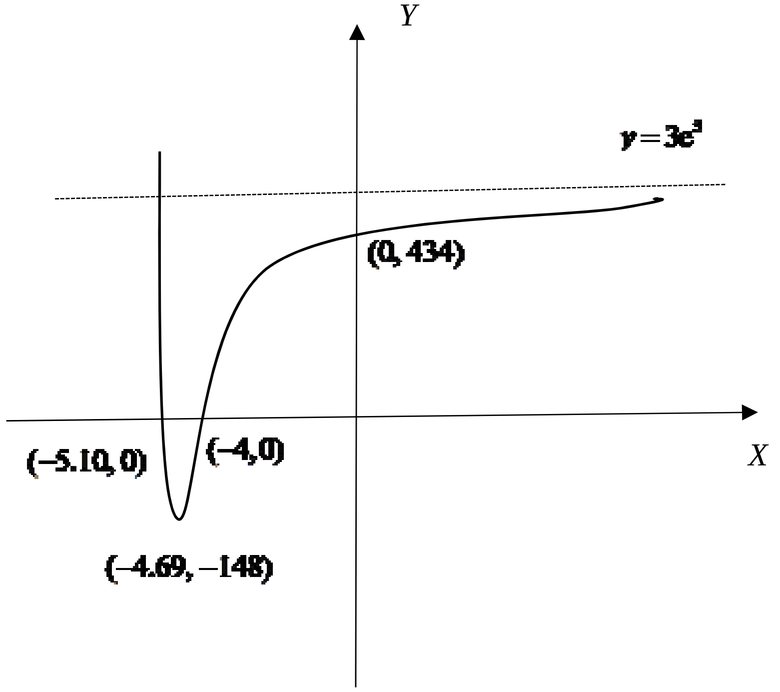
no	Solution
1.	$ax^2+4ax+7a>2x+2 \quad \Rightarrow \quad ax^2+(4a-2)x+\left(7a-2\right)>0$ For $Ax^2+Bx+C>0$ for all real values of x, we must have: $A>0$ and $B^2-4AC<0$ That is, $a>0$ and $(4a-2)^2-4a(7a-2)<0$ $(4a-2)^2-4a(7a-2)<0 \quad \Rightarrow \quad 16a^2-16a+4-28a^2+8a<0$ $\Rightarrow \quad -12a^2-8a+4<0 \quad \Rightarrow \quad 3a^2+2a+1>0$ $\Rightarrow \quad 3a^2+2a-1>0$ $\Rightarrow \quad -12a^2-8a+4<0 \quad \Rightarrow \quad 3a^2+2a-1>0$ $\Rightarrow \quad (3a-1)(a+1)>0$ Thus, $a>\frac{1}{3}$ or $a<-1$ Since $a>0$, we have $a>\frac{1}{3}$
2	(a) Total area = 2 + 4 + 7 = 13 (b) $\int_0^{10} f(x)-g(x)\,\mathrm{d}x$ $=\int_0^a f(x)-g(x)\,\mathrm{d}x+\int_a^b f(x)-g(x)\,\mathrm{d}x+\int_b^{10} f(x)-g(x)\,\mathrm{d}x$ $=\int_0^a f(x)-g(x)\,\mathrm{d}x-\int_a^b g(x)-f(x)\,\mathrm{d}x+\int_b^{10} f(x)-g(x)\,\mathrm{d}x$ $=2-4+7$ $=5$
3	$u_n=an^2+bn+c$ When n = 1, $3=a+b+c$...........(1) n = 2, $1=4a+2b+c$...........(2) n =3, $-5=9a+3b+c$...........(3) Using GC, a = -2, b = 4, c = 1. $u_n=-2n^2+4n+1$

	$-2n^2+4n+1>2$ $2n^2-4n+1<0$ $2(n^2-2n)+1<0$ $2(n-1)^2-2+1<0$ $2(n-1)^2-1<0$ $1-\frac{1}{\sqrt{2}}<n<1+\frac{1}{\sqrt{2}}$ $\text{Since } n\in\mathbf{Z}^+, n=1$
4	(i) $f'(x)=4e^{1-x}-2e^{-3-2x}$ At stationary point, $\frac{dy}{dx}=0$ $4e^{1-x+1}-2e^{-3-2x}=0$ $4e^{1-x}=2e^{-3-2x}$ $\frac{e^{-3-2x}}{e^{1-x}}=\frac{4}{2}$ $e^{-3-2x-1+x}=2$ $e^{-x-4}=2$ $-x-4=\ln 2$ $\therefore x=-\ln 2-4$ (ii) $y=\int 4e^{1-x}-2e^{-3-2x}\ dx$ $=4\int e^{1-x}\ dx-2\int e^{-3-2x}\ dx$ $=4\left(\frac{e^{1-x}}{-1}\right)-2\left(\frac{e^{-3-2x}}{-2}\right)+C$ $y=-4e^{1-x}+e^{-3-2x}+C$ Given that the curve cut the x-axis at $x=-4$, so $0=-4e^{1+4}+e^{-3+8}+C$ $0=-4e^5+e^5+C$ $C=3e^5$ Equation of curve: $y=-4e^{1-x}+e^{-3-2x}+3e^5$

