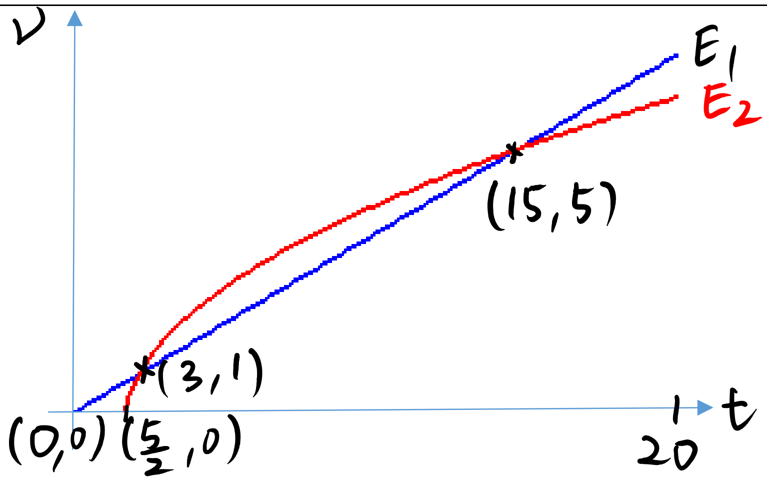
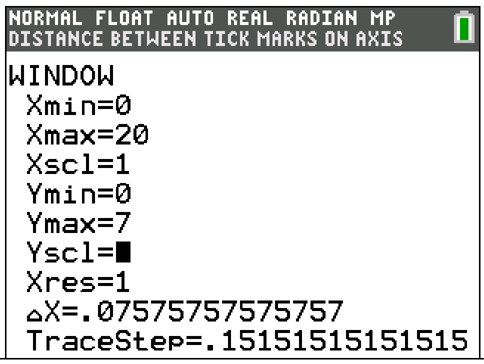
Qn	Suggested Solution
1	Let a be the distance in km, b the time taken in minutes, c be the value of promo discount that caleb had. $\begin{cases} 3.2+0.55a+0.29b=15.6 \\ 3+0.8a=6.6+2c \\ 3+0.45a+0.2b=9.4+c \end{cases}$ $\begin{cases} 0.55a+0.29b=12.4 \\ 0.8a-2c=3.6 \\ 0.45a+0.2b-c=6.4 \end{cases}$ Using GC, $a=12, b=20, c=3$ Hence the time taken was <u>20 minutes</u> and the distance travelled was <u>12 km</u> .
2i	Equation of asymptote: $y = \ln 2$ Coordinates of point of intersection with y-axis: $(0, 1 + \ln 2)$
ii	Using GC, x-coordinate = 1.70438 = 1.7044 (4 d.p.)
iii	Area $= \int_0^{1.70438} \left(\ln 2 + 2^{-x} - 1\right) \mathrm{d}x$ $= 0.477006$ $= 0.477 \text{ unit}^2$
3(a)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\pi^2}{\sqrt{(3-\pi x)}}\right)$ $= \pi^2\left(-\dfrac{1}{2}\right)(3-\pi x)^{-\frac{3}{2}}(-\pi)$ $= \dfrac{\pi^3}{2}(3-\pi x)^{-\frac{3}{2}}$ $= \dfrac{\pi^3}{2\sqrt{(3-\pi x)^3}}$
(b)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{-x+2\ln x}\right)$ $= (\mathrm{e}^{-x+2\ln x})(-1+\frac{2}{x})$ $= (\mathrm{e}^{-x})(\mathrm{e}^{\ln x^2})(-1+\frac{2}{x})$ $= x^2\mathrm{e}^{-x}(-1+\frac{2}{x})$ $= x\mathrm{e}^{-x}(2-x)$ (shown) Alternatively (use of product rule) $\mathrm{e}^{-x+2\ln x} = \mathrm{e}^{-x}.\, \mathrm{e}^{2\ln x} = \mathrm{e}^{-x}.\, \mathrm{e}^{\ln x^2} = x$ $\therefore \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{-x+2\ln x}\right)$ $= \dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^2\mathrm{e}^{-x}\right)$ $= \mathrm{e}^{-x}(2x) + x^2\left(-\mathrm{e}^{-x}\right)$ $= x\mathrm{e}^{-x}(2-x)$

