

TEMASEK JUNIOR COLLEGE, SINGAPORE

JC 2

Preliminary Examination 2017



## MATHEMATICS

**8865/01**

### Higher 1

**29 August 2017**

Additional Materials: Answer paper

**3 hours**

List of Formulae (MF26)

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### READ THESE INSTRUCTIONS FIRST

Write your Civics group and name on all the work that you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

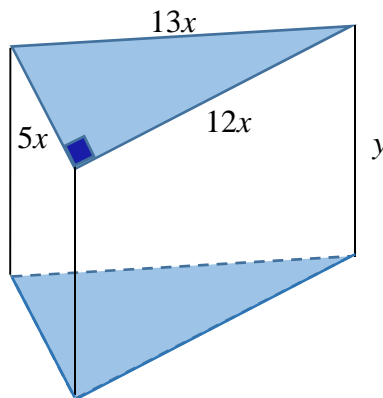
At the end of the examination, fasten all your work securely together.

This document consists of **5** printed pages.

**[Turn over**

## Section A: Pure Mathematics [40 marks]

- 1 The gradient of a curve  $C$  is given by  $\frac{dy}{dx} = 2k + 3 - \frac{k+1}{(x-1)^2}$ , where  $k \in \mathbb{R}$ . Find the set of values of  $k$  for which  $C$  has 2 distinct turning points. [4]
- 2 (a) Differentiate  $\frac{e^x - 1}{e^{2x}}$ . [2]
- (b) Find  $\int (2x\sqrt{2x+1} + \sqrt{2x+1}) dx$ . [3]
- 3 The curve  $C$  has equation  $y = \ln x$ ,  $x > 0$ .
- (i) Find the equation of the tangent to  $C$  at the point where  $x = e^2$ , leaving your answer in terms of  $e$ . [3]
- (ii) Sketch the graph of  $C$  and the tangent to  $C$  at the point where  $x = e^2$  on the same diagram, stating the coordinates of intersection with the axes and the equation of any asymptote(s). [2]
- (iii) By using the result  $\int \ln x dx = x \ln x - x + c$ , where  $c$  is an arbitrary constant, find the area of the region bounded by  $C$ , the tangent to  $C$  at the point where  $x = e^2$ , the  $x$ -axis and the  $y$ -axis. Give your answer in terms of  $e$ . [3]
- 4 A prism with a cross-section in the shape of a right-angled triangle has dimensions (in cm) as shown in the diagram below.



The volume of the prism is  $7200 \text{ cm}^3$ . Show that the surface area of the prism is given by

$$S = \frac{7200}{x} + 60x^2 \text{ cm}^2. \quad [3]$$

Without using a calculator, find in surd form the value of  $x$  that gives a stationary value of  $S$ . Hence state, with a reason, whether  $S$  is a maximum or minimum. [4]

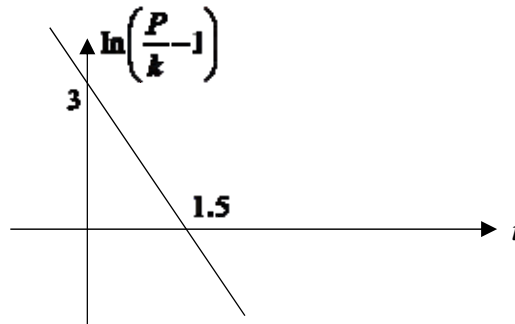
It is also given that  $x$  is decreasing at  $0.5 \text{ cm/s}$ , find the rate at which the surface area is decreasing when  $x = 5 \text{ cm}$ . [2]

- 5 The profit  $P$  (in thousands dollars) of a company after the start of a promotion “Clearance Sale” can be modelled by the equation

$$P = k(1 + be^{-rt}),$$

where  $t$  is the number of days elapsed since the start of the promotion and  $b$ ,  $r$  and  $k$  are positive constants.

- (i) Express  $\ln\left(\frac{P}{k} - 1\right)$  in terms of  $b$ ,  $r$  and  $t$  [1]
- (ii) The graph of  $\ln\left(\frac{P}{k} - 1\right)$  against  $t$  is given below.



By using the graph above, show that  $b = e^3$  and  $r = 2$ . [3]

- (iii) Using differentiation, show that  $\frac{dP}{dt} < 0$  for  $t \geq 0$ . Hence explain why the maximum profit occurs at  $t = 0$ . Given that the maximum profit is \$42171, find the value of  $k$ , correct to nearest integer value. [5]

The “Clearance Sale” ends after a week and another promotion “Happy Sale” takes place immediately after. The “Happy Sale” lasts for 3 weeks and the profit during “Happy Sale” can be modelled by the equation

$$P = a - \frac{1}{6}(t - 14)^2, \text{ for } 7 \leq t \leq 28.$$

- (iv) Given that the company’s maximum profit during “Happy Sale” is \$18000, find the value of  $a$ . [2]
- (v) Find the total profit of the company in 4 weeks, correct to the nearest dollars. [3]

### Section B: Statistics [60 marks]

- 6 Find the number of 6-letter passwords that can be formed using the letters from the word SINGAPORE if
- (i) repetitions of letters are not allowed. [1]
- (ii) at least two vowels must be chosen and repetition of letters are not allowed. [2]
- (iii) three distinct vowels and three distinct consonants are chosen, and vowels and consonants must alternate? [3]

- 7** A fixed number,  $n$ , of taxis entering VICOM is observed and the number of those taxis that fails inspection is denoted by  $X$ .

