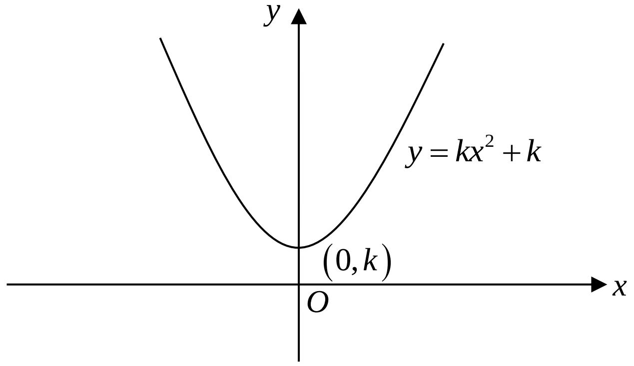
2017 H1 MATH (8865/01) JC 2 PRELIM – SOLUTIONS

Qn	Solution
1	**System of Linear Equations**
	Let x = price of black pomfret per kilogram (in dollars) y = price of sea bass per kilogram (in dollars) z = price of golden snapper per kilogram (in dollars) For Azel: $0.55x+0.45y+1.45z=38.77$ For Brenda: $0.60x+0.58y+1.6z=44.18$ For Cathy: $0.4x+0.75y+1.7z=45.81$ Using GC, $x=14.90$, $y=18.00$, $z=15.50$ Total price paid by Dillon = $\$(\left(0.7\times14.90\right)+\left(0.34\times18.00\right)+\left(1.42\times15.50\right))$ = \$ 38.56

Qn	Solution
2	**Techniques of Differentiation and Integration**
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(2x^2+1\right)\right)=\frac{4x}{2x^2+1}$ $\int_1^3 \frac{x}{2x^2+1}\,\mathrm{d}x=\frac{1}{4}\int_1^3\frac{4x}{2x^2+1}\,\mathrm{d}x$ $=\frac{1}{4}\left[\ln\left(2x^2+1\right)\right]_1^3$ $=\frac{1}{4}\left(\ln19-\ln3\right)$ $=\frac{1}{4}\ln\frac{19}{3}$ $\therefore a=\frac{1}{4},\ b=\frac{19}{3}$

Qn	Solution
3	**Exponential and Logarithm (Equation of Tangent)**
	$\ln y = (2-x)^2$ $y = \mathrm{e}^{(2-x)^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -2(2-x)\mathrm{e}^{(2-x)^2}$ When $x = 3$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 2\mathrm{e}$, $y = \mathrm{e}$. Equation of tangent is $y - \mathrm{e} = 2\mathrm{e}(x-3)$ $\Rightarrow y = 2\mathrm{e}x - 6\mathrm{e} + \mathrm{e}$ $\Rightarrow y = 2\mathrm{e}x - 5\mathrm{e}$

Qn	Solution
4	**Curve Sketching and Application of Integration**
(i)	
(ii)	$kx^2 + k = 4x + 3$ $kx^2 - 4x + k - 3 = 0$ Discriminant > 0 $(-4)^2 - 4(k)(k-3) > 0$ $16 - 4k^2 + 12k > 0$ $k^2 - 3k - 4 < 0$ $(k-4)(k+1) < 0$ $-1 < k < 4$ Since $k > 0$, $0 < k < 4$
(iii)	When $k = 3$ $3x^2 - 4x + 3 - 3 = 0$ $x(3x-4) = 0$ $x = 0$ or $x = \frac{4}{3}$
(iv)	$\int_0^{\frac{4}{3}} 4x + 3 - (3x^2 + 3)\ \mathrm{d}x$ $= \int_0^{\frac{4}{3}} 4x - 3x^2\ \mathrm{d}x$ $= \left[2x^2 - x^3\right]_0^{\frac{4}{3}}$ $= \left[2\left(\frac{4}{3}\right)^2 - \left(\frac{4}{3}\right)^3\right]$ $= \frac{32}{27}$

