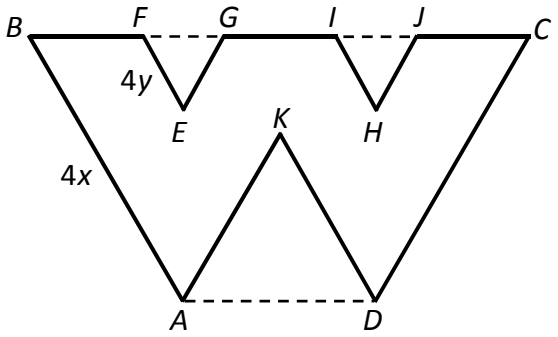
RI H1 Mathematics 2017 Prelim Exam Paper Question

1 Differentiate

(a) $\ln\{7\sqrt{(1+6x^2)}\}$, [2]

(b) $\dfrac{1}{6(1-7x)^2}$. [2]

2 A designer wishes to make a logo for her client. Her design for the logo is shown in the diagram below, which is formed by removing three triangles *EFG*, *HIJ* and *AKD* from a trapezium *ABCD*.



For aesthetic purpose, her design has the following features:

(1) Angles *ABC* and *BCD* are both 60°.

(2) Triangles *EFG* and *HIJ* are identical and equilateral.

(3) The points *F*, *G*, *I* and *J* are on *BC* such that $BF = GI = JC$.

(4) Triangle *AKD* is also equilateral, and the point *K* lies on the intersection of the line segments *AI* and *GD*.

It is given that $AB = CD = 4x$ mm and $EF = 4y$ mm.

(i) Show that $BF = 2(x - y)$ mm. [1]

(ii) Show that $AD = 2(x + y)$ mm. [1]

The perimeter and area of *ABFEGIHJCDKA* is 566 mm and $4655\sqrt{3}$ mm^2 respectively.

(iii) Find the values of x and y. [4]

3 **Do not use a calculator in answering this question.**

(a) Solve the inequality $\dfrac{(x-4)(x^2-4x+4)}{x+4} \geq 0$. [4]

(b) Find the range of values of k for which $(k-1)x^2 - 2x + k + 2 < 0$ for all real values

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	of x. [4]
4	A jug containing liquid is taken from a refrigerator and placed in a room with a constant temperature of 25 °C. The temperature of the liquid θ °C after time t minutes is given by $$\theta = 25 - Ae^{kt},$$ where A and k are real constants. Initially the temperature of the liquid is 9 °C. After 20 minutes, the temperature of the liquid increases to 17 °C. **(i)** Find the value of A and show that $k = \frac{1}{20}\ln\frac{1}{2}$. [4] **(ii)** Find the temperature of the liquid after 25 minutes. [1] **(iii)** Find the exact duration it takes for the temperature of the liquid to increase from 17 °C to 23 °C. [2] **(iv)** State what happens to θ for large values of t. [1] **(v)** Sketch a graph of θ against t. [2]
5	The curve C_1 has equation $y = \ln(1 + x)$ and the curve C_2 has equation $y = \ln 2 + 1 - x$. **(i)** Sketch the graphs of C_1 and C_2 on the same diagram, stating the equations of any asymptotes and the exact coordinates of any points where the curves crosses the axes. [3] **(ii)** Verify that C_1 and C_2 intersect at $x = 1$. [1] **(iii)** Find, correct to 2 decimal places, the area of the finite region bounded by C_1, C_2 and the x-axis. [3] **(iv)** Use integration to find the exact area of the finite region bounded by C_1, C_2 and the y-axis. Leave your answer in the form $A + B\ln 2$, where A and B are constants to be determined. [5]
6	It is given that $X \sim N(\mu, 7)$ and $P(X < 7) = 0.7$. **(i)** Find the value of μ correct to 3 decimal places. [2] **(ii)** Find $P(X_1 < X_2 + 1)$, where X_1 and X_2 are independent observations of X. [2]
7	A traditional bakery produces two types of biscuits – one with sweet fillings and one

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with salted fillings. The biscuits are sold in packs of 8, and each pack has a random selection of the two types of biscuits.

