2016 SH2 H2 Mathematics Preliminary Examination Paper 1 Suggested Solutions

Qn No.	Solution
1	Let \$$x$ be the price of a regular cup of coffee. \$$y$ be the price of a medium cup of coffee. \$$z$ be the price of a large cup of coffee. $5x + 3y + 2z = 20.90$ (1) $3x + 4y + z = 17.10$ (2) There is a 12.5% discount given to customer C since she has bought more than 12 cups. Hence $2x + 8y + 4z = \frac{100}{87.5} \times 28 = 32$ (3) Using GC, $x = 1.80$, $y = 2.30$, $z = 2.50$

Qn No.	Solution
2 (i)	$\dfrac{3x^2+14}{(x+1)(x+2)} \geq 2$ $\Rightarrow \dfrac{3x^2+14}{(x+1)(x+2)} - 2 \geq 0,\ x \neq -1, -2$ $\Rightarrow \dfrac{3x^2+14-(2x^2+6x+4)}{(x+1)(x+2)} \geq 0$ $\Rightarrow \dfrac{x^2-6x+10}{(x+1)(x+2)} \geq 0$ $\Rightarrow (x+1)(x+2)(x^2-6x+10) \geq 0, \qquad x \neq -1, -2.$ **Method 1: Completing the square** $\Rightarrow (x+1)(x+2)\left[(x-3)^2+1\right] \geq 0, \qquad x \neq -1, -2.$ Since $(x-3)^2+1 \geq 0$ for all values of *x*, $\Rightarrow (x+1)(x+2) \geq 0$ and $x \neq -1, -2$. $\Rightarrow x < -2$ or $x > -1$. **Method 2: Using both the discriminant and coefficient of x^2** For $x^2-6x+10$, discriminant $= 6^2 - 4(1)(10) = -4 < 0$ and the coefficient of x^2 is positive. Hence the graph of $y = x^2-6x+10$ lies entirely above the *x*-axis, which implies that $x^2-6x+10 \geq 0$ for all values of *x*.
2 (ii)	$\dfrac{3x^2+14}{(\lvert x\rvert-1)(\lvert x\rvert-2)} \geq 2$ $\dfrac{3(-\lvert x\rvert)^2+14}{(-\lvert x\rvert+1)(-\lvert x\rvert+2)} \geq 2$ Therefore $-\lvert x\rvert < -2$ or $-\lvert x\rvert > -1$ $\Rightarrow \lvert x\rvert > 2$ or $\lvert x\rvert < 1$ $\Rightarrow x < -2$ or $-1 < x < 1$ or $x > 2$

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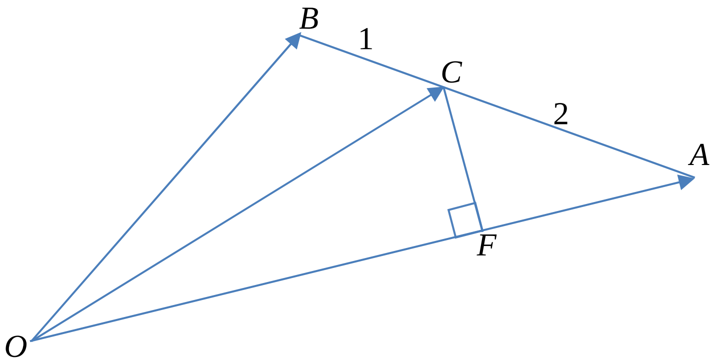
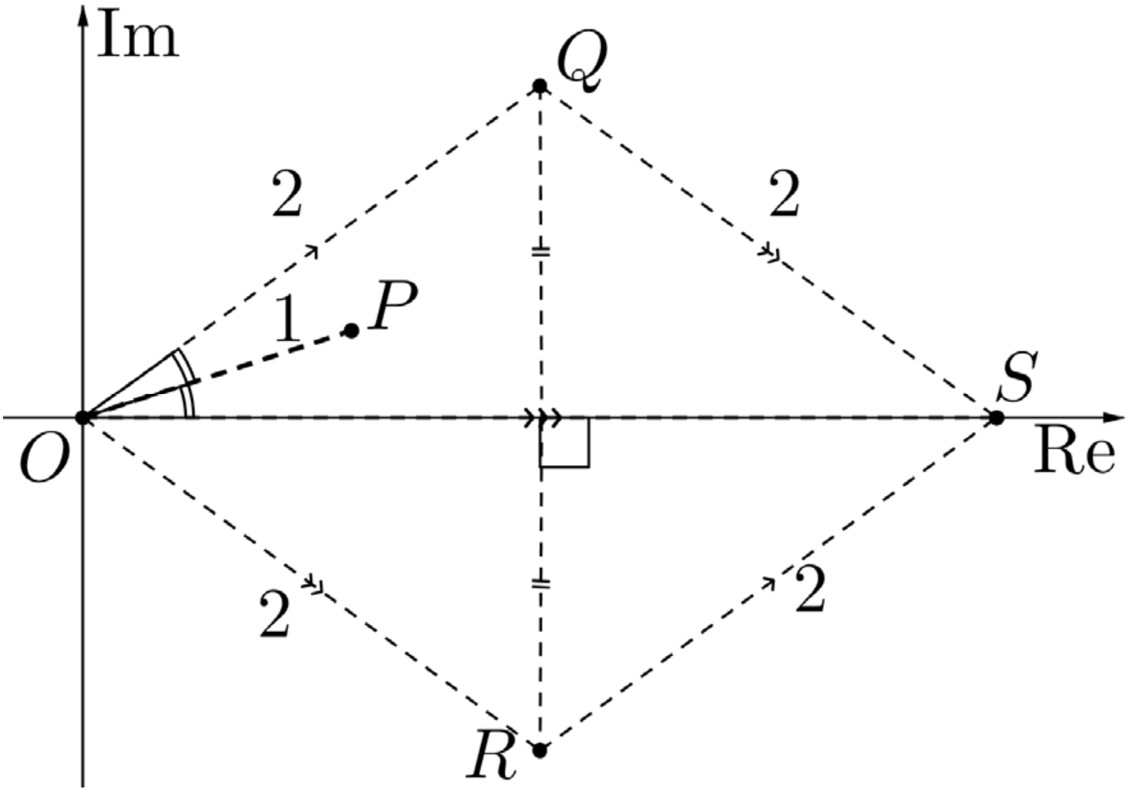
3	 By ratio theorem, $\overrightarrow{OC} = \dfrac{2\mathbf{b}+\mathbf{a}}{3}$.
	$\mathbf{a}\cdot\mathbf{b} = \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos 60^\circ = \dfrac{1}{2}\lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert$ Length of projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}$ $OF = \left\lvert \overrightarrow{OC}\cdot\dfrac{\overrightarrow{OA}}{\lvert\overrightarrow{OA}\rvert} \right\rvert$ $= \dfrac{1}{3}\left\lvert \dfrac{(2\mathbf{b}+\mathbf{a})\cdot\mathbf{a}}{\lvert\mathbf{a}\rvert} \right\rvert$ $= \dfrac{1}{3}\left\lvert \dfrac{2\mathbf{b}\cdot\mathbf{a}+\mathbf{a}\cdot\mathbf{a}}{\lvert\mathbf{a}\rvert} \right\rvert$ $= \dfrac{1}{3}\left\lvert \dfrac{2\mathbf{b}\cdot\mathbf{a}+\lvert\mathbf{a}\rvert^2}{\lvert\mathbf{a}\rvert} \right\rvert$ $= \dfrac{1}{3}\left(\dfrac{2\lvert\mathbf{a}\cdot\mathbf{b}\rvert}{\lvert\mathbf{a}\rvert} + \lvert\mathbf{a}\rvert \right)$ $= \dfrac{1}{3}\left(\dfrac{2\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos 60^\circ}{\lvert\mathbf{a}\rvert} + \lvert\mathbf{a}\rvert \right)$ $= \dfrac{1}{3}\left(\dfrac{2\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}{2\lvert\mathbf{a}\rvert} + \lvert\mathbf{a}\rvert \right)$ $= \dfrac{1}{3}\left(\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert\right)$

