1	Let \$$x$ be the entry rates for toddlers. Let \$$y$ be the entry rates for children. Let \$$z$ be the entry rates for adults. $0.75(2y+2z)=22.50$ $2y+2z=30 \quad ----(1)$ $0.8(x+y+2z)=20$ $x+y+2z=25 \quad ----(2)$ $0.5(x+3y+3z)+15=41.25$ $x+3y+3z=52.50 \quad ----(3)$ Using GC to solve eq (1), (2) and (3) $x=7.5$ $y=12.5$ $z=2.5$ $\therefore$ the entry rates are \$7.50 (Toddler), \$12.50 (children) and \$2.50 (adult) respectively.
2(i)	(a) Let $y=\dfrac{3}{\sqrt{(2x-7)^3}}=3(2x-7)^{-\frac{3}{2}}$ $\dfrac{dy}{dx}=3(2x-7)^{-\frac{5}{2}}\cdot\left(\dfrac{-3}{2}\right)\cdot(2)$ $=-9(2x-7)^{-\frac{5}{2}} \qquad \left(\text{or } -\dfrac{9}{(2x-7)^{\frac{5}{2}}}\right)$ (b) $\displaystyle\int_1^3 \frac{1}{e^{4t-3}}+\frac{1}{2t-1}\,dt$ $\displaystyle=\int_1^3 e^{-4t+3}+\frac{1}{2t-1}\,dt$ $\displaystyle=\left[-\frac{1}{4}e^{-4t+3}+\frac{1}{2}\ln(2t-1)\right]_1^3$ $\displaystyle=-\frac{1}{4}e^{-9}+\frac{1}{2}\ln(5)-\left(-\frac{1}{4}e^{-1}+\frac{1}{2}\ln(1)\right)$ $\displaystyle=-\frac{1}{4}\left(e^{-9}-e^{-1}\right)+\frac{1}{2}\ln 5$

3

(i)

$$3-10e^{2x}-8e^{4x}=0$$

Let $u=e^{2x}$. Then

$\Rightarrow \quad 3-10u-8u^2=0$

$\Rightarrow \quad (2u+3)(1-4u)=0$

$\Rightarrow \quad u=-\frac{3}{2} \quad$ or $\quad u=\frac{1}{4}$

$\Rightarrow \quad e^{2x}=-\frac{3}{2} \quad$ or $\quad e^{2x}=\frac{1}{4}$

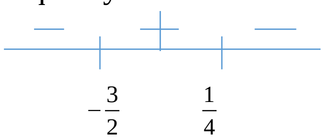
(rej $\because e^{2x}>0$)

$\therefore \quad 2x=\ln\frac{1}{4} \Rightarrow x=\frac{1}{2}\ln\frac{1}{4}=-\ln 2$

$a=-1$ and $b=2$

(ii)

For the inequality $3-10e^{2x}-8e^{4x}\le 0$,

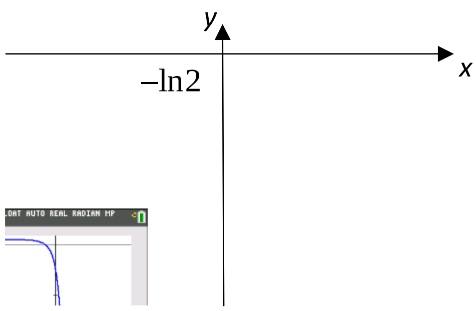


$e^{2x}\le -\frac{3}{2} \quad$ or $\quad e^{2x}\ge \frac{1}{4}$

(rej $\because e^{2x}>0$)

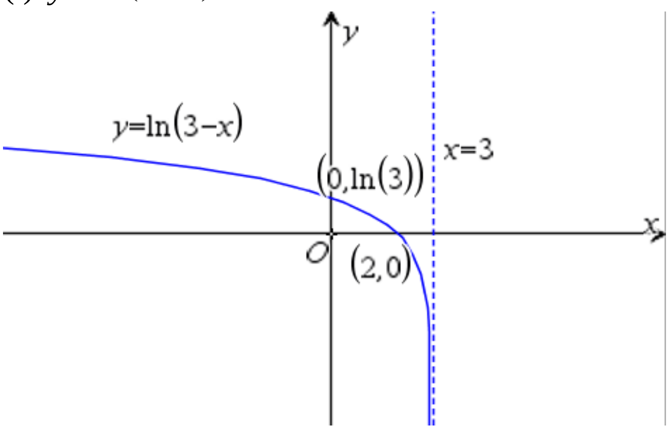
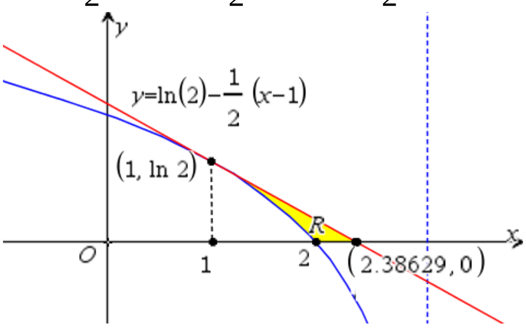
$\therefore \quad x\ge -\ln 2$

