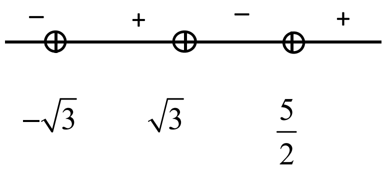
2016 ACJC JC2 H2 Mathematics 9740

Preliminary Examination Paper 1 Solutions

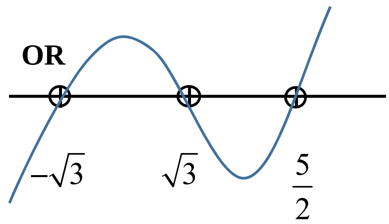
Qn	Solution	Remarks

1

$$\frac{2w-5}{w^2-3}>0$$



OR



$-\sqrt{3}<w<\sqrt{3}$ or $w>\frac{5}{2}$

$$\frac{\left(2|y|-5\right)\sin x}{y^2-3}\le 0,$$

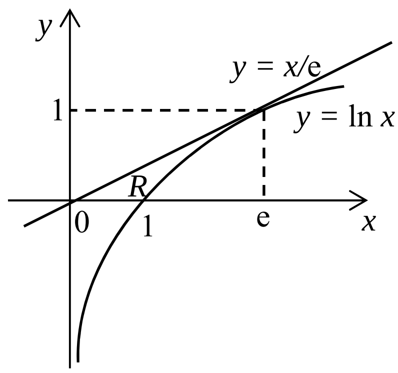
Since $\sin x<0$ for $\pi<x\le\frac{3\pi}{2}$, $\quad \frac{2|y|-5}{y^2-3}\ge 0$

From above, $\quad -\sqrt{3}<|y|<\sqrt{3}$ or $|y|\ge\frac{5}{2}$

$0\le|y|<\sqrt{3}$ or $|y|\ge\frac{5}{2}$

$-\sqrt{3}<y<\sqrt{3}$ or $y\ge\frac{5}{2}$ or $y\le-\frac{5}{2}$

2



Method 1 - Integration

Volume of solid S

$$=\pi\int_0^e\left(\frac{x}{e}\right)^2 dx-\pi\int_1^e(\ln x)^2\,dx$$

$$=\frac{\pi}{e^2}\int_0^e x^2dx-\pi\int_1^e(\ln x)^2\,dx$$

$$=\frac{\pi}{e^2}\left[\frac{x^3}{3}\right]_0^e-\pi\int_1^e(\ln x)^2\,dx$$

Method 2 – Volume of Cone

Volume of solid S

$$=\text{Vol of cone}-\pi\int_1^e(\ln x)^2\,dx$$

$$=\frac{1}{3}\pi(1)^2(e)-\pi\int_1^e(\ln x)^2\,dx$$

$$=\frac{1}{3}\pi e-\pi\int_1^e(\ln x)^2\,dx$$

	$=\frac{\pi}{e^2}\left[\frac{e^3}{3}\right]-\pi\int_1^e(\ln x)^2\,dx$ $=\frac{1}{3}\pi e-\pi\int_1^e(\ln x)^2\,dx$ $\int_1^e(\ln x)^2\,dx$ $=\left[x(\ln x)^2\right]_1^e-2\int_1^e\ln x\,dx$ $\quad u_1=(\ln x)^2 \quad \frac{dv_1}{dx}=1$ $=\left[x(\ln x)^2\right]_1^e-2\left(\left[x\ln x\right]_1^e-\int_1^e 1\,dx\right)$ $\quad \frac{du_1}{dx}=2(\ln x)(\frac{1}{x}) \quad v_1=x$ $=\left[x(\ln x)^2\right]_1^e-2\left[x\ln x\right]_1^e+2\int_1^e 1\,dx$ $\quad u_2=\ln x \quad \frac{dv_2}{dx}=1$ $=\left[x(\ln x)^2\right]_1^e-2\left[e\ln e-\ln 1\right]_1^e+2\left[x\right]_1^e$ $\quad \frac{du_2}{dx}=\frac{1}{x} \quad v_2=x$ $=\left[x(\ln x)^2\right]_1^e-2\left[e\ln e-\ln 1\right]+2\left[e-1\right]$ $=e-2e+2e-2$ $=e-2$ Hence, volume of solid S $=\frac{1}{3}\pi e-\pi(e-2)$ $=\frac{1}{3}\pi e-\pi e+2\pi$ $=2\pi-\frac{2}{3}\pi e$ $=\frac{2}{3}\pi(3-e)$	
3 (i)	 2 500 000 000 = 1000 000$(2)^{n-1}$ 2^{n-1} = 2500 n-1 = $\frac{\ln 2500}{\ln 2}$ = 11.2877 n = 12. 2877 His net worth will first exceed 2.5 billion when n = 13 The year 1993 + (13 -1)(1) = 2005 or 1993 + 13 – 1 = 2005	
3 (ii)	100 000 + 1000 + 1000(1.5) + 1000(1.5^2) + ...(15 terms) = 100 000 + $\frac{1000(1.5^{15}-1)}{1.5-1}$ = \$ 973 787.7808 = \$ 973 788	
4 (a)	 $g(x)=\frac{1}{\left(\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right)\left(\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right)}=\frac{1}{\left(\frac{1}{2}\cos^2 x-\frac{1}{2}\sin^2 x\right)}=$	