Qn No.	Solution
4 (a)	**Method 1A** $2w-z=6\text{i} \quad \cdots\cdots(1)$ $wz=\frac{13}{2} \quad \cdots\cdots(2) \quad \Rightarrow w=\frac{13}{2z} \quad \cdots\cdots(3)$ Substituting (3) into (1), $2\left(\frac{13}{2z}\right)-z=6\text{i}$ $\frac{13}{z}-z=6\text{i}$ $13-z^2=6\text{i}z$ $z^2+6\text{i}z-13=0$ $z^2+6\text{i}z+(3\text{i})^2-(3\text{i})^2-13=0$ $(z+3\text{i})^2=4$ $z=2-3\text{i} \text{ or } -2-3\text{i}$ When $z=2-3\text{i}$, $w=\frac{13}{2(2-3\text{i})}=1+\frac{3}{2}\text{i}$ When $z=-2-3\text{i}$, $w=\frac{13}{2(-2-3\text{i})}=-1+\frac{3}{2}\text{i}$ $\therefore w=1+\frac{3}{2}\text{i},\ z=2-3\text{i} \text{ or } w=-1+\frac{3}{2}\text{i},\ z=-2-3\text{i}$ **Method 1B** $2w-z=6\text{i} \quad \cdots\cdots(1)$ $wz=\frac{13}{2} \quad \cdots\cdots(2) \quad \Rightarrow z=\frac{13}{2w} \quad \cdots\cdots(3)$ Substituting (3) into (1), $2w-\frac{13}{2w}=6\text{i}$ $4w^2-13=12\text{i}w$ $4w^2-12\text{i}w-13=0$ $4\left(w^2-3\text{i}w-\frac{9}{4}\right)+9-13=0$ $4\left(w-\frac{3\text{i}}{2}\right)^2-4=0$ $\left(w-\frac{3\text{i}}{2}\right)^2=1 \Rightarrow w=1+\frac{3\text{i}}{2} \text{ or } -1+\frac{3\text{i}}{2}$ When $w=1+\frac{3}{2}\text{i}$, $z=\frac{13}{2\left(1+\frac{3}{2}\text{i}\right)}=2-3\text{i}$ When $w=-1+\frac{3}{2}\text{i}$, $z=\frac{13}{2\left(-1+\frac{3}{2}\text{i}\right)}=-2-3\text{i}$ $\therefore w=1+\frac{3}{2}\text{i},\ z=2-3\text{i} \text{ or } w=-1+\frac{3}{2}\text{i},\ z=-2-3\text{i}$

