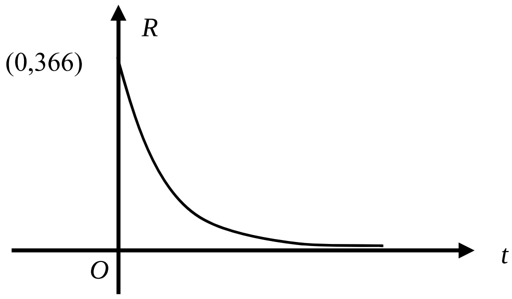
YISHUN JUNIOR COLLEG5E
Mathematics Department

PRELIM SOLUTION

Subject : JC2 H1 MATHEMATICS 8864 **Date** :

Qn	Solution
1(i)	**(a)** $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(5\ln\left(1-3x^2\right)\right)=5\left(\dfrac{1}{1-3x^2}\right)\left(-6x\right)$ $=-\dfrac{30x}{1-3x^2}$ **(b)** $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{\left(2x+3\right)^2}\right)=\dfrac{\mathrm{d}}{\mathrm{d}x}\left(2x+3\right)^{-2}$ $=-2(2x+3)^{-3}(2)$ $=-4(2x+3)^{-3}$
(ii)	$\displaystyle\int_1^3 x^3\left(\frac{1}{x}-1\right)^2\,\mathrm{d}x=\int_1^3\left(x-2x^2+x^3\right)\mathrm{d}x$ $=\left[\dfrac{x^2}{2}-\dfrac{2x^3}{3}+\dfrac{x^4}{4}\right]_1^3$ $=\left(\dfrac{3^2}{2}-\dfrac{2(3)^3}{3}+\dfrac{(3)^4}{4}\right)-\left(\dfrac{1^2}{2}-\dfrac{2(1)^3}{3}+\dfrac{(1)^4}{4}\right)$ $=\dfrac{27}{4}-\dfrac{1}{12}$ $=\dfrac{20}{3}$
2	$\left(-k-2\right)^2-4(k)(4k)<0$ and $k<0$ $k^2+4k+4-16k^2<0$ $-15k^2+4k+4<0$ $(5k+2)(3k-2)>0$ $k<-\dfrac{2}{5}$ or $k>\dfrac{2}{3}$ Since $k<0$, $\therefore\ k<-\dfrac{2}{5}$

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>3
(i)</td><td>
</td></tr>
<tr><td>(ii)</td><td>From GC, when $t = 20$,
$\frac{\mathrm{d}R}{\mathrm{d}t} = -4.96$
The rate of decrease is 4.96 micrograms/litre per min</td></tr>
<tr><td>(iii)</td><td>$40 = 366e^{-0.0998t}$
$e^{-0.0998t} = \frac{40}{366}$
$-0.0998t = \ln\left(\frac{40}{366}\right)$
$t = 22.18$
$t \approx 22\text{mins}$

Alternative solution:
Draw graph of y = 40 and find intersection points.</td></tr>
<tr><td></td><td>$P = -0.03x^3 + 0.1x^2 + x - 0.1$
$\frac{\mathrm{d}P}{\mathrm{d}x} = -0.09x^2 + 0.2x + 1$
For maximum P, $\frac{\mathrm{d}P}{\mathrm{d}x}=0$
$-0.09x^2 + 0.2x + 1=0$
$x = 4.624753 \ \ (x > 0)$