	$\frac{1}{\frac{1}{2}(\cos^2 x-\sin^2 x)} = \frac{2}{\cos 2x} \approx \frac{2}{(1-\frac{(2x)^2}{2})} = \frac{2}{1-2x^2} = 2(1-2x^2)^{-1}$ $\approx 2(1+2x^2) = 2+4x^2$ m must be **sufficiently** small for g(x) $\approx 2+ax+bx^2$	
4 (b)	$(1-x^2)^{-1/2} = 1+\frac{\left(-\frac{1}{2}\right)}{1}\left(-x^2\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(-x^2\right)^2+$ $\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\left(-x^2\right)^3 = 1+\frac{1}{2}x^2+\frac{3}{8}x^4+\frac{5}{16}x^6+.....$ $\cos^{-1}x=\int\frac{-1}{\sqrt{1-x^2}}\,dx=-\int(1+\frac{1}{2}x^2+\frac{3}{8}x^4+\frac{5}{16}x^6+....)\,dx$ $\approx -(x+\frac{1}{6}x^3+\frac{3}{40}x^5+\frac{5}{112}x^7)+C$ When $x=0$, $\cos^{-1}0=\frac{\pi}{2}=C$ $\cos^{-1}(x)=-(x+\frac{1}{6}x^3+\frac{3}{40}x^5)+\frac{\pi}{2}$	
5	$\frac{1}{3^r}\left(\frac{u_{r+1}}{3}-u_r\right)=2r$ $\sum_{r=1}^{n}\frac{u_{r+1}}{3^{r+1}}-\frac{u_r}{3^r}=2\sum_{r=1}^{n}r$ $\frac{u_2}{3^2}-\frac{u_1}{3^1}$ $+\frac{u_3}{3^3}-\frac{u_2}{3^2}$ $+...$ $+\frac{u_n}{3^n}-\frac{u_{n-1}}{3^{n-1}}$ $+\frac{u_{n+1}}{3^{n+1}}-\frac{u_n}{3^n}=2\left(\frac{n}{2}(1+n)\right)$ $\frac{u_{n+1}}{3^{n+1}}-\frac{1}{3}=n(n+1)$ $u_{n+1}=3^{n+1}\left(n(n+1)+\frac{1}{3}\right)=3^n\left(3n^2+3n+1\right)$	
6	Rate of change = rate of growth – rate of decrease Rate of change = rate of earning interest – rate of withdrawal $\frac{dM}{dt}=0.05M-4000$	

$$\int \frac{1}{0.05M-4000}\,dM = \int 1\,dt$$

$$\frac{1}{0.05}\ln\left|0.05M-4000\right| = t + C$$

$$\ln\left|0.05M-4000\right| = \frac{t}{20}+\frac{C}{20}$$

$$\left|0.05M-4000\right| = e^{\frac{t}{20}+\frac{C}{20}}$$

$0.05M - 4000 = Ae^{\frac{t}{20}}$ where $A = \pm e^{\frac{C}{20}}$

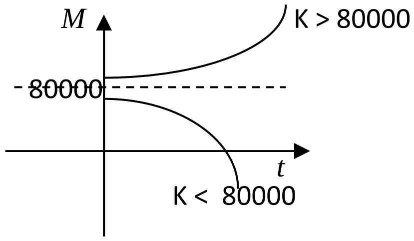
$M = 80000 + 20Ae^{\frac{t}{20}}$

When $t = 0$, $M = K$

$K = 80000 + 20A$

$20A = K - 80000$

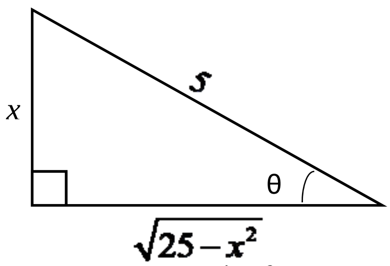
Hence $M = 80000 + (K-80000)e^{\frac{t}{20}}$



