H_0 and conclude that there is insufficient evidence at 10% level, that the mean mass of the Health and Fitness Club members has decreased. (or that the trainer's claim is invalid.)

(iii)

There is 0.15584 probability of drawing **a random sample of 30** Health and Fitness Club members with **sample mean less than 84.2 kg**, assuming that the population mean weight is 85 kg.

(iv)

$$H_0: \mu = 85$$

$$H_1: \mu < 85$$

Test at 10% level significance level.

Assuming that H_0 is true,

Since n is large, by the Central Limit Theorem, $\bar{X} \sim N\left(85, \frac{5^2}{30}\right)$ approximately.

Test statistic: $Z = \frac{\bar{X} - 85}{5/\sqrt{30}} \sim N(0, 1)$ approximately.

Since the null hypothesis is rejected,

$\Rightarrow z$ -value falls inside critical region

$\Rightarrow z$ -value < -1.28155

$$\Rightarrow \frac{m-85}{\frac{5}{\sqrt{30}}} < -1.28155$$

$$m-85 < -1.1699$$

$$m < 83.83$$

$$\therefore 0 < m < 83.83$$

Alternate method

$$\text{Using } \bar{X} \sim N\left(85, \frac{5^2}{30}\right)$$

$$P(\bar{X} \leq m) < 0.1$$

$$\Rightarrow m < 83.83$$

$$\therefore 0 < m < 83.83$$