Or use graphical method



Therefore,

$\therefore \quad x\ge -\ln 2$

4	(i) $y=\ln(3-x)$  (ii) $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{3-x}$ At point P, $x=1$; $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}$; $y=\ln(2)$ Eqn of tangent at P: $y-\ln(2)=-\frac{1}{2}(x-1)$ $y=\ln(2)-\frac{1}{2}(x-1)$ $y=-\frac{1}{2}x+\ln 2+\frac{1}{2}$ (iii) $y=-\frac{1}{2}x+\ln 2+\frac{1}{2}$ $y=\ln 2-\frac{1}{2}(x-1)$  The area of R = Area of triangle – area under the curve $=\frac{1}{2}(2.38629-1)(\ln 2)-\int_1^2 \ln(3-x)\,\mathrm{d}x$ $=0.0941586528$ ≈ 0.0942 units^2

5

(i)

$x = t^3 - 13t^2 + 40t + 35$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 3t^2 - 26t + 40$$

For min value of x, $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$

$3t^2 - 26t + 40 = 0$

$(3t-20)(t-2) = 0$

$\Rightarrow t = 2 \;\; or \;\; t = \frac{20}{3}$

t	2^-	2	2^+
Sign of $\frac{dx}{dt}$	$+$	0	$-$
slope			

t	$\left(\frac{20}{3}\right)^-$	$\frac{20}{3}$	$\left(\frac{20}{3}\right)^+$
Sign of $\frac{dx}{dt}$	$-$	0	$+$
slope			

Therefore, x is a minimum when $t = \frac{20}{3}$.

Or

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 6t - 26$$

When $t = 2$, $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 6(2) - 26 = -14 < 0$

When $t = \frac{20}{3}$, $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 6\left(\frac{20}{3}\right) - 26 = 14 > 0$

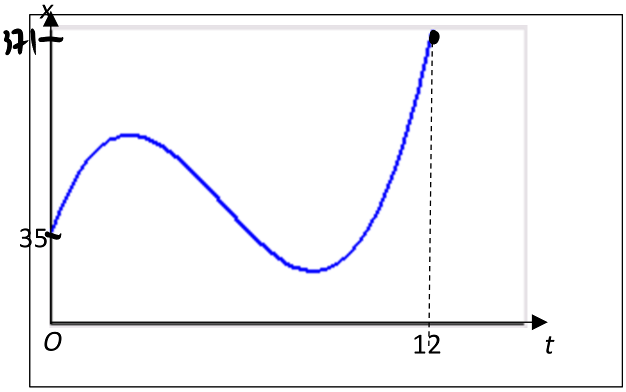
Therefore, x is a minimum when $t = \frac{20}{3}$.

$$x = \left(\frac{20}{3}\right)^3 - 13\left(\frac{20}{3}\right)^2 + 40\left(\frac{20}{3}\right) + 35$$

$= 20.185$

$x = 20.2 \quad$ or $\quad \frac{545}{27}$

(ii)



(iii)

Area of the region

$$= \int_0^{12} t^3 - 13t^2 + 40t + 35 \ \mathrm{d}t$$

$$= \left[\frac{t^4}{4} - 13\frac{t^3}{3} + 20t^2 + 35t \right]_0^{12}$$

$$= \frac{1}{4}(12)^4 - \frac{13}{3}(12)^3 + 20(12)^2 + 35(20) - 0$$

$$= 996$$

(or using GC to solve)

The total manufacturing cost to manufacture the smart phones for a period of 12 months is $996 million.