Money is completely withdrawn if $K < 80000$

7(i) Let $x = 5\sin\theta$

$$\frac{dx}{d\theta} = 5\cos\theta$$



$$\int \sqrt{25-x^2}\,dx$$

$$= \int \sqrt{25-(5\sin\theta)^2}(5\cos\theta)d\theta$$

$$= \int \sqrt{25-25\sin^2\theta}(5\cos\theta)d\theta$$

$$= \int \sqrt{25(1-\sin^2\theta)}(5\cos\theta)d\theta$$

$$= \int \sqrt{25(\cos^2\theta)}(5\cos\theta)d\theta$$

$$= \int (5\cos\theta)^2 d\theta$$

$$= 25\int \cos^2\theta\, d\theta$$

$$= \frac{25}{2}\int 1+\cos 2\theta\, d\theta$$

$$= \frac{25}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + c$$

$$= \frac{25}{2}\left(\theta + \sin\theta\cos\theta\right) + c$$

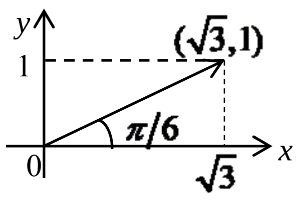
$$= \frac{25}{2}\left(\sin^{-1}\frac{x}{5} + \left(\frac{x}{5}\right)\left(\frac{\sqrt{25-x^2}}{5}\right)\right) + c$$

Let $x = 5\sin\theta$

$$\sin\theta = \frac{x}{5}$$

$$\theta = \sin^{-1}\frac{x}{5}$$

$$\cos\theta = \frac{1}{5}\sqrt{25-x^2}$$

	$=\dfrac{25}{2}\sin^{-1}\dfrac{x}{5}+\dfrac{1}{2}x\sqrt{25-x^2}+c$	
7(ii)	$x^2+(y-b)^2=25$ $(y-b)^2=25-x^2$ Since $y<0$, $y-b<0$, $y-b=-\sqrt{25-x^2}$ $y=b-\sqrt{25-x^2}$ Area of region R $=\left\lvert\int_0^a b-\sqrt{25-x^2}\,dx\right\rvert$ $=-\int_0^a b-\sqrt{25-x^2}\,dx$ $=\int_0^a\sqrt{25-x^2}\ dx-\int_0^a b\ dx$ $=\left[\dfrac{25}{2}\sin^{-1}\dfrac{x}{5}+\dfrac{1}{2}x\sqrt{25-x^2}\right]_0^a-\left[bx\right]_0^a$ $=\dfrac{25}{2}\sin^{-1}\dfrac{a}{5}+\dfrac{1}{2}a\sqrt{25-a^2}-ab$ $=\dfrac{1}{2}a\sqrt{25-a^2}+\dfrac{25}{2}\sin^{-1}\dfrac{a}{5}-a\sqrt{25-a^2}$ $=\dfrac{25}{2}\sin^{-1}\dfrac{a}{5}-\dfrac{1}{2}a\sqrt{25-a^2}$. Substituting $x = a$ and $y = 0$ into the equation $x^2+(y-b)^2=25$, we have $a^2+(0-b)^2=25$ $b=\sqrt{25-a^2}$	
8 (i) **8 (ii)**	$z=k+i$ $z^2=(k+i)^2=k^2+2(k)(i)+(i)^2=(k^2-1)+(2k)i$ $z^3=(k+i)^3=k^3+3(k)^2(i)+3(k)(i)^2+(i)^3$ $=(k^3-3k)+(3k^2-1)i$ $z^3-iz^2-2z-4i=0$ $[(k^3-3k)+(3k^2-1)i]-i[(k^2-1)+(2k)i]-2[k+i]-4i=0$ $[(k^3-3k)+2k-2k]+i[(3k^2-1)-(k^2-1)-2-4]=0$ $(k^3-3k)+i(2k^2-6)=0$ $k(k^2-3)=0$ and $2k^2-6=0$ ($k=0$ or $k=\pm\sqrt{3}$) and $k=\pm\sqrt{3}$ Hence, $k=\pm\sqrt{3}$ $z=\sqrt{3}+i\quad(\because k>0)$ $\lvert z\rvert=\sqrt{1+3}=2$ $\arg(z)=\dfrac{\pi}{6}$  	

