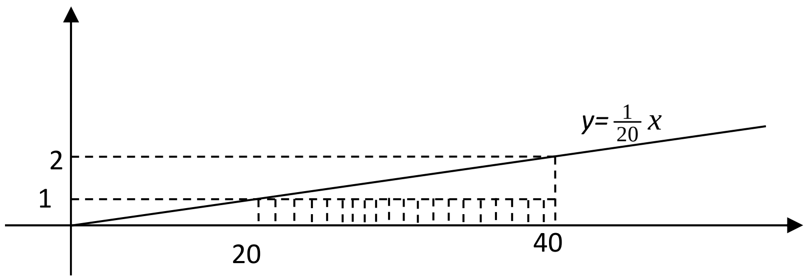
Anglo-Chinese Junior College
H2 Mathematics 9740
2016 JC 2 Pelim Paper 2 Solutions

Qn	Solutions	Remarks
1	 Number of squares for $20 \le x \le 40$ is 20	

Column	Range of x	Number of squares
1	$20 \le x \le 40$	20
2	$40 \le x \le 60$	40
3	$60 \le x \le 80$	60
	.	
	.	
n	$380 \le x \le 400$	

Number of columns = $n = \dfrac{380}{20} = 19$ for $0 \le x \le 400$

OR 380 = 20 + (n-1)(20) $\Rightarrow n$ =19

OR 400 = 40 + (n-1)(20) $\Rightarrow n$ =19

AP sequence : 20, 40, 60, …….. 19 terms

Number of complete squares in region R for $0 \le x \le 400$ is

$\dfrac{19}{2}\left[2(20)+18(20)\right] = 3800$

Region R is symmetrical about the line x = 400.

Total number of squares in region R = 3800 x 2 = 7600

2(i)

$x = a\cos^3 t, \quad y = a\sin^3 t \quad for \quad 0 \le t \le \pi$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t} = -\tan t$$

$Let \ \dfrac{dy}{dx} = 0$

$-\tan t = 0 \Rightarrow t = 0, \pi$

Points on curve where tangent is parallel to x-axis are $(a,0)$ and $(-a,0)$

Let $\dfrac{dx}{dy} = 0 \Rightarrow \dfrac{1}{-\tan t} = 0 \Rightarrow t = \dfrac{\pi}{2}$

Point on curve where tangent is parallel to y-axis are $(0,a)$

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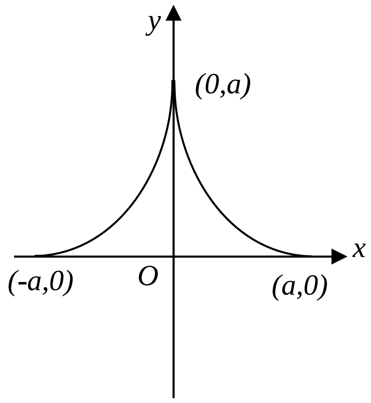
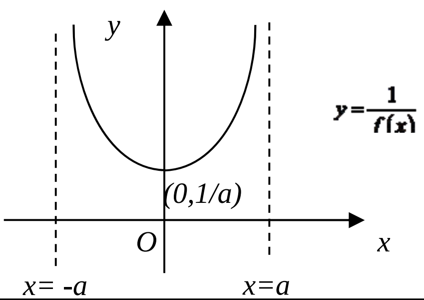
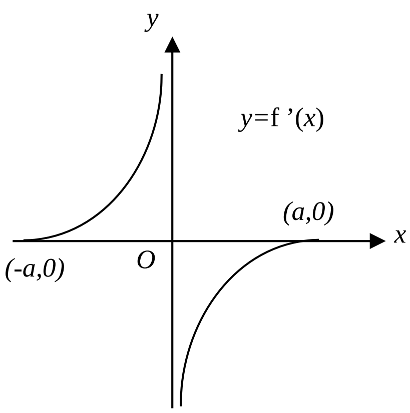
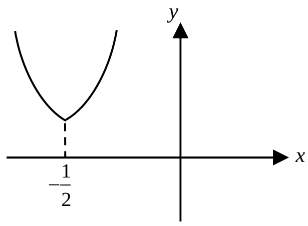
2 (ii)		
2 (iii) (a)		
2 (iii) (b)		
2 (iv)	Equation of tangent at point P with parameter p: $y-a\sin^3 p=-\tan p(x-a\cos^3 p)$ $y\cos p-a\sin^3 p\cos p=-\sin p(x-a\cos^3 p)$ $x\sin p+y\cos p=\left(a\sin p\cos p\right)(\cos^2 p+\sin^2 p)$ $x\sin p+y\cos p=a\sin p\cos p\ \ (shown)$	
2 (v)	Given $p=\frac{\pi}{3}$, gradient of tangent at P is $-\tan\frac{\pi}{3}=-\sqrt{3}$ $\therefore$ gradient of tangent at Q $=\frac{1}{\sqrt{3}}$ Value of parmeter at $Q:-\tan t=\frac{1}{\sqrt{3}}\Rightarrow t=\frac{5\pi}{6}$	