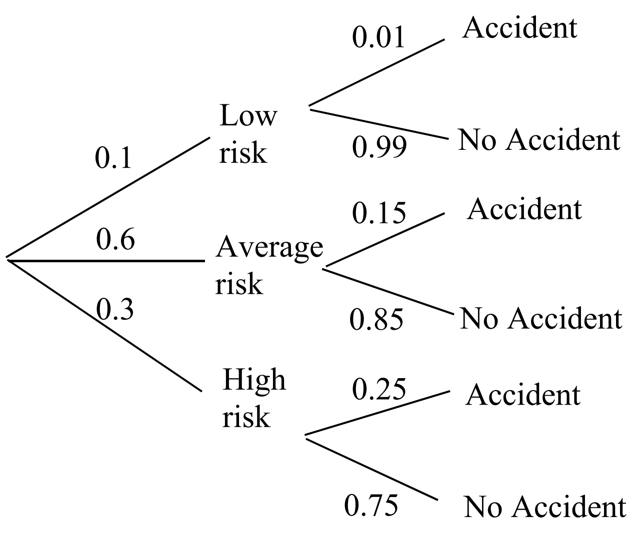
	$\int_1^{e} x e^{-x+a}(x-2)\,dx$ $= -e^{a}\int_1^{e} x e^{-x}(2-x)\,dx$ $= -e^{a}\left[e^{-x+2\ln x}\right]_1^{e}$ $= -e^{a}\left[e^{-e+2\ln e} - e^{-1+2\ln 1}\right]$ $= -e^{a}\left[e^{-e+2} - e^{-1}\right]$ $= e^{a-1} - e^{a-e+2}$
4(i)	Let r be the radius of water surface area Using similar triangles, $\frac{r}{6} = \frac{x}{15} \Rightarrow r = \frac{2}{5}x$ Volume of water, $V = \frac{1}{3}\pi\left(\frac{2x}{5}\right)^2 x$ $= \frac{4}{75}\pi x^3$ (shown)
(ii)	Given $\frac{dV}{dt} = 8$ From part (i), $\frac{dV}{dx} = \frac{4}{75}(3)\pi x^2 = \frac{4\pi x^2}{25}$ Using Chain Rule, $\frac{dx}{dt} = \frac{dx}{dV}\cdot\frac{dV}{dt} = \frac{25}{4\pi x^2}\times 8 = \frac{50}{\pi x^2}$ When $x = 5$, $\frac{dx}{dt} = \frac{50}{\pi(5)^2} = \frac{2}{\pi}$ cm/s The rate of increase of the depth of water is $\frac{2}{\pi}$ cm/s when x is 5 cm.
b	Let the height of the cylinder be h. By similar triangles, $\frac{r}{6} = \frac{15-h}{15} \Rightarrow h = 15 - \frac{5}{2}r$

	Total surface area of the cylinder, $A = 2\pi r^2 + 2\pi rh$ $= 2\pi r^2 + 2\pi r\left(15 - \frac{5}{2}r\right)$ $= 30\pi r - 3\pi r^2$ (shown) $\frac{dA}{dr} = 30\pi - 6\pi r$ $\frac{dA}{dr} = 0$ $30\pi - 6\pi r = 0 \Rightarrow r = 5$ $\frac{d^2A}{dr^2} = -6\pi < 0$ Total surface area is a maximum when $r = 5$. $\therefore$ maximum value of the total surface area of the cylinder $= 30\pi(5) - 3\pi(25) = 75\pi \text{ cm}^2$
5i	 
ii	Points of intersection are at $t = 3$ and $t = 15$ Hence duration = $15 - 3 =$ <u>12 minutes</u>
iii	Let d_1 and d_2 be the distance travelled by E_1 and E_2 respectively.

