HWA CHONG INSTITUTION
2016 JC2 PRELIMINARY EXAMINATION

MATHEMATICS
Higher 2

9740/01

Paper 1

Wednesday **14 September 2016** **3 hours**

Additional materials: Answer paper
List of Formula (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and CT class on all the work you hand in, including the Cover Page which is found on Page 2.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Do not write anything on the List of Formula (MF15).

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, place the completed cover page on top of your answer scripts and fasten all your work securely together with the string provided.

This question paper consists of 6 printed pages.

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MATHEMATICS

9740
14 September 2016
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Name:

CT: 1 5

COVER PAGE

1. Write your name, CT group and calculator model(s) in the spaces provided.
2. Arrange your answers in numerical order.
3. Detach this cover page and place it on top of your answer paper and fasten them securely together with the string provided.

For Examiner's Use			
Question No.	**Marks Obtained**	**Total Marks**	**Remarks**
1		4	
2		5	
3		8	
4		9	
5		9	
6		9	
7		10	
8		10	
9		11	
10		12	
11		13	
TOTAL		**100**	

Graphing Calculator Model:

Scientific Calculator Model:

1. A sequence follows the recurrence relation

$$U_{n+1} - U_n = 2\cos\frac{(2n+1)x}{2}\sin\frac{x}{2}\ ,\ U_1 = \sin x \text{ for } n = 1, 2, 3, \ldots .$$

Prove by mathematical induction that $U_n = \sin(nx)$ for all positive integer n. [4]

2. Solve the inequality $\dfrac{2}{4(x+1)^2+1} > 1.$ [2]

Hence find $\displaystyle\int_{-1}^{\frac{\sqrt{3}-2}{2}} \left|1 - \frac{2}{4(x+1)^2+1}\right| \mathrm{d}x,$ leaving your answer in exact form. [3]

3. Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$ with $\mathbf{a}$ not parallel to $\mathbf{b}$. The point P is on AB produced with $AP : AB = 3:1$ and the position vector of point Q is $2\mathbf{a}$.

(a) Find the position vector of the point of intersection of lines OB and PQ, giving your answer in terms of $\mathbf{b}$. [4]

(b) It is given that $\mathbf{a} \times \mathbf{b} = 2\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$ and the point $C(0,\ 3,\ 4)$ does not lie on the plane OAB. Find the foot of the perpendicular from C to the plane OAB. [4]

4. Prove that $\dfrac{2n+1}{\sqrt{n^2+2n}+\sqrt{n^2-1}} = \sqrt{n^2+2n} - \sqrt{n^2-1}.$ [2]

Hence find $\displaystyle\sum_{n=1}^{N} \frac{2n+1}{\sqrt{n^2+2n}+\sqrt{n^2-1}}$. [3]

(a) Deduce the value of $\displaystyle\sum_{n=2}^{N} \frac{2n-1}{\sqrt{n^2-2n}+\sqrt{n^2-1}}$. [3]

(b) Show that $\displaystyle\sum_{n=1}^{N} \frac{2n+1}{2n-1} > \sqrt{N^2+2N}$. [1]

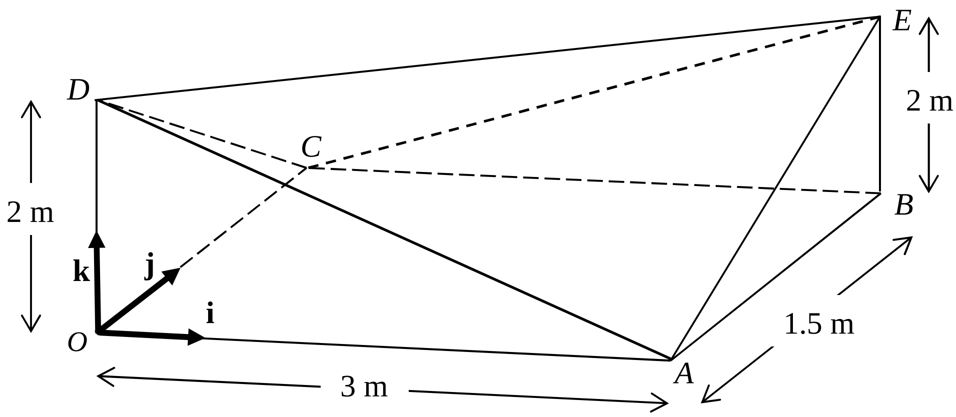
5. Sketch on a single Argand diagram, the loci defined by $-\frac{\pi}{4} < \arg(z+1+2\mathrm{i}) \le \frac{\pi}{4}$ and $|(2+\mathrm{i})w+5| \le \sqrt{5}$. [4]

(i) Find the minimum value of $\arg(w)$. [2]

(ii) Find the minimum value of $|z-w|$. [2]

(iii) Given that $\arg(z-w) < \theta,\ -\pi < \theta \le \pi$, state the minimum value of θ. [1]

6. A group of boys want to set up a camping tent. They lay down a rectangular tarp *OABC* on the horizontal ground with *OA* = 3 m and *AB* = 1.5 m and secure the points *D* and *E* vertically above *O* and *B* respectively, such that $OD = BE = 2$ m.



Assume that the tent takes the shape as shown above with 6 triangular surfaces and a rectangular base. The point *O* is taken as the origin and the unit vectors **i**, **j** and **k** are taken to be in the direction of $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively.