3

(i) Sketch y=f(x)



h(x) has an inverse if $x \le -\frac{1}{2}$. Greatest value of k is $-\frac{1}{2}$

$y - 1 = e^{|2x+1|}$

$\ln(y - 1) = |2x+1|$

$2x + 1 = \pm\ln(y-1)$

$x = -\frac{1}{2} \pm \frac{1}{2}\ln(y-1)$

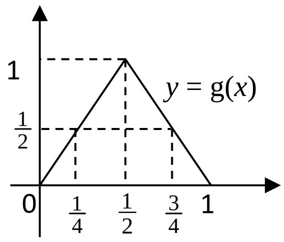
$x = -\frac{1}{2} - \frac{1}{2}\ln(y-1)$ since $x \le -\frac{1}{2}$

$h^{-1}(x) = -\frac{1}{2} - \frac{1}{2}\ln(x-1)$

$h(-\frac{1}{2}) = 2$

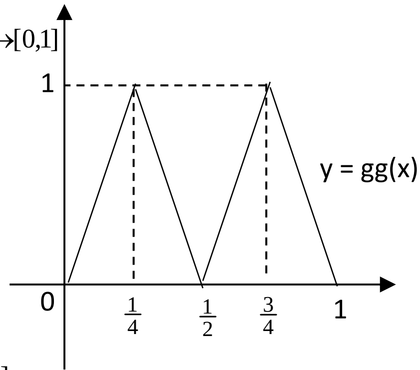
Domain of $h^{-1}(x)$ = Range of h(x) = $[2, \infty)$

(ii)



Composite function exist because
$R_g = [0,1] \subseteq D_g = [0,1]$

(iii) $[0, \frac{1}{4}] \xrightarrow{g} [0, \frac{1}{2}] \xrightarrow{g} [0,1]$



$[\frac{1}{4}, \frac{1}{2}] \xrightarrow{g} [\frac{1}{2}, 1] \xrightarrow{g} [0,1]$

$[\frac{1}{2}, \frac{3}{4}] \xrightarrow{g} [\frac{1}{2}, 1] \xrightarrow{g} [0,1]$

