Candidate Name: _____ Class: _____

PIONEER JUNIOR COLLEGE
While I Live, I Learn

JC2 PRELIMINARY EXAM
Higher 2

MATHEMATICS **9740/02**
Paper 2 **16 Sep 2016**
3 hours

Additional Materials: Cover page
Answer papers
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your full name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 Two complex numbers a and b are given by $2+3\mathrm{i}$ and $-4-5\mathrm{i}$ respectively.

(i) On a single Argand diagram, sketch the loci

(a) $|2z-a-b|=|a-b|$,

(b) $0 \le \arg(z-b) \le \arg(a-b)$. [4]

(ii) Find range of arg(z) where z is the complex number satisfies the relations in part **(i)**. [4]

2 A curve C has equation $y=\dfrac{ax}{x-1}$ where $a>0$.

(i) By writing the equation of C as $y=A+\dfrac{B}{x-1}$, state a sequence of transformations which transform the graph of $y=\dfrac{1}{x}$ to C. [3]

(ii) Sketch C, giving the equations of any asymptotes and the coordinates of any points of intersection with the axes. [2]

(iii) The region R is bounded by C, the lines $x=2$, $x=4$ and $y=a$. Find the exact volume in terms of a when R is rotated through 2π radians about the x-axis. [3]

(iv) The region S is bounded by $y=\dfrac{a}{x}$, the lines $x=1$, $x=3$ and $y=0$. State the exact volume in terms of a when S is rotated through 2π radians about the line $y=-a$. [1]

3 The function f is defined by $\mathrm{f}(x) = \begin{cases} \frac{x}{2} & \text{if } x \leq 0, \\ 2\sin x & \text{if } 0 < x \leq 4. \end{cases}$

(i) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(ii) If the domain of f is restricted to $x \leq k$, state the largest value of k, in exact form, for which the function f^{-1} exist. [1]

(iii) Using the domain from part **(ii)**, define f^{-1} in a similar form. [4]

(iv) Solve $\mathrm{f}^{-1}(x) = \mathrm{f}(x)$. [2]

In the rest of the question, the domain of f is as originally defined.

The function g is defined by $\mathrm{g} : x \mapsto -x^3, x \in \mathbb{R}, x > 0$.

(v) Find an expression for $\mathrm{fg}(x)$. [2]

4 **(i)** Differentiate $\tan^{-1}\left(\frac{\sqrt{3}}{2}x\right)$ with respect to x. [2]

(ii) Find the binomial expansion for $\frac{1}{3x^2+4}$ up to and including the term in x^6, giving the coefficients as exact fractions in their simplest form. Find the set of values of x for which the expansion is valid. [5]

(iii) Hence, find the first four non-zero terms of the Maclaurin series for $\tan^{-1}\left(\frac{\sqrt{3}}{2}x\right)$. Give the coefficients as exact fractions in their simplest form. [5]

Section B: Statistics [60 marks]

5 A pharmaceutical company has invented a new drug for diabetic patients and wishes to carry out a trial of the new drug involving 5% of the patients from a local hospital.

(i) Explain how a systematic sample could be carried out. [2]

(ii) State one disadvantage of systematic sampling in this context and name a more appropriate sampling method. [2]

6 Given that $\mathrm{P}(A \mid B') = 3\mathrm{P}(A \mid B)$ and $\mathrm{P}(B') = 4\mathrm{P}(B)$.

(i) Show that $\mathrm{P}(B') = \frac{4}{5}$. [2]

(ii) Using $\mathrm{P}(A \cap B') = \mathrm{P}(A) - \mathrm{P}(A \cap B)$, find $\mathrm{P}(B' \mid A)$. [3]

7 The sales department of a company consists of 3 teams led by Mrs Wong, Miss Tan and Mr Lim. Each team is made up of 1 team leader and 5 sales executives. The number of male and female sales executives within each team is given in the table below:

	Team *A*	Team *B*	Team *C*
Team Leader	Mrs Wong	Miss Tan	Mr Lim
Number of Male Executive(s)	3	4	0
Number of Female Executive(s)	2	1	5

A taskforce is to be formed by selecting 7 representatives from the 18 members of the department. Find the number of different taskforces that can be formed if the taskforce must include

(i) Miss Tan and 1 other team leader, [2]

(ii) more females than males, [2]

(iii) at least 1 representative from each team. [3]

8 In this question you should state clearly the values of the parameters of any normal distribution you use.

The mass in kilograms of an Atlantic salmon is a normally distributed continuous random variable X with mean μ and standard deviation σ.

