H1 Mathematics   **8865/01**

Paper 1   **14 September 2017**

3 Hours

Additional Materials: Writing paper

Graph Paper

List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **8** printed pages.

[Turn Over

Section A: Pure Mathematics [40 marks]

1   Azel, Brenda, Cathy and Dillion went to the wet market to buy three different kinds of fish. As there were no receipts provided, they did not know how much they paid for the individual prices of fish per kilogram. However, Azel, Brenda and Cathy can remember the total amount that they each paid. The weights of the different kinds of fish and the total amounts paid are shown in the following table.

	Azel	Brenda	Cathy	Dillon
Black pomfret (kg)	0.55	0.60	0.40	0.70
Sea Bass (kg)	0.45	0.58	0.75	0.34
Golden snapper (kg)	1.45	1.60	1.70	1.42
Total amounts paid in dollars	38.77	44.18	45.81	

Assuming that, for each kind of fish, the price per kilogram paid by them is the same, calculate the total amount that Dillon paid. [5]

2   Differentiate $\ln\left(2x^2+1\right)$ with respect to x. Hence find the exact value of $\displaystyle\int_{1}^{3} \frac{x}{2x^2+1}\,dx$, leaving your answer in the form $a\ln b$ where a and b are constants to be determined. [5]

3   Find the exact equation of the tangent to the curve $\ln y = (2-x)^2$ at the point where $x = 3$. [5]

4 A curve C has equation $y = kx^2 + k$, where k is a positive constant.
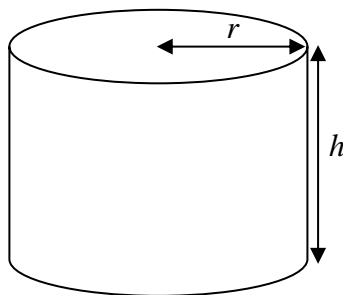
   (i) Sketch C. [2]

   (ii) Find the range of values of k for which the line $y = 4x + 3$ intersects the curve C at 2 distinct points. [4]

It is given that $k = 3$.

   (iii) Find the exact x-coordinates of the points of intersection between the line $y = 4x + 3$ and the curve C. [2]

   (iv) Hence find the exact area bounded by the line $y = 4x + 3$ and the curve C. [4]

5

A factory decides to design a closed cylindrical water tank of radius r cm and height h cm to hold a maximum of 16000π cm^3 of water. The outer surface area (including the base and lid) of the tank is to be coated with a layer of paint. It is assumed that the thickness of the cylindrical water tank is negligible.

   (i) Show that the outer surface area of the tank, S cm^2 is given by

$$S = \frac{32000\pi}{r} + 2\pi r^2.$$
[3]

   (ii) In order to reduce the amount of paint used, the factory wishes to minimize the value of S. Using differentiation, find the value of r for which S is minimized. Hence find the minimum value of S. [5]

   (iii) Sketch, in this context, the graph of S against r. [2]

   (iv) The tank with minimum value of S is being manufactured. Water is being poured into this cylindrical tank at a constant rate of 1000 cm^3 per minute. Find the rate of change of the depth of water. [3]

Section B: Statistics [60 marks]

6 Five numbers 1, 3, 6, 7 and 8 are used to form a five-digit number. If each number can only be used once, find the number of ways such that the five-digit number is

 (i) formed without further restrictions, [1]

(ii) odd and between 30000 and 80000. [3]

It is now given that all five numbers can be used with repetitions. Find the number of ways to form the five-digit number if it must be even. [2]

7 A manufacturer produces a large number of mugs everyday and the mugs are sold in batches of 50. On average, a proportion p of the mugs are defective. The random variable X is the number of defective mugs in a randomly chosen batch of 50. It is assumed that X has the distribution $B(50, p)$.

 (i) Given that $P(X = 0 \text{ or } 1) = 0.15$, formulate an equation in p and find the value of p. [3]

For the rest of the question, use $p = 0.06$.

In order to ensure the highest quality in their product, the manufacturer decides to carry out a quality control test using one of the following methods:

Method A: Select 10 mugs from the batch at random and accept the batch if there are fewer than 3 defective mugs, otherwise reject the batch.

Method B: Select 5 mugs from the batch at random and accept the batch if there is no defective mug, reject if there are two or more defective mugs, otherwise select another 5 mugs at random from the batch. If the second sample is drawn, accept the batch if there are fewer than 2 defective mugs, otherwise reject the batch.

