

INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
in preparation for General Certificate of Education Advanced Level
Higher 1

CANDIDATE
NAME

CLASS

INDEX NUMBER

MATHEMATICS

8864/01

Paper 1

28 August 2017

3 hours

Additional Materials: Answer Paper
 Cover Page
 Graph Paper
 List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper. You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **6** printed pages.



Section A: Pure Mathematics [35 marks]

1 (i) Differentiate $\frac{3}{\sqrt{(2x-7)^3}}$. [2]

(ii) Find $\int \frac{1}{e^{4t-3}} + \frac{1}{2t-1} dt$. [3]

2 By considering $u = e^{2x}$ or otherwise, solve the equation

$$3 - 10e^{2x} - 8e^{4x} = 0,$$

leaving your answer in the form $a \ln b$, where a and b are integers to be determined.

[4]

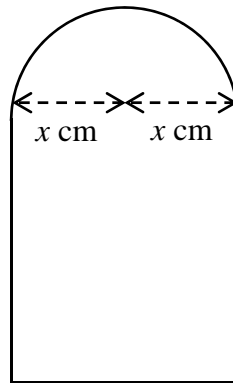
Hence solve the inequality

$$3 - 10e^{2x} - 8e^{4x} \leq 0,$$

leaving your answer in exact form.

[2]

3 A window frame is in the shape of a rectangle joined to a semicircle of radius x cm.



(i) If the window frame is made using 300 cm of framework with negligible thickness,

show that the total area of the window is given by $A = 300x - 2x^2 - \frac{1}{2}\pi x^2$. [2]

(ii) Using differentiation, find the maximum area of the window. Leave your answer correct to the nearest integer. [4]

4 A curve C has equation $y = \ln(3 - x)$.

- (i) Sketch C , indicating clearly the exact coordinates of any points of intersection with the axes and the equation of asymptote, if any. [2]
- (ii) Find the equation of the tangent to C at the point P where $x = 1$, giving your answer in the form $y = mx + c$, where m and c are exact constants to be determined. [3]
- (iii) R is the region bounded by the tangent at P , the curve C and the x -axis. By sketching the equation of the tangent at P on the diagram in part (i), indicate the region R . Hence find the numerical value of the area of region R . [4]

5 The curve C has equation $y = x^4 - 4x^3 + \frac{9}{2}x^2 - 2x + 2$.

- (i) Find $\frac{dy}{dx}$. Hence find the coordinates of the stationary points on the curve. [3]
- (ii) Use a non-calculator method to determine the nature of each of the stationary points. [2]
- (iii) Sketch the graph of C , stating the coordinates of the stationary points and any points where the curve crosses the axes. [2]
- (iv) By adding a suitable graph in your diagram in part (iii), solve the inequality $x^4 - 4x^3 + \frac{9}{2}x^2 + x - 9 < 0$, giving your answer correct to 4 decimal places. [2]

Section B: Statistics [60 marks]

- 6** The Physical Education (PE) Department of Sunflower Secondary School intends to carry out a survey to investigate the number of hours of exercise each student spends per week. The school has a total of 1200 students studying in four different levels. On one particular school day, a PE teacher selects a random sample of 80 students from those who enter the school via the main gate by

- choosing at random one of the first 15 students who enter the main gate,
- then choosing every 15th student after the first student is chosen.

- (i) State the sampling method used by the PE teacher. [1]
- (ii) State one disadvantage of the sampling method used in this context. [1]
- (iii) Describe briefly how, in this case, the PE teacher might choose a more appropriate random sample. [3]

- 7** At a lucky draw booth, each contestant will roll an unbiased die. If the die shows a “6”, the contestant will pick a counter at random from Box A. Otherwise, he will pick a counter at random from Box B. Box A contains 3 red counters, 2 green counters and 3 yellow counters. Box B contains 5 red counters, 3 green counters and 2 yellow counters.