The mean number of biscuits with sweet fillings in each pack is 3.2.

(i) Find the probability that a randomly chosen pack contains no biscuits with sweet fillings. [2]

(ii) Show that the probability that a randomly chosen pack contains at least four biscuits with sweet fillings is 0.406 correct to 3 significant figures. [2]

A customer buys 18 packs of biscuits for a wedding.

(iii) Find the probability that at most 9 of these packs contains at least four biscuits with sweet fillings. [2]

8 Box *A* contains 2 green marbles and 6 red marbles. Box *B* contains 4 green marbles and 2 red marbles. Two fair dice are tossed at the same time. Box *A* is selected if at least one '6' is shown. Otherwise, box *B* is selected. One marble is then chosen from the selected box and its colour noted.

Draw a tree diagram to represent this situation. [4]

Find the probability that

(i) the marble chosen is green, [2]

(ii) the marble chosen is from box *A*, given that its colour is red. [2]

Give your answers as a fraction in its lowest term.

9 Caffeine is said to affect our sleep at night. In a student research study, different amounts of caffeine, *x* grams, were given to a test subject for 8 consecutive evenings and the times, *t* minutes, for the test subject to fall asleep at night were recorded.

The results are given in the table below.

Day	1	2	3	4	5	6	7	8
x	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
t	15	16	20	23	25	24	30	35

(i) Draw a scatter diagram to illustrate the data. [2]

(ii) Calculate the equation of the regression line of t on x, and draw this line on your scatter diagram. [1]

(iii) Find $\bar{x}$ and $\bar{t}$, and mark the point $(\bar{x}, \bar{t})$ on your scatter diagram. [1]

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(iv) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(v) Use the regression line in part **(ii)** to predict the time taken for the test subject to fall asleep when 1.00 grams of caffeine was given. Comment on the validity of this prediction. [2]

10 A farmer grows watermelons. He claims that the average mass of watermelons in his farm is at least 10 kg. To test this claim, a random sample of 70 watermelons is checked and the masses of watermelons, x kg, are summarised as follows:

$$\sum(x - 10) = -28, \ \sum(x - 10)^2 = 267.$$

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) Test at the 5 % significance level whether the farmer's claim is valid. [4]

The farming process is improved and the new population variance is known to be 3.31 kg^2. A new random sample of 70 watermelons is checked and the total mass of this sample is m kg. A test at the 5 % significance level shows that there is sufficient evidence to suggest that the population mean mass of watermelons is more than 10 kg.

(iii) Find the range of possible values of m. [3]

11 A pet shop owner carries African bullfrogs. The masses, in kg, of the male and female frogs are independent and are normally distributed with means and standard deviations as shown in the following table:

	Mean mass	Standard deviation
Male	1.4	0.28
Female	0.7	0.14

(i) A male and a female bullfrog are chosen at random. Find the probability that the mass of the female frog is greater than the mass of the male frog, stating clearly the mean and variance of any distribution that you use. [3]

The owner wishes to build a tank for up to four frogs. If he uses material X, the total mass of the frogs must not exceed the maximum safety limit of 4.5 kg.

(ii) Two male and two female bullfrogs are chosen at random. Find the probability that their total mass do not exceed the maximum safety limit of 4.5 kg, stating clearly the mean and variance of any distribution that you use.

If the owner uses material Y, the maximum safety limit of the tank is improved to L kg.

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	(iii) It is 95 % certain that four male bullfrogs chosen at random have a total mass not exceeding the maximum safety limit of L kg. Find, correct to 1 decimal place, the least value of L. [4]
12	Find the number of ways in which the letters of the word SECTION can be arranged if **(i)** the letters are not in alphabetical order, [1] **(ii)** the consonants (S, C, T, N) and vowels (E, I, O) must alternate, [2] **(iii)** all the vowels are together, [2] **(iv)** all the vowels are separated, [2] **(v)** there must be exactly two letters between the two letters E and O. [2] Find, as a fraction in its lowest term, the probability that after arranging the letters of the word SECTION, there is at least one consonant and at least one vowel between the two letters E and O. [4]