



**NANYANG JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION**

Higher 2

MATHEMATICS

9740/02

Paper 2

22nd September 2016

3 Hours

Additional Materials: Cover Sheet
 Answer Papers
 List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **6** printed pages.

Section A: Pure Mathematics [40 marks]

- 1** The first four terms of a sequence of numbers are 10, 6, 5 and 7. S_n is the sum of the first n terms of this sequence. Given that S_n is a cubic polynomial in n , find S_n in terms of n . [4]

Show that $U_n = \frac{3}{2}n^2 - \frac{17}{2}n + 17$, where U_n denotes the n^{th} term of the sequence. [2]

Find the set of values of n for which $S_n < 3U_n$. [2]

- 2** On separate diagrams, draw sketches of the graphs of

(i) $y = \frac{x^2(3-x)}{1+x}$,

(ii) $y^2 = \frac{x^2(3-x)}{1+x}$,

including the coordinates of the points where the graphs cross the axes and the equations of any asymptotes. You should show the features of the graphs at the points where it crosses the x -axis clearly.

Show that the area of the region enclosed by the graph in (ii) may be expressed in the

form $2 \int_0^3 \frac{3x - x^2}{\sqrt{4 - (x-1)^2}} dx$.

By using the substitution $x - 1 = 2 \sin \theta$, evaluate this area exactly. [10]

- 3** (a) Solve $z^3 - 2(2-i)z^2 + (8-3i)z - 5+i = 0$, given that one of the three roots is real. [5]

(b) The complex number u is given by $u = \cos \theta + i \sin \theta$, where $0 < \theta < \frac{\pi}{2}$.

(i) Show that $1 - u^2 = -2iu \sin \theta$ and hence find the modulus and argument of $1 - u^2$ in terms of θ . [4]

(ii) Given that $(1 - u^2)^{10}$ is real and negative, find the possible values of θ in terms of π . [3]

- 4** [In this question, you may use the result that for a circle with radius r , a sector with angle θ has arc length $r\theta$ and area $\frac{1}{2}r^2\theta$.]
- (a) A circle of radius r is divided into 16 sectors of decreasing arc length. Let L_n and A_n be the arc length and the area of the n th sector respectively. Suppose L_n is an arithmetic sequence with first term r and common difference d .
- (i) Show that $d = \left(\frac{\pi-8}{60}\right)r$. [2]
- (ii) Show that A_n is an arithmetic sequence. [3]
- (b) Let G_n be the area of a sector of a circle with radius a . Suppose that G_n is a geometric sequence with first term a and common ratio r , where $0 < r < 1$.
- (i) If N sectors are needed to form the circle, show that r satisfies the equation
- $$r^N - \pi ar + (\pi a - 1) = 0. \quad [3]$$
- (ii) If an infinite number of sectors are needed to form the circle, find r in terms of a . [2]

Section B: Statistics [60 marks]

- 5** A company sells a certain brand of baby milk powder and would like to gather feedback on their product. Explain why quota sampling is appropriate in this situation and describe briefly how a sample of 50 could be chosen using quota sampling. [3]
- The company wishes to randomly reward 5 customers with free milk vouchers through a lucky draw. Suppose that 2000 customers qualify for the draw, show that there will be equal probability of a particular customer being the first to be selected or the third to be selected for the free milk vouchers. [2]

- 6** The mass, in grams, of an ice-cube has the distribution $N(\mu, \sigma^2)$. The mean mass of a random sample of n ice-cubes is denoted by \bar{X} . It is given that $P(\bar{X} < 35.0) = 0.97725$ and $P(\bar{X} \geq 20.0) = 0.84134$.

(i) Obtain an expression for σ in terms of n . [3]

(ii) Find $P(\bar{X} > 32)$. [2]

Assume now that the mass of an ice-cube has the distribution $N(25, 50)$.