- (a) A contestant will win a prize if a yellow counter is picked.
- (i) Draw a tree diagram to represent this situation. [2]
- (ii) Find the probability that a contestant wins a prize. [2]
- (iii) Given that the contestant wins a prize, find the probability that it came from Box A. [2]
- (b) The rule of winning a prize has now changed. Each contestant needs to pick two counters, without replacement, instead of one. A contestant will win a prize if both counters picked are yellow. Find the probability that a contestant wins a prize. [2]

- 8** In a neighbourhood, it is known that 9% of the residents use the bicycle-sharing platform, ShareBike. A sample of n residents is selected at random and the number of residents who use ShareBike in the sample is denoted by the random variable X .

- (i) State, in context, an assumption needed for X to be well modelled by a binomial distribution. [1]
- (ii) Explain why the assumption stated in part (i) may not hold in this context. [1]

Assume now the assumption stated in part (i) does in fact hold.

- (iii) Find the greatest value of n such that the probability that there is at least 1 resident using ShareBike is less than 0.99. [3]
- (iv) Given that $n = 40$, find the probability that more than 3 but at most 5 residents use ShareBike. [2]
- (v) Given instead that $n = 70$, using a suitable approximation, find the probability that less than 7 residents use ShareBike. [4]

- 9** Two electrical components, Type A and Type B, have lifespans of A weeks and B weeks respectively. It is given that A and B are independent random variables with distributions $N(43, 8^2)$ and $N(40, 6^2)$ respectively.

- (i) Find the probability that the total lifespan of 3 randomly chosen Type A components is shorter than thrice the lifespan of a randomly chosen Type B component. [4]
- (ii) Find the probability of the event that both the lifespan of a randomly chosen Type A component exceeds 38 weeks and the total lifespan of 2 randomly chosen Type B components exceeds 82 weeks. [4]
- (iii) Find the probability that the total lifespan of a randomly chosen Type A component and 2 randomly chosen Type B components exceeds 120 weeks. [3]
- (iv) Explain why the answer in part (ii) is smaller than the answer in part (iii). [1]

- 10** Mr Lee recorded the length of time, t minutes, taken to travel to work when leaving home x minutes after 7 am on 10 mornings over two weeks. The results are as follows.

x	0	5	10	15	20	25	30	35	40	45
t	15	19	30	28	32	39	30	48	53	62

- (i) Plot a scatter diagram on graph paper for this data, labelling the axes, using a scale of 2 cm to represent 10 minutes on the t -axis and an appropriate scale for the x -axis. [2]
- (ii) Suggest a reason why one of the data points does not seem to follow the trend and indicate the corresponding point on your diagram by labelling it P . [2]

Omit the point P .

- (iii) Calculate the product moment correlation coefficient and comment on this value. [2]
- (iv) Find the equation of the least squares regression line of t on x , writing your answer in the form $t = ax + b$. [1]
- (v) Sketch the regression line on your scatter diagram and interpret the meaning of the value of a in the context of the question. [2]
- (vi) Mr Lee needs to arrive at work no later than 8.30 am. Estimate, to the nearest minute, the latest time that he has to leave home without arriving late at work. [3]

- 11** A large group of Health and Fitness Club members is known to have a mean mass of 85 kg. The trainer claims that the mean mass of his members has decreased under his strict routine. To investigate his claim, the mass, x kg, of 30 randomly chosen members are collated and the results are summarised below.

$$\sum x = 2526, \quad \sum (x - \bar{x})^2 = 544$$

- (i) Find unbiased estimates of the population mean and variance. [3]
- (ii) Test at the 10% level of significance whether the trainer's claim is valid. [5]
- (iii) State the meaning of the p -value obtained in part (ii). [1]
- (iv) The trainer makes some adjustments to his training routine and the new population standard deviation is known to be 5 kg. A new sample of 30 members is randomly chosen and the mean mass of this sample is m kg. At the 10% level of significance, find the range of values of m for the trainer's claim to be valid, giving your answer correct to 2 decimal places. [3]