VICTORIA JUNIOR COLLEGE
Preliminary Examination
Higher 2

MATHEMATICS **9740/02**
Paper 1
Wednesday **8am – 11am**

21 September 2016 **3 hours**

READ THESE INSTRUCTIONS FIRST

Write your name and CT group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of 5 printed pages

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Section A: Pure Mathematics [40 marks]

1 When an object moves through a fluid, it experiences a force that slows it down. This force is called the drag force. At low speeds, it is known that the drag force causes the rate of change in the speed of the object to be proportional to its speed. You may assume that the experiment described below is carried out at low speeds and the only factor that affects the speed is the drag force.

An experiment is conducted to find out how the speed of an object changes as it moves through a certain fluid. When the speed of the object slows down to a speed of D m s^{-1}, a sensor is triggered and the subsequent speeds of the object are recorded.

(i) Show that the speed of the object, v m s^{-1}, at t s after the sensor is triggered, is given by

$v = De^{-pt}$, where p is a positive constant. [4]

(ii) On a single diagram, sketch the curves, C_1 and C_2, of v against t corresponding to $p = e$ and $p = \frac{1}{e}$.

State a single transformation the maps C_1 onto C_2. [3]

2 Given that $y = \sqrt{\left(e^x \cos^2 x\right)}$, show that $2y\frac{dy}{dx} = y^2 - e^x \sin 2x$. [2]

(i) Find the series expansion of y in ascending powers of x up to and including the term in x^2. [3]

(ii) Hence, or otherwise, find the series expansion of $\frac{1}{\sqrt{\left(e^x \cos^2 x\right)}}$ in ascending powers of x up to and including the term in x^2. [3]

3 **(a)** The points A, B, C and D represent the complex numbers $-2+5i$, z_1, $4+i$ and z_2 respectively. Given that $ABCD$ is a square, labelled in an anti-clockwise direction, show that $z_1 = -1$. Find z_2. [4]

(b) Show that the equations $z^5 = z^*$ and $|z| = 1$ can be reduced to $z^n = 1$, where n is a positive integer to be determined. Find all possible values of z in the form $re^{i\theta}$, where $r > 0$ and $0 \le \theta < 2\pi$. [4]

Given further that $0 < \arg(z) < \frac{\pi}{2}$, find the smallest positive real number k for $\frac{(1+i)}{z^k}$ to be purely imaginary. [4]

4 **(a)** Referred to the origin *O*, the points *A* and *B* have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. The points *C* on *AB* and *D* on *OB* are such that $2AC = CB$ and $2OD = 3DB$. Show that a vector equation of the line *m* passing through *C* and *D* can be written as

$$\mathbf{r} = \frac{3}{5}\mathbf{b} + \lambda(5\mathbf{a} - 2\mathbf{b}),\ \lambda \in \mathbb{R}\ . \qquad [4]$$

It is given that $|\mathbf{a}| = 2, |\mathbf{b}| = 5$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $60°$. The point *F* on *m* is such that *F* is nearest to *O*. Show that the position vector of *F* can be written as $k(5\mathbf{a} + 2\mathbf{b})$, where *k* is a constant to be found. [4]

(b) Plane π has equation $3x + 2y + 5z = 45$.

Obtain a vector equation of π in the form

$$\mathbf{r} = \mathbf{t} + \lambda\mathbf{u} + \mu\mathbf{v},\ \lambda, \mu \in \mathbb{R}\ ,$$

given that $\mathbf{t}$ and $\mathbf{u}$ are of the form $p\mathbf{i} + p\mathbf{j}$ and $2\mathbf{i} + q\mathbf{j}$ respectively, where p and q are constants to be determined, and $\mathbf{u}$ is perpendicular to $\mathbf{v}$. [5]

Section B: Statistics [60 marks]

5 The head of the Physical Education department of a school wants to gather students' views about the school's efforts in promoting student participation in physical activities. On a particular afternoon, he surveys the first 30 students who turn up at the school gymnasium.

(i) Explain why the above method may not be suitable for the purpose of his survey. [1]

(ii) Describe another sampling method that would yield a sample that is more appropriate in this context. [2]

6 Numbers in this question are formed using only the digits 1, 2, 6, 7 and 9.

(i) How many 4-digit numbers can be formed if repetition of digits is allowed? [1]

(ii) How many even numbers between 10,000 and 30,000 can be formed, if each digit can only be used once? [2]

(iii) A "trick" number is a 6-digit number formed using exactly 3 different digits, and that each digit is smaller than or equal to the following digit. How many "trick" numbers can be formed? [e.g. 127777 and 667799 are "trick" numbers, 111122 and 192992 are not "trick" numbers.] [5]

7 Box A contains 10 red, 8 blue and 7 green balls. Box B contains 2 white and 3 black balls. All the balls are indistinguishable except for their colours. Three balls are taken from Box A and two balls are taken from Box B, at random and without replacement.

