1) $x =$ Number of 6-blade packages sold $y =$ Number of 12-blade packages sold $z =$ Number of 24-blade packages sold $x + y + z = 12$ ------ (1) $6x + 12y + 24z = 162$ ------ (2) $2x + 3y + 4z = 35$ ------ (3) $x = 5 \quad y = 3 \quad z = 4$	
2) $y = \frac{2}{1+x^2} - 1$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4x}{(1+x^2)^2}$ The coordinates of points A, *B* and *C* are $A(0,1)$, $B(1,0)$ and $C(-1,0)$ At *B*, $\frac{\mathrm{d}y}{\mathrm{d}x} = -1$ Thus equation of tangent at *B* is $y = -x + 1$. When $x = 0$, $y = -0 + 1$ = 1. Thus the tangent at *B* passes through $A(0,1)$. At *C*, $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ Thus equation of tangent at *C* is $y = x + 1$. When $x = 0$, $y = 0 + 1$ = 1. Thus the tangent at *C* passes through $A(0,1)$.	
3(a)(i) $\frac{\mathrm{d}}{\mathrm{d}x} \ln\left(\frac{5x+2}{4x^2}\right)$ $= \frac{\mathrm{d}}{\mathrm{d}x}\left[\ln(5x+2) - \ln 4x^2\right]$ $= \frac{5}{5x+2} - \frac{2}{x}$	
(ii) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{e}^{\sqrt{x}}\right] = \frac{\mathrm{e}^{\sqrt{x}}}{2\sqrt{x}}$	

(b)

$$\int_1^4 \frac{e^{\sqrt{x}+\ln 3}}{\sqrt{x}}\,dx = \int_1^4 \frac{e^{\sqrt{x}}e^{\ln 3}}{\sqrt{x}}\,dx$$

$$= \int_1^4 \frac{3e^{\sqrt{x}}}{\sqrt{x}}\,dx$$

$$= 6\int_1^4 \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$$

$$= 6\left[e^{\sqrt{x}}\right]_1^4$$

$$= 6e^{\sqrt{4}} - 6e$$

$$= 6e(e-1)$$

4(i)

$$\frac{dV}{dt} = 0.0012e^{0.24t} - 0.0594e^{-0.12t}$$

$$0.0012e^{0.24t} - 0.0594e^{-0.12t} = 0$$

$$0.0012e^{0.24t} = 0.0594e^{-0.12t}$$

$$\frac{e^{0.24t}}{e^{-0.12t}} = \frac{0.0594}{0.0012}$$

$$e^{0.36t} = 49.5$$

$$0.36t = \ln 49.5$$

$$t = 10.839$$

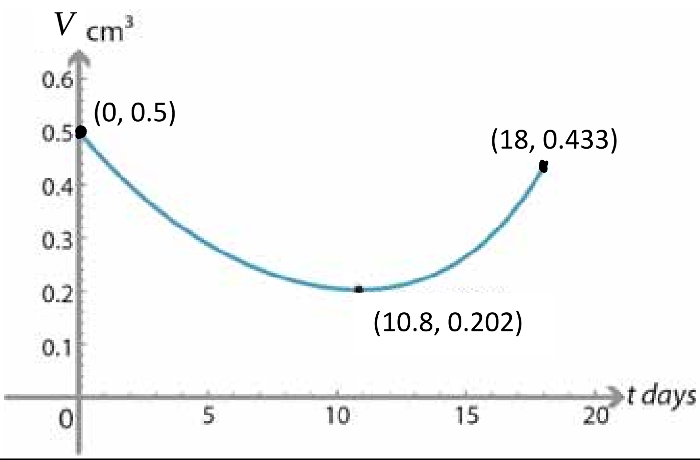
$$V = 0.202$$

sign of $\frac{dV}{dt}$

$-$ $+$

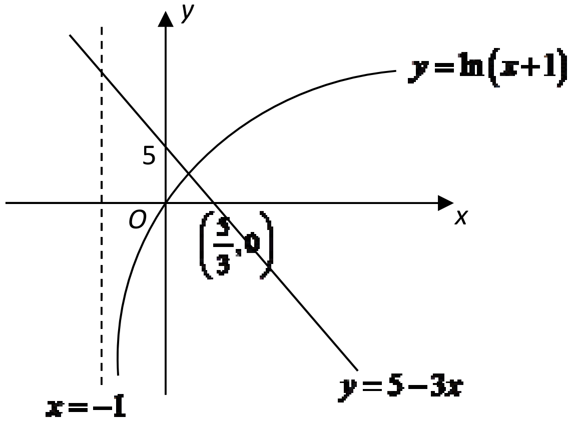
10.839

(ii)



(iii) From GC, $\frac{dV}{dt} = -0.0286$

Rate of decrease = 0.0286



5(a)(i)