(iv)

$$P = 45 + 20\ln(3x+4)$$

$$\frac{dP}{dx} = \frac{60}{3x+4}$$

When $t = 8$, $x = 35$

$$\frac{\mathrm{d}P}{\mathrm{d}x} = \frac{60}{3(35)+4} = \frac{60}{109}$$

(v)

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\mathrm{d}P}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$$

When $x = 8$,

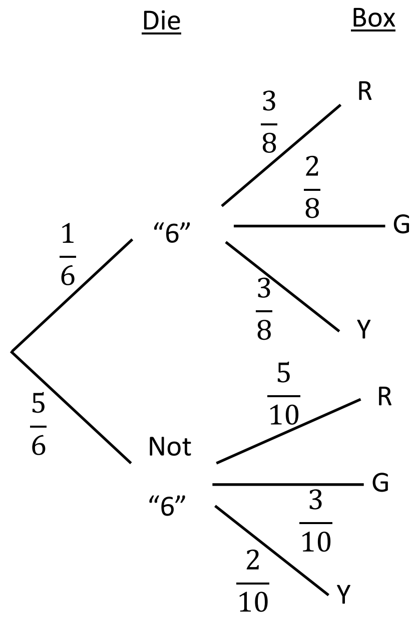
$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{60}{109} \times [3(8)^2 - 26(8) + 40]$$

$$= \frac{1440}{109} \quad \text{or } 13.2$$

The rate of increase in profit when $t = 8$ is $13.2 million per month.

6	(i) Number of ways $= 4\times3\times2\times2$ $= 48$ Required probability $= \dfrac{48}{{}^5P_4}$ $= \dfrac{48}{120}$ $= 0.4$ (ii) Case 1 : 1st digit is '4' Number of ways $= 1\times3\times1\times2$ $= 6$ Case 2 : 1st digit is '3' or '5' Number of ways $= 2\times3\times2\times2$ $= 24$ Total number of ways $= 24+6 = 30$ Required probability $= \dfrac{30}{48} = \dfrac{5}{8}$ **<u>Alternative method</u>** $\mathrm{P}(\text{greater than 3000} \mid \text{number is even})$ $= \dfrac{\mathrm{P}(\text{greater than 3000} \cap \text{number is even})}{\mathrm{P}(\text{number is even})}$ $= \dfrac{\frac{24+6}{120}}{0.4}$ $= \dfrac{5}{8}$

<table>
<tr><td>7</td><td>



(a)(ii)

$$P(\text{wins a prize}) = \left(\frac{1}{6}\times\frac{3}{8}\right)+\left(\frac{5}{6}\times\frac{2}{10}\right) = \frac{11}{48}$$

(a)(iii)

$$P(\text{from box A} \mid \text{wins the prize}) = \frac{\frac{1}{6}\times\frac{3}{8}}{\frac{11}{48}} = \frac{3}{11}$$

(b)

$$P(\text{wins the grand prize}) = \left(\frac{1}{6}\times\frac{3}{8}\times\frac{2}{7}\right)+\left(\frac{5}{6}\times\frac{2}{10}\times\frac{1}{9}\right) = \frac{55}{1512}$$

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<tr><td>8</td><td>

(i) The assumption is that the event of a resident using ShareBike or not is independent of any other residents in the neighbourhood.

(ii) The assumption may not hold as usually families may use ShareBike together as they are going for the activity together.