Qn No.	Solution
4 (a)	**Method 2A** $2w-z=6\text{i} \quad \cdots\cdots(1) \quad \Rightarrow z=6\text{i}-2w \cdots\cdots(3)$ $wz=\dfrac{13}{2} \quad \cdots\cdots(2)$ Substituting (3) into (2), $w(6\text{i}-2w)=\dfrac{13}{2}$ $w(6\text{i}-2w)=\dfrac{13}{2}$ $4w^2-12\text{i}w=13$ $4w^2-12\text{i}w-13=0$ $4\left(w^2-3\text{i}w-\dfrac{9}{4}\right)+9-13=0$ $4\left(w-\dfrac{3\text{i}}{2}\right)^2-4=0$ $\left(w-\dfrac{3\text{i}}{2}\right)^2=1 \Rightarrow w=1+\dfrac{3\text{i}}{2}$ or $-1+\dfrac{3\text{i}}{2}$ When $w=1+\dfrac{3}{2}\text{i}$, $z=\dfrac{13}{2\left(1+\dfrac{3}{2}\text{i}\right)}=2-3\text{i}$ When $w=-1+\dfrac{3}{2}\text{i}$, $z=\dfrac{13}{2\left(-1+\dfrac{3}{2}\text{i}\right)}=-2-3\text{i}$ $\therefore w=1+\dfrac{3}{2}\text{i}, z=2-3\text{i}$ or $w=-1+\dfrac{3}{2}\text{i}, z=-2-3\text{i}$ **Method 2B** $2w-z=6\text{i} \quad \cdots\cdots(1) \quad \Rightarrow 2w=6\text{i}+z \cdots\cdots(3)$ $wz=\dfrac{13}{2} \quad \cdots\cdots(2)$ Substituting (3) into (2), $(6\text{i}+z)z=13$ $6\text{i}z+z^2=13$ $z^2+6\text{i}z-13=0$ $z^2+6\text{i}z+(3\text{i})^2-(3\text{i})^2-13=0$ $(z+3\text{i})^2=4$ $z=2-3\text{i}$ or $-2-3\text{i}$ When $z=2-3\text{i}$, $w=\dfrac{13}{2(2-3\text{i})}=1+\dfrac{3}{2}\text{i}$ When $z=-2-3\text{i}$, $w=\dfrac{13}{2(-2-3\text{i})}=-1+\dfrac{3}{2}\text{i}$ $\therefore w=1+\dfrac{3}{2}\text{i}, z=2-3\text{i}$ or $w=-1+\dfrac{3}{2}\text{i}, z=-2-3\text{i}$

Qn No.	Solution
4 (a)	**Method 3** $2w-z=6\text{i} \quad \cdots\cdots(1)$ $wz=\dfrac{13}{2} \quad \cdots\cdots(2)$ $(2)\Rightarrow w=\dfrac{13}{2z} \quad \cdots\cdots(3)$ Substituting (3) into (1), $2\left(\dfrac{13}{2z}\right)-z=6\text{i}$ $\dfrac{13}{z}-z=6\text{i}$ $13-z^2=6\text{i}z$ $z^2+6\text{i}z-13=0$ Let $z=x+y\text{i}$. Then $(x+\text{i}y)^2+6\text{i}(x+\text{i}y)-13=0$ $x^2-y^2+2\text{i}xy+6\text{i}x-6y-13=0$ $x^2-y^2-6y-13+\text{i}(2xy+6x)=0$ Comparing imaginary parts, $2xy+6x=0$ $x(y+3)=0$ $x=0$ or $y=-3$ If $x=0$, comparing real parts, $-y^2-6y-13=0$ $y^2+6y+13=0$ $(y+3)^2+4=0$ Therefore, there is no solution if $x=0$. If $y=-3$, comparing real parts, $x^2-3^2-6(-3)-13=0$ $x^2-4=0$ $x=\pm 2$ When $x=2$, $z=2-3\text{i}$, $w=\dfrac{13}{2(2-3\text{i})}=1+\dfrac{3}{2}\text{i}$ When $x=-2$, $z=-2-3\text{i}$, $w=\dfrac{13}{2(-2-3\text{i})}=-1+\dfrac{3}{2}\text{i}$ $\therefore w=1+\dfrac{3}{2}\text{i},\ z=2-3\text{i}$ or $w=-1+\dfrac{3}{2}\text{i},\ z=-2-3\text{i}$

