

TEMASEK JUNIOR COLLEGE, SINGAPORE

Preliminary Examination 2016
Higher 2

MATHEMATICS

9740/02

Paper 2

14 September 2016

Additional Materials: Answer Paper
 List of Formulae (MF15)
 Graph Paper

3 hours

READ THESE INSTRUCTIONS FIRST

Write your Civics group and name on all the work that you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

At the end of the examination, fasten all your work securely together.

This document consists of **6** printed pages.

Section A: Pure Mathematics [40 marks]

- 1** A sequence u_1, u_2, u_3, \dots is such that $u_1 = 2$ and

$$u_n = u_{n-1} - \frac{1}{n(n-1)}, \text{ for all integers } n \geq 2.$$

- (i) By expressing $\frac{1}{r(r-1)}$ in partial fractions or otherwise, find $\sum_{r=2}^n \frac{1}{r(r-1)}$ in terms of n . [4]

- (ii) By considering $\sum_{r=2}^n (u_r - u_{r-1})$ and using the result in part (i), show that for all integers $n \geq 2$, u_n can be expressed in the form $a + \frac{b}{n}$, where a and b are constants to be determined. [3]

ii

Let $\frac{1}{r(r-1)} = \frac{A}{r} + \frac{B}{r-1}$

$$A(r-1) + Br = 1$$

Sub $r=0 \Rightarrow A=-1$

Sub $r=1 \Rightarrow B=1$

So $\frac{1}{r(r-1)} = \frac{1}{r-1} - \frac{1}{r}$

$$\sum_{r=2}^n \frac{1}{r(r-1)} = \sum_{r=2}^n \left(\frac{1}{r-1} - \frac{1}{r} \right)$$

$$= \begin{array}{c} \frac{1}{1} - \cancel{\frac{1}{2}} \\ + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} \\ \vdots \\ \cancel{\frac{1}{n-2}} - \cancel{\frac{1}{n-1}} \\ + \cancel{\frac{1}{n-1}} - \frac{1}{n} \end{array}$$

$$= 1 - \frac{1}{n}$$

1ii	$\sum_{r=2}^n (u_r - u_{r-1}) = -\sum_{r=1}^n \frac{1}{r(r-1)}$ $ \begin{array}{rcl} & \cancel{u_2} & - u_1 \\ + & \cancel{u_3} & - \cancel{u_2} \\ & \vdots & \\ & \cancel{u_{n-1}} & - u_{n-2} \\ + & \cancel{u_n} & - \cancel{u_{n-1}} \end{array} = -\left(1 - \frac{1}{n}\right) $ $u_n = u_1 - 1 + \frac{1}{n} = 1 + \frac{1}{n}$
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- 2 Solve the equation $w^4 + 2 - 2\sqrt{3}i = 0$, giving the roots in the form $re^{i\theta}$, where $-\pi < \theta \leq \pi$ and $r > 0$. [4]

The roots represented by w_1 and w_2 are such that $-\frac{\pi}{2} < \arg(w_1) < 0$ and

$$\frac{\pi}{2} < \arg(w_2) < \pi.$$

The complex number z satisfies the relations $|z - w_1| \geq |z - w_2|$ and

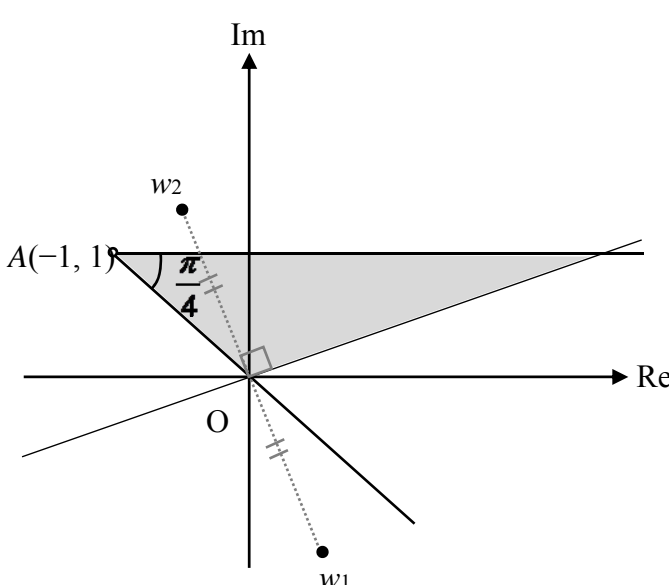
$$-\frac{\pi}{4} \leq \arg[z - (-1 + i)] \leq 0.$$