(iii)



(iv)

$$f'(x) = 4e^{-x+1} - 2e^{-2x-3}$$

Gradient of the tangent at the point (-4, 0)

$$= f'(-4) = 4e^{4+1} - 2e^{8-3} = 2e^5$$

Equation of the tangent at the point (-4, 0):

$$y - 0 = 2e^5(x+4)$$
$$y = 2e^5 x + 8e^5$$

(v)

Area required

$$= \int_{-4}^{0} \left(2e^5 x + 8e^5\right) - \left(-4e^{1-x} + e^{-3-2x} + 3e^5\right) \, dx$$
$$= \int_{-4}^{0} 2e^5 x + 5e^5 + 4e^{1-x} - e^{-3-2x} \, dx$$
$$= \left[\frac{2e^5}{2} x^2 + 5e^5 x + \frac{4}{-1} e^{1-x} - \frac{1}{-2} e^{-3-2x} \right]_{-4}^{0}$$

$$= \left[e^5 x^2 + 5e^5 x - 4e^{1-x} + \frac{1}{2} e^{-3-2x} \right]_{-4}^{0}$$

$$= \left(0 + 0 - 4e^1 + \frac{1}{2} e^{-3} \right) - \left(16e^5 - 20e^5 - 4e^5 + \frac{1}{2} e^{-3+8} \right)$$

$$= \frac{1}{2} e^{-3} + \frac{15}{2} e^5 - 4e$$

5

(i) $S(24) = 50e^{-0.04 \times 24} = 19.1$ (thousand papers)

(ii) The total circulation in the 24 hrs

$$\int_0^{24} 50e^{-0.04t}\, dt = \left[\frac{50}{-0.04} e^{-0.041t} \right]_0^{24} = \left[-1250e^{-0.04t} \right]_0^{24}$$

$$= \left(-1250e^{-0.96} \right) - (-1250) = 771.38$$

(iii) Assume the number of viewers at t th hour follows a continuous random variable.

(iv) To increase the circulation from 32 thousand papers to 50 thousand papers, the amoubt spent on advertising will be 50-32 = 18 cents.

Hence total revenue = 50 x (0.5) = 25 (thousand dollars)

(iv) Let R be the hourly profit,

$$R = 16000 + 180x - 10x^2$$

$$\frac{dR}{dx} = 180 - 20x$$

For max R, $\frac{dR}{dx} = 0$

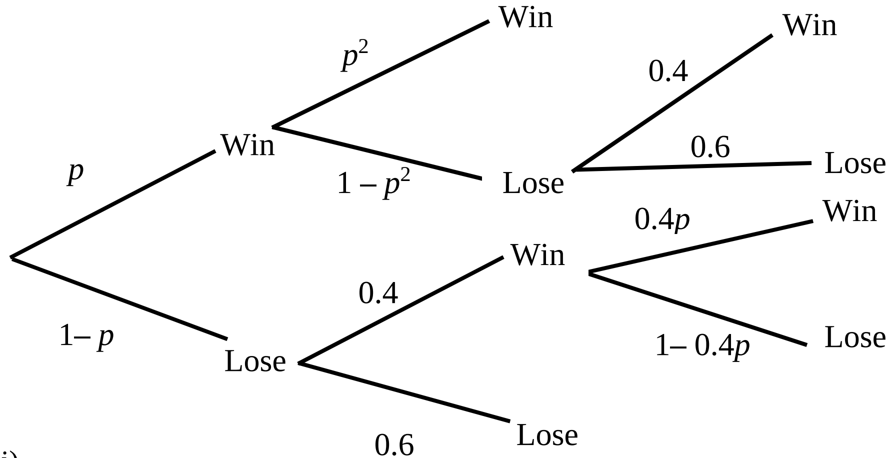
$$x = \frac{180}{20} = 9$$

x	9^-	9	9^+
$\frac{dR}{dx}$	+ve	0	-ve
slope	/	-	\

When x = 9, R is maximum

Maximum $R = 16000 + 180(9) - 10(9)^2 = 16810$

= 1.68 (thousand dollars)