(i) State, in context, two assumptions needed for  $X$  to be well modelled by a binomial distribution. [2]

Assume now that  $X$  has the distribution  $B(n, p)$ .

(ii) Given that  $n = 20$  and  $p = 0.15$ , find  $P(X = 2 \text{ or } 3)$ . [2]

(iii) Given that  $n = 10$  and  $p = 0.3$ , find  $P(3 \leq X < 6)$ . [2]

(iv) Given that  $n = 3$  and  $P(X \leq 1) = 0.5$ , show that  $4p^3 - 6p^2 + 1 = 0$ . Hence find the value of  $p$ . [3]

- 8** The number of hours,  $x$ , spent daily on revision for mathematics and the marks,  $y$ , obtained for the mathematics year-end examination are recorded for 10 randomly selected students. The results are given in the following table.

$x$	1.3	2.1	1.1	2.3	2.7	1.2	3.2	3.4	3.0	2.5
$y$	68	74	64	76	75	66	85	81	86	75

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the equation of the regression line of  $y$  on  $x$ , in the form  $y = mx + c$ , giving the values of  $m$  and  $c$  correct to 4 significant figures. Sketch this line on your scatter diagram. [2]

(iv) Use the equation of your regression line to estimate the marks obtained by a student who spends 1.5 hours a day on revision for mathematics. Comment on the reliability of your estimate. [3]

- 9** A bag contains 3 black balls and 5 white balls. Paul draws a ball at random from the bag and notes the colour. If a black ball is selected, Paul returns it to the bag and adds an additional black ball into the bag. If a white ball is selected, Paul does not return it to the bag but adds 2 black balls into the bag. Paul then draws a ball at random from the bag again and notes the colour.

Draw a tree diagram to represent the information of the two draws. [3]

(i) Show that the probability that Paul selects a black ball on both draws is  $\frac{1}{6}$ . [1]

(ii) Find the probability that Paul selects a white ball either on his first or second draw, or both. [2]

(iii) Find the probability that Paul selects a white ball on his first draw, given that he selects a black ball on his second draw. [3]

- 10** A company claims that their electric-powered V1 cars is designed to travel a mean distance of 500 km on one full charge. To test this claim, a random sample of 80 V1 cars is taken and the distance,  $x$  km, travelled on one full charge are summarised by

$$\sum (x - 500) = 46 \quad \text{and} \quad \sum (x - 500)^2 = 460.$$

- (i) Find the unbiased estimates of the population mean and variance. [3]
- (ii) Suggest a reason why, in this context, the given data is summarised in terms of  $(x - 500)$  instead of  $x$ . [1]
- (iii) Test, at the 5% level of significance, whether the company's claim is valid. [4]
- (iv) State, with a reason, whether it is necessary to assume a normal distribution for the test to be valid. [1]

The company introduces a new electric-powered V2 car and claims that the V2 can travel more than the mean distance of the V1 on one full charge. The population variance of the V2 cars is known to be  $13 \text{ km}^2$ . A random sample of 50 V2 cars is taken.

- (v) Find the set of values within which the mean distance of this sample must lie, such that there is not enough evidence from the sample to support the company's claim at the 1% level of significance. [4]

- 11** Three friends Anand, Beng and Charlie goes racing regularly at the Temasek Circuit, which offers a standard route on Track 1 or a more challenging route on Track 2. The time taken, in minutes, taken by them to complete a round on Tracks 1 and 2 have independent normal distributions with means and standard deviations as shown in the following table.

	Track	Mean	Standard deviation
Anand	1	3.15	0.21
Beng	1	3.18	0.10
Charlie	2	4.22	0.15

- (i) Find the probability that Beng takes less than 3.15 minutes to complete a randomly chosen round on Track 1. [1]
- (ii) Find the probability that Anand and Beng each takes less than 3.15 minutes to complete a randomly chosen round on Track 1. [2]
- (iii) Find the probability that Beng is faster than Anand in completing a randomly chosen round on Track 1. [3]
- (iv) Find the probability that out of 8 complete rounds on Track 2, there are more than 4 rounds in which Charlie takes less than 4.20 minutes to complete. [3]

Temasek Circuit charges customers \$18 per minute on Track 1 and \$22 per minute on Track 2. On a particular day, Beng completes 10 rounds on Track 1 and Charlie completes 8 rounds on Track 2.

- (v) Find the probability that Beng and Charlie pay a total of less than \$1300. [5]

**End of Paper**