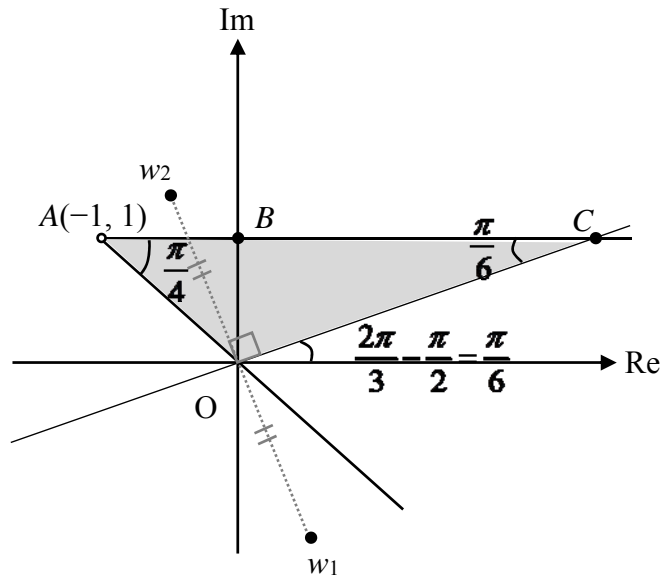
On an Argand diagram, sketch the region R in which the points representing z can lie.

[3]

Find the exact area of R .

[3]

2	$w^4 + 2 - 2\sqrt{3}i = 0 \Rightarrow w^4 = -2 + 2\sqrt{3}i$ $= 4e^{i\frac{2\pi}{3}}$ $= 4e^{i\left(\frac{2\pi}{3} + 2k\pi\right)}$ $\Rightarrow w = (4)^{\frac{1}{4}} \left[e^{i\left(\frac{2\pi}{3} + 2k\pi\right)} \right]^{\frac{1}{4}} = \sqrt{2}e^{i\left(\frac{\pi}{6} + \frac{k\pi}{2}\right)}, \quad \text{where } k = 0, \pm 1, -2$ $\therefore w = \sqrt{2}e^{i\frac{\pi}{6}}, \sqrt{2}e^{i\frac{2\pi}{3}}, \sqrt{2}e^{i\left(-\frac{5\pi}{6}\right)}, \sqrt{2}e^{i\left(-\frac{\pi}{3}\right)}$
	$\therefore w_1 = \sqrt{2}e^{i\left(-\frac{\pi}{3}\right)}, w_2 = \sqrt{2}e^{i\frac{2\pi}{3}}$ 



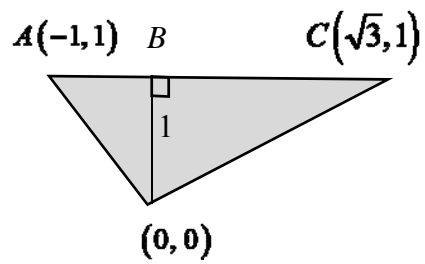
$$\tan \frac{\pi}{6} = \frac{1}{BC}$$

$$BC = \sqrt{3}$$

Area of shaded region

$$= \frac{1}{2}(1 + \sqrt{3})(1)$$

$$= \frac{1 + \sqrt{3}}{2}$$



- 3 The line l has equation $\frac{x+1}{-1} = \frac{z+6}{2}$, $y = 4$ and the point A has coordinates $(-1, 3, -5)$.

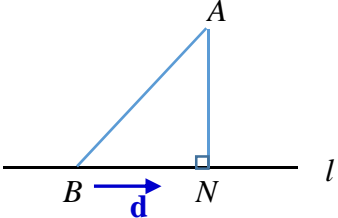
(i) Find the position vector of the foot of the perpendicular from A to l . [3]

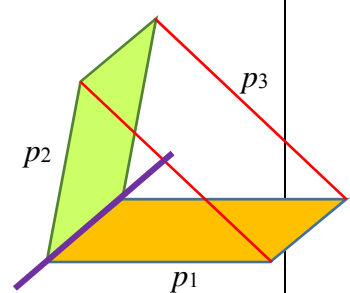
(ii) Plane p_1 contains l and A . Show that the equation of p_1 is $2x + y + z = -4$. [3]

Given that the plane p_2 has equation $x + 2y + cz = -5$ where c is a negative constant, and that the acute angle between p_1 and p_2 is 60° , find the value of c . [3]

(iii) Find the equation of the line of intersection, m , between p_1 and p_2 . [1]