$[\frac{3}{4}, 1] \xrightarrow{g} [0, \frac{1}{2}] \xrightarrow{g} [0,1]$

	(iv) $[0,1] \xrightarrow{g} [0,1] \xrightarrow{f} \left[e+1,\ e^3+1\right]$ $R_{fg} = \left[e+1,\ e^3+1\right]$	
4 (i)	$\begin{pmatrix}1\\1\\b\end{pmatrix}\cdot\begin{pmatrix}-3\\4\\1\end{pmatrix}=0$ $-3+4+b=0$ $b=-1$ **Alternatively,** $\begin{pmatrix}1\\1\\b\end{pmatrix}\times\begin{pmatrix}2\\3\\-6\end{pmatrix}=k\begin{pmatrix}-3\\4\\1\end{pmatrix}$ $\begin{pmatrix}-6-3b\\6+2b\\1\end{pmatrix}=k\begin{pmatrix}-3\\4\\1\end{pmatrix}$ Equating **k** component, $k=1.$ Equating **i** component, $-6-3b=-3k$ $\therefore -3b=-3(1)+6$ $b=-1$ (Shown)	
4 (ii)	Line RS: $\mathbf{r}=\begin{pmatrix}-2\\4\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-1\end{pmatrix},\ \lambda\in\mathbb{R}$ Substitute this line to p_1, $\begin{pmatrix}-2+\lambda\\4+\lambda\\1-\lambda\end{pmatrix}\cdot\begin{pmatrix}2\\3\\-6\end{pmatrix}=10$ $\lambda=\frac{8}{11}$ $\overrightarrow{OS}=\begin{pmatrix}-2+\lambda\\4+\lambda\\1-\lambda\end{pmatrix}=\begin{pmatrix}-\frac{14}{11}\\\frac{52}{11}\\\frac{3}{11}\end{pmatrix}$	

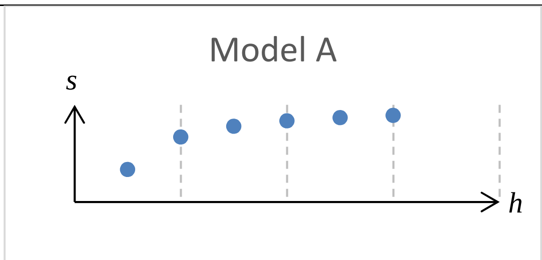
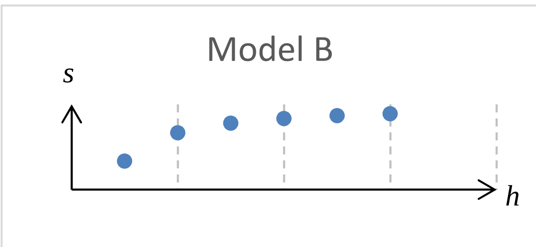
4 (iii)	$p_1: \quad r.\dfrac{\begin{pmatrix}2\\3\\-6\end{pmatrix}}{7}=\dfrac{10}{7}$ $p_2: \quad r.\dfrac{\begin{pmatrix}2\\3\\-6\end{pmatrix}}{7}=\dfrac{-a}{7}$ distance between two planes : $\dfrac{8}{7}=\dfrac{10-(-a)}{7}$ or $\dfrac{8}{7}=\dfrac{(-a)-10}{7}$ a = -2 or a = -18	
4 (iv)	$p_1: \quad \begin{pmatrix}5\\2\\c\end{pmatrix}.\begin{pmatrix}2\\3\\-6\end{pmatrix}=10$ $10+6-6c=10$ $c=1$	
4 (v)	Let point A be (5, 0, 0) which is a point on line l. $PF=\dfrac{\left\lvert \overline{AP}\times\begin{pmatrix}-3\\4\\1\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}-3\\4\\1\end{pmatrix}\right\rvert}=\dfrac{\left\lvert\begin{pmatrix}0\\2\\1\end{pmatrix}\times\begin{pmatrix}-3\\4\\1\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}-3\\4\\1\end{pmatrix}\right\rvert}=\dfrac{\left\lvert\begin{pmatrix}-2\\-3\\6\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}-3\\4\\1\end{pmatrix}\right\rvert}$ $=\dfrac{7}{\sqrt{26}}=1.3728$ $QF=2\left(\dfrac{8}{7}\right)=\dfrac{16}{7}$ Area PFQ = $\dfrac{1}{2}(1.3728)\left(\dfrac{16}{7}\right)=1.5689=1.57$	