$6x+\ln(x+1)^2<10$

$6x+2\ln(x+1)<10$

$3x+\ln(x+1)<5 \Rightarrow \ln(x+1)<5-3x$

Point of intersection is at $x=1.3779$ (3sf).
Hence, from graph, solution of inequality: $-1<x<1.38$

(ii)
$\int_0^{1.3779}\left(5-3x-\ln(x+1)\right)\,dx+\int_{1.3779}^{3}\left(\ln(x+1)-(5-3x)\right)\,dx$

≈ 7.76459

≈ 7.76

(b)

$4x^2+(5-3x)^2=4k^2$

$13x^2-30x+25-4k^2=0$

$b^2-4ac>0$

$30^2-4(13)\left(25-4k^2\right)>0$

$208k^2-400>0$

$13k^2-25>0$

$\left(\sqrt{13}k+5\right)\left(\sqrt{13}k-5\right)>0$

$k<-\frac{5}{\sqrt{13}}$ or $k>\frac{5}{\sqrt{13}}$

$\left\{k\in\mathbb{R}:k<-\frac{5}{\sqrt{13}} \text{ or } k>\frac{5}{\sqrt{13}}\right\}$

6(i)

Total number of ways of selecting 4 chocolates

$= {}^{14}C_4 = 1001$

Number of ways of selecting 2 soft centres (and 2 hard)

$= {}^{8}C_2 \times {}^{6}C_2 = 420$

$$\text{P}(\text{ 2 soft}) = \frac{420}{1001}$$

$$= \frac{60}{143} \quad \text{or} \;\; 0.420$$

(ii)

Number of ways of selecting at most 3 soft centres

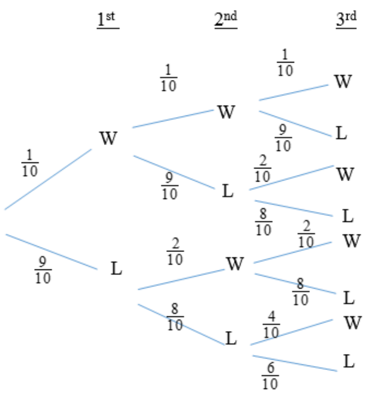
$= 1001 - {}^{8}C_4 \times {}^{6}C_0 = 420$

$= 931$

$$\text{P}(\text{at most 3 soft}) = \frac{931}{1001}$$

$$= \frac{133}{143} \quad \text{or} \;\; 0.930$$

7)



$\text{P}(A' \cap B)$ is the probability of the player winning the first 2 stages and lose the 3rd stage.

$$\text{P}(A' \cap B) = \frac{1}{10} \times \frac{1}{10} \times \frac{9}{10} = \frac{9}{1000}$$

$$\text{P}(A') = \frac{9 + 72 + 144 + 432}{1000} = \frac{657}{1000}$$

$\mathrm{P}(B) = \dfrac{9+18+36}{1000} = \dfrac{63}{1000} = 0.063$

$\mathrm{P}(A')\times \mathrm{P}(B) = 0.0414 \neq \mathrm{P}(A'\cap B)$

$\therefore A'$ and B are not independent

$\Rightarrow$ A and B are not independent.

8) Let X g be the mass of a cabbage.

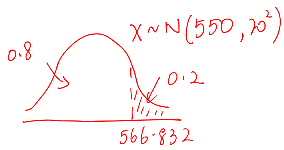
$X \sim \mathrm{N}(550, 20^2)$

(i) $\mathrm{P}(X > 575) = 0.10565 \approx 0.106$

(ii) $\mathrm{P}(X > m) \leq 0.2$

$m \geq 566.832$

Smallest mass is 566.8 g.



(iii) Let \$$C$ be the cost of a cabbage. $C = \dfrac{0.60}{100}X$

$\mathrm{E}(C) = \dfrac{0.60}{100}\times 550 = 3.3$

$\mathrm{Var}(C) = \left(\dfrac{0.60}{100}\right)^2 \times 20^2 = 0.0144$

$C \sim \mathrm{N}(3.3, 0.0144)$

$\mathrm{P}(C < 3.20) = 0.20233 \approx 0.202$

(iv) Let Y g be the mass of half a cabbage. $Y = \dfrac{1}{2}X$

$Y \sim \mathrm{N}\left(\dfrac{550}{2}, \dfrac{20^2}{4}\right)$

$Y_1 + Y_2 - X \sim \mathrm{N}\left(\dfrac{550}{2}+\dfrac{550}{2}-550, \dfrac{20^2}{4}+\dfrac{20^2}{4}+20^2\right)$

$Y_1 + Y_2 - X \sim \mathrm{N}(0, 600)$

$\mathrm{P}(0 < Y_1 + Y_2 - X < 50) = 0.47939 \approx 0.479$

9) Not every student will have equal chance of being selected as those who do not go to the canteen during lunch break will have no chance of being interviewed.

(i) The probability of a student supporting candidate A is constant for all the students. A student supporting candidate A is independent of whether other students will support candidate A.