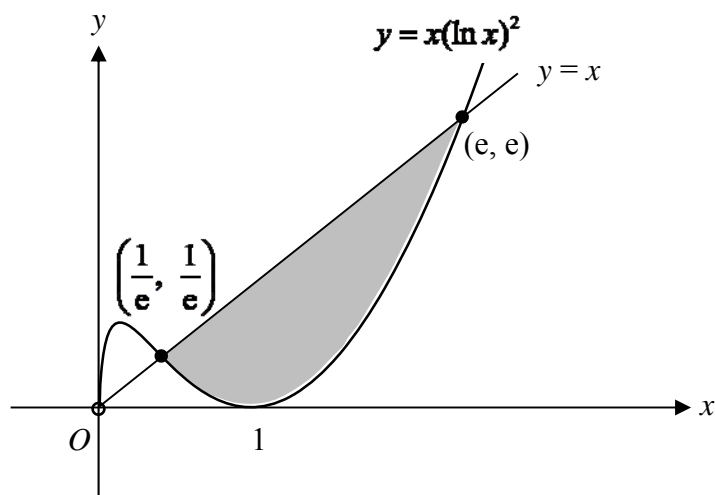
(iv) The plane p_3 has equation $3x + \alpha y + 7z = \beta$ where α and β are constants. Given that the planes p_1 , p_2 and p_3 have no common point, what can be said about the values of α and β ? [2]

<p>3i</p>	<p>Let $\lambda = \frac{x+1}{-1} = \frac{z+6}{2}$, $y = 4$ $x = -1 - \lambda$, $z = -6 + 2\lambda$, $y = 4$</p> <p>$l: \mathbf{r} = \begin{pmatrix} -1 \\ 4 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$, $\lambda \in \mathbb{R}$</p> <p>Let N be the foot of the perpendicular from A to l.</p> <p>$\overrightarrow{ON} = \begin{pmatrix} -1 \\ 4 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 + \lambda \\ 4 \\ -6 + 2\lambda \end{pmatrix}$ for some $\lambda \in \mathbb{R}$</p> <p>$\Rightarrow \overrightarrow{AN} = \begin{pmatrix} -\lambda \\ 1 \\ -1 + 2\lambda \end{pmatrix}$</p> <p>$\overrightarrow{AN} \perp l$ i.e. $\begin{pmatrix} -\lambda \\ 1 \\ -1 + 2\lambda \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} = 0 \Rightarrow \lambda = \frac{2}{5}$</p> <p>Thus $\overrightarrow{ON} = \frac{1}{5} \begin{pmatrix} -7 \\ 20 \\ -26 \end{pmatrix}$</p> 
<p>3ii</p>	<p>Let B be the point on l with coordinates $(-1, 4, 6)$</p> <p>$\overrightarrow{BA} = \begin{pmatrix} -1 \\ 3 \\ -5 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \\ -6 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ or $\overrightarrow{AN} = -\frac{1}{5} \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix}$</p>

	<p>A normal to p_1 is $\begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} \times \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} = 5 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$</p> <p>Equation of p_1 is $\mathbf{r} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = -4$ i.e. $2x + y + z = -4$</p> <p>Equation of p_2 is $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 2 \\ a \end{pmatrix} = -5$</p> <p>$\cos 60^\circ = \frac{\left \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ c \end{pmatrix} \right }{\sqrt{6}\sqrt{5+c^2}}$</p> <p>$\frac{1}{2} = \frac{ 4+c }{\sqrt{6}\sqrt{5+c^2}}$</p> <p>$30 + 6c^2 = 4(c^2 + 8c + 16)$</p> <p>$c^2 - 16c - 17 = 0$</p> <p>$(c - 17)(c + 1) = 0$</p> <p>$c = 17$ (rejected since $c < 0$) or $c = -1$</p>
3iii	<p>$p_1: 2x + y + z = -4$</p> <p>$p_2: x + 2y + cz = -5$</p> <p>Using GC, $x = -1 - t$ $y = -2 + t$ $z = t$</p> <p>Equation of m is $\mathbf{r} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, t \in \mathbb{R}$</p>
3iv	<p>Since the <u>3 planes are not parallel</u> and <u>they have no common point</u>, m is parallel to p_3 but not contained in p_3.</p> <p>m is perpendicular to \mathbf{n}_3: $\begin{pmatrix} 3 \\ \alpha \\ 7 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = 0 \Rightarrow \alpha = -4$</p> <p>Also $(-1, -2, 0)$ on m does not lie in p_3: $3x + \alpha y + 7z = \beta$ Thus $3(-1) + (-4)(-2) + 7(0) \neq \beta \Rightarrow \beta \neq 5$</p> 

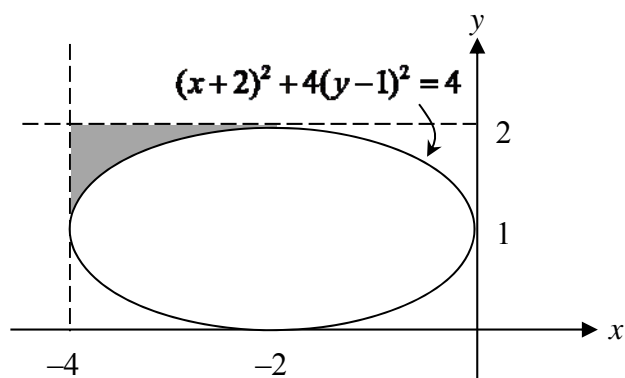
- 4 (a) The diagram below shows the graphs of $y = x(\ln x)^2$ and $y = x$. The two graphs intersect at the points $\left(\frac{1}{e}, \frac{1}{e}\right)$ and (e, e) .