<table>
<tr><th>Qn No.</th><th>Solution</th></tr>
<tr><td>4 (b)</td><td>

Let $p = \mathrm{e}^{\mathrm{i}\alpha}$. Then $q = 2\mathrm{e}^{\mathrm{i}(2\alpha)}$.
Let S represent
$q^* + 2p^2 = 2\mathrm{e}^{\mathrm{i}(-2\alpha)} + 2\mathrm{e}^{\mathrm{i}(2\alpha)} = 2\,\mathrm{Re}\left(\mathrm{e}^{\mathrm{i}(2\alpha)}\right) = 2\cos 2\alpha$,
i.e. S represents the point whose real coordinate is twice that of Q, and imaginary coordinate zero.</td></tr>
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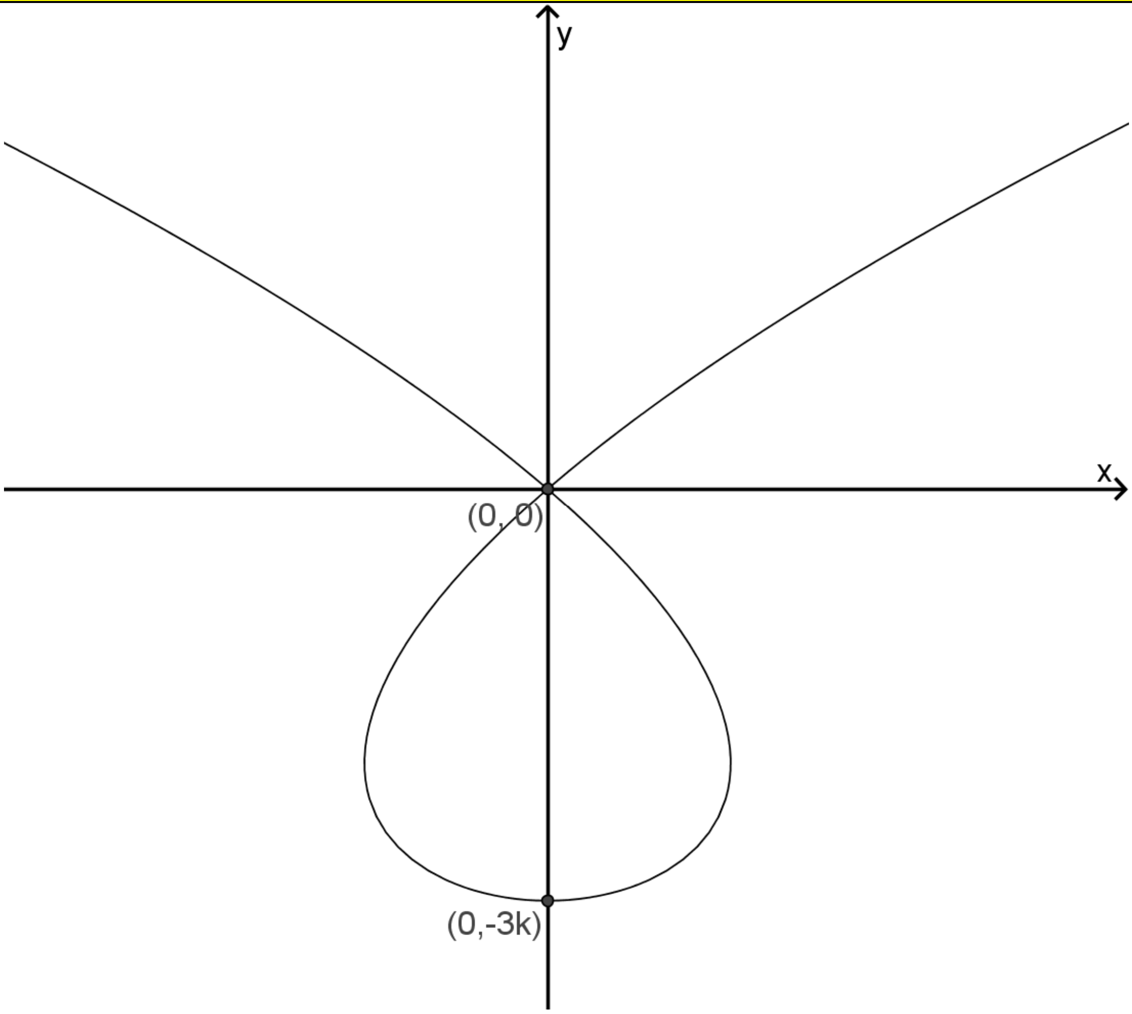
Qn No.	Solution
5	$V=x^2h+\frac{1}{3}x^2\left(\frac{1}{2}x\right)=x^2\left(h+\frac{1}{6}x\right)$ $h=\frac{V}{x^2}-\frac{1}{6}x$ $S=4xh+x^2+4\left(\frac{1}{2}\right)x\left(\frac{1}{\sqrt{2}}x\right)$ $\quad=4xh+\left(1+\sqrt{2}\right)x^2$ Substituting $h=\frac{V}{x^2}-\frac{1}{6}x$: $S=4x\left(\frac{V}{x^2}-\frac{1}{6}x\right)+\left(1+\sqrt{2}\right)x^2$ $\quad=\frac{4V}{x}+\left(\frac{1}{3}+\sqrt{2}\right)x^2$ For minimum surface area: $\frac{\mathrm{d}S}{\mathrm{d}x}=-\frac{4V}{x^2}+2\left(\frac{1}{3}+\sqrt{2}\right)x=0$. $\frac{4V}{x^2}=2\left(\frac{1}{3}+\sqrt{2}\right)x$ $2V=\left(\frac{1}{3}+\sqrt{2}\right)x^3$ $6V=\left(1+3\sqrt{2}\right)x^3$ $x^3=\frac{6V}{1+3\sqrt{2}}$ $x=\left(\frac{6V}{1+3\sqrt{2}}\right)^{\frac{1}{3}}=1.0460049\left(V^{\frac{1}{3}}\right)$ $\quad=1.046\left(V^{\frac{1}{3}}\right)$ (3d.p) Min $S=A$ $=\frac{4V}{1.0460049\left(V^{\frac{1}{3}}\right)}+\left(\frac{1}{3}+\sqrt{2}\right)\left(1.0460049\left(V^{\frac{1}{3}}\right)\right)^2$ $=\left(\frac{4}{1.0460049}+\left(\frac{1}{3}+\sqrt{2}\right)(1.0460049)^2\right)\left(V^{\frac{2}{3}}\right)$ $=5.736110\left(V^{\frac{2}{3}}\right)$ $=5.736\left(V^{\frac{2}{3}}\right)$ (3d.p) Remark: 1st or 2nd derivative test is not necessary as it is given in the question that the surface area had a minimum value.

<table>
<tr><th>Qn No.</th><th>Solution</th></tr>
<tr><td>6 (i)</td><td>



To find *y*-intercept,
let *x* = 0:

$$t^3 - kt = 0$$
$$t\left(t^2 - k\right) = 0$$
$$t = 0 \quad \text{or} \quad t = \pm\sqrt{k}$$
$$y = -3k \qquad y = 0$$
$$\therefore \left(0, -3k\right) \qquad \left(0, 0\right)$$

To find *x*-intercept,
let *y* = 0:

$$3\left(t^2 - k\right) = 0$$
$$t = \pm\sqrt{k}$$
$$x = 0$$
$$\therefore \left(0, 0\right)$$

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<tr><td>6 (ii)</td><td>

$$x = t^3 - kt \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = 3t^2 - k$$
$$y = 3\left(t^2 - k\right) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = 6t$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}} = \frac{6t}{3t^2 - k}$$

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Qn No.	Solution
6 (iii)	When $t=-\sqrt{\frac{k}{3}}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6t}{3t^2-k}$ is undefined since $3t^2-k=3\left(\frac{k}{3}\right)-k=0$. Hence, the tangent is a vertical line with equation $x=t^3-kt$ $=t\left(t^2-k\right)$ $=\left(-\sqrt{\frac{k}{3}}\right)\left(\frac{k}{3}-k\right)$ $=\frac{2}{3}k\sqrt{\frac{k}{3}}$
6 (iv)	The tangent passes through the point $\left(\frac{2}{3}k,k\right)$, therefore $x=\frac{2}{3}k\sqrt{\frac{k}{3}}$ $\frac{2}{3}k=\frac{2}{3}k\sqrt{\frac{k}{3}}$ $k\left(1-\sqrt{\frac{k}{3}}\right)=0$ $k=0$ or $1-\sqrt{\frac{k}{3}}=0$ (NA since $k>0$) $\frac{k}{3}=1$ $k=3$

