Candidate Name: ________________________ Class: _____________

PIONEER JUNIOR COLLEGE
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JC2 PRELIMINARY EXAMINATION
Higher 1

MATHEMATICS **8865/01**
Paper 1
13 September 2017
3 hours

Additional Materials: Cover page
Answer papers
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your full name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1. A researcher prepares three types of food samples X, Y and Z for his experiment. Each food sample weighs 20 grams and contains three types of ingredients, namely, fibre, wheat and sweetener. The amount of fibre, wheat and sweetener in each sample is given below.

Sample Type	Amount in grams for each 20 grams of sample		
	Fibre	Wheat	Sweetener
X	12	6	2
Y	8	6	6
Z	6	14	0

The researcher wants to prepare a new food sample type T by mixing different amounts of sample types X, Y and Z such that in 20 grams of sample type T, there are 8.8 grams of fibre, 7.6 grams of wheat and 3.6 grams of sweetener.

Determine the amount, in grams, of samples types X, Y and Z in 20 grams of sample type T. [4]

2. Mr Woo purchased x kg of cherries from fruit stall A for \$$a$. He bought 1 kg less cherries from fruit stall B for \$$a$. He realised that fruit stall B charged him more by \$5 per kg. By considering the difference in the unit price of the cherries bought from the two fruit stalls or otherwise, show that $5x^2 - 5x - a = 0$. [2]

Find the maximum weight of cherries that Mr Woo can buy from fruit stall A if he does not want to spend more than \$50 on cherries. Give your answer to the nearest integer. [3]

3. **(a)** Differentiate $e^{x^2} + \dfrac{1}{px^2+1}$ with respect to x, where p is a constant. [2]

(b) Find $\displaystyle\int \frac{2x-1}{x+3}\,dx$. [3]

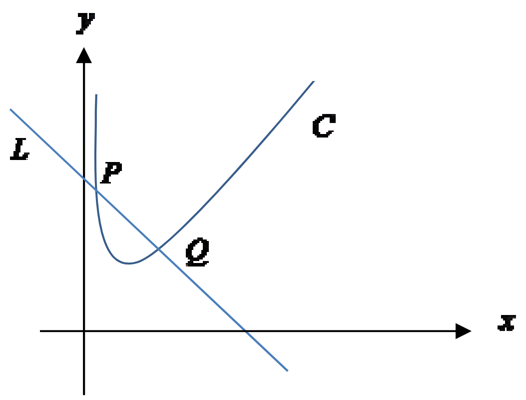
4. The curve C has equation $y = qx - \ln(2x^2+1)$, where q is a positive constant.

(i) Find, in terms of q, the equation of the tangent to C at the point where $x=1$. Give your answer in the form $y = ax + b$, with a and b in exact forms. [3]

(ii) Find the exact value of q such that C has 1 stationary point. [3]

(iii) Using the value of q found in **(ii)**, find the equation of the tangent which is parallel to the x-axis. [2]

5.



The diagram shows the curve C with equation $y=\left(\sqrt{x}+\dfrac{2k}{\sqrt{x}}\right)^2$ and the line L with equation $y=13k-x$, where k is a positive constant. The graphs intersect at P and Q as shown. Show that the x-coordinates of P and Q are $\dfrac{1}{2}k$ and $4k$ respectively. [2]

Hence find, in terms of k, the area of the region bounded by C, L, the x-axis and the line $x=k$. [4]

6. The number of customers (in thousands), C, of a new company is believed to be modelled by the equation

$$C=8(1-e^{-kt}),$$

where t is the number of years from the time the company starts its operation and k is a positive constant.

(i) Given that the company has 7 thousand customers at the end of the 3rd year of operation, determine the exact value of k, giving your answer in the simplest form. [3]

Using the value of k found in **(i)**,

(ii) sketch the graph of C against t, stating the equations of any asymptotes, [2]

(iii) find the exact value of $\dfrac{dC}{dt}$ when $t=2$, simplifying your answer. Give an interpretation of the value you have found, in the context of the question. [3]

At the end of the 6th year of operation, the number of customers of the company is now believed to be modelled by the equation

$$C=-0.05t^2+0.7t+5.475,$$

where $t\geq 6$.

(iv) Use differentiation to find the value of t, where $t\geq 6$, which gives the maximum value of C. Hence, find the maximum value of C. [4]

[Turn over

Section B: Statistics [60 marks]

7. A group of 10 students consisting of 6 females and 4 males bought tickets to attend a concert. If the tickets were for a particular row of 10 adjacent seats, find the number of possible seating arrangements when

(i) the first and last seats were occupied by students of the same gender, [3]

(ii) one particular student did not turn up for the concert. [1]

8. At a stall in a fun-fair, games of chance are played, where at most 1 prize is won per game. The probability that a prize is won in each game is 0.1. For each day, 80 games are played. The random variable X is the number of prizes being won on a particular day.

(i) Find $\mathrm{P}(X > 5)$. [1]

The stall is opened for n days and on each day, 80 games are played.

(ii) If n = 10, find the probability that there are 4 days with at most 5 prizes being won each day. [2]

(iii) If n is large, using a suitable approximation, find the minimum value of n such that the probability that the average number of prizes being won each day exceeds 8.5 is less than 0.1. [4]

9. In a box containing a large number of apples, 15% of the apples are rotten. A random sample of 20 apples is drawn to inspect.

(i) Explain the significance of the phrase ‘large number’ in the first sentence of the question. [1]

(ii) Find the probability that there is at least 1 but less than 4 rotten apples in the random sample. [3]

A box containing large number of apples is chosen for export if there is no rotten apple from the random sample. If there is at least 1 but less than 4 rotten apples in the random sample, another random sample of 10 apples is drawn from the box to inspect. If there is no rotten apple in the second random sample, the box will be chosen for export. Otherwise, the box will not be chosen for export.