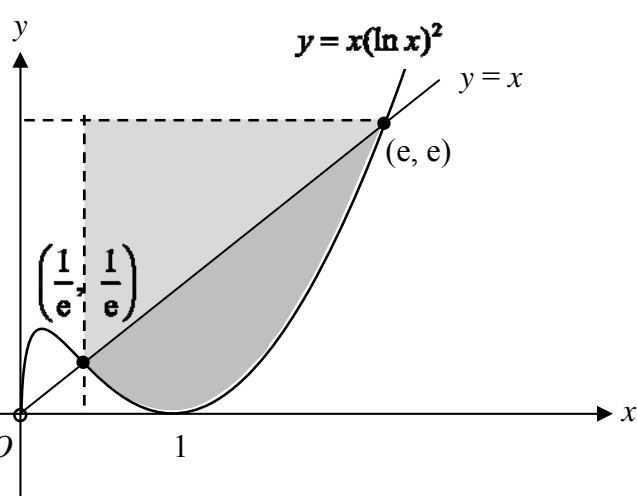
Find the exact area of the shaded region bounded by the graphs of $y = x(\ln x)^2$ and $y = x$. [5]



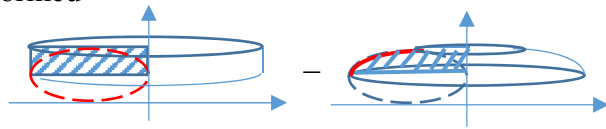
Hence, without integrating, find the exact area of the region bounded by the graphs of $y = x(\ln x)^2$ and the lines $y = e$ and $x = \frac{1}{e}$. [2]

- (b) Find the volume of the solid formed when the shaded region bounded by the lines $x = -4$, $y = 2$ and the ellipse $(x+2)^2 + 4(y-1)^2 = 4$ is rotated through 2π radians about the y-axis. Give your answer correct to 1 decimal place. [4]



4a	<p>Area of shaded region</p> $= \frac{1}{2} \left(e + \frac{1}{e} \right) \left(e - \frac{1}{e} \right) - \int_{\frac{1}{e}}^e x (\ln x)^2 dx$ $= \frac{1}{2} \left(e^2 - \frac{1}{e^2} \right) - \left(\left[\frac{x^2}{2} (\ln x)^2 \right]_{\frac{1}{e}}^e - \int_{\frac{1}{e}}^e \frac{x^2}{2} \frac{2 \ln x}{x} dx \right)$ $= \frac{1}{2} \left(e^2 - \frac{1}{e^2} \right) - \left(\left[\frac{e^2}{2} - \frac{1}{2e^2} \right] - \int_{\frac{1}{e}}^e x \ln x dx \right)$ $= \frac{1}{2} \left(e^2 - \frac{1}{e^2} \right) - \frac{1}{2} \left(e^2 - \frac{1}{e^2} \right) + \left(\left[\frac{x^2}{2} \ln x \right]_{\frac{1}{e}}^e - \int_{\frac{1}{e}}^e \frac{x^2}{2} \frac{1}{x} dx \right)$ $= \frac{1}{2} \left(e^2 + \frac{1}{e^2} \right) - \frac{1}{2} \left[\frac{x^2}{2} \right]_{\frac{1}{e}}^e$ $= \frac{1}{2} \left(e^2 + \frac{1}{e^2} \right) - \frac{1}{2} \left(\frac{e^2}{2} - \frac{1}{2e^2} \right)$ $= \frac{1}{4} \left(e^2 + \frac{3}{e^2} \right)$  <p>Area = $\frac{1}{4} \left(e^2 + \frac{3}{e^2} \right) + \frac{1}{2} \left(e - \frac{1}{e} \right)^2$</p>
4b	<p>Given $(x+2)^2 + 4(y-1)^2 = 4$</p> $(x+2)^2 = 4[1 - (y-1)^2] \Rightarrow x+2 = \pm 2\sqrt{1 - (y-1)^2}$ <p>The shaded region is bounded by the section of the ellipse where $x \leq -2$.</p> <p>Hence $x = -2 - 2\sqrt{1 - (y-1)^2}$</p>

Volume of solid formed



$$= \pi 4^2 (2-1) - \pi \int_1^2 x^2 \, dy$$

$$= 16\pi - \pi \int_1^2 \left(-2 - 2\sqrt{1-(y-1)^2} \right)^2 \, dy$$

$$= 9.6 \text{ units}^3$$

Section B: Statistics [60 marks]

- 5 An apartment block has 24 two-bedroom apartments and 88 four-bedroom apartments. A surveyor wishes to conduct interviews on 35 households in this block to learn about their household expenditure. She decides to use stratified sampling across apartment types in the block, assuming that only one household occupies each apartment.