An ice dispenser discharges 15 ice cubes each time into a cup. State the distribution of the mass of a discharge of 15 ice cubes. [1]

(iii) Find the mass exceeded by 10% of these discharges, correct to 1 decimal place. [2]

(iv) Find the probability that the mass of the first discharge of ice-cubes is more than the second discharge. [2]

- 7** A team of 5 men and 5 women is to be picked from 8 men and 9 women such that two of the 9 women, Ann and Lucy, must both be selected or not at all. Find the number of ways in which this can be done. [2]

Assume now the team is selected and Ann, Carrie and Lucy are included.

(i) The selected team is to form a queue. Find the number of possible arrangements if Ann and Lucy are to occupy both the second and the sixth positions and no two people of the same gender are to stand next to each other. [3]

(ii) On another occasion, the selected team is required to be seated at a round table with 10 chairs of different colours. If only Carrie can be seated between Ann and Lucy, find the number of possible arrangements. [3]

- 8** Two teams, the Ramblers and the Strollers, meet annually for a quiz which always has a winner. If the Ramblers wins the quiz, the probability of them winning the following year is 0.7. If the Strollers wins the quiz, the probability of them winning the following year is 0.5.

The Ramblers won the quiz in 2015.

- (i) Find the probability that the Strollers will win in 2018. [2]
- (ii) If the Strollers were to win in 2018, what is the probability that it will be their first win for at least three years since 2015? [2]
- (iii) Assuming that the Strollers wins in 2018, find the smallest value of n such that the probability of the Ramblers winning the quiz for n consecutive years after 2018 is less than 5%. [3]

- 9** It is believed that the probability p of a randomly chosen pregnant woman giving birth to a Down Syndrome child is related to the woman's age x , in years. The table gives observed values of p for 6 different values of x .

x	20	25	30	35	40	45
p	0.00023	0.00067	0.00125	0.00333	0.01000	0.03330

- (i) Sketch the scatter diagram for the given data. [1]
- (ii) Find, correct to 4 decimal places, the product moment correlation coefficient between
 - (a) p and x ,
 - (b) $\ln p$ and x ,
 - (c) p and x^2 . [2]
- (iii) Using the most appropriate case from part (ii), find the equation which best models the probability of a pregnant woman giving birth to a Down Syndrome child at different ages. [2]
- (iv) Hence, estimate the expected number of children with Down Syndrome that will be born to 5000 randomly chosen pregnant women of age 32. [2]

- 10** At an early stage in analysing the marks, x , scored by a large number of candidates in an examination paper, the Examination Board takes the scores from a random sample of 250 candidates. The results are summarised as follows:

$$\sum x = 11872 \quad \text{and} \quad \sum x^2 = 646193$$

- (i) Calculate unbiased estimates of the population mean and variance to 3 decimal places. [2]
 - (ii) In a 1-tail test of the null hypothesis $\mu = 49.5$, the alternative hypothesis is accepted. State the alternative hypothesis and find an inequality satisfied by the significance level of the test. [4]
 - (iii) It is subsequently found that the population mean and standard deviation for the examination paper are 45.292 and 18.761 respectively. Find the probability that in a random sample of size 250, the sample mean is at least as high as the one found in the sample above. [2]
- 11** On a typical weekday morning, customers arrive at the post office independently and at a rate of 3 per 10 minute period.
- (i) State, in context, a condition needed for the number of customers who arrived at the post office during a randomly chosen period of 30 minutes to be well modelled by a Poisson distribution. [1]
 - (ii) Find the probability that no more than 4 customers arrive between 11.00 a.m. and 11.30 a.m. [2]
 - (iii) The period from 11.00 a.m. to 11.30 a.m. on a Tuesday morning is divided into 6 periods of 5 minutes each. Find the probability that no customers arrive in at most one of these periods. [2]
- The post office opens for 3.5 hours each in the morning and afternoon and it is noted that on a typical weekday afternoon, customers arrive at the post office independently and at a rate of 1 per 10 minute period. Arrivals of customers take place independently at random times.
- (iv) Show that the probability that the number of customers who arrived in the afternoon is within one standard deviation from the mean is 0.675, correct to 3 decimal places. [3]
 - (v) Find the probability that more than 38 customers arrived in a morning given that a total of 40 customers arrived in a day. [4]
 - (vi) Using a suitable approximation, estimate the probability that more than 100 customers arrive at the post office in a day. [3]