(iii) Find the probability that a randomly chosen box is chosen for export. [2]

(iv) Four boxes each containing a large number of apples are chosen for inspection. If it is known that the first box is chosen for export, find the probability that exactly two out of the four boxes are chosen for export. [2]

10. A magazine claims that the average time a child spends outdoors is no longer than 14 hours a week. To verify this, Henry randomly selects 50 children and the amount of time that each child spends outdoors in a particular week, x hours, is recorded. The results are summarised as follows.

$$\sum(x-14)=3.9 \qquad \sum(x-14)^2=2.7$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Suggest a reason why, in this context, the given data is summarised in terms of $(x-14)$ rather than x. [1]

(iii) Test at the 1% significance level whether the claim made in the magazine is valid. [5]

11. **(a)** Seven pairs of values of variables x and y are measured where x and y are positive values. Draw a sketch of a possible scatter diagram for each of the following cases:

(i) the product moment correlation coefficient is approximately zero, [1]

(ii) the product moment correlation coefficient is approximately 0.8. [1]

(b) A study on how the trade-in value p, in thousand dollars, of a particular make of used car depreciates with the age of the car t, in years, is conducted. The data for 7 cars is collected and shown in the following table.

Age, t	2	3	4	5	6	7	8
Trade-in value, p	54.0	50.1	45.3	38.6	35.1	33.5	30.4

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of this question. [2]

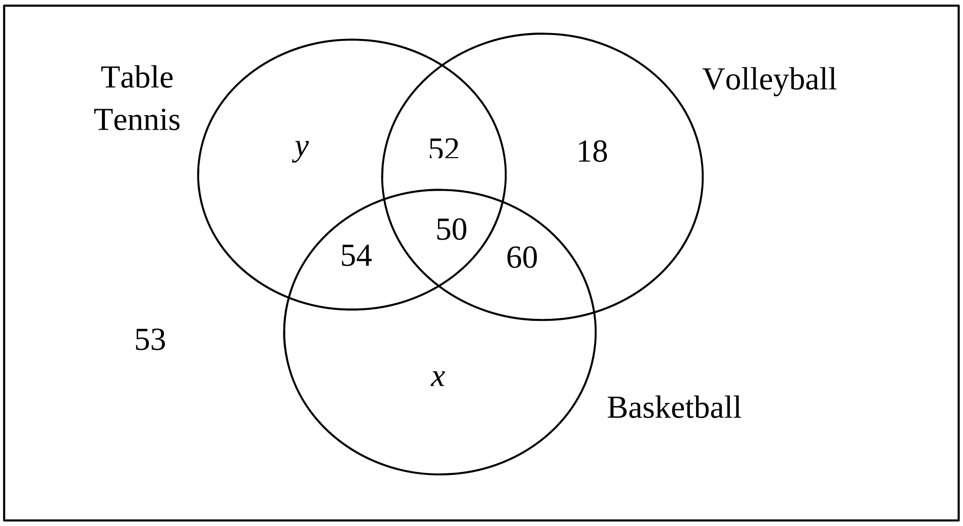
(iii) Find the equation of the regression line of p on t in the form $p=mt+c$, giving the values of m and c correct to 2 decimal places. [1]

(iv) The data for a second sample of another 6 cars is obtained. The regression lines of p on t and of t on p for the second sample are given respectively as:

$$p=61.45-4.19t \quad \text{and} \quad t=14.39-0.23p.$$

Calculate the mean trade-in value and mean age for the combined sample of 13 cars. [3]

12.



A group of students are surveyed on the types of sport(s) they can play out of the three sports namely table tennis, volleyball and basketball. The numbers of students who can play the different sport(s) are shown in the Venn diagram. The number of students who can play table tennis only is y and the number of students who can play basketball only is x. One of the students is chosen at random.

T is the event that the student can play table tennis.
B is the event that the student can play basketball.
V is the event that the student can play volleyball.

(i) Write down the expressions for P(T) and P(V) in terms of x and y. Given that T and V are independent, show that $13y - 17x = 199$. [3]

(ii) Given that $\mathrm{P}(T \cup B) = \dfrac{379}{450}$, find the values of x and y. [3]

Using the values of x and y found in **(ii)**, find

(iii) $\mathrm{P}(B \cap (T \cup V))$, [1]

(iv) $\mathrm{P}(T \mid V)$. [1]

Three students from the whole group are chosen at random.

(v) Find the probability that among the three students, one can play exactly two sports out of the three sports (table tennis, volleyball and basketball), the other one can play table tennis only and the remaining one cannot play any of the sports. [3]

13. The masses, in kilograms, of cod fish and salmon fish sold by a fishmonger are normally distributed. The means and standard deviations of these distributions, and the selling prices, in \$ per kilogram, of cod fish and salmon fish are shown in the following table.

	Mean (kg)	Standard deviation (kg)	Selling price (\$ per kg)
Cod Fish	a	0.1	68
Salmon Fish	0.6	0.15	30

(i) Find the probability that a randomly chosen cod fish has mass less than $(0.2 + a)$ kg. [2]

(ii) It is known that 20% of the cod fish sold by the fishmonger have a mass of at least 0.5 kg. Find the value of a. [3]

Use $a = 0.4$ for the rest of the question.

(iii) Find the probability that the total mass of 4 randomly chosen cod fish is within $\pm$ 0.1 kg of twice the mass of a randomly chosen salmon fish. [4]

(iv) Find the probability that a randomly chosen cod fish has a selling price exceeding \$25 and a randomly chosen salmon fish has a selling price exceeding \$15. [2]

(v) State an assumption needed for your calculations in **(iii)** and **(iv)**. [1]

\- **End of paper** -