



VICTORIA JUNIOR COLLEGE

JC 2 PRELIMINARY EXAMINATION 2017

H1 MATHEMATICS

8865/01

Paper 1

3 hours

Additional Materials: Answer Paper
Graph Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and CT group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

Section A: Pure Mathematics (40 marks)

1 A certain brand of razor blades comes in packages of 6, 12 and 24 blades, costing \$2, \$3 and \$4 per package, respectively. A store sold 12 packages of razor blades containing a total of 162 blades for \$35. How many packages of each type were sold? [4]

2 The equation of a curve is $y = \frac{2}{1+x^2} - 1$. The curve intersects the y -axis at A and the x -axis at B and C . Show that both tangents to the curve at B and C pass through A . [5]

3 (a) Differentiate $\ln\left(\frac{5x+2}{4x^2}\right)$. [3]

(b) (i) Differentiate $e^{\sqrt{x}}$. [1]

(ii) Use a non-calculator method to find the exact value of $\int_1^4 \frac{e^{\sqrt{x}+\ln 3}}{\sqrt{x}} dx$. [4]

4 Malignant tumours respond to chemotherapy. A medical experiment was conducted, in which mice with tumours were given a chemotherapeutic drug. The tumour volume $V \text{ cm}^3$ after t days can be modelled by the equation

$$V = 0.005e^{0.24t} + 0.495e^{-0.12t}, \text{ for } 0 \leq t \leq 18.$$

(i) Use differentiation and this model to find the minimum value of V , justifying that this value is a minimum. [6]

(ii) Sketch the graph of V against t . [2]

(iii) Find the rate at which the tumour volume is decreasing when $t = 5$. [1]

5 Sketch the graphs of $y = 5 - 3x$ and $y = \ln(x+1)$ on the same diagram. [3]

(i) Hence solve the inequality $6x + \ln(x+1)^2 < 10$. [3]

(ii) Find the area of the finite regions bounded by the two graphs, the y -axis and the line $x = 3$. [4]

The line $y = 5 - 3x$ intersects the curve $4x^2 + y^2 = 4k^2$ at two distinct points. Find the set of possible values of k in an exact form. [4]

Section B: Statistics (60 marks)

6 A box contains 6 chocolates with hard centres and 8 chocolates with soft centres. Four chocolates are chosen at random.

(i) Calculate the probability that two of the chocolates have hard centres. [3]

(ii) Calculate the probability that at most three of the chocolates have soft centres. [2]

7 In a computer game, a player is required to play 3 stages. The probability of a player winning the first stage is $\frac{1}{10}$. For each of the second and third stage, the probability of a player winning is

- same as the probability of winning the preceding stage if he won that stage,
- twice the probability of winning the preceding stage if he lost that stage.

Draw a tree diagram to represent this information. [3]

A is the event: The player won his third stage.

B is the event: The player won exactly 2 stages.

Describe what is meant by the probability $P(A \cap B)$ and find this probability. [2]

Hence, or otherwise, determine if A and B are independent. [4]

8 The masses of cabbages sold at a certain supermarket were normally distributed with mean 550 grams and standard deviation 20 grams. The selling price for the cabbages is \$0.60 per 100 grams.

(i) Find the probability that a randomly chosen cabbage has a mass exceeding 575 grams. [2]

(ii) Find the smallest mass exceeded by at most 20% of the cabbages, giving your answer correct to 1 decimal place. [2]

(iii) Write down the mean and variance of the cost of a cabbage and find the probability that a randomly chosen cabbage costs less than \$3.20. [3]

Some shoppers gave feedback that the cabbages are too big. The supermarket now sells the cabbages as a whole and also by halves.

(iv) Find the probability that the total masses of two randomly chosen halves is more than the mass of a randomly chosen whole cabbage by less than 50 grams. [4]

[Turn over

- 9 In an opinion poll before the election of student council president in a Junior College, 30 students were interviewed when they were having their lunch break in the canteen. Explain why this sample may not be representative of the population. [1]

A random sample of 30 students were chosen and interviewed. Let X be the number of students from this sample who support candidate A .

- (i) State, in context, two assumptions needed for X to be well modelled by a binomial distribution. [2]

Assume that $X \sim B\left(30, \frac{4}{9}\right)$. Find

- (ii) the most likely number of students who support candidate A , [2]
 (iii) the probability that X is within 1 standard deviation from its mean. [4]

It is known that $p\%$ of the students support candidate B and that the probability of at most 1 student from a random sample of 30 support candidate B is 0.3. Show that p satisfies the equation $(1 - 0.01p)^{29}(1 + 0.29p) = 0.3$ and hence find the value of p , correct to 2 decimal places. [4]

- 10 A farmer claims that the average yield of the apple trees in his plantation has increased from 98.5 kg after using a new fertiliser. The yields of a random sample of 25 apple trees after using the new fertiliser are summarised as follows, where x kg denotes the yield of an apple tree in his plantation.

$$\bar{x} = 99.7 \quad \sum (x - \bar{x})^2 = 178$$

- (i) Test, at 5% level of significance, whether the farmer's claim should be rejected. [4]
 (ii) State, with a reason, whether it is necessary to assume a normal distribution for the test to be valid. [1]
 (iii) State the meaning of '5% level of significance' in the context of this question. [1]

The yields of pear trees in the same plantation were known to follow a normal distribution with mean 102 kg and variance 7.49 kg^2 . After using the new fertiliser, the yields of a random sample of 20 pear trees were recorded. Using a 5% significance test, it was found that there was no significant difference in the mean yield of the pear trees.

- (iv) Find the set of values of the mean yield of this sample. [3]
 (v) State an assumption needed for your calculation in part (iv). [1]
 (vi) State, giving a reason, what you can conclude from the test if the level of significance is changed to 1%. [1]

- 11** The Mathematics score, x , and the English score, y , of 8 Primary Four students during a year end examination are given in the following table.

Student	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>
x	37	41	49	52	53	57	72	75
y	73	64	53	65	50	57	65	45

- (i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]
- (ii) Find the product moment correlation coefficient. [1]
- (iii) The least squares regression line of y on x is used to calculate an estimate of the English score for a student who scored 48 for Mathematics. State, with reasons, whether the estimate will be a reliable one. [2]

It is discovered that student G 's English score is wrongly recorded.

- (iv) Find student G 's English score if the correct equation of the regression line of y on x is $y = 88.722 - 0.57976x$. [3]
- (v) Based on the revised data with the correct English score for student G as obtained in (iv), calculate an estimate of a student's Mathematics score if his English score is 75. Comment on the reliability of the estimate. [3]

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