NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 2

MATHEMATICS **9740/02**

Paper 2 **14 September 2016**

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)
Cover Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, registration number, subject tutorial group, on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in the brackets [ ] at the end of each question or part question.

National Junior College

Section A: Pure Mathematics [40 marks]

1 **(a)** Kenny took a loan of \$9600 from a friend, and arranged to pay his loan fully in a period of exactly 48 months. To fulfil this arrangement, he paid \$$a$ on the last day of the first month, and on the last day of each subsequent month, he paid \$$d$ more than in the previous month. However, due to financial difficulties, Kenny stopped his payments after his 40th payment, and as a result he still had exactly \$2400 left unpaid.

In which month did Kenny first pay at least \$130 on the last day of that month? [5]

(b) **(i)** Explain why the series $1+\mathrm{e}^{-2x}+\mathrm{e}^{-4x}+\cdots$ converges for any positive real number x, and express the sum to infinity in terms of x. [2]

(ii) Given that $x = 10$, find the least value of n such that $S - S_n < S(10^{-100})$, where S and S_n represent the sum to infinity and the sum of the first n terms of the series respectively. [3]

2 The functions f and g are defined by

$$\mathrm{f}: x \mapsto x^2 - 4x + 3, \text{ for } x \in \mathbb{R},\ x \le a \text{ and}$$

$$\mathrm{g}: x \mapsto \tan^{-1}(2x+1), \text{ for } x \in \mathbb{R},\ x > -2,$$

where a is a constant.

(a) If $a = 2$, solve the equation $\mathrm{f}(x) = x$ exactly. [2]

(b) If $a = 3$,

(i) give a reason why f has no inverse. [2]

(ii) Prove that the composite function gf exists and state the rule, domain and exact range of the composite function. [6]

3 The point A has coordinates $(2q, 0, 2)$, where q is a constant, and the planes p_1, p_2 have equations $x + y = 4$ and $3x + 2y - 5z = 7$ respectively.

(i) Find the coordinates of the foot of perpendicular from A to p_1. Express your answer in terms of q. [3]

(ii) The point B is the mirror image of A in p_1. If B lies in p_2, find the value of q. [4]

(iii) p_1 and p_2 intersect in a line l. Find a vector equation of l. [1]

Another plane p_3 has equation $\lambda x + z = \mu$, where λ and μ are constants.

(iv) Given that the three planes have no point in common, what can be said about the values of λ and μ? [2]

4 The complex number z satisfies the relation $|z-3|=5$.

(i) Illustrate this relation in an Argand diagram. [2]

(ii) Find the largest possible value of $\arg(z+3-3\mathrm{i})$. [3]

It is further given that z also satisfies the relation $|z-4\mathrm{i}|=|z-6+4\mathrm{i}|$.

(iii) Illustrate this relation in the same diagram as your sketch in part **(i)**. Find the possible values of z exactly. [5]

Section B: Statistics [60 marks]

5 A school comprises a large number of students. A sample comprising 2% of the student population is to be selected to take part in a survey on their opinions about the school facilities.

(a) Describe briefly how this sample can be obtained via systematic sampling. [2]

(b) Give one advantage and one disadvantage of quota sampling in this context. [2]

6 The continuous random variable X has the distribution $\mathrm{N}(\mu,\sigma^2)$. It is known that $\mathrm{P}(X<17.7)=0.15$ and $\mathrm{P}(X>21.9)=0.2$. Calculate the values of μ and σ. [4]

7 A group of 15 student councillors comprises 6 from the House Committee, 5 from the Liaison Committee and 4 from the Welfare Committee. Two particular student councillors, Louis and Lionel, are from the House Committee and the Liaison Committee respectively.

The group stand in a circle to have a meeting. Find the number of possible arrangements if

(i) no two student councillors from the House Committee stand next to each other. [2]

(ii) student councillors from the same committee must stand next to one another and Louis and Lionel must stand next to each other. [2]

The group is to form a Task Force of 10 student councillors to organise a school activity. Find the number of possible ways the Task Force may be formed if the Task Force must include at least 1 student councillor from each of the 3 committees. [3]

8 The table below shows the ages of teak trees, x years, with trunk diameters, y inches. It can be assumed that the diameters of teak trees depend on their ages.