- (i) Give an advantage of this sampling method. [1]
- (ii) Describe how a stratified sample can be obtained. [3]

5i	Stratified sampling ensures that households occupying two-bedroom apartments and households occupying four-bedroom apartments are proportionately represented in the sample.
5ii	<p>Total number of units = $24 + 88 = 112$</p> <p>Number of 2-bedroom households needed = $\frac{24}{112} \times 35 = 7.5$</p> <p>Number of 4-bedroom households needed = $\frac{88}{112} \times 35 = 27.5$</p> <p>She should choose 8 two-bedroom apartments and 27 four-bedroom apartments (or 7 two-bedroom apartments and 28 four-bedroom apartments)</p> <p>Using the list of apartment numbers as sampling frames for the two types of apartments, she would use simple random sampling (or systematic sampling) to select the 2-bedroom apartments and 4-bedroom apartments to be interviewed.</p>

- 6 A mathematician arranges all eight letters in the word PARALLEL to form different 8-letter code words. Find the number of different code words that can be formed if

- (i) the code words start with an L and end with an A, [1]
- (ii) the 2 A's are not adjacent to each other, [3]
- (iii) there is exactly one letter between the first and second L, and exactly one letter between the second and third L. [3]

6i	<p>Number of ways $= \frac{6!}{2!} = 360$ P AA R LLL E</p>
6ii	<p>Method 1 (By Slotting)</p> <p>Number of ways $= \binom{7}{2} \frac{6!}{3!} = 2520$</p> <p>Method 2 (Complement)</p> <p>Number of ways to arrange letters without restriction $= \frac{8!}{2!3!} = 3360$</p> <p>Number of ways to arrange letters with both A's together $= \frac{7!}{3!} = 840$</p> <p>Total number of ways $= \frac{8!}{2!3!} - \frac{7!}{3!} = 2520$</p>
6iii	<p>Number of ways $= 4 \times \frac{5!}{2!} = 240$ L L L _ _ _</p> <p>Method 2</p> <p>Number of ways $= \binom{5}{2} \times 2! \times \frac{4!}{2!} = 240$</p> <p>Number of ways $= \frac{{}^5C_1 \times {}^4C_1 \times 4!}{2!} = 240$</p> <p>Method 3</p> <p>Case 1: LALAL _ _ _ Number of ways $= 4! = 24$</p> <p>Case 2: LXLAL _ _ _ or LALXL _ _ _ Number of ways $= 2 \times \binom{3}{1} \times 4! = 144$</p> <p>Case 3: LXLYL _ _ _ or LYLXL _ _ _ Number of ways $= 2 \times \binom{3}{2} \times \frac{4!}{2!} = 72$</p> <p>Total = 240</p>

- 7 Anand, Beng and Charlie are selling cupcakes to raise funds for the charity, Boys And Girls Understand Singapore (BAGUS). Anand will bake 60% of the cupcakes, Beng will bake 40% of the cupcakes and Charlie will spread frosting on all the cupcakes.

20% of Anand's cupcakes will turn out flawed, while 12% of Beng's cupcakes will turn out flawed. Charlie has a 6% chance of spreading the frosting badly on any cupcake, regardless of whether it is flawed or otherwise.

If a cupcake is flawed or has frosting spread badly (or both), it is considered sub-standard. Otherwise, a cupcake is considered "good".

Find the probability that

- (i) a randomly chosen cupcake is "good", [2]
- (ii) a randomly chosen cupcake is either baked by Anand or is sub-standard but not both, [3]
- (iii) a randomly chosen cupcake is baked by Anand given that it is sub-standard. [3]

7i	<p>P(a randomly chosen cupcake is "good")</p> $= 0.6 \times 0.8 \times 0.94 + 0.4 \times 0.88 \times 0.94$ $= 0.78208 = 0.782 \text{ (3sf)}$
7ii	<p>P(cupcake is either baked by Anand, is sub-standard but not both)</p> <div data-bbox="517 1227 984 1503" data-label="Diagram"> </div> $= P(\text{cupcake is baked by Anand and is good}) + P(\text{cupcake is baked by Beng and is sub-standard})$ $= 0.6 \times 0.8 \times 0.94 + 0.4 \times (0.12 + 0.88 \times 0.06)$ $= 0.520 \text{ (3sf)}$ <p>Method 2</p> $= 0.6 \times 0.8 \times 0.94 + 0.4 \times (1 - 0.88 \times 0.94)$ $= 0.520 \text{ (3sf)}$