	$\mathrm{d}_1 = \frac{1}{2}(20)(\frac{1}{3}(20)) = \frac{200}{3}\,\mathrm{m}$ (or 66.7 m) $\mathrm{d}_2 = \int_{\frac{5}{2}}^{20} \sqrt{(2t-5)}\,\mathrm{d}t$ $= \left[\frac{2}{3}\frac{(2t-5)^{\frac{3}{2}}}{2}\right]_{\frac{5}{2}}^{20}$ $= \frac{1}{3}\sqrt{35^3}\,\mathrm{m}$ (or 69.0 m) Since $\mathrm{d}_2 > \mathrm{d}_1$, E_2 travelled a longer distance.
iv	Let v_1 and v_2 denote the speeds of E_1 and E_2. To have the same acceleration, $\frac{\mathrm{d}v_1}{\mathrm{d}t} = \frac{\mathrm{d}v_2}{\mathrm{d}t}$ $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{3}t\right) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\sqrt{(2t-5)}\right)$ $\frac{1}{3} = \frac{1}{2}(2t-5)^{-\frac{1}{2}}(2)$ $\frac{1}{3} = \frac{1}{\sqrt{(2t-5)}}$ $2t-5=9$ $t=7$ Hence the time at which they have the same acceleration is <u>00 07</u>
	Suppose the speeds of both particles is the same, ie. $v_1 = v_3$ $\frac{1}{3}t = \sqrt{(a-(t-5)^2)}$ $\frac{1}{9}t^2 = a-(t-5)^2$ $\frac{1}{9}t^2 = a - t^2 + 10t - 25$ $\frac{10}{9}t^2 - 10t + (25-a) = 0$ For the velocities to be always different, $10^2 - 4(\frac{10}{9})(25-a) < 0$ $100 - \frac{1000}{9} + \frac{40}{9}a < 0$ $a < \frac{5}{2}$ since a is positive, set of values of $a = \{a \in \square^{+} : a < \frac{5}{2}\}$

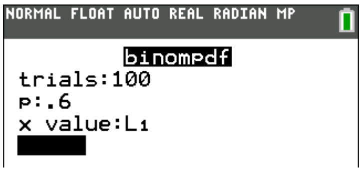
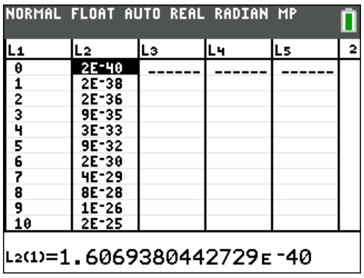
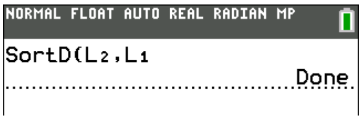
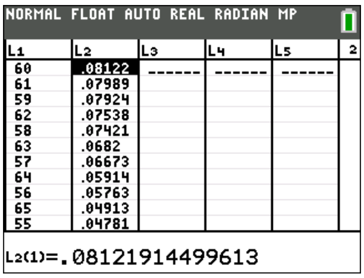
6i	Weights of people in the village are independent of each other.
(ii)	$\bar{x} = 1.965$ (from GC) Since $n=5$, $np \approx 1.965 \Rightarrow p \approx 0.393$

7(i)	No. of ways = 9! = 362880
ii)	No. of ways $= 9! - 7 \times 2 \times 7!$ $= 9! - 70560$ $= 292\ 320$
	Probability $= \dfrac{2 \times 2}{4!}$ $= \dfrac{1}{6}$ Probability $= \text{P(Tan siblings sit between parents} \mid \text{Wong family takes Row L)}$ $= \dfrac{\text{P(Tan siblings sit between parents and Wong family takes Row L)}}{\text{P(Wong family takes Row L)}}$ $= \dfrac{\frac{5! \times 2 \times 2}{9!}}{\frac{5!4!}{9!}}$ $= \frac{1}{6}$

<table>
<tr><td>8
(i)</td><td>P(holder is not involved in any accident | the holder is classified as “average” risk) = 0.85</td></tr>
<tr><td>(ii)</td><td>


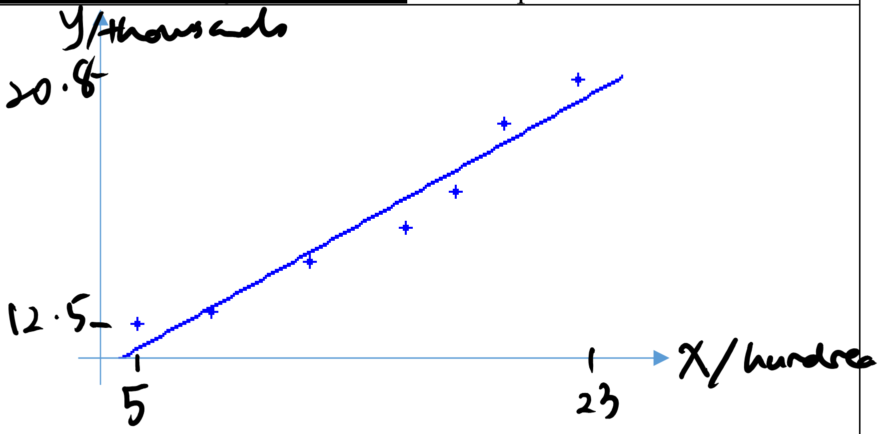
Probability of a randomly chosen policy holder not involved in any car accident
= (0.1)(0.99) + (0.6)(0.85) + (0.3)(0.75)
= 0.834</td></tr>
<tr><td>(iii)</td><td>P(policy holder is “low risk” | has met at least one car accident)</td></tr>
</table>