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<u>Method 1: By Polar Form & Trigonometry</u>

$$z = 2e^{i\pi/6} = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$
$$z^n = 2^n e^{in\pi/6} = 2^n\left(\cos\frac{n\pi}{6} + i\sin\frac{n\pi}{6}\right)$$
$$z^n \text{ is real } \Leftrightarrow \sin\frac{n\pi}{6} = 0$$
$$\Leftrightarrow \frac{n\pi}{6} = k\pi,\ \text{where } k \in \mathbf{Z}$$
$$\Leftrightarrow n = 6k,\ \text{where } k \in \mathbf{Z}$$

Hence, $n = 0,\ \pm 6,\ \pm 12,\ \pm 18,\ ...$

<u>Method 2: By Properties of arg(z)</u>

$$\arg(z^n) = n\arg(z) = \frac{n\pi}{6}$$

z^n is real, the point representing z^n on the Argand diagram is on the x-axis.

Thus, $\arg(z^n) = \frac{n\pi}{6} = k\pi$, where $k \in \mathbf{Z}$

$\therefore n = 6k$, where $k \in \mathbf{Z}$

i.e. $n = 0,\ \pm 6,\ \pm 12,\ \pm 18,\ ...$

Given $\left|z^n\right| > 100$.

$\left|z^n\right| = \left|z\right|^n = 2^n$

Hence, $2^n > 100$

But n is a multiple of 6. We then have

$2^6 = 64 < 100$

$2^{12} = 4096 > 100$

The least value of n is then 12.

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Let P_n be the statement $\sum_{r=1}^{n} \frac{3r+1}{r(r+1)(r+2)} = \frac{n(7n+9)}{4(n+1)(n+2)}$ for all integers $n \geq 1$.

When n = 1, LHS = $\sum_{r=1}^{1} \frac{3r+1}{r(r+1)(r+2)} = \frac{3+1}{1(2)(3)} = \frac{2}{3}$

RHS = $\frac{7+9}{4(2)(3)} = \frac{2}{3}$ = LHS

$\therefore$ P_1 is true.

