YISHUN JUNIOR COLLEGE

Mathematics Department

PRELIM SOLUTION

Subject : JC2 H1 MATHEMATICS 8865 **Date** :

Qn	Solution
1(i)	(a) $\frac{\mathrm{d}}{\mathrm{d}x}\left(5\ln\left(1-3x^2\right)\right)=5\left(\frac{1}{1-3x^2}\right)(-6x)$ $=-\frac{30x}{1-3x^2}$ (b) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{(2x+3)^2}\right)=\frac{\mathrm{d}}{\mathrm{d}x}(2x+3)^{-2}$ $=-2(2x+3)^{-3}(2)$ $=-4(2x+3)^{-3}$
(ii)	$\int_1^3 x^3\left(\frac{1}{x}-1\right)^2\,\mathrm{d}x=\int_1^3\left(x-2x^2+x^3\right)\mathrm{d}x$ $=\left[\frac{x^2}{2}-\frac{2x^3}{3}+\frac{x^4}{4}\right]_1^3$ $=\left(\frac{3^2}{2}-\frac{2(3)^3}{3}+\frac{(3)^4}{4}\right)-\left(\frac{1^2}{2}-\frac{2(1)^3}{3}+\frac{(1)^4}{4}\right)$ $=\frac{27}{4}-\frac{1}{12}$ $=\frac{20}{3}$
2	$(-k-2)^2-4(k)(4k)<0$ and $k<0$ $k^2+4k+4-16k^2<0$ $-15k^2+4k+4<0$ $k<0$ and $(5k+2)(3k-2)>0$ $k<-\frac{2}{5}$ or $k>\frac{2}{3}$ Since $k<0$, $\therefore\ k<-\frac{2}{5}$

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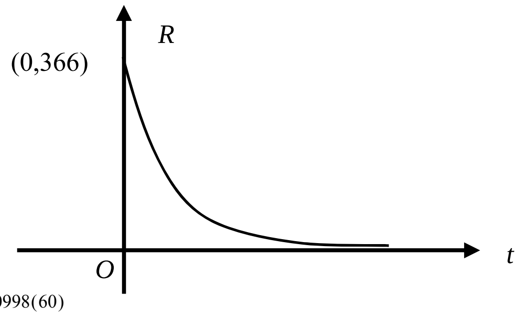
(i)



(ii)

$R = 366e^{-0.0998(60)}$

$\quad = 0.918$

The concentration is 0.918 micrograms/litre

(iii)

From GC, when $t = 20$,

$\dfrac{dR}{dt} = -4.96$

The rate of decrease is 4.96 micrograms/litre per min

(iv)

$40 = 366e^{-0.0998t}$

$e^{-0.0998t} = \dfrac{40}{366}$

$-0.0998t = \ln\left(\dfrac{40}{366}\right)$

$t = 22.18$

$t \approx 22\text{mins}$

Alternative solution:

Draw graph of y = 40 and find intersection points.