	$= \dfrac{\text{P(holder is classified as "low" risk and met with at least 1 accident)}}{\text{P(holder meets with at least 1 accident)}}$ $= \dfrac{0.1(0.01)}{1-0.834}$ $= \dfrac{1}{166} = 0.00602$ (3 s.f.)
(iv)	Probability = 2(0.834)(1-0.834) =0.276888 (exact)

9 (i)	Let X denote the no. of questions he can answer correctly out of n. $X \sim \mathrm{B}(n, 0.6)$ if $n = 100$, Variance $= npq = 100(0.6)(0.4) = 24$ (verified)
ii	$X \sim \mathrm{B}(100, 0.6)$      $\mathrm{P}(X = 59) = 0.07924$ $\mathrm{P}(X = 60) = 0.08122$ $\mathrm{P}(X = 61) = 0.07989$ The most probable number of questions answered correctly is <u>60.</u>
(iii)	Required probability $= \mathrm{P}(X \geq 50)$ $= 1 - \mathrm{P}(X \leq 49)$ $= 0.98324$ (to 5sf) $= 0.983$ (to 3sf) (shown)
iv	Let Y denote the no. of exams out of m that he passed. $Y \sim \mathrm{B}(m, 0.983)$

	$\mathrm{P}(Y=m)\le 0.904$ $\binom{m}{m}0.983^m(1-0.983)^0 \le 0.904$ $0.983^m \le 0.904$ $m\lg 0.983 \le \lg 0.904$ $m \ge 5.88621$ least $m=6$
	$X \sim \mathrm{B}(100, 0.6)$ $\mathrm{E}(X) = 60$ $\mathrm{Var}(X) = 24$ By CLT, since $n = 40$ is large, $\overline{X} \sim \mathrm{N}\left(60, \frac{24}{40}\right)$ approximately $\mathrm{P}(\overline{X} \le 58) = 0.0049117 = 0.00491$ (3s.f.)

10 (i)	Let Y be the score of Group Y students. $\mathrm{P}(Y \ge a) \ge 0.6$ $\mathrm{P}(Y < a) < 0.6$ Thus $a < 32.733$ The maximum mark is 32.7
(ii)	$\mathrm{E}\left(Y_1+Y_2+Y_3+Y_4-3X\right) = 4\mathrm{E}\left(Y\right)-3\mathrm{E}\left(X\right) = -29$ $\mathrm{Var}\left(Y_1+Y_2+Y_3+Y_4-3X\right) = 4\mathrm{Var}\left(Y\right)+9\mathrm{Var}\left(X\right) = 280$ $\therefore Y_1+Y_2+Y_3+Y_4-3X \sim \mathrm{N}\left(-29, 280\right)$ $\mathrm{P}(Y_1+Y_2+Y_3+Y_4<3X) = \mathrm{P}(Y_1+Y_2+Y_3+Y_4-3X<0) = 0.958$
(iii)	$\overline{M} = \dfrac{X_1+\ldots+X_{20}+Y_1+\ldots+Y_{20}}{40}$ $\mathrm{E}\left(\overline{M}\right) = \dfrac{20\mathrm{E}(X)+20\mathrm{E}(Y)}{40} = \dfrac{1}{2}\left(\mathrm{E}(X)+\mathrm{E}(Y)\right) = 44.5$ Let $\sigma^2 = \mathrm{Var}\left(\overline{M}\right)$ $= \dfrac{1}{1600}\left(20\mathrm{Var}(X)+20\mathrm{Var}(Y)\right)$ $= \dfrac{1}{80}\left(\mathrm{Var}(X)+\mathrm{Var}(Y)\right) = 0.5625$ $\overline{M} \sim N\left(44.5, 0.5625\right)$ Since $\mathrm{P}(-k<\overline{M}-44.5<k)=0.9545$ $\therefore 44.5-k=43.000$ $\Rightarrow k=1.50$ (3 s.f)  **<u>Alternative</u>**