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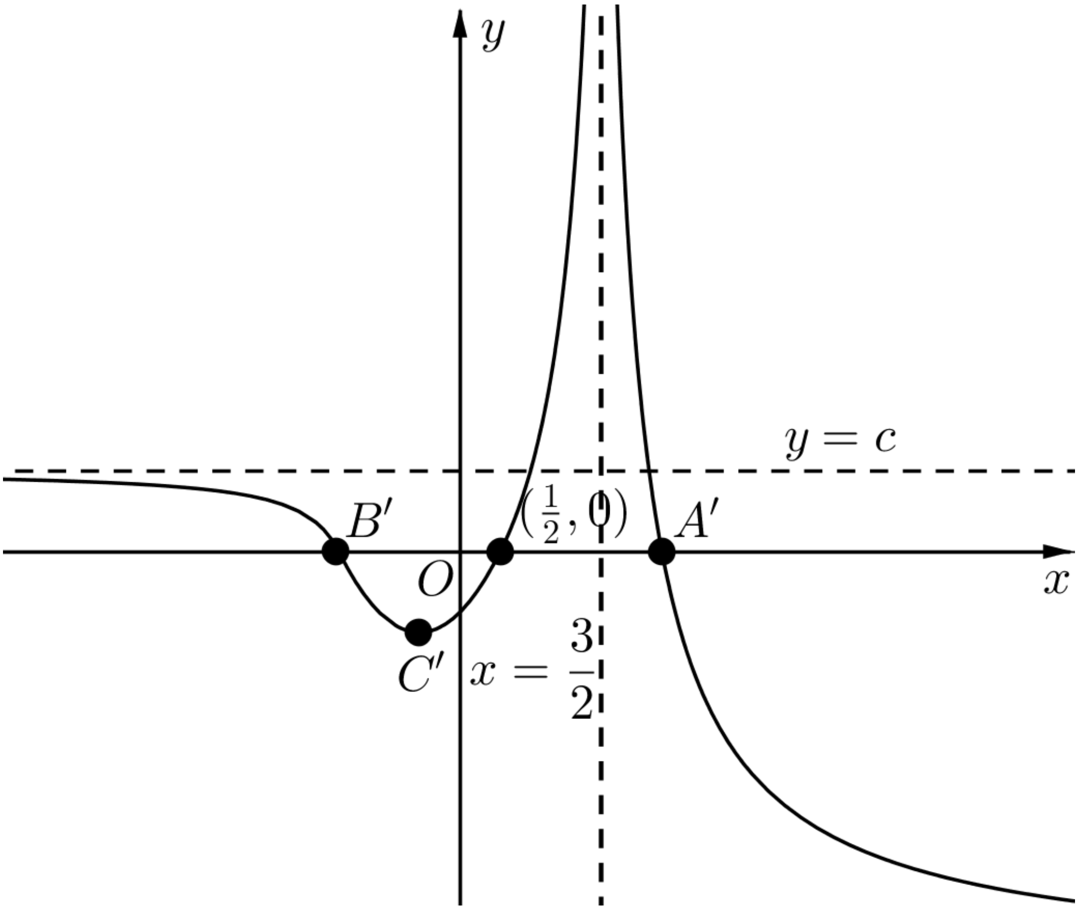
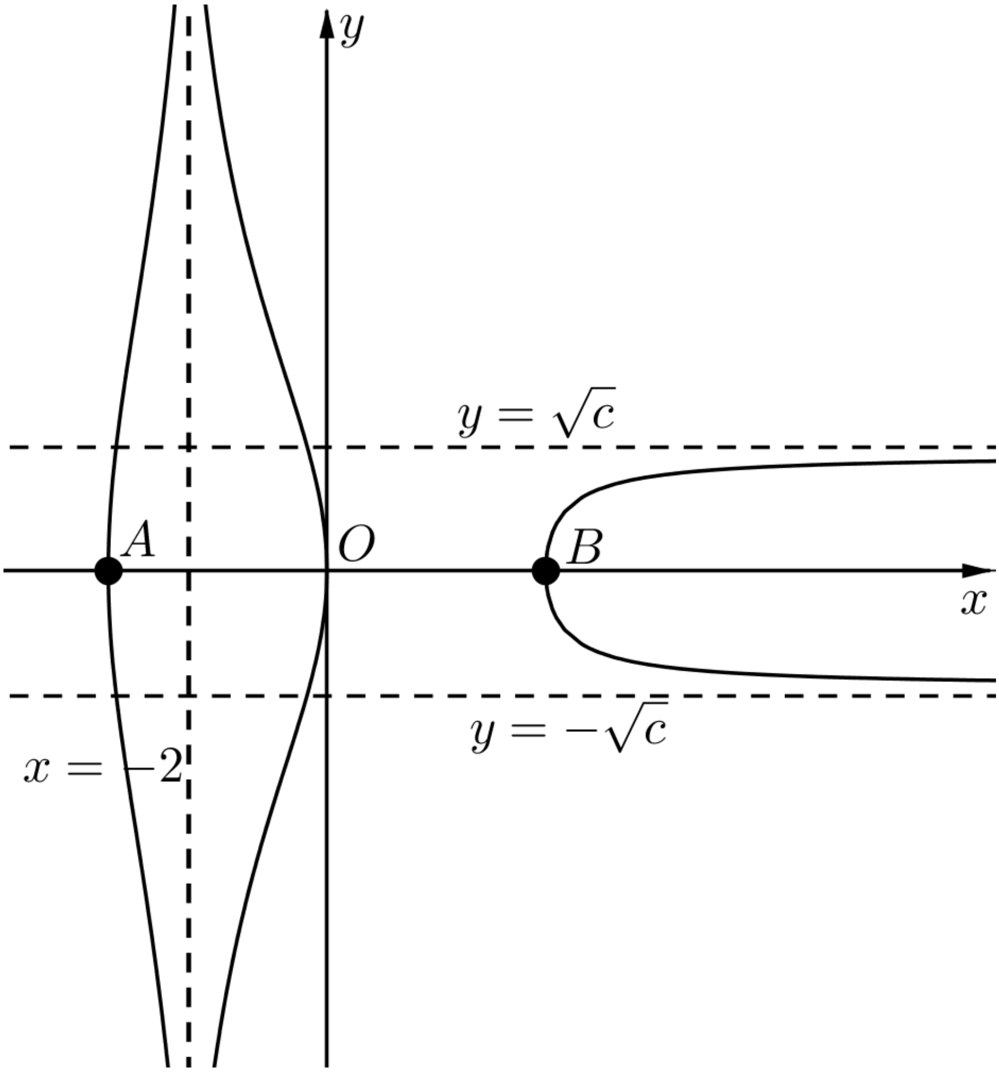
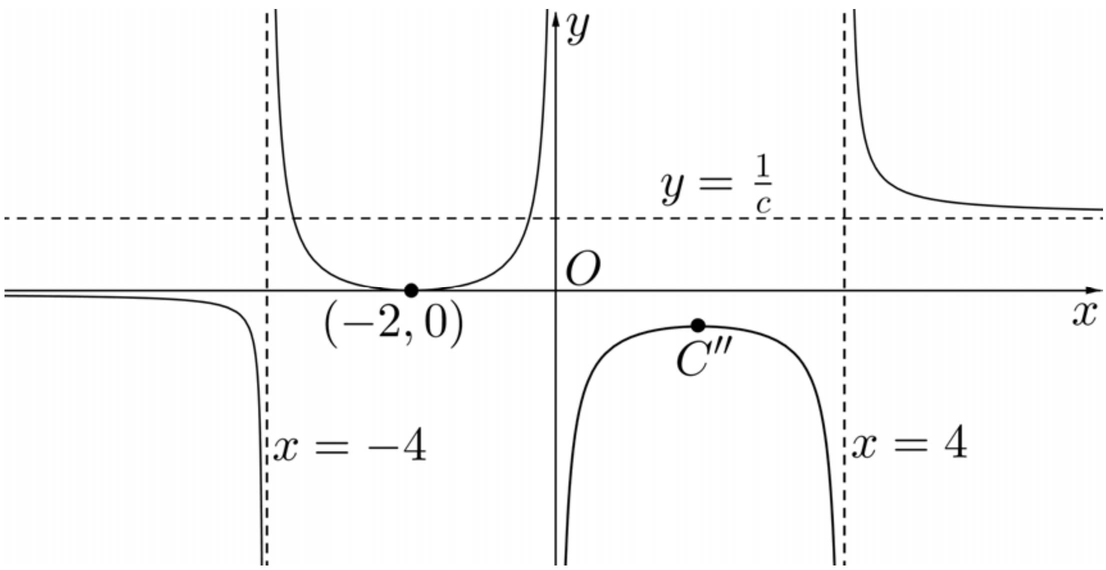
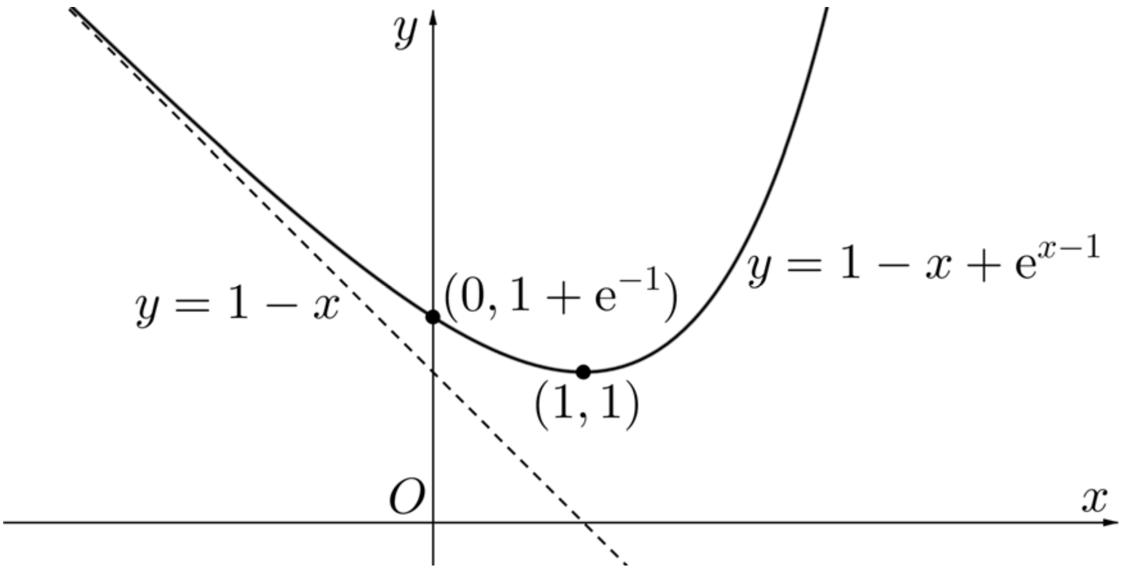
7 (i)	$y=\ln\left(\sec x\right)$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{\sec x}\left(\sec x\tan x\right)=\tan x$ $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\sec^2 x$ $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=2\sec x\left(\sec x\tan x\right)$ $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=2\left(\sec^2 x\right)\left(\tan x\right)=2\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)$ (shown)
7 (ii)	$\dfrac{\mathrm{d}^4y}{\mathrm{d}x^4}=2\left(\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)+2\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)^2$ When $x=0$, $y=\ln\left(\sec 0\right)=\ln 1=0$ $\qquad$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\tan 