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DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 1)

8865/01

Paper 1

September 2017

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Total
Score													
Max Score	4	5	8	10	13	3	6	7	10	9	10	15	100

Section A : Pure Mathematics [40 marks]

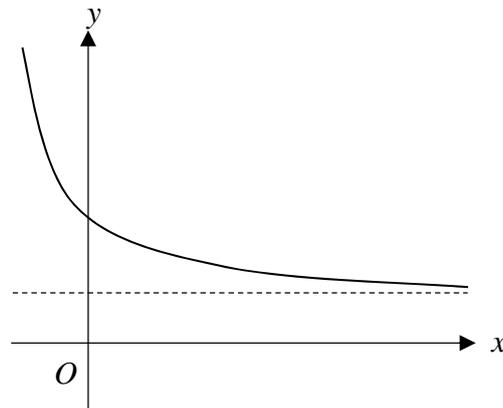
- 1** Abel, Ben and Caleb travelled from Raffles Place to Bishan using the same route. They hired private transport from different car companies, Car X, Y and Z, respectively. Ben and Caleb had cash vouchers which could be used to offset their final fares. Ben's cash voucher was twice as much in value as Caleb's.

The fare comprises 3 components: A fixed base fare, the distance travelled and the time taken for the journey. The rates of the different companies are shown in the following table.

	Car X	Car Y	Car Z
Base fare (\$)	3.20	3.00	3.00
Per kilometre (\$)	0.55	0.80	0.45
Per minute (\$)	0.29	0	0.20

Abel, Ben and Caleb paid \$15.60, \$6.60 and \$9.40 respectively. Find the distance travelled and the time taken from Raffles Place to Bishan. Assume that the traffic is the same for all 3 journeys. [4]

2



The diagram shows the curve C with equation $y = \ln 2 + 2^{-x}$.

- (i) State the exact equation of the asymptote and the exact coordinates of the point of intersection with the y -axis. [2]
- (ii) Find the x -coordinate of the point of intersection of C and the line $y = 1$, giving your answer correct to 4 decimal places. [1]
- (iii) Write down as an integral an expression for the area of the region bounded by C , $y = 1$ and the y -axis and evaluate this integral. [2]

- 3** (a) Differentiate $\frac{\pi^2}{\sqrt{(3-\pi x)}}$. [2]

- (b) Show that $\frac{d}{dx}(e^{-x+2\ln x}) = xe^{-x}(2-x)$. [3]

Hence find the exact solution of $\int_1^e xe^{-x+a}(x-2) dx$ in terms of the constant a . [3]

- 4 [It is given that the volume of a cone is $\frac{1}{3}\pi r^2 h$ where h is the vertical height and r the radius of the circular base of the cone.]

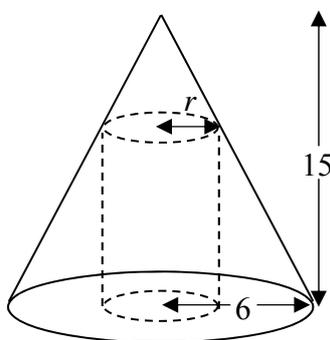
A hollow cone has a base radius 6 cm and height 15 cm. It is made of material with negligible thickness.

- (a) The cone is inverted. Initially, the cone is empty and water is poured into it at a rate of $8 \text{ cm}^3 \text{ s}^{-1}$. The depth of water in the cone is x cm at time t seconds.

(i) Show that the volume $V \text{ cm}^3$ of the water in the cone is given by $V = \frac{4}{75}\pi x^3$. [2]

- (ii) Find the exact rate of increase of the depth of water at the instant when the depth is 5 cm. [3]

(b)



The same cone is now placed in an upright position and a solid cylinder is to be inscribed in the cone (see diagram). Show that the total surface area $A \text{ cm}^2$ of the cylinder of radius r cm is given by $A = 30\pi r - 3\pi r^2$, and hence find the exact value of the maximum A . [3]

- 5 At time 0000, a physicist observes the behaviour of 2 particles E_1 and E_2 for a period of 20 minutes. E_2 is stationary for a few minutes before it starts moving. The speed v , in m/min after t min, of E_1 and E_2 satisfies the equation $v = \frac{1}{3}t$ and $v = \sqrt{(2t - 5)}$ respectively.

- (i) On the same diagram, sketch the speed-time graphs of E_1 and E_2 during the period of observation of 20 minutes, stating the exact coordinates of any points of intersection with the axes and points of intersection of the two graphs. [2]
- (ii) State the duration for which E_2 moves faster than E_1 . [1]
- (iii) The distance travelled is represented by the area under a speed-time graph. Determine which particle travels a longer distance for the period of observation of 20 minutes. [4]
- (iv) The derivative of speed with respect to time is known as acceleration. Without using a calculator, find the time at which E_1 and E_2 have the same acceleration. [3]

The speed of a third particle E_3 satisfies the equation $v = \sqrt{a - (t - 5)^2}$, where a is a positive constant. Find the set of values of a , given that E_1 and E_3 will not travel at the same speed at any time t . Show your working clearly. [3]

Section B : Statistics [60 marks]

- 6 A sample of 5 people is chosen from a village of large population.
- (i) The number of people in the sample who are underweight is denoted by X . State, in context, the assumption required for X to be well modelled by a binomial distribution. [1]
- (ii) On average, the proportion of people in the village who are underweight is p . A total of 1000 samples of 5 people are chosen at random from the village and the results are shown in the table below.

x	0	1	2	3	4	5
Number of samples	93	252	349	220	75	11

Using the above results, find \bar{x} . Hence estimate the value of p . [2]

- 7 The Tan family has 2 children while the Wong family has 3 children. The children, together with both their parents, catch a movie at the cinema. At the ticket counter, they realise only 2 rows of consecutive empty seats are left, where Row L has 5 seats and Row M has 4 seats.