6	(i) No of way = 10! = 3628800 P(no 2 basketball players are next to one another) (ii) $= \dfrac{7!\binom{8}{3}3!}{3628800} = \dfrac{7}{15} = 0.467$ P(tennis players are together|basketball players are separated) (iii) $= \dfrac{\left(\dfrac{4!4!\binom{5}{3}3!}{3628800}\right)}{\dfrac{7}{15}} = \dfrac{1}{49} = 0.0204$
7	2 assumptions 1. Whether the student wear spectacles is independent of one another. 2. The probability of selecting a student wearing spectacle is constant. (ii)S~B(15,*p*) $P(S=0 \text{ or } S=1)=0.3$ $P(S=0)+P(S=1)=0.3$ $(1-p)^{15}+15p(1-p)^{14}=0.3$ $(1-p)^{15}+15p(1-p)^{14}-0.3=0$ Using GC, $p=0.155$ $P(2 \le S < 8)$=0.699 (iii) Let X be no. of the students wearing spectacles out of 30 students. $X \sim B(30, 0.7)$ Since n = 50 is large, by Central Limit theorem, $\overline{X} \sim N(21, \frac{6.3}{50})$ approximately $P(\overline{X} \le 20) = 0.00242$
8(i)	 (ii)

$\mathrm{P}(\mathrm{WW})+\mathrm{P}(\mathrm{WLW})+\mathrm{P}(\mathrm{LWW})=0.6528$

$p^3+p\left(1-p^2\right)(0.4)+(1-p)(0.4)(0.4p)=0.6528$

$0.6p^3-0.16p^2+0.56p-0.6528=0$

Using GC, $p=0.8$

(iii) P(wins second set | wins the game)

$=\dfrac{0.64^3+0.36(0.4)(0.4\times0.64)}{0.64^3+0.64\left(1-0.64^2\right)(0.4)+0.36(0.4)(0.4\times0.64)} \quad =\dfrac{0.299008}{0.4501504}$

= 0.664

9i

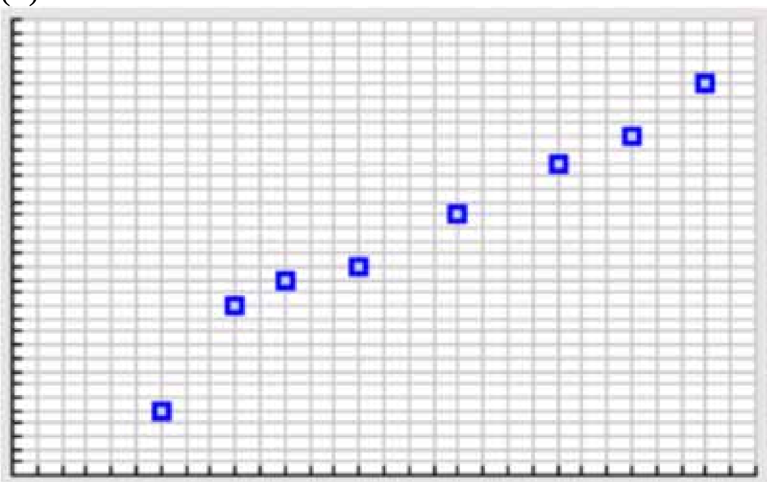
$\bar{x}=16.625$

Since the point $(\bar{x},\bar{y})$ will pass through the regression line y on x, $\bar{y}=18.619$.

$\bar{y}=\dfrac{5+13+15+a+20+24+26+30}{8}$

$a=16$

(ii)

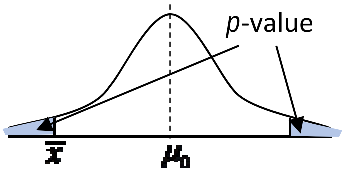


(iii) r = 0.9799 (4 dp)

The r value is close to 1, shows that most of the points lie close to the best fit line through the curve. This shows that there is strong positive linear correlation between the growth rate of the organisms and the temperature.