0=0$ $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\sec^2 0=1$ $\qquad$ $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=2(1)(0)=0$ $\dfrac{\mathrm{d}^4y}{\mathrm{d}x^3}=2\left(0\right)\left(0\right)+2\left(1\right)^2=2$ Thus, Maclaurin series for y is $y\approx+0x+1\left(\dfrac{x^2}{2!}\right)+0x^3+2\left(\dfrac{x^4}{4!}\right)$ $=\dfrac{1}{2}x^2+\dfrac{1}{12}x^4$
7 (iii)	$\dfrac{1}{12}x^2+\ln\left(\sec x\right)=\cos 2x$ Using the above results and the first three terms of Maclaurin Series of $\cos 2x$, $\dfrac{1}{12}x^2+\dfrac{1}{2}x^2+\dfrac{1}{12}x^4\approx1-\dfrac{1}{2!}\left(2x\right)^2+\dfrac{1}{4!}\left(2x\right)^4$ $\dfrac{7}{12}x^2+\dfrac{1}{12}x^4=1-2x^2+\dfrac{2}{3}x^4$ $\dfrac{7}{12}x^4-\dfrac{31}{12}x^2+1=0$ $7x^4-31x^2+12=0$ $\left(7x^2-3\right)\left(x^2-4\right)=0$ $x^2=\dfrac{3}{7}$ or $x^2=4$ $x=\pm\sqrt{\dfrac{3}{7}}$ or ±2 Since α is positive and close to zero, $\alpha=\sqrt{\dfrac{3}{7}}$