Find the number of different possible arrangements if

- (i) there are no restrictions, [1]
- (ii) the Tan siblings must not sit together. [3]

The 9 of them are randomly seated. Find the probability that Tan siblings sit between their parents given that the Wong family takes Row L. [2]

- 8 The insurance company Adiva classifies 10% of their car policy holders as ‘low risk’, 60% as ‘average risk’ and 30% as ‘high risk’. Its statistical database has shown that of those classified as ‘low risk’, ‘average risk’ and ‘high risk’, 1%, 15% and 25% are involved in at least one accident respectively.

Find the probability that a randomly chosen policy holder

- (i) is not involved in any accident if the policy holder is classified as “average risk”, [1]
- (ii) is not involved in any accident, [2]
- (iii) is classified as “low risk” if the policy holder is involved in at least one accident. [2]

Two policy holders are chosen at random.

- (iv) Find the probability that one is not involved in any accident while the other is involved in at least one accident. [2]

- 9 Students pursuing a particular university course are required to take m modules in each semester. At the end of each module, the students have to take an examination which comprises n questions. It may be assumed that for each examination, the number of questions answered correctly by a randomly chosen student follows a binomial distribution $B(n, 0.6)$ with variance 24.

(i) Verify that $n = 100$. [1]

To pass a module, a student must answer at least 50 questions correctly in the examination.

(ii) Find the most probable number of questions that a randomly chosen student answers correctly in an examination. [2]

(iii) Show that the probability that a randomly chosen student passes a module is 0.983. [1]

(iv) Given that a randomly chosen student is at most 90.4% confident of passing all his modules in a semester, find the least value of m . [3]

Forty students in this course are randomly selected and their marks for a particular examination are recorded. Use a suitable approximation to find the probability that on average at most 58 questions are answered correctly. [3]

- 10 The students in a college are separated into two groups of comparable sizes, Group X and Group Y. The marks for their Mathematics examination are normally distributed with means and variances as shown in the following table.

	Mean	Variance
Group X	55	20
Group Y	34	25

- (i) Find, to 1 decimal place, the maximum passing mark if at least 60% of students from Group Y are to pass the examination. [2]
- (ii) Find the probability that the total marks of 4 students from Group Y is less than three times the mark of a student from Group X. State clearly the mean and variance of the distribution you use in your calculation. [3]
- (iii) The marks of 40 students, with 20 each randomly selected from Group X and Group Y, are used to compute a new mean score, \bar{M} . Given that $P(-k < \bar{M} - 44.5 < k) = 0.9545$, find the value of k . [3]

State a necessary assumption for your calculations to hold in parts (ii) and (iii). [1]

- 11** A retail manager of a large electrical appliances store wants to study the relationship between advertising expenditure, x hundred dollars, and the sales of their refrigerators, y thousand dollars, on a monthly basis. The data is shown in the following table.

x	5	8	12	16	18	20	23
y	12.5	12.9	14.6	15.8	17.0	19.3	20.8

- (i) Draw a scatter diagram to illustrate the above data and calculate the product moment correlation coefficient between x and y . [3]
- (ii) Find the equation of the regression line of y on x , in the form $y = a + bx$. Sketch this line on your scatter diagram. [2]
- (iii) Use a suitable regression line to estimate the advertising expenditure of a particular month if \$15000 was made from the sale of refrigerators. Comment on the reliability of this estimate. [3]
- (iv) Explain the meaning of b in the context of this question. [1]
- (v) There is an additional expenditure of \$200 for every month of advertising. Without any further calculations, state any change you would expect in the values of the constant b found in part (ii). [1]
- 12** (a) The centre thickness, X micrometres, of soft contact lenses from a certain company is a normally distributed random variable with mean μ . The company claims that the centre thickness of their lenses is at most 30 micrometres. A random sample of 60 contact lenses is measured. The results are summarised as follows.

$$\sum(x - 30) = 24 \quad \sum(x - 30)^2 = 144$$

Test, at the $2\frac{1}{2}\%$ significance level, whether the claim is justified. [6]

Explain, in the context of the question, the meaning of “at the $2\frac{1}{2}\%$ significance level”. [1]

- (b) The company Snatch provides a ride-hailing service comprising taxis and private cars in Singapore. Snatch claims that the mean waiting time for a passenger from the booking time to the time of the vehicle’s arrival is 7 minutes. A random sample of 30 passengers’ waiting times is obtained and the standard deviation of the sample is 2 minutes. A hypothesis test conducted concludes that there is sufficient evidence at the 1% significance level to reject the claim.
- (i) State appropriate hypotheses and the distribution of the test statistic used. [3]
- (ii) Find the range of values of the sample mean waiting time, \bar{t} . [3]
- (iii) A hypothesis test is conducted at the 1% significance level whether the mean waiting time of passengers is more than 7 minutes. Using the existing sample, deduce the conclusion of this test if the sample mean waiting time is more than 7 minutes. [2]