	<p>Method 3</p> <p> $= P(\text{baked by Anand}) + P(\text{sub-standard}) - 2 P(\text{baked by Anand and sub-standard})$ $= 0.6 + (1 - 0.78208) - 2 (0.6 \times (1 - 0.8 \times 0.94))$ $= 0.520 \text{ (3sf)}$ </p>
7iii	<p> $P(\text{cupcake baked by Anand} \mid \text{cupcake is sub-standard})$ $= \frac{P(\text{cupcake is sub-standard and baked by Anand})}{P(\text{cupcake is sub-standard})}$ $= \frac{0.6 \times (0.2 + 0.8 \times 0.06)}{1 - 0.78208} \quad \text{or} \quad \frac{0.6 \times (1 - 0.8 \times 0.94)}{1 - 0.78208}$ $= 0.683 \text{ (3sf)}$ </p>

- 8 The rate of growth, x units per hour, of a particular family of bacteria is believed to depend in some way on the controlled temperature T °C. Experiments were undertaken in the laboratory to investigate this and the results were tabulated as follows:

T	10	20	30	40	50	60	70	80
X	33	37	41	48	55	65	78	94

Draw a scatter diagram for these values, labelling the axes clearly. [1]

State, with a reason, which of the following model is most appropriate for the given data,

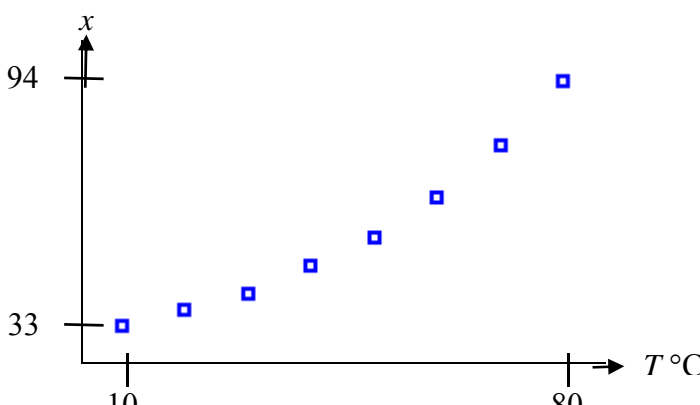
(A) $x = a + bT$ (B) $x = ae^{bT}$ (C) $x = a + b \ln T$

where a and b are constants, and $b > 0$. [1]

In addition, when $T = 90$, the rate of growth of bacteria was k units per hour.

Use the most appropriate model identified for the subsequent parts of this question.

- (i) A suitable linear regression line was constructed based on all 9 pairs of transformed data, including the additional pair of data. If the values of a and b were determined to be 27.06 and 0.01497 respectively, find the value of k , correct to the nearest whole number. [4]
- (ii) Find the product moment correlation coefficient for all 9 pairs of transformed data. [1]
- (iii) Use the regression line in (i) to predict the growth rate of the bacteria when the temperature is 105 °C, giving your answer to the nearest whole number. Comment on the reliability of this prediction. [2]

8	
8	<p>Model B is the most appropriate as the curve is concave upwards and x increases with T, which is the same as the shape shown in the scatter diagram.</p> <p>OR</p> <p>Model B is the most appropriate since x increases at an increasing rate as T increases, which is the same as the shape shown in the scatter diagram.</p>

8i	<p>Regression line for Model B:</p> $\ln x = \ln a + bT$ $\ln x = \ln(27.06) + 0.01497T$ $\bar{T} = \frac{10 + 20 + 30 + 40 + 50 + 60 + 70 + 80 + 90}{9} = 50 \text{ and}$ $\frac{\sum \ln x}{9}$ $= \frac{\ln 33 + \ln 37 + \ln 41 + \ln 48 + \ln 55 + \ln 65 + \ln 78 + \ln 94 + \ln k}{9}$ $= \frac{31.77392262 + \ln k}{9}$ <p>Since $(\frac{\sum \ln x}{9}, \bar{T})$ lies on the regression line,</p> $\frac{31.77392262 + \ln k}{9} = \ln(27.06) + 0.01497(50)$ $\Rightarrow k = 104 \text{ (nearest whole number)}$
8ii	From GC, $r = 0.997$
8iii	<p>When $T = 65$,</p> $\ln x = \ln(27.06) + 0.01497(105)$ $x = 130 \text{ (nearest whole number)}$ <p>The estimated growth rate cannot be taken as reliable as the temperature 105°C, from which the value of $x = 130$ is computed from, lies outside the data range of T i.e. $[10, 90]$.</p>

- 9 Small defects in a twill weave and satin weave occur randomly and independently at a constant mean rate of 1.2 defects and 0.8 defects per square metre respectively.