Qn No.	Solution
8 (a)	 Coordinates of A', B' and C' are $\left(\frac{5}{2},0\right)$, $\left(-\frac{3}{2},0\right)$ and $\left(\frac{1-a}{2},\frac{b}{2}\right)$

Qn No.	Solution
8 (b)	
8 (c)	 Coordinates of C'' are $\left(a,-\dfrac{2}{b}\right)$.

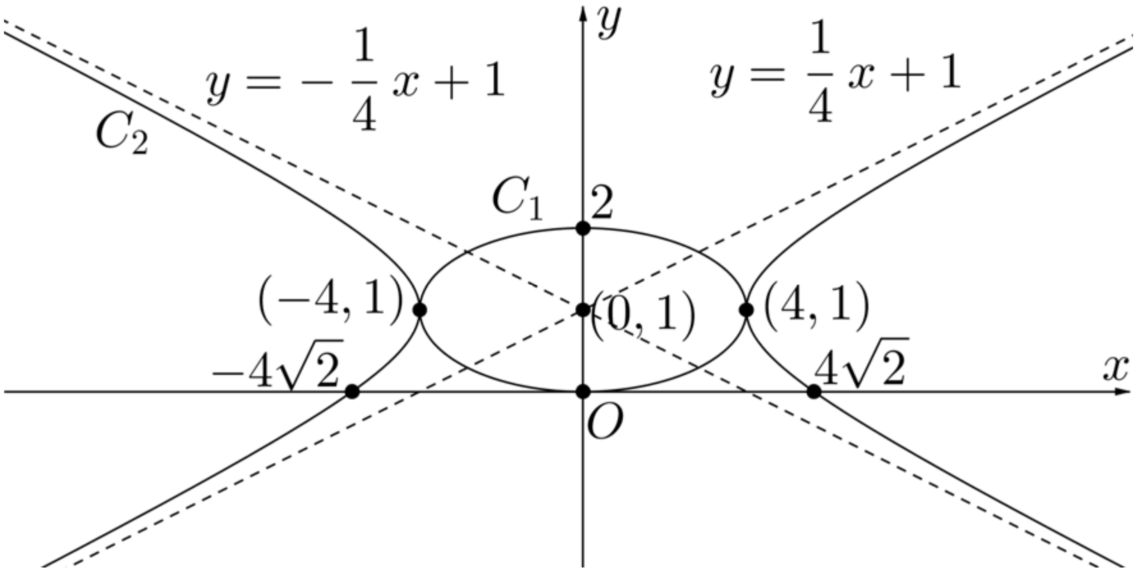
Qn No.	Solution
9 (i)	$y=z-x\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}z}{\mathrm{d}x}-1$ The $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}-1=\frac{x-2}{y}$ becomes $\frac{1}{z-x}\left(\frac{\mathrm{d}z}{\mathrm{d}x}-1\right)-1=\frac{x-2}{z-x}\Rightarrow\frac{\mathrm{d}z}{\mathrm{d}x}-1-(z-x)=x-2$ $\Rightarrow\frac{\mathrm{d}z}{\mathrm{d}x}=z-1$ $\Rightarrow\int\frac{1}{z-1}\,\mathrm{d}z=\int 1\,\mathrm{d}x$ $\Rightarrow\ln\lvert z-1\rvert=x+c$ $\Rightarrow\lvert z-1\rvert=\mathrm{e}^{x+c}=A\mathrm{e}^{x}$ $\Rightarrow z-1=\pm A\mathrm{e}^{x}=B\mathrm{e}^{x}$ $\Rightarrow y=1-x+B\mathrm{e}^{x}$ Since the solution curve passes through (1,1), then $1=1-1+B\mathrm{e}\Rightarrow B=\mathrm{e}^{-1}$. Hence the equation of curve is $y=1-x+\mathrm{e}^{x-1}$.
9 (ii)	