$P = -0.03x^3 + 0.1x^2 + x - 0.1$

$\dfrac{dP}{dx} = -0.09x^2 + 0.2x + 1$

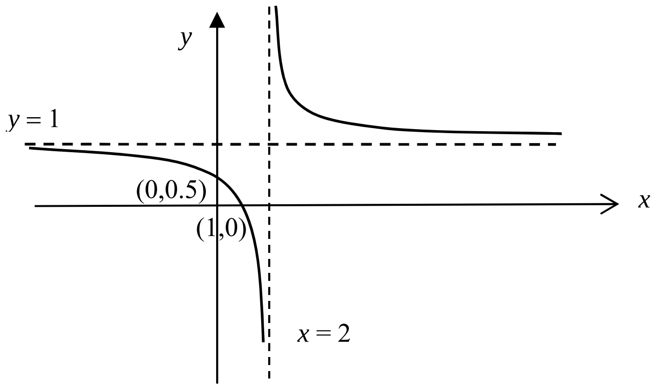
For maximum P, $\dfrac{dP}{dx}$=0

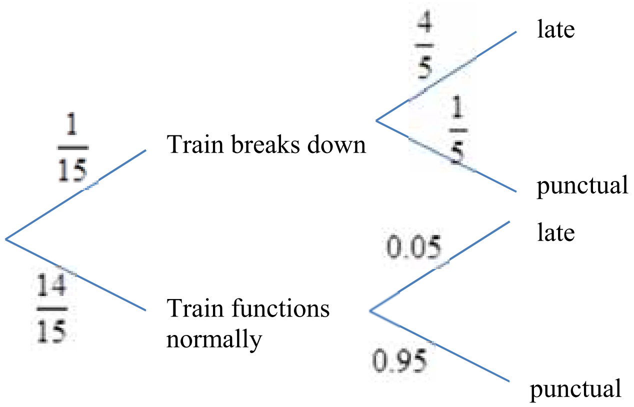
$-0.09x^2 + 0.2x + 1$=0

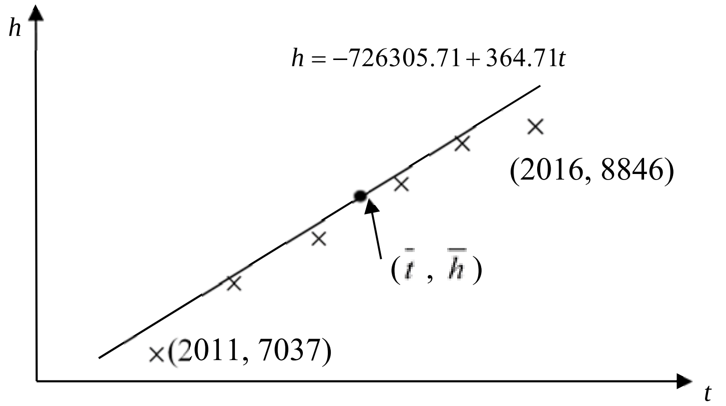
$x = 4.624753 \;\; (x > 0)$

x	4.624753^-	4.624753	4.624753^+
$\dfrac{dP}{dx}$	+ve	0	-ve
slope			

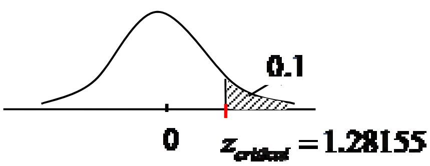
Thus, P is maximum when the number of bottles is 46248.

4	 $\frac{1}{x-2}+1=x+3$ $1+(x-2)=x^2+x-6$ $x^2=5$ $x=\pm\sqrt{5}$ Area of the region $=\int_{-\sqrt{5}}^{0}\left(x+3-\left(\frac{1}{x-2}+1\right)\right)\mathrm{d}x$ $=\left[\frac{x^2}{2}+2x-\ln\|x-2\|\right]_{-\sqrt{5}}^{0}$ $=-\ln 2-\left(\frac{5}{2}-2\sqrt{5}-\ln\|-\sqrt{5}-2\|\right)$ $=-\ln 2-\left(\frac{5}{2}-2\sqrt{5}-\ln\left(2+\sqrt{5}\right)\right)$ $=-\ln 2-\frac{5}{2}+2\sqrt{5}+\ln\left(2+\sqrt{5}\right)$
5	$y=x^3-2e^{-x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2+2e^{-x}$ When $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=3(1)^2+2e^{-1}$ and $y=1-2e^{-1}$ Equation of tangent: $y-\left(1-\frac{2}{e}\right)=\left(3+\frac{2}{e}\right)(x-1)$ $y=\left(3+\frac{2}{e}\right)x-2-\frac{4}{e}$
6	$6x+3y+4z=93.80$ $5x+3y+4z=85.30$ $4x+2y+4z=70.40$ From GC, x = 8.50, y = 6.40, z = 5.90 The usual retail prices of 1 packet of milo, 1 packet of cereal and 1 packet of coffee are \$8.50, \$6.40 and \$5.90 respectively.

7(a)(i)	$\mathrm{P}(A\cup B)=1-\mathrm{P}(A'\cap B')$ $=\dfrac{9}{17}$ $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$ $\dfrac{9}{17}=\dfrac{1}{2}+\mathrm{P}(B)-\dfrac{1}{34}$ $\Rightarrow \mathrm{P}(B)=\dfrac{1}{17}$ (Shown) Smallest value of n is 34.
(ii)	$\mathrm{P}(A\mid B)=\dfrac{\frac{1}{34}}{\frac{1}{17}}=\dfrac{1}{2}$
(b)(i)	
(ii)	P (train functions normally | late) $=\dfrac{\frac{14}{15}(0.05)}{0.1}=\dfrac{7}{15}\approx 0.467$
8(i)	No of teams = ${}^{12}C_5+{}^{3}C_1\,{}^{12}C_4$ $=2277$
(ii)	No of teams = ${}^{15}C_5-{}^{11}C_5$ $=2541$
9(i)	Let X be the random variable 'number of smartphone users with anti-virus software A installed on their smartphones out of 20 users' $X\sim \mathrm{B}(20, 0.37)$ $\mathrm{P}(X\geq 8)=1-\mathrm{P}(X\leq 7)$ ≈ 0.47346 $=0.473$ (3 sig fig)
(ii)	Let W be the 'number of samples with at least eight users with anti-virus software A installed on their smartphone out of 50 samples' $W\sim \mathrm{B}(50, 0.47346)$ $\mathrm{P}(W<30)=\mathrm{P}(W\leq 29)$ $\approx 0.95061 \quad = 0.951$ (3 sig fig)