(i) Find the probability that there are exactly 9 defects in 7 square metres of twill weave. [2]

(ii) A box is made from 2 square metres of twill weave and 3 square metres of satin weave, chosen independently. A box is considered “faulty” if there are more than 10 defects. Show that the probability that a randomly chosen box is “faulty” is 0.0104, correct to 3 significant figures. [2]

(iii) A random batch of 50 boxes is delivered to a customer once every week. The customer can reject the entire batch if there are at least 2 “faulty” boxes in the batch. Using a suitable approximation, find the probability that the customer will reject the entire batch in a randomly selected week. [3]
Hence estimate the probability that the customer will reject 2 batches in a randomly selected period of 4 weeks. [2]

9i	<p>Let T be the number of defects in 7 m^2 of twill weave. $T \sim P_o(1.2 \times 7)$, i.e. $T \sim P_o(8.4)$ $P(T = 9) = 0.129025899 = 0.129$</p>
9ii	<p>Let W be the number of defects in a box. $W \sim P_o(2 \times 1.2 + 3 \times 0.8)$, i.e. $W \sim P_o(4.8)$ $P(\text{box is faulty}) = P(W > 10)$ $= 1 - P(W \leq 10)$ $= 0.0104$ (shown)</p>
9iii	<p>Let F be the number of “faulty” boxes in a random batch of 50 boxes. $F \sim B(50, 0.0104)$ Since n is large and $np = 0.52 < 5$, $F \sim P_o(0.52)$ approximately. $P(\text{customer rejects the entire batch})$ $= P(F \geq 2)$ $= 1 - P(F \leq 1)$ ≈ 0.096329 $= 0.0963$ or 0.0966 (3 sig fig) $\text{Required probability} \approx \binom{4}{2} (0.096329)^2 (1 - 0.096329)^2$ $= 0.0455$ or 0.0457 (3 sig fig) Alternative Solution: Let X be the number of batches out of 4 that are rejected. $X \sim B(4, 0.096329)$ $P(X = 2) = 0.0455$ or 0.0457</p>

- 10 Newmob is a mobile phone service provider which sells several brands of mobile phones. uPhones and Samseng phones are sold at a subsidy to its subscribers. Each subscriber can either buy one uPhone or one Samseng phone or both one uPhone and one Samseng phone. The probability that a randomly chosen subscriber buys a uPhone is 0.3, and independently, the probability that the subscriber buys a Samseng phone is p .

(i) In a random sample of 50 subscribers, the probability that at most 20 subscribers buy a Samseng phone is twice the probability that exactly 15 subscribers buy a uPhone. Find the value of p . [3]

For the remainder of this question, you may take the value of p to be 0.4.

(ii) In a random sample of 50 subscribers, find the probability that the number of subscribers who buy a uPhone is greater than the expected number of subscribers who buy a Samseng phone. [2]

(iii) Each subsidy for the uPhone costs Newmob \$280 and each subsidy for a Samseng phone costs \$200. Newmob has 1000 subscribers. Using suitable approximations, find the probability that the total subsidy given by Newmob for uPhone purchases exceeds the total subsidy given for Samseng phone purchases. [5]

10i Let U and S be the number of subscribers who purchased a uPhone and a Samseng phone respectively in a sample of size n .

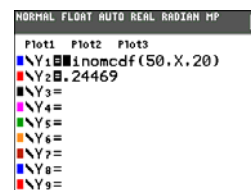
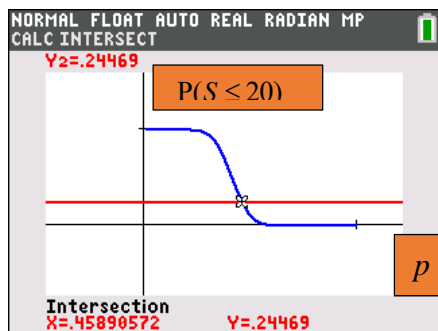
$$U \sim B(n, 0.3) \quad S \sim B(n, p)$$

When $n = 50$,

$$P(S \leq 20) = 2P(U = 15)$$

$$P(S \leq 20) = 0.24469$$

From GC,



OR

P(S ≤ 20)			AL RADIAN MP	
X				
.4585	.24651	.24469		
.4586	.24606	.24469		
.4587	.24561	.24469		
.4588	.24516	.24469		
.4589	.24472	.24469		
.459	.24427	.24469		
.4591	.24382	.24469		
.4592	.24338	.24469		
.4593	.24293	.24469		
.4594	.24249	.24469		
.4595	.24204	.24469		