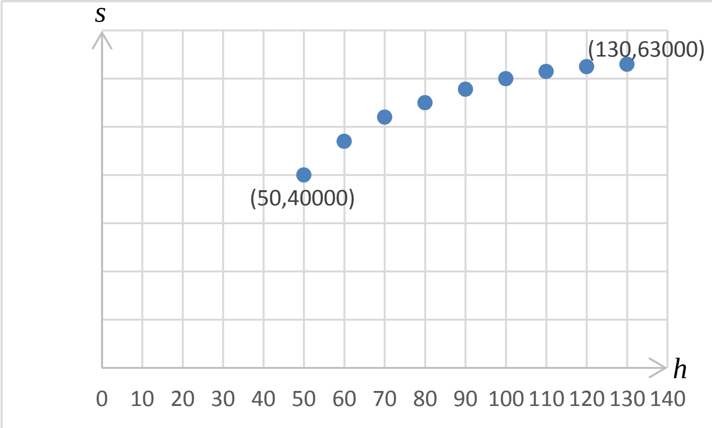
	<u>Alternative method:</u> $\overrightarrow{OF} = \begin{pmatrix} 5-3\lambda \\ 4\lambda \\ \lambda \end{pmatrix}$ $\overrightarrow{FP} = \overrightarrow{OP} - \overrightarrow{OF} = \begin{pmatrix} 3\lambda \\ 2-4\lambda \\ 1-\lambda \end{pmatrix}$ Since $\overrightarrow{FP} \perp \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 3\lambda \\ 2-4\lambda \\ 1-\lambda \end{pmatrix} . \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix} = 0$ $-9\lambda + 8 - 16\lambda + 1 - \lambda = 0$ $\lambda = \frac{9}{26}$ $\therefore \left\|\overrightarrow{FP}\right\| = \left\| \begin{pmatrix} (3)\left(\frac{9}{26}\right) \\ 2-(4)\left(\frac{9}{26}\right) \\ 1-\left(\frac{9}{26}\right) \end{pmatrix} \right\| = \left\| \begin{pmatrix} \frac{27}{26} \\ \frac{8}{13} \\ \frac{17}{26} \end{pmatrix} \right\| = 1.3728$ $QF = 2\left(\frac{8}{7}\right) = \frac{16}{7}$ Area PFQ = $\frac{1}{2}(1.3728)\left(\frac{16}{7}\right) = 1.5689 = 1.57$	
5 (i)	Number the club members in order from 1 to 15000 according to the name list. (Alphabetical order) Since $k = \frac{15000}{500} = 30$, select a member randomly from the name list. Thereafter, select every 30th member cycling to the start of the list if the end of list is reached until we form a sample of 500 members.	
5(ii)	There is a bias when the name list of the members of the fitness club have a **periodic** or **cyclic** pattern i.e. there is some pattern in the way the names is arranged and the pattern coincides in some way with the sampling interval of 30.	
6	Let the random variable L be the length of a randomly chosen glass panel. Let the random variable B be the breadth of a randomly	

	chosen glass panel. Let $X = 2L_1 + 2B_1 + 2L_2 + 2B_2$ $\therefore X \sim N(1800, 2.32)$ Let $\overline{L} = \dfrac{L_1 + L_2 + L_3 + ... + L_n}{n}$ $\therefore \overline{L} \sim N\left(300, \dfrac{0.5^2}{n}\right)$ $X - \overline{L} \sim N\left(1500, 2.32 + \dfrac{0.5^2}{n}\right)$ $\mathrm{P}\left(X - \overline{L} > 1501\right) < 0.2576$ $\mathrm{P}\left(Z > \dfrac{1501-1500}{\sqrt{2.32 + \dfrac{0.5^2}{n}}}\right) < 0.2576$ $\mathrm{P}\left(Z < \dfrac{1}{\sqrt{2.32 + \dfrac{0.5^2}{n}}}\right) > 0.7424$ $\dfrac{1}{\sqrt{2.32 + \dfrac{0.5^2}{n}}} > 0.6507622042$ $\sqrt{2.32 + \dfrac{0.5^2}{n}} < 1.536659618$ $2.32 + \dfrac{0.5^2}{n} < 2.361322781$ $\dfrac{0.5^2}{n} < 0.0413227807$ $n > 6.049931679$ Least $n = 7$.	
7 (i)	Let the random variable X be the number of guests checking into the hotel in a given hour. $\mathrm{P}(X \le 4) = 0.7064384499 = 0.706$ (3s.f)	