Qn	Solution
5	**Maxima/Minima and Connected Rate of Change**

(i)

$16000\pi = \pi r^2 h$

$h = \dfrac{16000}{r^2}$ --- (1)

$S = 2\pi r h + 2\pi r^2$ --- (2)

Substitute (1) into (2),

$S = 2\pi r\left(\dfrac{16000}{r^2}\right) + 2\pi r^2$

$S = \dfrac{32000\pi}{r} + 2\pi r^2$ (shown)

(ii)

For minimum S, $\dfrac{dS}{dr} = 0$.

$$\therefore \frac{dS}{dr} = -\frac{32000\pi}{r^2} + 4\pi r = 0$$

$$4\pi r = \frac{32000\pi}{r^2}$$

$$r^3 = 8000$$

$$r = \sqrt[3]{8000}$$

$$r = 20$$

For $r = 20$,

r	20^-	20	20^+
$\dfrac{dS}{dr}$	$-$	0	$+$
Slope of curve	\	—	/

$\therefore S$ is minimum when $r = 20$.

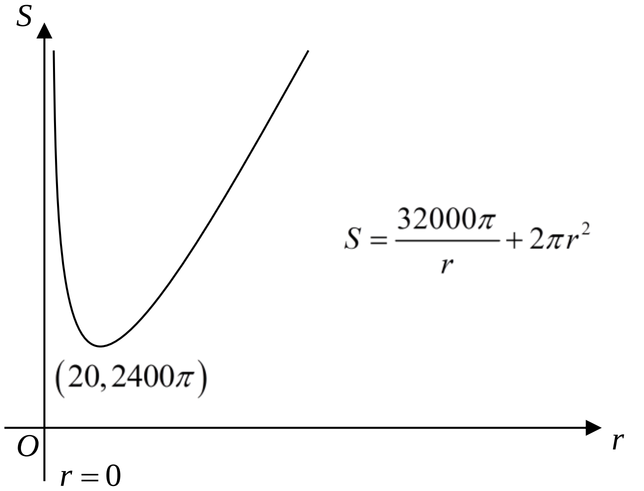
When $r = 20$,

$S = \dfrac{32000\pi}{20} + 2\pi(20)^2$

$= 1600\pi + 800\pi$

$= 2400\pi \text{ cm}^2$ or 7540 cm^2 (3s.f)

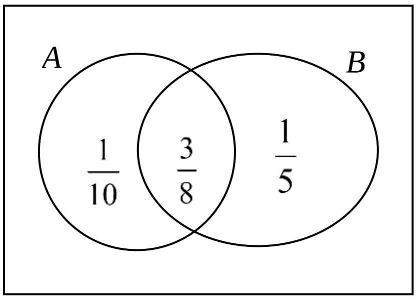
(iii)



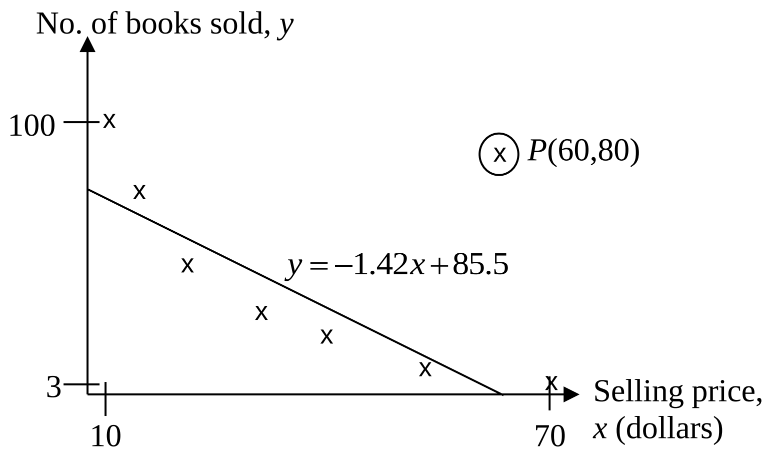
(iv)	Given: $\frac{dV}{dt} = 1000 \text{ cm}^3/\text{min}$ $V = \pi r^2 h$ $V = \pi(20)^2 h$ $V = 400\pi h$ $\frac{dV}{dh} = 400\pi$ Chain rule: $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$ $= (1000) \times \frac{1}{400\pi}$ $= 0.796 \text{ cm / min}$ (3 s.f)