<table>
<tr><td></td><td>$\overline{M} \sim N\left(44.5, \sigma^2\right)$
Since $\mathrm{P}(\lvert\overline{M} - 44.5\rvert < 2\sigma) = 0.9545$
$\therefore k = 2\sigma = 2\sqrt{0.5625} = 1.50$ (3sf)</td></tr>
<tr><td></td><td><u>Marks obtained by the students</u> are independent of one another.</td></tr>
<tr><td>11
i
ii</td><td>

r = 0.97139 = 0.971 (3 s.f.)

The equation of y on x :
$y = 9.3484 + 0.46531x$
$y = 9.35 + 0.465x$ (3 s.f.)</td></tr>
<tr><td>iii</td><td>Since x is the independent variable, y on x should be used for the estimation.
For $y = 15$,
$x = 12.146 = 12$
The advertising expenditure is <u>$12,000.</u>

This estimate is reliable because :
- r is close to 1 which indicates a strong positive linear correlation between x and y.
- $y = 15$ is within the given data range (interpolation), $12.5 < y < 20.8$.</td></tr>
<tr><td>iv</td><td>b is the gradient of the regression line which indicates that with every $100 spent on advertising in a month, there is an increase of $465 in the sale of refrigerators.</td></tr>
<tr><td>v</td><td>There would be no change to b.</td></tr>
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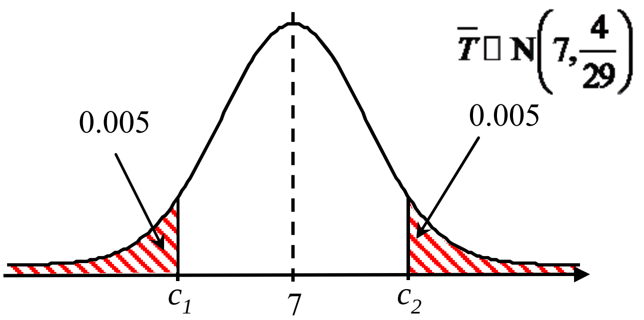
12 **a**	$\bar{x} = \frac{\Sigma(x-30)}{60} + 30 = 30.4$ $s^2 = \frac{1}{59}\left[\Sigma(x-30)^2 - \frac{(\Sigma(x-30))^2}{60}\right] = 2.2780 \text{ (5 s.f.)}$ $H_0 : \mu = 30$ $H_1 : \mu > 30$ Conduct a 1-tail test at $2\frac{1}{2}$% significance level. Under H_0, $\bar{X} \sim N(30, \frac{2.2780}{60})$ approximately. Using a z-test, p-value = $P(\bar{X} > 30.4) = 0.020043 = 0.0200$ (3 s.f.) Since p-value < 0.025, we reject Ho and conclude that there is sufficient evidence at $2\frac{1}{2}$% significance level that the mean centre thickness of the soft contact lenses are more than 30 um. Ie. The claim is not justified.
	It means that there is a probability of 0.025 of wrongly rejecting the claim that the mean centre thickness of the soft contact lenses is at most 30 um.
b **(i)**	Let μ be the mean of X. $H_0 : \mu = 7$ $H_1 : \mu \neq 7$ $s^2 = \frac{30}{29}(\text{sample variance}) = \frac{30}{29}(4) = \frac{120}{29}$ Under H_0, since the sample size is large, the test statistic is $\bar{T} \sim N\left(7, \frac{4}{29}\right)$ approximately by Central Limit Theorem.
(ii)	Since the claim is rejected i.e. to reject H_0 at 1% significance level.  From GC, $c_1 = 6.04$ and $c_2 = 7.96$. $\bar{t} \leq 6.04$ or $\bar{t} \geq 7.96$ (3 .s.f)

(iii)	From the two tail test, we know that $\text{p-value (two tail)} \le 0.01$. For a one-tail test, $\text{p-value(one tail)} = \dfrac{\text{p-value (two tail)}}{2} \le 0.005 < 0.01$, therefore we reject H_0 and conclude that there is sufficient evidence at 1% significance level to say that mean waiting time is more than 7 minutes.