7 (ii)	Required probability $=\left[\mathrm{P}\left(X\le 4\right)\right]\times\left[\mathrm{P}\left(X=0\right)\right]^2\times\frac{3!}{2!}$ $=0.0015822508$ $=0.00158$ (3s.f)	
7 (iii)	Let the random variable Y be the number of guests checking into the hotel in a day. $Y \sim P_O\left(86.4\right)$ Since $\lambda>10$, $Y\sim N\left(86.4, 86.4\right)$ approx. $\mathrm{P}\left(85<X<90\right)\overset{c.c}{\approx}\mathrm{P}\left(85.5<X<89.5\right)=0.169$ (3s.f)	
7 (iv)	Let the random variable W be the number of one-hour blocks in a day, which has not more than 4 guests checking into the hotel. $W\sim B\left(24, 0.7064384499\right)$ $\mathrm{P}\left(W\ge n\right)<0.124$ $\mathrm{P}\left(W\le n-1\right)>0.876$ Using G.C, $\mathrm{P}\left(W\le 19\right)-0.876>-2.797\times10^{-4}$ $\mathrm{P}\left(W\le 20\right)-0.876>0.7534$ $\therefore n-1=20$ Least $n=21$.	
7 (v)	This is because the **mean** number of guests checking into the hotel **per hour** is **unlikely to be constant throughout the year**. The number of guests checking into the hotel is likely to vary across different months in a year due to **seasonal fluctuations** caused by factors such as the **holiday seasons**, etc. Hence a Poisson distribution may not be a good model.	
8 (i)	Let random variable X be a randomly chosen Hono's lunch waiting time at North Vista outlet. To test $H_0:\ \mu=60$ Against $H_1:\ \mu<60$ at 10% level of significance. Under H_0, $T=\frac{\overline{X}-\mu_0}{s/\sqrt{n}}\sim t(n-1)$, i.e. $T=\frac{\overline{X}-60}{s/\sqrt{10}}\sim t(9)$	

	Value of test statistic: $t = \dfrac{\overline{x} - \mu_0}{s / \sqrt{n}}$ $= \dfrac{52.1 - 60}{14.9328 / \sqrt{10}} = -1.6729597$ *p-value* = $= 0.064332 < 0.1$ $\therefore$ Reject H_0 and conclude that there is sufficient evidence at 10% level of significance that the average waiting time during lunch periods at North Vista is less than one hour.	
8 (ii)	Unbiased estimate of population variance is $s^2 = \left(\dfrac{n}{n-1}\right)(\text{sample variance}) = \left(\dfrac{56}{55}\right)(69.8) = 71.06909$ To test $H_0: \ \mu = 60$ Against $H_1: \ \mu \neq 60$ at 5% level of significance. Under H_0, $Z = \dfrac{\overline{T} - 60}{s/\sqrt{56}} \sim N(0,1)$ *approx.*, Value of test statistic: $z = \dfrac{\overline{t} - 60}{\sqrt{71.06909} / \sqrt{56}} = \dfrac{\overline{t} - 60}{1.12654}$ Since the null hypothesis is rejected, $\dfrac{\overline{t} - 60}{1.12654} < -1.959964$ or $\dfrac{\overline{t} - 60}{1.12654} > 1.959964$ $\overline{t} < 57.792$ or $\overline{t} > 62.208$	
9 (a) (i)	$p\,(0.72) + (1 - p)\,(0.28) = 0.44p + 0.28$	
(ii)	$(0.44p + 0.28)(0.6) + 1 - (0.44p + 0.28)$ $= 0.888 - 0.176p$ Since $0 < p < 1$, $0 > -0.176p > -0.176$ $0.888 > 0.888 - 0.176p > 0.888 - 0.176$ $0.888 > 0.888 - 0.176p > 0.712$ $\therefore 0.712 < 0.888 - 0.176p < 0.888$	