Assume that P_k is true for some positive integer k, $k \geq 1$,

i.e. $\sum_{r=1}^{k} \frac{3r+1}{r(r+1)(r+2)} = \frac{k(7k+9)}{4(k+1)(k+2)}$

Need to prove P_{k+1} is true,

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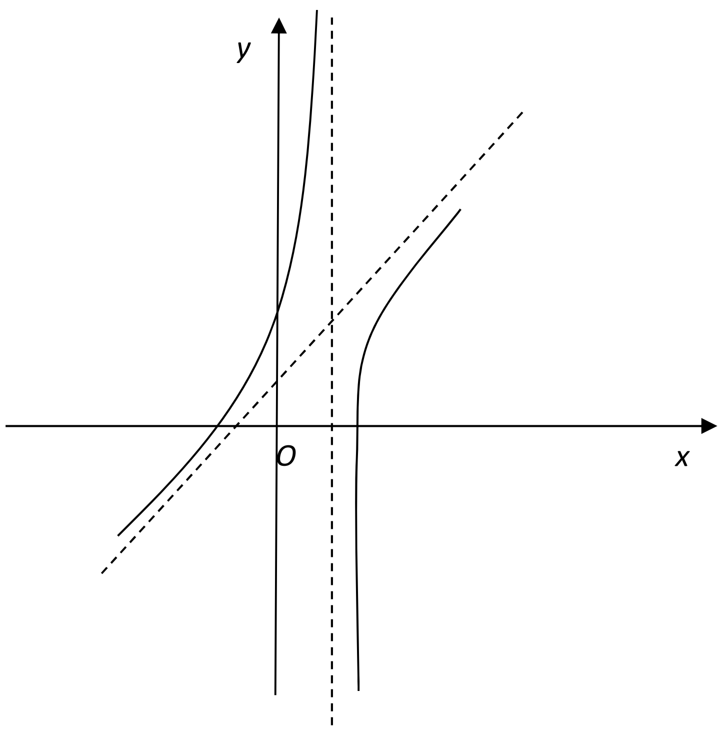
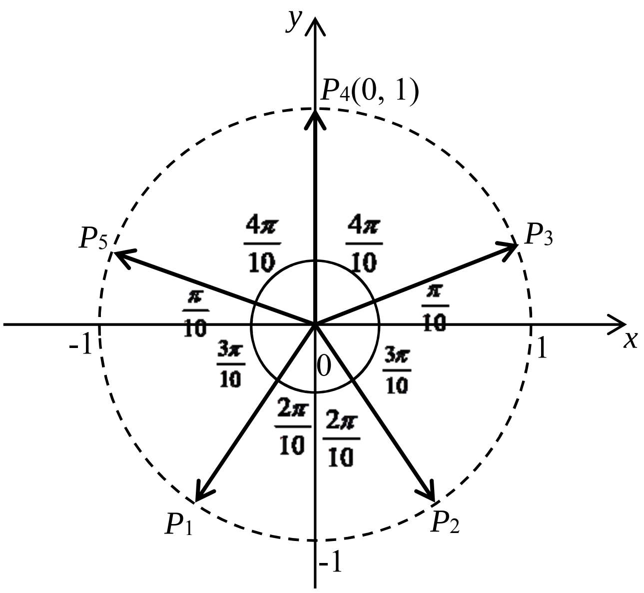
	i.e. $\displaystyle\sum_{r=1}^{k+1}\frac{3r+1}{r(r+1)(r+2)}=\frac{(k+1)(7k+16)}{4(k+2)(k+3)}$. LHS of $\mathrm{P}_{k+1}=\displaystyle\sum_{r=1}^{k+1}\frac{3r+1}{r(r+1)(r+2)}$ $=\displaystyle\sum_{r=1}^{k}\frac{3r+1}{r(r+1)(r+2)}+\frac{3(k+1)+1}{(k+1)(k+2)(k+3)}$ $=\dfrac{k(7k+9)}{4(k+1)(k+2)}+\dfrac{3k+4}{(k+1)(k+2)(k+3)}$ $=\dfrac{7k^3+30k^2+39k+16}{4(k+1)(k+2)(k+3)}$ $=\dfrac{(7k^2+23k+16)(k+1)}{4(k+1)(k+2)(k+3)}$ $=\dfrac{(k+1)(7k+16)}{4(k+2)(k+3)}=\text{RHS}$ $\therefore$ P_k is true $\Rightarrow$ P_{k+1} is true. Since P_1 is true, and P_k is true $\Rightarrow$ P_{k+1} is true, by the Principle of Mathematical Induction, $\displaystyle\sum_{r=1}^{n}\frac{3r+1}{r(r+1)(r+2)}=\frac{n(7n+9)}{4(n+1)(n+2)}$ is true for all integers $n\geq 1$.	
9 (i)	$\dfrac{n(7n+9)}{4(n+1)(n+2)}=\dfrac{7}{4}-\dfrac{6n+7}{2(n+1)(n+2)}$ Since n is a positive integer, $\dfrac{6n+7}{2(n+1)(n+2)}>0$ $\therefore \displaystyle\sum_{r=1}^{n}\frac{3r+1}{r(r+1)(r+2)}=\frac{n(7n+9)}{4(n+1)(n+2)}=\frac{7}{4}-\frac{6n+7}{2(n+1)(n+2)}<\frac{7}{4}$ Alternatively, $\displaystyle\sum_{r=1}^{n}\frac{3r+1}{r(r+1)(r+2)}=\sum_{r=1}^{n}\frac{1}{2r}+\frac{2}{r+1}-\frac{5}{2(r+2)}=....=\frac{7}{4}-\frac{1}{2(n+1)}-\frac{5}{2(r+2)}<\frac{7}{4}$ Since n is a positive integer, $-\dfrac{-1}{2(n+1)}-\dfrac{5}{2(n+2)}<0$	

9 (ii)	$(r+1)^3 = r^3 + 3r^2 + 3r + 1$ $r(r+1)(r+2) = r^3 + 3r^2 + 2r$ $\therefore (r+1)^3 > r(r+1)(r+2)$ $\sum_{r=1}^{n} \frac{3r}{(r+1)^3} < \sum_{r=1}^{n} \frac{3r}{r(r+1)(r+2)} < \sum_{r=1}^{n} \frac{3r+1}{r(r+1)(r+2)} < \frac{7}{4}.$	
10	$y = \frac{x^2 - 4k^2}{x-k}$ where k is a constant such that $k \neq 0$ $xy - ky = x^2 - 4k^2$ $x^2 - xy + (ky - 4k^2) = 0$ x is real $\Rightarrow$ discriminant ≥ 0 $y^2 - 4\left(ky - 4k^2\right) \geq 0$ $y^2 - 4ky + 16k^2 \geq 0$ $\left(y - 2k\right)^2 + 12k^2 \geq 0$ This inequality is true for all values of y. Therefore y can take the set of all real numbers. *Alternative Method:* $\frac{dy}{dx} = 0$ $\frac{x^2 - 2xk + 4k^2}{\left(x-k\right)^2} = 0$ $x^2 - 2xk + 4k^2 = 0$ discriminant = $-12k^2 < 0$ no real roots. Hence, no turning pts. $y \in \mathbb{R}$.	