<table>
<tr><td>x</td><td>4.624753^-</td><td>4.624753</td><td>4.624753^+</td></tr>
<tr><td>$\frac{\mathrm{d}P}{\mathrm{d}x}$</td><td>+ve</td><td>0</td><td>–ve</td></tr>
<tr><td>slope</td><td>/</td><td>—</td><td>\</td></tr>
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Thus, P is maximum when the number of bottles is 46248.</td></tr>
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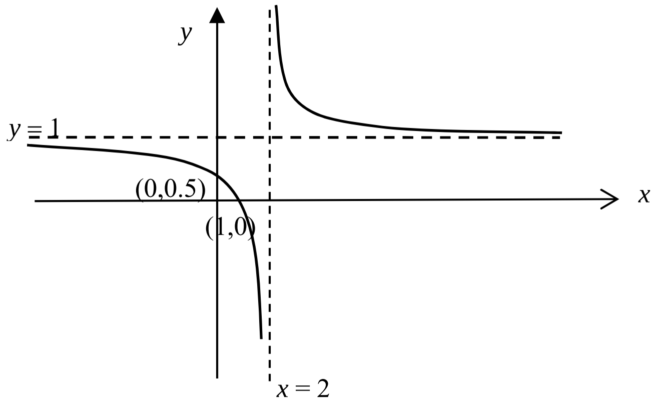
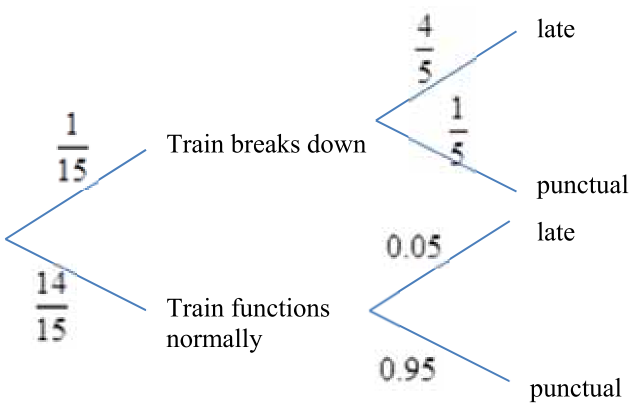
Qn	Solution
4	 $\dfrac{1}{x-2}+1=x+3$ $1+(x-2)=x^2+x-6$ $x^2=5$ $x=\pm\sqrt{5}$ Area of the region = $\int_{-\sqrt{5}}^{0}\left(x+3-\left(\dfrac{1}{x-2}+1\right)\right)\mathrm{d}x$ $=\left[\dfrac{x^2}{2}+2x-\ln\lvert x-2\rvert\right]_{-\sqrt{5}}^{0}$ $=-\ln 2-\left(\dfrac{5}{2}-2\sqrt{5}-\ln\lvert -\sqrt{5}-2\rvert\right)$ $=-\ln 2-\left(\dfrac{5}{2}-2\sqrt{5}-\ln\left(2+\sqrt{5}\right)\right)$ $=-\ln 2-\dfrac{5}{2}+2\sqrt{5}+\ln\left(2+\sqrt{5}\right)$
5	$y=x^3-2e^{-x}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=3x^2+2e^{-x}$ When $x=1$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=3(1)^2+2e^{-1}$ and $y=1-2e^{-1}$ Equation of tangent: $y-\left(1-\dfrac{2}{e}\right)=\left(3+\dfrac{2}{e}\right)(x-1)$ $y=\left(3+\dfrac{2}{e}\right)x-2-\dfrac{4}{e}$

Qn	Solution
6(a)(i)	$P(A\cup B)=1-P(A'\cap B')$ $=\frac{9}{17}$ $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $\frac{9}{17}=\frac{1}{2}+P(B)-\frac{1}{34}$ $\Rightarrow P(B)=\frac{1}{17}$ (Shown) Smallest value of n is 34.
(ii)	$P(A\mid B)=\dfrac{\frac{1}{34}}{\frac{1}{17}}=\frac{1}{2}$
(b)(i)	
(ii)	P (train functions normally | late) $=\dfrac{\frac{14}{15}(0.05)}{0.1}$ ≈ 0.467
7(i)	Let X be the random variable 'number of smartphone users with anti-virus software A installed on their smartphones out of 20 users' $X\sim B(20, 0.37)$ $P(X\geq 8)=1-P(X\leq 7)$ ≈ 0.47346 $=0.473$ (3 sig fig)
(ii)	Let W be the 'number of samples with at least eight users with anti-virus software A installed on their smartphone out of 50 samples' $W\sim B(50, 0.47346)$ Since n is large, $np=23.673>5$, $n(1-p)=26.327>5$ $W\sim N(23.673, 12.465)$ approx $P(W<30)\to P(W<29.5)$ using c.c. ≈ 0.95057 $=0.951$ (3 sig fig)

Qn	Solution
(iii)	Let Y be the 'number of smartphone users who did not have any anti-virus software installed, out of n' $Y \sim \mathrm{B}(n, 0.07)$ $\mathrm{P}(Y \le 1) < 0.5$ $\mathrm{P}(Y = 0) + \mathrm{P}(Y = 1) < 0.5$ ${}^{n}C_0 (0.07)^0 (0.93)^n + {}^{n}C_1 (0.07)(0.93)^{n-1} < 0.5$ $(0.93)^n + n(0.07)(0.93)^{n-1} < 0.5$ $(0.93)^{n-1}(0.93 + 0.07n) < 0.5$ (shown) Using GC, When $n = 23$, $(0.93)^{n-1}(0.93+0.07n) = 0.5146 > 0.5$ When $n = 24$, $(0.93)^{n-1}(0.93+0.07n) = 0.4918 < 0.5$ Therefore, least $n = 24$
8	Number the customers from 1 to 3000. $k = \frac{3000}{60} = 50$ Randomly choose the first customer from the first 50 customers. Thereafter, select every 50th customer until 60 customers are chosen. Using a stratified sample will take into consideration the different opinions from all the different strata (for example, age group), hence resulting in a sample which is more representative of the population.
9(i)	$X \sim N(50, 10^2)$ Required Prob = $\left[\mathrm{P}(X > 60)\right]^2 \approx (0.158655)^2$ $= 0.02517$ ≈ 0.0252 (3 s.f)
(ii)	Let μ be the population mean time taken (min) the company has to achieve $X \sim N(\mu, 10^2)$ $\mathrm{P}(X < 60) \ge 0.95$ $\mathrm{P}(Z < \frac{60-\mu}{10}) \ge 0.95$ $\frac{60-\mu}{10} \ge 1.64485$ $\mu \le 43.552$ Maximum $\mu = 43.5$
(iii)	Let W be the amount of electricity (kWh) used in a month by a household $W \sim N(522, 26^2)$ Total charge per month, $B = 0.21W \sim N(109.62, 29.8116)$ $P(100 < B < 120) = 0.932$ (3 s.f)
(iv)	$T = B_1 + B_2 \sim N(219.24, 59.6232)$ $P(T \ge d) > 0.9$ $1 - P(T < d) > 0.9$ $P(T < d) < 0.1$ $d < 209.344$