(ii) By considering the probability of accepting the batch in each method, justify which method should the manufacturer adopt to carry out the quality control test. [6]

8 **(a)** *A* and *B* are two events such that $P(A|B') = \dfrac{4}{17}$, $P(B) = \dfrac{23}{40}$ and $P(A \cap B) = \dfrac{3}{8}$.

By using a Venn diagram or otherwise, find $P(A' \cap B')$. [3]

Determine if *A* and *B* are independent events. [2]

(b) A basket contains 35 durians, of which 15 are MSW durians and 20 are D24 durians. Of the MSW durians, 4 are infested with maggots and of the D24 durians, 3 are infested with maggots. Two durians are chosen at random from the basket.

(i) Show that the probability that both are MSW durians and at least one durian is infested with maggots is $\dfrac{10}{119}$. [2]

(ii) Given that at least one durian is infested with maggots, find the probability that both are MSW durians. [3]

9   Peter is interested to find out how the sale of his books varies with the selling price. Over
a period of eight weeks, he sells the books at a different selling price, x (in dollars),
which is fixed for each week. He also records the number of books, y, sold in that week.
The results are summarised in the following table.

Week	1	2	3	4	5	6	7	8
Selling Price (x dollars)	70	60	50	35	26	17	12	10
Number of Books Sold (y)	3	80	8	20	30	50	75	100

(i)   Draw a scatter diagram to illustrate the above information, labelling the axes
clearly.   [2]

(ii)   On the scatter diagram, circle the data point which is an outlier and label it P. [1]

(iii)   Omitting the point P, find the product moment correlation coefficient and the least
square regression line $y = a + bx$. Sketch this line on your scatter diagram. You are
not required to find the axial intercepts.   [3]

(iv)   Use the least square regression line to estimate the number of books sold when the
selling price of the book for that week is $75. Comment on the reliability of this
estimate.   [2]

(v)   Comment on whether a linear model would be appropriate, referring to both the
scatter diagram (omitting the point P) and the context of the question.   [2]

10 **(a)** A chocolate company claims that, on average, the consumption of their dark chocolate over time can decrease one's cholesterol level by at least 20 mg/dL. Over a period of time, a random sample of 80 volunteers who consume this dark chocolate daily have their cholesterol level measured. The table below shows the decrease in the cholesterol level, measured in mg/dL for the 80 volunteers.

Decrease in the cholesterol level (in mg/dL)	17	18	19	20	21	22	23	24
Number of volunteers	8	15	13	18	14	6	4	2

Test the chocolate company's claim at 5% significance level. [6]

(b) The principal of Meridian Childcare Centre claims that a child at the centre spends an average of μ_0 hours on afternoon nap. A survey was conducted and the time, x hours, spent by 60 randomly chosen children on afternoon naps is as follows:

$$\sum x = 185, \quad \sum x^2 = 626$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Find the range of values of μ_0 such that the principal is confident, at 4% level of significance, that he did not indicate wrongly the mean time that the children spend on afternoon naps. [4]

[Question 11 is printed on the next page.]

[Turn Over

11 In a particular supermarket, fishes are priced according to their weight. The weight of a randomly chosen Black Tilapia fish has a normal distribution with mean μ grams and standard deviation 32 grams. It is found that 10% of the Black Tilapia fish weigh heavier than 541 grams.

 (i) Show that the mean weight of a Black Tilapia fish is 500 grams, correct to the nearest grams. [3]

 (ii) Find the probability that out of three randomly chosen Black Tilapia fish, two weigh between 440 grams and 550 grams and one weighs more than 550 grams. [3]

 (iii) The Black Tilapia fish is priced at $ a per kg. It is given that the probability of the total price of 2 randomly chosen Black Tilapia fish cost less than $6.90 is less than 0.84241, correct to 5 significant figures. Find the range of values of a. [4]

The supermarket also sells Grey Mullet fish. The weight of a randomly chosen Grey Mullet fish follows an independent normal distribution with mean weight 800 grams and standard deviation 50 grams.

 (iv) Find the probability that the average weight of two randomly chosen Black Tilapia fish and three randomly chosen Grey Mullet fish exceeds 690 grams. [3]

End of Paper