(i) Show that the line *DE* can be expressed as $\mathbf{r} = 2\mathbf{k} + \lambda(2\mathbf{i}+\mathbf{j}),\ \lambda \in \mathbb{R}$. [2]

(ii) Find the Cartesian equation of the plane *ADE*. [3]

(iii) Determine the acute angle between the planes *ADE* and *OABC*. Hence, or otherwise, find the acute angle between the planes *ADE* and *CDE*. [4]

7. The curve *C* has equation $y = \frac{x-2}{kx^2+x-2}$, where $k > 1$.

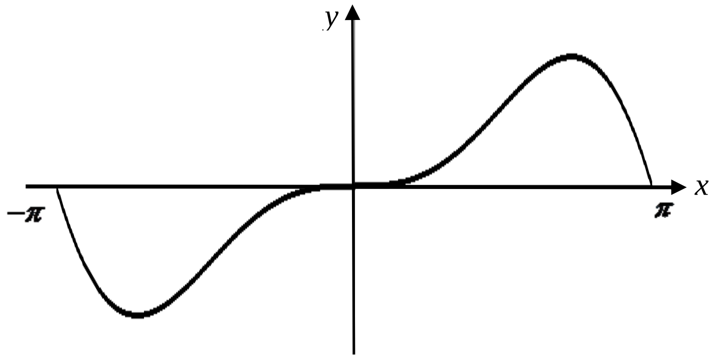
(i) Find the equation of the tangent at the point *A* where *C* cuts the *y*-axis. [2]

(ii) Sketch *C*, giving the equations of asymptotes, the coordinates of turning points and axial intercepts in terms of k, if any. [4]

(iii) Find the equation of the normal at the point *B* where *C* cuts the *x*-axis. Leave your answer in terms of k. [2]

(iv) Hence show that the value of the area bounded by the tangent at *A*, the normal at *B* and both the *x*- and *y*-axes is more than $\frac{15}{8}$ square units. [2]

8. The curve C (as shown in the diagram below) has equation $y = x^2 \sin x,\ -\pi \le x \le \pi$.



(i) Calculate the exact area of the region R enclosed by C and the x-axis. [4]

(ii) Sketch the curve with equation $(y+1)^2 - 4(x+2)^2 = 1$, showing clearly the coordinates of the turning points and the equation(s) of any asymptote(s). [2]

(iii) Hence find the volume of the solid generated when the region bounded by the 2 curves is rotated through 4 right angles about the x-axis. [4]

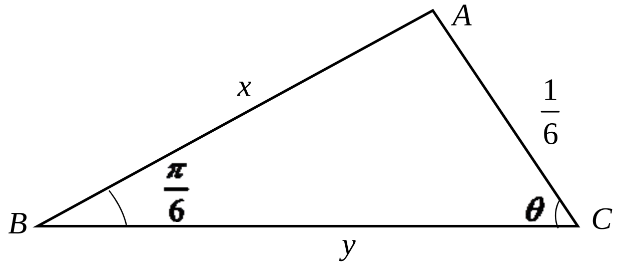
9. A manufacturer produces cylindrical containers using sheet metal of negligible thickness. The cylindrical container has an open top, and a base and curved sides made up of the sheet metal.

(a) **(i)** It is given that the volume of the cylindrical container is fixed at k cm^3. Show that when the amount of sheet metal used for the cylindrical container is a minimum, the ratio of its height to its radius is 1:1. [5]

(ii) A product designer proposed a new design where the height of the cylindrical container is always 2.5 times that of its radius. Given that the radius of a cylindrical container produced using the new design equals the radius of the container produced in part **(a)(i)** with minimum sheet metal. Find the ratio of the amount of sheet metal used in this new design to the minimum amount of sheet metal used in part **(a)(i)**. [2]

(b) To reduce cost, plastic with negligible thickness, instead of sheet metal is used to manufacture the new design cylindrical containers in part **(a)(ii)** using *injection blow moulding* technology. In the injection blow moulding process, it is assumed that the cylindrical containers increase in size proportionately with the height to radius ratio remaining constant at 5:2 throughout the process. If the volume of the cylindrical container increases at a rate of 80 cm^3 per second, find the rate of change of the surface area of the cylindrical container when its height is 50 cm. [4]

10.



In the triangle ABC, $AB = x$, $BC = y$, $AC = \frac{1}{6}$, angle $ABC = \frac{\pi}{6}$ radians and angle $ACB = \theta$ radians (see diagram).

(a) **(i)** Show that $\dfrac{x}{y} = \dfrac{2\sin\theta}{\cos\theta + \sqrt{3}\sin\theta}$. [3]

(ii) Given that θ is sufficiently small, express $\dfrac{x}{y}$ as a cubic polynomial in θ. [3]

(b) **(i)** Show that $\theta = \sin^{-1}(3x)$. [1]

(ii) Find the Maclaurin series for θ, up to and including the term in x^3. [5]

11. The functions f and g are defined by

$$\mathrm{f} : x \mapsto \frac{1}{2}\mathrm{e}^{1-x^2},\ x \in \mathbb{R},\ x \le 1 \text{ and}$$

$$\mathrm{g} : x \mapsto \sqrt{1 - \ln x},\ x \in \mathbb{R},\ 0 < x \le \mathrm{e}.$$

(i) Show that gf exists, and find the range of gf. [4]

(ii) Justify, with a reason, whether f^{-1} exists. [2]

(iii) The domain of f is restricted to $(-\infty, b]$ such that b is the largest value for which the inverse function f^{-1} exists. State the value of b and define f^{-1} clearly. [4]

(iv) The graph of $y = \mathrm{h}(x)$ is obtained by transforming the graph of $y = \mathrm{g}(x)$ in the following 2 steps.

Step 1: Scale parallel to the x-axis by a factor of 2.

Step 2: Reflect in the x-axis.

Define h in a similar form. [3]

End of Paper