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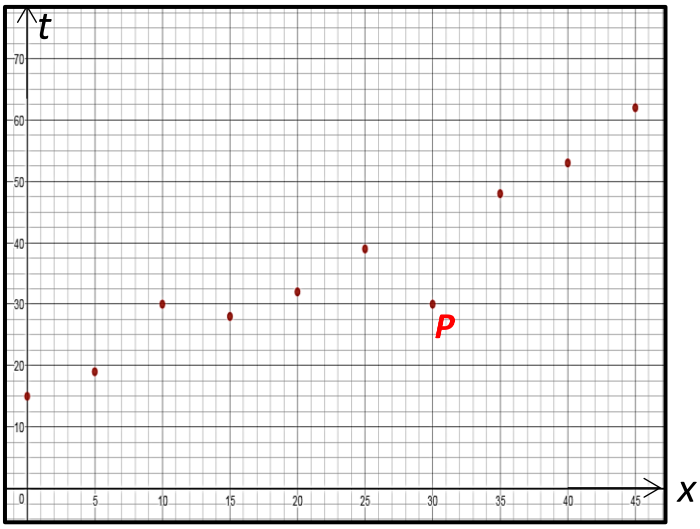
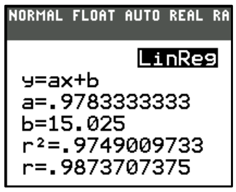
	(iii) $X \sim B(n, 0.09)$ $P(X \geq 1) < 0.99$ $P(X = 0) > 0.01$ $\binom{n}{0}(0.09)^0 (1-0.09)^n > 0.01$ $(0.91)^n > 0.01$ $n < \dfrac{\ln 0.01}{\ln 0.91}$ $n < 48.830$ $\therefore$ Greatest value of n is 48. **<u>Alternative method</u>** Using GC, When $n = 47$, $P(X = 0) = 0.0119$ (> 0.01) When $n = 48$, $P(X = 0) = 0.0108$ (> 0.01) When $n = 49$, $P(X = 0) = 0.0098$ (<0.01) $\therefore$ Greatest value of n is 48. (iv) $X \sim B(20, 0.09)$ $P(2 < X \leq 5) = P(X \leq 5) - P(X \leq 2)$ $= 0.260$ (3 s.f.) (v) $E(X) = 20 \times 0.09 = 1.8$ $Var(X) = 20 \times 0.09 \times (1-0.09) = 1.638$ Sample mean $\bar{X} = \dfrac{X_1 + X_2 + ...X_{40}}{40}$ Since $n = 40$ is sufficiently large, by Central Limit Theorem, $\bar{X} \sim N\left(1.8, \dfrac{1.638}{40}\right)$ approximately. $P\left(\bar{X} > 2\right) = 0.161$ (3 s.f.)
9	(i) $A \sim N\left(43, 8^2\right)$ $B \sim N\left(40, 6^2\right)$ $P\left(A_1 + A_2 + A_3 < 3B\right) = P\left(A_1 + A_2 + A_3 - 3B < 0\right)$ Let $S = A_1 + A_2 + A_5 - 3B$ $E(S) = 3(43) - 3(40) = 9$ $Var(S) = 3\left(8^2\right) + 3^2\left(6^2\right) = 516$ $P\left(S < 0\right) = 0.346$ (3 s.f.)

	(ii) $B \sim \mathrm{N}\left(40, 6^2\right)$ $\mathrm{E}\left(B_1 + B_2\right) = 2(40) = 80$ $\mathrm{Var}\left(B_1 + B_2\right) = 2\left(6^2\right) = 72$ $B_1 + B_2 \sim \mathrm{N}(80, 72)$ Required probability $= \mathrm{P}\left(A > 38\right) \times \mathrm{P}\left(B_1 + B_2 > 82\right)$ $= 0.299$ (3 s.f.) (iii) $\mathrm{E}\left(A + B_1 + B_2\right) = 43 + 2(40) = 123$ $\mathrm{Var}\left(A + B_1 + B_2\right) = 8^2 + 2\left(6^2\right) = 136$ $A + B_1 + B_2 \sim \mathrm{N}(123, 136)$ Required probability $= \mathrm{P}\left(A + B_1 + B_2 > 120\right)$ $= 0.602$ (3 s.f.) (iv) Because the case in (ii) is a proper subset of the case in (iii). For eg, Part iii contains cases whereby the lifespan of component A may not exceed 38 weeks (eg. 36 weeks) but total lifespan of 2 components of B exceeds 82 weeks (eg. 84 weeks), and yet the total lifespan is more than 120 weeks.
10	(i)  (ii) Acceptable reasons: The traffic condition on the road was good (Lesser cars on the road, no traffic jam) and thus he required much shorter travelling time though he left home only at 7.30am. It was a public holiday/school holiday/Sunday and yet Mr Lee has to work.

(iii)

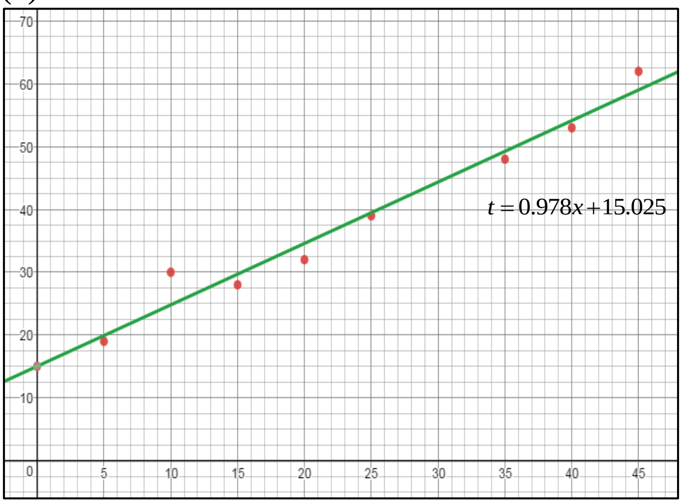
$r \approx 0.987$

The pmcc is close to 1, indicating a strong positive linear correlation between x and t. I.e. the later Mr Lee leaves home after 7 am, the longer the travelling time would take.