Qn	Solution
6	**Permutations and Combinations**
(i)	Number of ways = $5! = 120$
(ii)	<u>Case 1 (first digit 3 or 7)</u> Number of ways $= 2 \times 2 \times 3! = 24$ <u>Case 2 (first digit 6)</u> Number of ways $= 3 \times 3! = 18$ Total number of ways $= 24 + 18 = 42$
	Number of ways = $5^4 \times 2 = 1250$

Qn	Solution
7	**Binomial Distribution**
(i)	Let X be the number of mugs, out of 50, that are defective. $X \sim \mathrm{B}(50, p)$ $\mathrm{P}(X = 0 \text{ or } 1) = 0.15$ $\mathrm{P}(X = 0) + \mathrm{P}(X = 1) = 0.15$ $\binom{50}{0} p^0 (1-p)^{50} + \binom{50}{1} p^1 (1-p)^{49} = 0.15$ $(1-p)^{50} + 50p(1-p)^{49} = 0.15$ Using GC, $p = 0.0659$ (3 s.f.)
(ii)	**Method A:** Let Y be the number of mugs, out of 10, that are defective. $Y \sim \mathrm{B}(10, 0.06)$ $\mathrm{P}(\text{accept the batch})$ $= \mathrm{P}(Y < 3)$ $= \mathrm{P}(Y \le 2)$ $= 0.981$ (3 s.f) **Method B:** Let W be the number of mugs, out of 5, that are defective. $W \sim \mathrm{B}(5, 0.06)$ $\mathrm{P}(\text{accept the batch})$ $= \mathrm{P}(W = 0) + \mathrm{P}(W = 1)\mathrm{P}(W < 2)$ $= \mathrm{P}(W = 0) + \mathrm{P}(W = 1)\mathrm{P}(W \le 1)$ $= 0.73390 + (0.23422)(0.96813)$ $= 0.961$ (3 s.f) Since the **probability** of accepting the batch is **higher for Method A**, the manufacturer should adopt **<u>Method A</u>** to carry out the quality control test as it will shorten the process of the quality control test. Since the **probability** of accepting the batch is **lower for Method B**, the manufacturer should adopt **<u>Method B</u>** to carry out the quality control test as it ensures a more stringent quality control test.

Qn	Solution
8	**Probability**
(a)	$\mathrm{P}\left(A \mid B'\right)=\frac{4}{17}$ $\frac{\mathrm{P}\left(A\cap B'\right)}{\mathrm{P}\left(B'\right)}=\frac{4}{17}$ $\mathrm{P}\left(A\cap B'\right)=\frac{4}{17}\left(1-\frac{23}{40}\right)$ $=\frac{1}{10}$  $\mathrm{P}\left(A'\cap B'\right)=1-\mathrm{P}\left(A\cup B\right)$ $=1-\left(\frac{1}{10}+\frac{23}{40}\right)$ $=\frac{13}{40}$
	$\mathrm{P}\left(A\cap B\right)=\frac{3}{8}$ $\mathrm{P}\left(A\right)=\frac{1}{10}+\frac{3}{8}=\frac{19}{40}$ $\mathrm{P}\left(A\right)\mathrm{P}\left(B\right)=\left(\frac{19}{40}\right)\left(\frac{23}{40}\right)=\frac{437}{1600}$ Since $\mathrm{P}\left(A\cap B\right)\neq \mathrm{P}\left(A\right)\mathrm{P}\left(B\right)$, A and B are not independent. (Alternative method) $\mathrm{P}\left(A\right)=\frac{1}{10}+\frac{3}{8}=\frac{19}{40}$ $\mathrm{P}\left(A \mid B'\right)=\frac{4}{17}$ Since $\mathrm{P}\left(A \mid B'\right)\neq \mathrm{P}\left(A\right)$, A and B' are not independent events, therefore A and B are not independent events.