(ii) $X \sim \mathrm{B}\left(30, \dfrac{4}{9}\right)$

x	$P(X=x)$
12	0.13058
13	0.14465
14	0.14051

$\therefore$ The most likely number of students who support candidate A is 13.

(iii) mean $= 30\times\frac{4}{9}=\frac{40}{3}$

standard deviation $=\sqrt{30\times\frac{4}{9}\times\left(1-\frac{4}{9}\right)}=2.7217$

$P\left(\frac{40}{3}-2.7217<X<\frac{40}{3}+2.7217\right)$

$= P(11\le X\le 16)=P(X\le 16)-P(X\le 10) = 0.72867\approx 0.729$

Let Y be number of students out of 30 who support candidate B.

$Y \sim B\left(30, \frac{p}{100}\right)$

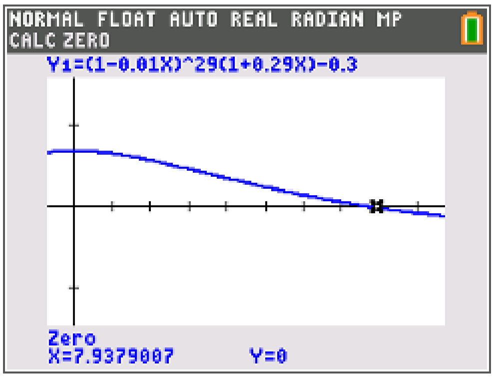
$P(Y\le 1)=P(Y=0)+P(Y=1)$

$=(1-0.01p)^{30}+30\times 0.01p\times(1-0.01p)^{29}$

$=(1-0.01p)^{29}(1-0.01p+0.3p)$

$=(1-0.01p)^{29}(1+0.29p)$

$=0.3$



$p\approx 7.94$ (correct to 2 d.p.)

10) Let μ kg be the population mean yield of an apple tree.

(i) $H_0: \mu = 98.5$

$H_1: \mu > 98.5$

Level of significance : 5%

Test Statistic: When H_0 is true, $Z = \dfrac{\overline{X} - 98.5}{\frac{S}{\sqrt{25}}} \sim N(0, 1)$ approximately

Computation : $\overline{x} = 99.7$, $\quad s^2 = \dfrac{n}{n-1}\left(\dfrac{\sum (x - \overline{x})^2}{n}\right) = \dfrac{178}{24}$

p – value = 0.01379154

Conclusion : Since p – value = 0.0138 < 0.05, H_0 is rejected at 5% level of significance. There is sufficient evidence to conclude that the farmer's claim should not be rejected.

(ii) It is not necessary as the sample size is 25 which is sufficiently large. Central Limit Theorem can be applied and $\overline{X}$ is approximately normal.

(iii) 5% level of significance means that there is a 0.05 probability that the test will conclude that there is an increase in the mean yield of the apple trees when the mean yield is actually 98.5 kg.

(iv) $H_0 : \mu = 102$

$H_1 : \mu \neq 102$

Level of significance : 5%

Rejection region: $z \le -1.95996$ or $z \ge 1.95996$

Conclusion : H_0 is not rejected.

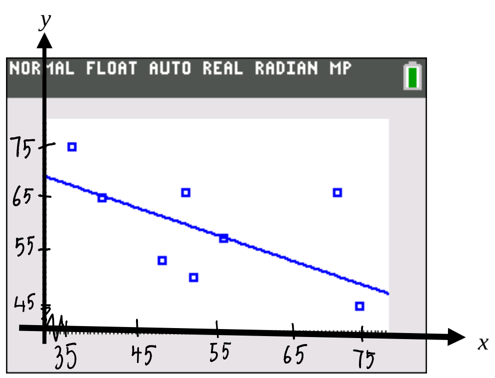
$$z = \frac{\overline{x} - 102}{\sqrt{\frac{7.49}{20}}} \quad \Rightarrow -1.95996 < \frac{\overline{x} - 102}{\sqrt{\frac{7.49}{20}}} < 1.95996$$

$\therefore \{\, \overline{x} \in \mathbb{R} : 101 < \overline{x} < 103 \,\}$

(v) We need to assume that the population variance remained unchanged.

(vi) Since H_0 is not rejected, p – value > 0.05 $\Rightarrow$ p – value > 0.01. Hence H_0 will also not be rejected at 1% level of significance.

11)(i)



(i) $r = -0.53382$

(ii)	The estimate is unreliable as from the scatter diagram, the points do not seem to lie close to straight line and r is not close to – 1.	
(iii)	Let the score be p. $\overline{x}=\frac{436}{8}$, $\overline{y}=\frac{407+p}{8}$ $\frac{407+p}{8}=88.722-0.57976\left(\frac{436}{8}\right)$ $\therefore$ p = 50 (nearest integer)	
(iv)	From GC, x = 121.07 – 1.1653y When y = 75, x = 34 (nearest integer) The estimate is not reliable as y = 75 is outside the given data range $45\le y\le 73$.	