(b)	Since *n* is large, by Central Limit Theorem, $\overline{X}=\dfrac{X_1+X_2+\ldots+X_{50}}{50}\sim N\left(4.8,\ \dfrac{1.44}{50}\right)$ approx. Required probability $=\mathrm{P}\left(\overline{X}<5\right)=0.8807035=0.881$ (3s.f)	
(ci)	If Aaron is the goalkeeper: ${}^6C_4\,{}^5C_4\,{}^5C_2=750$ If Aaron is the forward: ${}^6C_4\,{}^5C_4\,{}^5C_1=375$ Number of ways required = 1125	
(cii)	Number of ways required = $5!6!8!$ = 3483648000	
(ciii)	Required probability $=\dfrac{\mathrm{P}(\text{goalkeepers are opposite \& defenders with 2 particular midfielders are together})}{\mathrm{P}(\text{defenders with 2 particular midfielders are together})}$ $=\dfrac{8!2!8!}{(11-1)!8!}=\dfrac{2!}{10\times 9}=\dfrac{1}{45}$	
10 (i)	  	

10 (ii)		
10 (iii)	A Linear model will not be approriate. This is because the scatter diagram indicates that as h increases, s is increasing at a decreasing rate which is not a linear realtionship. Furthermore, a linear model will mean that the product's monthly sales in Singapore will increase indefinitely with the increase of the number of promoters. This is not realistic in the context of the question as the product's monthly sales will likely slow down and perhaps decrease due to market stauration.	
10 (iv)	Correlation coefficent of s on $\ln(h)$ = 0.981 Correlation coefficent of s on $1/h$ = -0.998 Since correlation coefficent of s on $1/h$ is stronger, hence use least square regression line of s on $1/h$. Least square regression line of s on $1/h$: $s = 78531.62777 - \dfrac{1896285.284}{h}$ $s = 78531.628 - \dfrac{1896285.284}{h}\ (3\text{ d.p})$ when $s = 75000$, $75000 = 78531.62777 - \dfrac{1896285.284}{h}$ $h = 536.9437001 \approx 537$ (nearest integer).	

10 (v)	The estimate is not reliable as s=75000 does not lie within $40000 \le s \le 63000$. Hence, we are extrapolating.	
10 (vi)	$s = 78531.62777 - \dfrac{1896285.284}{h}$ $\dfrac{s}{1.34} = \dfrac{78531.62777}{1.34} - \dfrac{1896285.284}{1.34h}$ $\dfrac{s}{1.34} = 58605.692365671 - \dfrac{1415138.2716417}{h}$ $\Rightarrow\ u = 58605.692 - \dfrac{1415138.272}{h}$ (3 dp) where u = monthly sales in US dollars (i.e. $u = \dfrac{s}{1.34}$)	
11 (i)	The probability that each salesman is successful in closing a deal is assumed to be constant.	
11 (ii)	<u>Assumption: Deals closed are independent of one another</u> Deals closed may not be independent of one another as customers may collaborate to buy cars as a group for better bargaining power.	
11 (iii)	$X \sim \mathrm{B}(60, 0.2)$ Since $np = 12 > 5$ and $nq = 48 > 5$, $X \sim \mathrm{N}(12, 9.6)$ *approx.* $Y \sim \mathrm{B}(50, 0.3)$ Since $np = 15 > 5$ and $nq = 35 > 5$, $Y \sim \mathrm{N}(15, 10.5)$ *approx.* $X + Y \sim \mathrm{N}(27, 20.1)$ *approx.* $\mathrm{P}(X+Y > 20) \overset{c.c}{\approx} \mathrm{P}(X+Y > 20.5) = 0.926\ (3\text{s.f})$	

11 (iv)	$\mathrm{P}(C=30)=0.03014$ ${}^{60}C_{30}\times p^{30}\times(1-p)^{30}=0.03014$ $p^2-p+0.2399991029=0$ $\therefore\ p=0.6$ or 0.3999955 Since $p<0.5$, $p=0.4$ (1 d.p)	
11 (v)	$C\sim \mathrm{B}(60,0.05)$ Since $n=60>50$ such that $np=3<5$, $C\sim P_O(3)$ *approx.* $\mathrm{P}(C>4)=0.185$ (3s.f)	