Mr Wong guesses that there are at least 1 red ball and exactly 2 black balls taken, while Mr Tan guesses that all the balls taken are of different colours.

(i) Show that the probability that Mr Wong is correct is 0.241, correct to 3 significant figures. [3]

(ii) Find the probability that Mr Tan is correct. [2]

(iii) Find the probability that Mr Wong is correct, given that Mr Tan is wrong. [3]

8 A shop sells two brands of refrigerators which are in the same price range. The number of Tahichi refrigerators sold per week is a random variable with the distribution Po(1.3) and the number of Sungsam refrigerators sold per week is a random variable with the distribution Po(1.1).

(i) Show that the probability of a total of at least 10 refrigerators being sold in a randomly chosen 4-week period is 0.491, correct to 3 significant figures. [3]

(ii) A 4-week period is called a "good" period if at least 10 refrigerators are sold. Find, using a suitable approximation, the probability that, in 52 randomly chosen 4-week periods, there are more than 25 but at most 32 "good" periods. [4]

(iii) State, in the context of this question, two assumptions needed for your calculations in part **(i)** to be valid. Explain why one of these assumptions may not hold in this context. [3]

9 The masses of grade A durians from a plantation are normally distributed with mean 1.96 kg and standard deviation 0.24 kg and the masses of grade B durians from the same plantation are normally distributed with mean 1.00 kg and standard deviation σ kg.

The probability that a randomly chosen grade B durian has a mass of more than 0.8 kg is 0.95. Show that $\sigma = 0.122$, correct to 3 significant figures. [3]

(i) 50 grade A and 1 grade B durians are randomly picked from this plantation. Find the probability that the average mass of the 50 grade A durians is more than twice the mass of the grade B durian. Explain whether there is a need to use Central Limit Theorem in your working. [4]

(ii) A wholesaler buys 50 grade B durians. Using a suitable approximation, find the probability that more than 47 of the durians will have a mass of more than 0.8 kg. [4]

10 An ice-cream shop owner in Singapore wishes to find out how the daily sales of ice-cream depend on the daily average temperature. The following data are collected over 10 days.

Day	1	2	3	4	5	6	7	8	9	10
Daily average temperature, t °C	24.0	25.1	26.2	31.0	28.4	34.0	27.2	32.9	33.5	29.5
No. of cups of ice creams sold in one day, x	100	130	140	171	158	179	150	176	178	163

(i) Without calculating the equation of the regression line of x on t, find the coordinates of a point that will lie on this line. [2]

(ii) Draw a scatter diagram to illustrate the data and find the product moment correlation coefficient between x and t. [2]

(iii) **Without any calculations**, explain whether a quadratic model is more appropriate than a linear model to fit the data. [1]

(iv) The model $x = a(34.2 - t)^2 + b$ is used to fit the data. Calculate the least squares estimates of a and b. [2]

(v) By using the values found in part **(iv)**, estimate the expected number of cups of ice creams sold in 1 day if the daily average temperature is 31.0°C. [1]

11 The mass X g, of one loaf of "Gardener" wholemeal bread is a random variable with mean μ g, which is claimed to be 400g. A random sample of 5 loaves of wholemeal bread has masses in g as follows,

371.3, 399.4, 402.3, 388.3, 400.4.

Carry out a test at the 5% significance level to determine whether this claim is valid, stating clearly any assumption made. [4]

Another random sample of 50 loaves of wholemeal bread is taken, with results summarised below,

$$\sum(x-400) = -102.4\,, \ \sum(x-400)^2 = 8030.2\,.$$

Using the second sample, another test was carried out at the k% significance level to determine the validity of the claim. Find the set of possible values of k for which the test concludes that the claim is incorrect.
Explain, in the context of the question, the meaning of "k% significance level". [5]

n hypothesis tests are carried out at 4% level of significance to test the validity of the claim.

Given that μ is indeed 400g, find the least value of n such that the probability of at most 1 test making a wrong conclusion is less than 0.05. [3]

[End of Paper]