	(iv) When $y = 18$, sub into $y = 0.993x + 2.11$ $18 = 0.993x + 2.11$ $x = 16.002 = 16.0\,^0\text{C}$. The estimate is reliable because the growth rate of the organism is within the data range and r is close to 1. (v) The estimate is unreliable as $x = 30^o c$ is out of the data range of x.
10	Unbiased estimate of population mean, $\bar{x} = \frac{-40}{50} + 80 = 79.2$ Unbiased estimate of population variance, $s^2 = \frac{1}{49}\left[450 - \frac{(-40)^2}{50}\right] = \frac{418}{49} = 8.53061 \approx 8.53$ (i) Let X denote the H1 Mathematics score of a student and μ the population mean H1 Mathematics score. To test $H_o : \mu = 80$ $H_1 : \mu < 80$ Level of significance: 5% Test statistic: Under H_o, $\bar{X} \sim N(80, \frac{8.53061}{50})$ approximately, by Central Limit Theorem since n is large Reject H_o if p-value ≤ 0.05 $\bar{x} = 79.2$, $n = 50$, $s = \sqrt{\frac{418}{49}}$ From GC, p-value = 0.026385 Since p-value $<$ 0.05, there is sufficient evidence to reject H_o and conclude at 5% significant level that there is sufficient evidence that the owner has overstated their mean H1 Mathematics score. (ii) Not the same conclusion Note $H_1 : \mu \ne 80$ (teacher's claim) and p-value = $0.026385 \times 2 = 0.05277 > 0.05$ Do not reject H_0, i.e insufficient evidence to support teacher's claim that the mean H1 Mathematics score is not 80 To test $H_o : \mu = 80$ $H_1 : \mu \ne 80$ at 5% level of significance Under H_o, $\bar{X} \sim N(80, \frac{5^2}{n})$ approximately, by Central Limit Theorem since n is large Reject H_o if p-value ≤ 0.05 owner's claim is valid => do not reject H_o $\therefore p$-value > 0.05

$\mathrm{P}\left(\bar{X} \le \bar{x}\right) > \dfrac{0.05}{2}$, i.e. $\mathrm{P}\left(\bar{X} \le \bar{x}\right) > 0.025$ or



$$\mathrm{P}\left(Z \le \frac{78.8-80}{5/\sqrt{n}} \right) > 0.025$$

$$\frac{78.8-80}{5/\sqrt{n}} > -1.95996$$

$$-0.24\sqrt{n} > -1.95996$$

$$\sqrt{n} < 8.1665$$

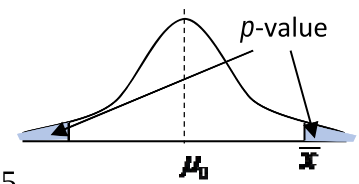
$$n < 66.692$$

$$n \le 66$$

$\mathrm{P}\left(\bar{X} \ge \bar{x}\right) > \dfrac{0.05}{2}$, i.e. $\mathrm{P}\left(\bar{X} \ge \bar{x}\right) > 0.025$

$$1-\mathrm{P}\left(\bar{X} < \bar{x}\right) > 0.025$$

$$\mathrm{P}\left(\bar{X} < \bar{x}\right) < 0.975$$



$$\mathrm{P}\left(Z < \frac{78.8-80}{5/\sqrt{n}} \right) < 0.975$$

$$\frac{78.8-80}{5/\sqrt{n}} < 1.95996$$

$$-0.24\sqrt{n} < 1.95996$$

$$\sqrt{n} > -8.1665$$

$$\sqrt{n} > 0$$

The largest n is 66

11i	$\mathrm{P}(H < 2k) = 0.95$

	$P(Z < \frac{2k-1.8}{b}) = 0.95$ $\frac{2k-1.8}{b} = 1.6448$.........(1) $P(H < k) = 0.25$ $P(Z < \frac{k-1.8}{b}) = 0.25$ $\frac{k-1.8}{b} = -0.67449$.........(2) Solving equation (1) and (2), $k = 1.39446$ $b = 0.601$ (ii) Let H be the mass of a randomly chosen honeydew and D be the mass of a randomly chosen durian. $H \sim N(1.8, 0.4^2)$ $D \sim N(1.5, 0.3^2)$ P(mass of the honeydew is more than 1.7 kg and the mass of the durian is less than 1.7 kg) = $P(H > 1.7) \times P(D < 1.7) = 0.448$ (iii) Probability = $P(H < 1.6) \times P(1.6 \le H \le 1.7) \times P(H > 1.7) \times 3! = 0.103$ (iv) $H_1 + H_2 \sim N(3.6, 0.32)$ $3D \sim N(4.5, 0.81)$ $3D - (H_1 + H_2) \sim N(0.9, 1.13)$ $P(-0.8 \le 3D - (H_1 + H_2) \le 0.8) = 0.408$ (v) $C_H = 3(H_1 + H_2) \sim N(10.8, 2.88)$ $C_H + C_D \sim N(25.8, 11.88)$ $P(C_H + C_D > 28.5) = 0.217$