X=.4585

$$p = 0.459 \text{ (3 sig fig)}$$

10ii	<p>Given $p = 0.4$, $n = 50$, $S \sim B(50, 0.4)$</p> <p>$E(S) = 50(0.4) = 20$</p> <p>$P(U > 20) = 1 - P(U \leq 20)$</p> <p>$= 0.0478$ (3 sig fig)</p>
10iii	<p>When $n = 1000$</p> <p>$U \sim B(1000, 0.3)$</p> <p>Since n is large, $np = 300 > 5$, $n(1 - p) = 700 > 5$</p> <p>$U \sim N(300, 210)$ approximately</p> <p>$S \sim B(1000, 0.4)$</p> <p>Since n is large, $np = 400 > 5$, $n(1 - p) = 600 > 5$</p> <p>$S \sim N(400, 240)$ approximately</p> <p>$280U - 200S \sim N(280(300) - 200(400), 280^2(210) + 200^2(240))$</p> <p>$280U - 200S \sim N(4000, 26064000)$</p> <p>$P(280U > 200S)$</p> <p>$= P(280U - 200S > 0)$</p> <p>$= P(280U - 200S > 0.5)$ (by continuity correction)</p> <p>$= 0.783$ (3 sig fig)</p>

- 11 The volume of a packet of soya bean milk is denoted by V ml and the population mean of V is denoted by μ ml. A random sample of 80 packets of soya bean milk is taken and the results are summarised by

$$\sum(v-250) = -217, \quad \sum(v-250)^2 = 30738.$$

Test, at the 4% significance level, whether μ is less than 250 ml. [5]

Explain, in the context of the question, the meaning of 'at the 4% significance level'. [1]

In another test, using the same data, and also at the 4% significance level, the hypotheses are as follows.

Null hypothesis: $\mu = \mu_0$

Alternative hypothesis: $\mu \neq \mu_0$

Given that the null hypothesis is rejected in favour of the alternative hypothesis, find the set of possible values of μ_0 . [4]

It is now given that $\mu = 250$ and the population variance is 385. A random sample of 50 packets of soya bean milk is taken and the total volume of the packets is denoted by T ml. By considering the approximate distribution of T and assuming that the volumes of all packets of soya bean milk are independent of one another, find $P(T > 12600)$. [3]

11i	<p>$H_0: \mu = 250$ $H_1: \mu < 250$ Level of significance: 4% $\bar{v} = \frac{-217}{80} + 250 = 247.2875$ $s^2 = \frac{1}{79} \left(30738 - \frac{(-217)^2}{80} \right) = 381.6378165$ Since $n = 80$ is large, by Central Limit Theorem, \bar{V} follows a normal distribution approximately. Test statistic: $\frac{\bar{V} - \mu}{\left(\frac{s}{\sqrt{n}} \right)} \sim N(0, 1)$ approximately If H_0 is true, $p\text{-value} = 0.107 > 0.04$, we do not reject H_0. (Or $z_{\text{calc}} = -1.24 < -1.751$) There is insufficient evidence at 4% level significance to conclude that $\mu < 250$. 'at the 4% significance level' means there is a 4% chance that we wrongly concluded that $\mu < 250$. (Or wrongly rejected $\mu = 250$ when it is true)</p>
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$$H_0: \mu = \mu_0$$

$$H_1: \mu \neq \mu_0$$

$$\text{If } H_0 \text{ is true, } z_{\text{cal}} = \frac{\bar{v} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)}$$

In order to reject H_0 , z_{cal} must lie in the critical region.

$$\frac{247.2875 - \mu_0}{\sqrt{\frac{381.6378165}{80}}} < -2.053749 \quad \text{or} \quad \frac{247.2875 - \mu_0}{\sqrt{\frac{381.6378165}{80}}} > 2.053749$$

$$\mu_0 > 251.77 \quad \text{or} \quad \mu_0 < 242.80$$

Set of possible values of $\mu_0 = \{ \mu_0 : \mu_0 < 243 \text{ or } \mu_0 > 252 \}$

(Also accept 242 other than 243)

$$T = V_1 + V_2 + V_3 + \dots + V_{50}$$

Given $E(V) = 250$ and $\text{Var}(V) = 385$

Since $n = 50$ is large, by Central Limit Theorem,

$T \sim N(50 \times 250, 50 \times 385) = N(12500, 19250)$ approximately.

$$P(T > 12600) = 0.236 \text{ (3 sig fig)}$$