(b) (i)	Let X be the event that a randomly chosen durian is MSW and it is infested with maggots. Let Y be the event that a randomly chosen durian is MSW and it is not infested with maggots. P(both are MSW durians and at least one durian is infested with maggots) $=\mathrm{P}(XY)+\mathrm{P}(YX)+\mathrm{P}(XX)$ $=\left(\frac{4}{35}\right)\left(\frac{11}{34}\right)\times 2+\left(\frac{4}{35}\right)\left(\frac{3}{34}\right)$ $=\frac{10}{119}$
(ii)	$\mathrm{P}(\text{at least one durian is infested with maggots})$ $=1-\mathrm{P}(\text{both durians are not infested with maggots})$ $=1-\left(\frac{28}{35}\right)\left(\frac{27}{34}\right)$ $=\frac{31}{85}$ Required probability $=\frac{\frac{10}{119}}{\frac{31}{85}}$ $=\frac{50}{217}$

Qn	Solution
9	**Correlation and Regression**
(i), (ii), (iii)	
(iii)	$r = -0.869$ $y = -1.4209x + 85.513$ $\;\;= -1.42x + 85.5$ (3 s.f.)
(iv)	When $x = 75$, $y = -1.4209(75) + 85.513$ $\;\;= -21.1$(3 s.f.) Since $x = 75$ lies outside the data range of *x*, the estimated value of *y* is not reliable since the linear relationship between *x* and *y* may not longer holds. Or Since the value of *y* (the number of books sold) cannot be negative, the estimated value of *y* is not reliable.
(v)	For a linear model, the number of books sold might fall below zero, hence a linear model might not be appropriate. From the scatter diagram, as the selling price of each book (*x*) increases, the number of books sold (*y*) decreases at a decreasing rate. Thus, a linear model might not be appropriate.

Qn	Solution
10	**Hypothesis Testing**
(a)	Let X be the decrease in cholesterol level for a randomly chosen volunteer (in mg/dL) Let μ denote the population mean decrease in cholesterol level in volunteers (in mg/dL) Since n = 80 is large, by Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$ approximately. Using GC, $\bar{x} = 19.7375$ (exact), $s^2 = 1.7628^2$ (5s.f) $\mathrm{H_0}$: $\mu = 20$ $\mathrm{H_1}$: $\mu < 20$ Test statistic: $Z = \frac{\bar{X} - \mu}{S/\sqrt{n}}$ Level of significance: 5% Reject $\mathrm{H_0}$ if $p-\text{value} < 0.05$ Under $\mathrm{H_0}$, using GC, $p-\text{value} = 0.0914$ (3 s.f) Conclusion: Since $p-\text{value} = 0.0914 > 0.05$, we **do not reject $\mathrm{H_0}$** and conclude that there is **insufficient evidence**, at the 5% significance level, that the mean decrease in cholesterol level is less than 20 mg/dL. Thus, the chocolate company's claim is valid at 5% level of significance.
(b)(i)	Unbiased estimate of μ is $\bar{x} = \frac{185}{60} = 3.08$ (3 s.f) Unbiased estimate of σ^2 is $s^2 = \frac{1}{59}\left[626 - \frac{185^2}{60}\right] = \frac{667}{708} = 0.942$ (3 s.f)
(ii)	Since n = 60 is large, by Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$ approximately $\mathrm{H_0}$: $\mu = \mu_0$ $\mathrm{H_1}$: $\mu \neq \mu_0$ Test statistic: $Z = \frac{\bar{X} - \mu}{S/\sqrt{n}}$ Level of significance: 4% Reject $\mathrm{H_0}$ if $z-\text{value} < -2.0537$ or $z-\text{value} > 2.0537$