10 (i)	$y=\dfrac{x^2-4k^2}{x-k}=x+k-\dfrac{3k^2}{x-k}$ Asymptotes: $y=x+k$ *and* $x=k$ Points of intercept with axes: $(0,4k)$, $(-2k,0)$, $(2k,0)$ 	
10 (ii)	$\int_{-1}^{1}\mathrm{f}\left(\lvert x\rvert\right)\,\mathrm{d}x=2\int_0^1\mathrm{f}\left(x\right)dx=2\int_0^1 x+k-\dfrac{3k^2}{x-k}\,\mathrm{d}x$ $=2\left[\dfrac{x^2}{2}+kx-3k^2\ln\lvert x-k\rvert\right]_0^1$ $=2\left[\dfrac{1}{2}+k-3k^2\ln\lvert 1-k\rvert+3k^2\ln\lvert -k\rvert\right]$ $=1+2k+6k^2\ln\dfrac{k}{k-1}.$	
10 (iii)	$y=\dfrac{x^2-4k^2}{x-k}\to y=\dfrac{\left(\dfrac{x}{2}\right)^2-4k^2}{\left(\dfrac{x}{2}\right)-k}\to y=\dfrac{x^2-16k^2}{2x-4k}$ $\to y=\dfrac{\left(x+2k\right)^2-16k^2}{2\left(x+2k\right)-4k}=\dfrac{\left(x+2k\right)^2-16k^2}{2x}$	

11 (i)	$\mathbf{a}\times\mathbf{b}=4\mathbf{a}\times\mathbf{c}$ $(\mathbf{a}\times\mathbf{b})-(4\mathbf{a}\times\mathbf{c})=0$ $(\mathbf{a}\times\mathbf{b})-(\mathbf{a}\times 4\mathbf{c})=0$ $\mathbf{a}\times(\mathbf{b}-4\mathbf{c})=0$ $\boldsymbol{a}$ is parallel to $\boldsymbol{b}-4\boldsymbol{c}$ $\boldsymbol{b}-4c=\alpha\,\boldsymbol{a}$	
(ii)	$\frac{1}{2}\lvert\mathbf{a}\times\mathbf{b}\rvert=\sqrt{126}$ $\frac{1}{2}\lvert 4\mathbf{a}\times\mathbf{c}\rvert=\sqrt{126}$ $\lvert\mathbf{a}\times\mathbf{c}\rvert=\frac{\sqrt{126}}{2}$ $\left\lvert\left(\frac{\mathbf{b}-4\mathbf{c}}{\sqrt{3}}\right)\times\mathbf{c}\right\rvert=\frac{\sqrt{126}}{2}$ $\lvert(\mathbf{b}\times\mathbf{c})-(4\mathbf{c}\times\mathbf{c})\rvert=\frac{\sqrt{3}\sqrt{126}}{2}$ $\lvert(\mathbf{b}\times\mathbf{c})\rvert=\frac{\sqrt{378}}{2}$ Alternatively, $\lvert\mathbf{b}\times\mathbf{c}\rvert=\left\lvert\mathbf{b}\times\left(\frac{\mathbf{b}-\sqrt{3}\mathbf{a}}{4}\right)\right\rvert=\frac{1}{4}\lvert\mathbf{b}\times\mathbf{b}-\mathbf{b}\times\sqrt{3}\mathbf{a}\rvert$ $=\frac{\sqrt{3}}{4}\lvert\mathbf{b}\times\mathbf{a}\rvert=\frac{\sqrt{3}}{2}.\frac{1}{2}\lvert\mathbf{b}\times\mathbf{a}\rvert=\frac{\sqrt{3}}{2}.\sqrt{126}$ $\therefore\quad\lvert(\mathbf{b}\times\mathbf{c})\rvert=\frac{\sqrt{378}}{2}$	
(iii)	Area of parallelogram with adjacent sides *OB* and *OC*.	
(iv)	$(\boldsymbol{b}-4\boldsymbol{c})\,.\,(\boldsymbol{b}-4\boldsymbol{c})=3\lvert\boldsymbol{a}\rvert^2$ $\lvert\boldsymbol{b}\rvert^2-8\,\boldsymbol{b}\,.\,\boldsymbol{c}+16\lvert\boldsymbol{c}\rvert^2=3\lvert\boldsymbol{a}\rvert^2$ $\boldsymbol{b}\,.\,\boldsymbol{c}=-\frac{10}{8}$ $\cos\theta=\frac{\boldsymbol{b}\,.\,\mathbf{c}}{\lvert\boldsymbol{b}\rvert\lvert\boldsymbol{c}\rvert}=\frac{-\frac{10}{8}}{1(2)}$ $\theta=128.7^\circ$	
12 (i)	$z^5-i=0$ $z^5=i$ $z^5=e^{i\pi/2}=e^{i(2k\pi+\frac{\pi}{2})}$, where $k\in\mathbf{Z}$ $z=e^{i(\frac{2k\pi}{5}+\frac{\pi}{10})}$ Putting $n=-2,-1,0,1,2$	