(i) It is known that $\mathrm{P}(X < 22) = 0.159$ and $\mathrm{P}(X > 31) = 0.106$. Show that $\mu = 26.0$ and $\sigma = 4.01$. [3]

(ii) In a random sample of 40 Atlantic salmon, estimate the probability that at least 35 of them have a mass of at most 31 kilograms. [3]

It is also known that the mass in kilograms of the Bluefin tuna has the distribution $\mathrm{N}(380, 10^2)$.

(iii) Find the probability that the average mass of 2 randomly chosen Bluefin tuna and 3 randomly chosen Atlantic salmon is at most 170 kg. [2]

9 The table gives the world record time, in seconds, for the 100 metres free style swimming event at the Olympic Games in the past years.

Year, x	1908	1920	1956	1968	1976	2000
Time, t	65.60	60.40	55.40	52.20	49.99	48.18

(i) Draw a scatter diagram to illustrate the data. [1]

(ii) Comment on whether a linear model would be appropriate, referring both to the scatter diagram and the context of the question. [2]

(iii) It is thought that the time can be modelled by one of the formulae $\ln t = a + bx$ or $\frac{1}{t} = a + bx$. Find, correct to 4 decimal places, the value of the product moment correlation coefficient between

(a) $\ln t$ and x,

(b) $\frac{1}{t}$ and x. [2]

(iv) Use your answers to part **(iii)** to explain which of $\ln t = a + bx$ or $\frac{1}{t} = a + bx$ is the better model. [1]

(v) The time corresponding to 1964 was added to obtain the equation with appropriate model chosen in part **(iv)** where $a = -0.09836$ and $b = 5.96846 \times 10^{-5}$. Find the time in 1964. [2]

10 A car manufacturer launches a new car model "Green Leaf" that is marketed to be environmentally friendly. It is claimed that the carbon emission of the "Green Leaf" is at most 80 g/km. The transport authority suspects that the figure is understated, and requests the manufacturer to submit test data from 20 units of the "Green Leaf". The test data submitted is as follows.

Carbon Emission (g/km)	78	79	80	81	82
No. of units	2	3	6	4	5

(i) Calculate unbiased estimates of the population mean and variance. [2]

(ii) Stating a necessary assumption, test at the 10% level of significance whether there is any evidence to doubt the manufacturer's claim. [4]

The transport authority subsequently decides to conduct their own test, and invites 10 owners of the "Green Leaf" to form a sample. The mean and variance of this sample is found to be 80.6 g/km and m^2 g^2/km^2 respectively.

(iii) Find the set of values of m for which the result of the test would be to reject the manufacturer's claim, at the 1% significance level. [3]

11 There are 2 main types of T-cells in the human body. T4-cells are "helper" cells that lead attacks against infections in the human body, while T8-cells are "suppressor" cells that kill cancer and virus infected cells in the human body. It is to be assumed that the number of T4-cells per 0.01 mm^3 of blood can be modelled by the distribution Po(5) and the number of T8-cells per 0.01 mm^3 of blood can be modelled by the independent distribution Po(1.5).

A patient is considered healthy if he or she has at least 4 T4-cells and at least 1 T8-cells in 0.01 mm^3 of blood.

(i) Find the probability that a randomly selected patient is healthy. [3]

(ii) Find the probability that only 1 out of 3 randomly selected patients is healthy. [2]

A patient is susceptible to infections if his or her T4-cells count falls below 3 per 0.01 mm^3 of blood.

(iii) Use a suitable approximation, which should be stated, to find the probability that, in 100 randomly selected patients, the number of patients susceptible to infections is between 20 and 50 inclusive. [4]

12 Mr Ouyang, a car manufacturer, finds that on average, 2% of his cars have faulty gearboxes. On a particular occasion, he selects n cars randomly for inspection, and the number of cars with faulty gearbox is denoted by the random variable C.

(i) State in context of this question, what must be assumed for C to be well modelled by a binomial distribution. [2]

(ii) Given that $n = 20$, find the probability that C is between 2 and 6. [2]

(iii) The probability that there are less than 2 cars with faulty gearbox in a sample of n cars is at most 0.95. Write down an inequality in terms of n, and find the least possible value of n. [3]

(iv) Mr Ouyang selects 100 batches of 20 cars. Estimate the probability that the average number of cars with faulty gearbox per batch is at least 0.3. [3]