	Under H_0, $z-\text{value} = \dfrac{3.0833-\mu_0}{\sqrt{0.94209/60}}$ Since the principal is confident that he did not indicate wrongly the mean time that the children took for afternoon naps at 4% level of significance, H_0 is not rejected. $-2.0537 < z-\text{value} < 2.0537$ $-2.0537 < \dfrac{3.0833-\mu_0}{\sqrt{0.94209/60}} < 2.0537$. $2.83 < \mu_0 < 3.34 \quad (3\text{s.f})$

Qn	Solution
11	**Normal and Sampling Distribution**
(i)	Let X be the weight of a randomly chosen Black Tilapia fish, in grams. $X \sim \mathrm{N}\left(\mu, 32^2\right)$ Given $\mathrm{P}\left(X > 541\right) = 0.10$ Standardizing, $Z \sim \mathrm{N}(0,1)$ $\mathrm{P}\left(X > 541\right) = 0.10$ $1 - \mathrm{P}\left(X \le 541\right) = 0.10$ $\mathrm{P}\left(X \le 541\right) = 0.9$ $\frac{541-\mu}{32} = 1.28155$ $\therefore \mu = 500$ (nearest gram)
(ii)	$X \sim \mathrm{N}\left(500, 32^2\right)$ $\mathrm{P}(440 < X < 550) \times \mathrm{P}(440 < X < 550) \times \mathrm{P}(X > 550) \times \frac{3!}{2!}$ $= \left(0.91052\right)^2 \times \left(0.059085\right) \times 3$ $= 0.147$ (3 s.f)
(iii)	$X \sim \mathrm{N}\left(500, 32^2\right)$ $\frac{a}{1000}\left(X_1 + X_2\right) \sim \mathrm{N}\left(\frac{a}{1000}(2)\left(500\right), \left(\frac{a}{1000}\right)^2 (2)(32^2)\right)$ $\frac{a}{1000}\left(X_1 + X_2\right) \sim \mathrm{N}\left(\mathrm{a},\ 0.002048a^2\right)$ $\mathrm{P}\left(\frac{a}{1000}\left(X_1 + X_2\right) < 6.9\right) < 0.84241$ $\mathrm{P}\left(Z < \frac{6.9-a}{\sqrt{0.002048a^2}}\right) < 0.84241$ $\frac{6.9-a}{\sqrt{0.002048a^2}} < 1.00441$ $6.9 - a < 0.045455a$ $1.045455a > 6.9$ $a > 6.60$ (3 s.f) Alternatively: $\mathrm{P}\left(\frac{a}{1000}\left(X_1 + X_2\right) < 6.9\right) < 0.84241$ Using GC, $\therefore a > 6.60\,(3\,\mathrm{s.f})$
(iv)	Let Y be the weight of a randomly chosen Grey Mullet fish, in grams. $Y \sim \mathrm{N}\left(800, 50^2\right)$ Let $T = \frac{X_1 + X_2 + Y_1 + Y_2 + Y_3}{5} \sim \mathrm{N}\left(\frac{1}{5}\left(2(500) + 3(800)\right),\ \frac{1}{5^2}\left(2(32^2) + 3(50^2)\right)\right)$

	$T \sim \mathrm{N}\left(680,\ 381.92\right)$ $\mathrm{P}(T > 690) = 0.304$ (3 s.f) **<u>Alternatively:</u>** $T = X_1 + X_2 + Y_1 + Y_2 + Y_3 \sim \mathrm{N}\left(2(500)+3(800),\ 2(32^2)+3(50^2)\right)$ $T \sim \mathrm{N}\left(3400,\ 9548\right)$ $\mathrm{P}(T > 690 \times 5) = \mathrm{P}(T > 3450) = 0.304$ (3 s.f)