	$z = e^{-\frac{7\pi}{10}i}, e^{-\frac{3\pi}{10}i}, e^{\frac{\pi}{10}i}, e^{\frac{\pi}{2}i}, e^{\frac{9\pi}{10}i}$ Given $-\pi < \arg(z_1) < \arg(z_2) < \arg(z_3) < \arg(z_4) < \arg(z_5) \le \pi$. i.e. $z_1 = e^{-\frac{7\pi}{10}i}, z_2 = e^{-\frac{3\pi}{10}i}, z_3 = e^{\frac{\pi}{10}i}, z_4 = e^{\frac{\pi}{2}i}, z_5 = e^{\frac{9\pi}{10}i}$ Let the points P_1, P_2, P_3, P_4 and P_5 on the Argand diagram represents the complex numbers z_1, z_2, z_3, z_4, and z_5.  	
12 (ii)	The locus $\lvert z - z_2 \rvert = \lvert z - z_3 \rvert$ is a perpendicular bisector of the line segment P_2P_3 where $P_2 \equiv z_2$ and $P_3 \equiv z_3$. Since OP_2P_3 is an isosceles triangle, the perpendicular cuts through the origin and bisect the angle P_2OP_3. Thus the perpendicular bisector is inclined at angle $\frac{\pi}{10}$ radian below the positive x-axis. Hence, the Cartesian equation of the locus $\lvert z - z_2 \rvert = \lvert z - z_3 \rvert$ is $y = -x\tan(\frac{\pi}{10})$ OR $y = x\tan(\frac{9\pi}{10})$ $\arg(z - z_1) = \arg(z_4)$	

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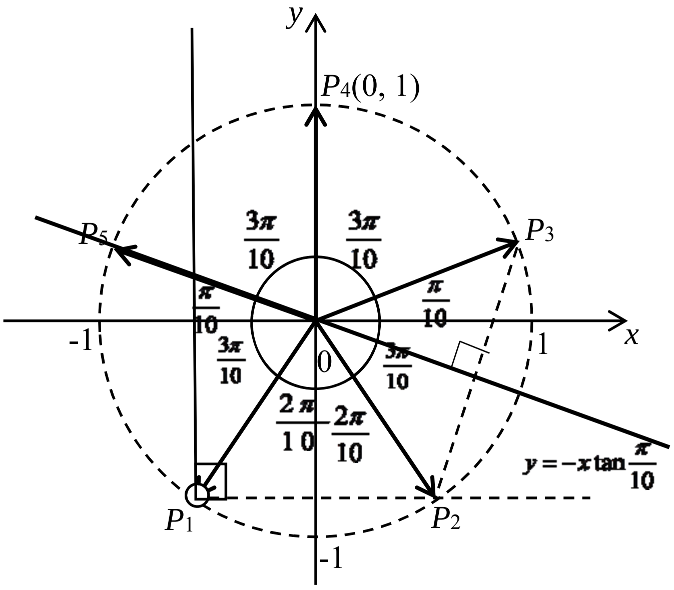
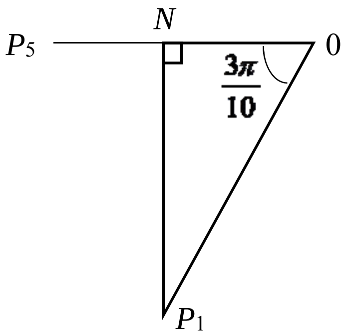
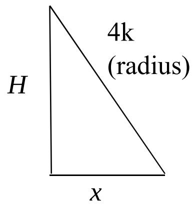
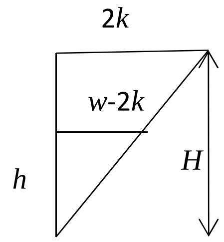