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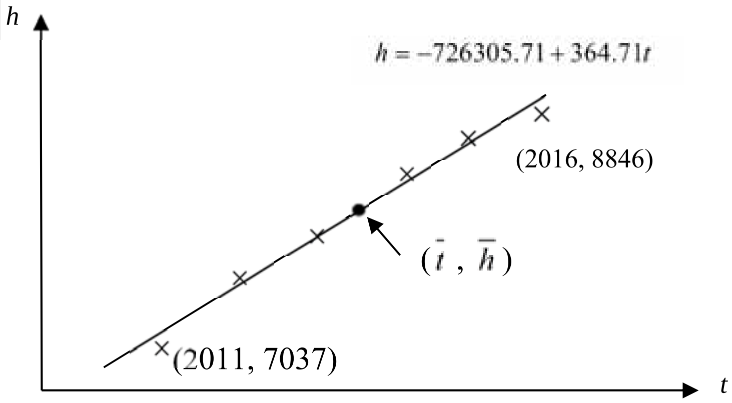
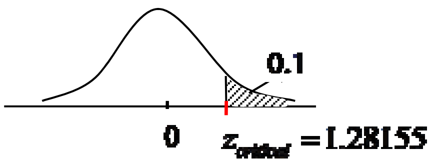
	Largest integral value of d is 209. Assume that the electricity used in each month is independent for a particular household.
(v)	Since $\mu - 3\sigma = 47 - 3(25) = -28 < 0$, Time taken to install a gas meter is impossible to be negative, Y is not well modelled by a normal distribution.
(vi)	Since sample size = 55 is large, $\bar{Y} = \frac{Y_1 + Y_2 + ... + Y_{55}}{55} \sim N\left(47, \frac{25^2}{55}\right)$ approx by CLT $P(\bar{Y} > 45) = 0.724$ (3 s.f.)
10(i)	
(ii)	$R \approx 0.992$ (3 s.f) There is a strong positive linear correlation between the median monthly household income from work and the year. As the year increases, the median monthly household income from work increases.
(iii)	$\bar{t} = 2013.5, \quad \bar{h} = 8046.5$
(iv)	$h = -726305.71 + 364.71t$ (2 d.p.)
(v)	When $h = 9700$, $9700 = -726305.71 + 364.71t$ $t = 2018.057$ Year: 2018 The estimate is not reliable since the estimate is obtained via extrapolation.
11(i)	Unbiased estimate of the population mean, $\bar{x} = \frac{6386}{150} = 42.573 \approx 42.6$ (3s.f.) Unbiased estimate of the population variance, $s^2 = \frac{1}{149}\left[277270 - \frac{6386^2}{150}\right] = 36.219 \approx 36.2$ (3 s.f.)
(ii)	$H_0 : \mu = 41$ $H_1 : \mu \neq 41$ Test at 5% significance level Under H_0, the test statistic $Z = \frac{\bar{X} - \mu}{S / \sqrt{n}} \sim N(0,1)$ approx. by CLT, where $\mu = 41, s = \sqrt{36.219}, \bar{x} = 42.573, \ n = 150$.

Qn	Solution
	By GC, p-value $= 0.00137\,(3\text{ s.f.})$. Since p-value < 0.05, we reject $\mathrm{H_0}$ and conclude that at 5% level, there is sufficient evidence that the claim is not valid.
(iii)	Since n is large, by Central Limit Theorem, the sample mean time spent by 150 customers is approximately normal. Hence it is not necessary to assume a normal distribution for the population for the test to be valid.
(iv)	There is a probability of 0.05 of concluding that the mean time spent by customers is not equal to 41 minutes when it is in fact 41 minutes.
(v)	$H_0 : \mu = 41$ $H_1 : \mu > 41$ (claim) Under $\mathrm{H_0}$, the test statistic $Z = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0,1)$ approx. by CLT, where $\mu = 41, \sigma = \sqrt{49.3}, \overline{x} = k,\ n = 40$.  Since $\mathrm{H_0}$ is not rejected, $\dfrac{k-41}{\sqrt{49.3}/\sqrt{40}} < 1.28155$ $k < 42.423$ $k < 42.4\,(3\text{ s.f.})$ Required set = $\{k \in \mathbb{R} : 0 < k < 42.4\}$