(iv) $t = 0.978x + 15.025$

(v)



$a = 0.978$ means that for every additional minute that Mr Lee delays in leaving home after 7am, his travelling time will increase by 0.978 minutes.

(vi)

<u>Method 1</u>:

There are 90 minutes from 7 am to 8.30 am.

$$x + t \le 90$$

$$x + (0.97833x + 15.025) \le 90$$

$$1.97833x \le 74.975$$

$$x \le 37.898$$

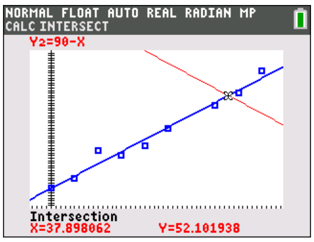
The largest possible value of x is 37 (correct to the nearest minute)

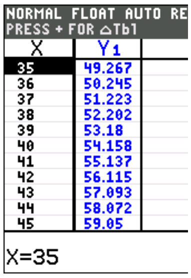
The latest time Mr Lee could leave home without being late for work is 7.37 am.

<u>Method 2</u>:

Sketch the line $x + t = 90$ and find x-coordinate of the point of intersection with the regression line.

If x = 38, Mr Lee will arrive late for work. Thus the latest time he needs to leave home is 7.37am.



	Method 3: **By Trial & Error, using GC** From part (vi), if $x = 40, t = 54.158, x + t > 90$ $x = 39, t = 53.18, \quad x + t > 90$ $x = 38, t = 52.202, \ x + t > 90$ $x = 37, t = 51.223, x + t < 90$ Thus the latest time he needs to leave home is 7.37am. 
11	(i) $\sum x = 2526, \qquad \sum (x - \bar{x})^2 = 544$ Unbiased estimates of the population mean μ is $\bar{x} = \frac{2526}{30} = 84.2$ Unbiased estimates of the population variance σ^2 is $s^2 = \frac{30}{29}\left[\frac{544}{30}\right]$ $= 18.75862$ $= 18.8 \ (3 \text{ s.f})$ (ii) $H_0: \ \mu = 85$ $H_1: \ \mu < 85$ Test at 10% significance level Assuming that H_0 is true, Since $n = 30$ is sufficiently large, by the Central Limit Theorem, $\bar{X} \sim \text{N}\left(85, \frac{s^2}{30}\right)$ approximately. Test statistic: $Z = \frac{\bar{X} - 85}{\sqrt{\frac{18.75862}{30}}} \sim \text{N}(0,1)$ approximately. Using GC, *p*-value = 0.15584 = 0.156 (3 s.f) $\left(\text{or } z = \frac{84.2 - 85}{\sqrt{\frac{18.75862}{30}}} = -1.0117\right)$ Since *p*-value = 0.15584 > 0.1 (or $z = -1.0117 > -1.28155$), we do not reject H_0 and conclude that there is insufficient evidence at 10% level, that the mean mass of the Health and Fitness Club members has decreased. (or that the trainer's claim is invalid.)

(iii)
There is 0.15584 probability of drawing **a random sample of 30** Health and Fitness Club members with **sample mean less than 84.2 kg**, assuming that the population mean weight is 85 kg.

(iv)
$H_0: \ \mu = 85$

$H_1: \ \mu < 85$

Test at 10% level significance level.

Assuming that H_0 is true,

Since n is large, by the Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(85, \frac{5^2}{30}\right)$ approximately.

Test statistic: $Z = \dfrac{\bar{X} - 85}{5/\sqrt{30}} \sim \mathrm{N}(0,1)$ approximately.

Since the null hypothesis is rejected,

$\Rightarrow$ z-value falls inside critical region

$\Rightarrow$ z-value < -1.28155

$$\Rightarrow \frac{m-85}{5/\sqrt{30}} < -1.28155$$

$$m - 85 < -1.1699$$

$$m < 83.83$$

$\therefore \quad 0 < m < 83.83$

Alternate method

Using $\bar{X} \sim \mathrm{N}\left(85, \frac{5^2}{30}\right)$

$\mathrm{P}\left(\bar{X} \le m\right) < 0.1$

$\Rightarrow m < 83.83$

$\therefore \quad 0 < m < 83.83$