$z \equiv \overrightarrow{OP}$
$z_1 \equiv \overrightarrow{OP_1}$, $z - z_1 \equiv \overrightarrow{OP} - \overrightarrow{OP_1}$ i.e. $z - z_1 \equiv \overrightarrow{P_1P}$
The least value of $|z - z_1|$ is the shortest distance from P_1 to the perpendicular bisector (line OP_5).
$\frac{P_1N}{OP_1} = \sin\frac{4\pi}{10}$
$P_1N = OP_1 \sin\frac{4\pi}{10}$
$= \sin\frac{2\pi}{5} = 0.951$


</td><td></td></tr>
<tr><td>12 (iii)</td><td>The locus $\arg(z - z_1) = \arg(z_4)$ is a half-line with its initial point at P_1 and above and excluding P_1, parallel to the y-axis. (Refer to diagram.)</td><td></td></tr>
<tr><td>12 (iii)</td><td>The intersection point between the 2 loci has the same x-coordinates as the point P_1, i.e. $x = \cos\frac{-7\pi}{10} = -\cos\frac{3\pi}{10}$.
Substituting $x = -\cos\frac{3\pi}{10}$ into the equation of the perpendicular bisector $y = -x\tan(\frac{\pi}{10})$, we have
$y = -(-\cos\frac{3\pi}{10})\tan\frac{\pi}{10} = \cos\frac{3\pi}{10}\tan\frac{\pi}{10} = 0.191$
$x = -\cos\frac{3\pi}{10} = -0.588$
Coordinates of the intersection point are $(-0.588, 0.191)$.</td><td></td></tr>
</table>

13 (i)	$S=\frac{1}{2}H\left(2x+8k\right)=\frac{1}{2}\left(2x+8k\right)\sqrt{\left(4k\right)^2-x^2}$ $=(x+4k)\sqrt{16k^2-x^2}\ (shown)$  	
(ii)	$\frac{dS}{dx}=\sqrt{16k^2-x^2}-\left(x+4k\right)x\left(16k^2-x^2\right)^{-\frac{1}{2}}$ $=\frac{-2x^2-4kx+16k^2}{\sqrt{16k^2-x^2}}=0$ $x^2+2kx-8k^2=0$ $\left(x-2k\right)\left(x+4k\right)=0$ $\Rightarrow x=2k \quad or \quad x=-4k\ (\text{N.A}, x>0)$ $\frac{dS}{dx}=\frac{-2x^2-4kx+16k^2}{\sqrt{16k^2-x^2}}=\frac{-2(x^2+2kx-8k^2)}{\sqrt{16k^2-x^2}}$ $\frac{d^2S}{dx^2}=-2\left[\frac{\sqrt{16k^2-x^2}\left(2x+2k\right)-(x^2+2kx-8k^2)\frac{1}{2}\left(16k^2-x^2\right)^{-1/2}(-2x)}{16k^2-x^2}\right]$ $when\ \ x=2k$ $\frac{d^2S}{dx^2}=-2\frac{\sqrt{12k^2}\,6k}{12k^2}=-\sqrt{12}<0$ Area of trapezium is maximum when *x=2k*.	
(iii)	Using similar triangles:  $\frac{h}{H}=\frac{w-2k}{2k}$ $When\ x=2k, H=\sqrt{12k^2}=2\sqrt{3}k$ $w=\frac{2kh}{H}+2k=\frac{h}{\sqrt{3}}+2k$ $\therefore V=\frac{3}{2}h\left(4k+2w\right)=\frac{3}{2}h\left(4k+2\left(\frac{h}{\sqrt{3}}+2k\right)\right)$ $=3h\left(4k+\frac{h}{\sqrt{3}}\right)(shown)$	

(iv)	$\frac{dV}{dh}=\left(4k+\frac{h}{\sqrt{3}}\right)3+3h\left(\frac{1}{\sqrt{3}}\right)$ $=12k+\frac{6h}{\sqrt{3}}$ $When\ h=\sqrt{3}k,\ \frac{dV}{dh}=18k$ $\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}=\frac{0.2}{18k}=\frac{1}{90k}m/s$	