(iii)	Let Y be the 'number of smartphone users who did not have any anti-virus software installed, out of n' $Y \sim \text{B}(n, 0.07)$ $\text{P}(Y \le 1) < 0.5$ $\text{P}(Y = 0) + \text{P}(Y = 1) < 0.5$ ${}^nC_0(0.07)^0(0.93)^n + {}^nC_1(0.07)(0.93)^{n-1} < 0.5$ $(0.93)^n + n(0.07)(0.93)^{n-1} < 0.5$ $(0.93)^{n-1}(0.93 + 0.07n) < 0.5$ (shown) Using GC, When $n = 23$, $(0.93)^{n-1}(0.93 + 0.07n) = 0.5146 > 0.5$ When $n = 24$, $(0.93)^{n-1}(0.93 + 0.07n) = 0.4918 < 0.5$ Therefore, least $n = 24$
10(i)	
(ii)	$r \approx 0.992$ (3 s.f.) There is a strong positive linear correlation between the median monthly household income from work and the year. As the year increases, the median monthly household income from work increases.
(iii)	$\bar{t} = 2013.5$, $\bar{h} = 8046.5$
(iv)	$h = -726305.71 + 364.71t$ (2 d.p.)
(v)	When $h = 9700$, $9700 = -726305.71 + 364.71t$ $t = 2018.057$ Year: 2018 Since the estimate is obtained via extrapolation, the estimate is not reliable.
11(i)	$X \sim N(50, 10^2)$ Required Prob = $[\text{P}(X > 60)]^2 \approx (0.158655)^2$ $= 0.02517$ ≈ 0.0252 (3 s.f)

(ii)	Let μ be the population mean time taken (min) the company has to achieve $X \sim N(\mu, 10^2)$ $P(X < 60) \geq 0.95$ $P(Z < \frac{60-\mu}{10}) \geq 0.95$ $\frac{60-\mu}{10} \geq 1.64485$ $\mu \leq 43.552$ Maximum $\mu = 43.5$
(iii)	Let W be the amount of electricity (kWh) used in a month by a household $W \sim N(522, 26^2)$ Total charge per month, $B = 0.21W \sim N(109.62, 29.8116)$ $P(100 < B < 120) = 0.932$ (3 s.f)
(iv)	$T = B_1 + B_2 \sim N(219.24, 59.6232)$ $P(T \geq d) > 0.9$ $1 - P(T < d) > 0.9$ $P(T < d) < 0.1$ $d < 209.344$ Largest integral value of d is 209. Assume that the electricity used in each month is independent for a particular household
(v)	Since $\mu - 3\sigma = 47 - 3(25) = -28 < 0$, Time taken to install a gas meter is impossible to be negative, Y is unlikely to be normally distributed.
(vi)	Since sample size=55 is large, $\bar{Y} = \frac{Y_1 + Y_2 + ... + Y_{55}}{55} \sim N\left(47, \frac{25^2}{55}\right)$ approx by CLT $P(\bar{Y} > 45) = 0.724$ (3 s.f.)
12(i)	Unbiased estimate of the population mean, $\bar{x} = \frac{6386}{150} = 42.573 \approx 42.6$ (3s.f.) Unbiased estimate of the population variance, $s^2 = \frac{1}{149}\left[277270 - \frac{6386^2}{150}\right] = 36.219 \approx 36.2$ (3 s.f.)
(ii)	$H_0 : \mu = 41$ $H_1 : \mu \neq 41$ Test at 5% significance level Under H_0, the test statistic $Z = \frac{\bar{X} - \mu}{S / \sqrt{n}} \sim N(0,1)$ approx. by CLT, where $\mu = 41, s = \sqrt{36.219}, \bar{x} = 42.573, n = 150$.

	By GC, p-value $= 0.00137\,(3\text{ s.f.})$. Since p-value < 0.05, we reject H_0 and conclude that at 5% level, there is sufficient evidence that the claim is not valid.
(iii)	Since n is large, by Central Limit Theorem, the sample mean time spent by 150 customers is approximately normal. Hence it is not necessary to assume a normal distribution for the population for the test to be valid.
(iv)	There is a probability of 0.05 of concluding that the mean time spent by customers is not equal to 41 minutes when it is in fact 41 minutes.
(v)	$H_0 : \mu = 41$ $H_1 : \mu > 41$ (claim) Under H_0, the test statistic $Z = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0,1)$ approx. by CLT, where $\mu = 41, \sigma = \sqrt{49.3}, \overline{x} = k,\ n = 40$.  Since H_0 is not rejected, $\dfrac{k-41}{\sqrt{49.3}/\sqrt{40}} < 1.28155$ $k < 42.423$ $k < 42.4\,(3\text{ s.f.})$ Required set = $\{k \in \mathbb{R} : 0 < k < 42.4\}$