Age x (years)	11	15	28	45	52	57	75	81	88	97
Diameter y (inches)	7.5	11.5	16	19	20.5	21	21.5	21.9	22.2	22.22

(i) Draw a scatter diagram for these values, labelling the axes. [2]

(ii) It is desired to predict the diameters of very old trees (of over hundred years old). Explain why, in this context, neither a linear nor a quadratic model is likely to be appropriate. [2]

(iii) Fit a model of the form $y = a - \dfrac{b}{x}$ to the data, and calculate the least squares estimates of a and b. Find the product moment correlation coefficient for this model. Use the equation that you have obtained to estimate the diameter of a 40 year-old teak tree, and comment on the reliability of your answer. [4]

9 It has been estimated that only 8% of the world's population has blue eyes. A group of 60 people are randomly selected from all over the world. The number of people in this group who have blue eyes is the random variable Y.

(i) State, in the context of this question, one assumption needed to model Y by a binomial distribution. [1]

Assume now that Y indeed follows a binomial distribution.

(ii) Find the probability that at least 5 but less than 21 people in the group will have blue eyes. [2]

(iii) Use a suitable approximation to find the probability that more than 9 people in the group have blue eyes. You should state the parameters of the distribution you have used. [3]

10 **(i)** Suppose a fair die is tossed twice. Calculate the probabilities that

(a) the sum of the scores of the two tosses is at least 8, and [1]

(b) the absolute difference between the scores of the two tosses is at least 4. [1]

In one round of a game, a player is to draw a ball, without replacement, from a box that contains 3 red balls and 4 white balls. If a red ball is drawn, the player will add the scores obtained from tossing a fair die twice. If a white ball is drawn, the player will take the absolute difference of the scores obtained from tossing a die twice.

The game ends if the sum of the scores is at least 8 or the absolute difference of the scores is at least 4. Else, the player will proceed to the second round of the game where the process of picking a ball from the box and tossing the die twice repeats.

(ii) Find the probability that the game ends at the first round. [2]

(iii) Suppose the game ends at the first round. Find the probability that a red ball is drawn.[2]

(iv) Find the probability that there are a total of 3 rounds of game played and exactly 2 white balls are selected. [3]

11 An accountant believes that the figures provided by a particular company for the amount of loans borrowed by its clients, \$$x$, are too low. He carries out an online survey for clients of this company. The responses from a random sample of 20 clients are summarised by

$$\sum x = 21350, \ \sum (x-\overline{x})^2 = 345900.$$

(i) Calculate unbiased estimates of the population mean and variance of the amount of loans borrowed by each client, correct to 1 decimal place. [2]

The company claims that its clients will borrow \$1000 on average.

(ii) Stating a necessary assumption, carry out a test at the 5% level of significance to determine whether the company has understated the mean amount of loans received by its clients. [5]

(iii) Explain, in the context of the question, the meaning of ‘at the 5% level of significance’. [1]

The responses from another random sample of n clients are collected. The sample mean value for this sample is the same as the sample mean value for the previously collected sample.

(iv) Given that the standard deviation of X is 250, and that the assumption you have made in part **(ii)** holds, calculate the range of values of n for which the null hypothesis would not be rejected at the 5% level of significance. [3]

12 Cars join an immigration checkpoint queue in a 1-hour period, such that no two cars join the queue at the same instant in time.

(i) State, in the context of this question, an assumption needed for the number of cars joining an immigration checkpoint queue in a 1-hour period to be well modelled by a Poisson distribution. [1]

Assume now that the number of cars joining an immigration checkpoint queue in a 1-hour period is a random variable with the distribution Po(23). It is further given that the number of cars leaving the same immigration checkpoint queue in a 1-hour period is a random variable with the distribution Po(27).

(ii) It is given that in a period of n minutes, the probability that at least one car leaves the queue exceeds 0.9. Write down an inequality in n. Hence find the least integer value of n. [4]

(iii) At 0900 on a certain morning there are 19 cars in the queue. Use appropriate approximations to find the probability that by 1100 there are at most 12 cars in the queue, stating the parameters of any distributions that you use. (You may assume that the queue does not become empty during this period.) [5]

(iv) Explain why a Poisson model for the number of cars joining an immigration checkpoint queue would probably not be valid if applied to a time period of several hours. [1]

– END OF PAPER –