Qn No.	Solution
10 (a)	$\dfrac{5x^2-2x+7}{(1-x)(2x^2+3)}=\dfrac{A}{1-x}+\dfrac{Bx+C}{2x^2+3}$, where $A=\dfrac{5-2+7}{2+3}=2$, $5x^2-2x+7=2(2x^2+3)+(Bx+C)(1-x)$ Comparing the coefficients of x^2 and x^0, we have $4-B=5\Rightarrow B=-1$, and $6+C=7\Rightarrow C=1$. Hence we get $\dfrac{5x^2-2x+7}{(1-x)(2x^2+3)}=\dfrac{2}{1-x}+\dfrac{-x+1}{2x^2+3}$ Therefore, $\displaystyle\int\frac{5x^2-2x+7}{(1-x)(2x^2+3)}\,\mathrm{d}x$ $\displaystyle=\int\frac{2}{1-x}+\frac{-x+1}{2x^2+3}\,\mathrm{d}x$ $\displaystyle=\int\frac{2}{1-x}\,\mathrm{d}x+\int\frac{-x+1}{2x^2+3}\,\mathrm{d}x$ $\displaystyle=\int\frac{2}{1-x}\,\mathrm{d}x-\int\frac{x}{2x^2+3}\,\mathrm{d}x+\int\frac{1}{2x^2+3}\,\mathrm{d}x$ $\displaystyle=-2\int\frac{-1}{1-x}\,\mathrm{d}x-\frac{1}{4}\int\frac{4x}{2x^2+3}\,\mathrm{d}x+\frac{1}{2}\int\frac{1}{x^2+\left(\sqrt{\tfrac{3}{2}}\right)^2}\,\mathrm{d}x$ $\displaystyle=-2\ln\lvert 1-x\rvert-\frac{1}{4}\ln\lvert 2x^2+3\rvert+\frac{1}{2}\cdot\frac{1}{\sqrt{\tfrac{3}{2}}}\tan^{-1}\left(\frac{x}{\sqrt{\tfrac{3}{2}}}\right)+c$ $\displaystyle=-2\ln\lvert 1-x\rvert-\frac{1}{4}\ln\lvert 2x^2+3\rvert+\frac{1}{\sqrt{6}}\tan^{-1}\left(\sqrt{\frac{2}{3}}x\right)+c$

Qn No.	Solution
10 (b) (i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left[\sin\left(\mathrm{e}^{-x}\right)\right]=-\mathrm{e}^{-x}\cos\left(\mathrm{e}^{-x}\right)$
10 (b) (ii)	$\int_0^n \mathrm{e}^{-2x}\cos\left(\mathrm{e}^{-x}\right)\mathrm{d}x$ $=\int_0^n \mathrm{e}^{-x}\left(\mathrm{e}^{-x}\cos\left(\mathrm{e}^{-x}\right)\right)\mathrm{d}x$ $=\left[\mathrm{e}^{-x}\left(-\sin\left(\mathrm{e}^{-x}\right)\right)\right]_0^n-\int_0^n\left(-\sin\left(\mathrm{e}^{-x}\right)\right)\left(-\mathrm{e}^{-x}\right)\mathrm{d}x$ $=\left[\mathrm{e}^{-n}\left(-\sin\left(\mathrm{e}^{-n}\right)\right)\right]-\left[\mathrm{e}^{0}\left(-\sin\left(\mathrm{e}^{0}\right)\right)\right]+\int_0^n -\mathrm{e}^{-x}\sin\left(\mathrm{e}^{-x}\right)\mathrm{d}x$ $=-\mathrm{e}^{-n}\sin\left(\mathrm{e}^{-n}\right)+\sin 1+\left[-\cos\left(\mathrm{e}^{-x}\right)\right]_0^n$ $=-\mathrm{e}^{-n}\sin\left(\mathrm{e}^{-n}\right)+\sin 1+\left[-\cos\left(\mathrm{e}^{-n}\right)+\cos\left(\mathrm{e}^{0}\right)\right]$ $=-\mathrm{e}^{-n}\sin\left(\mathrm{e}^{-n}\right)-\cos\left(\mathrm{e}^{-n}\right)+\sin 1+\cos 1$
10 (b) (iii)	$\int_0^\infty \mathrm{e}^{-2x}\cos\left(\mathrm{e}^{-x}\right)\mathrm{d}x$ $=\lim_{n\to\infty}\int_0^n \mathrm{e}^{-2x}\cos\left(\mathrm{e}^{-x}\right)\mathrm{d}x$ $=\lim_{n\to\infty}\left[-\mathrm{e}^{-n}\sin\left(\mathrm{e}^{-n}\right)-\cos\left(\mathrm{e}^{-n}\right)+\sin 1+\cos 1\right]$ $=\sin 1+\cos 1-1$

Qn No.	Solution
11 (a)	Let P_n be the statement $$\frac{2}{1^2\times 3^2}+\frac{3}{2^2\times 4^2}+\cdots+\frac{n+1}{n^2(n+2)^2}=\frac{5}{16}-\frac{1}{4(n+1)^2}-\frac{1}{4(n+2)^2}$$ for all $n\in\mathbf{Z}^+$. When n = 1, $\text{LHS}=\frac{2}{1^2\times 3^2}=\frac{2}{9}$ $\text{RHS}=\frac{5}{16}-\frac{1}{4(1+1)^2}-\frac{1}{4(1+2)^2}$ $=\frac{5}{16}-\frac{1}{16}-\frac{1}{36}=\frac{2}{9}=\text{LHS}$ Thus, P_1 is true. Assume P_k is true for some $k\in\mathbf{Z}^+$, i.e. $\frac{2}{1^2\times 3^2}+\frac{3}{2^2\times 4^2}+\cdots+\frac{k+1}{k^2(k+2)^2}=\frac{5}{16}-\frac{1}{4(k+1)^2}-\frac{1}{4(k+2)^2}$. Consider P_{k+1}: To show $\frac{2}{1^2\times 3^2}+\frac{3}{2^2\times 4^2}+\cdots+\frac{k+2}{(k+1)^2(k+3)^2}=\frac{5}{16}-\frac{1}{4(k+2)^2}-\frac{1}{4(k+3)^2}$. LHS of $P_{k+1}=\frac{2}{1^2\times 3^2}+\frac{3}{2^2\times 4^2}+\cdots+\frac{k+1}{k^2(k+2)^2}+\frac{k+2}{(k+1)^2(k+3)^2}$ $=\frac{5}{16}-\frac{1}{4(k+1)^2}-\frac{1}{4(k+2)^2}+\frac{k+2}{(k+1)^2(k+3)^2}$ $=\frac{5}{16}-\frac{1}{4(k+2)^2}+\frac{k+2}{(k+1)^2(k+3)^2}-\frac{1}{4(k+1)^2}$ $=\frac{5}{16}-\frac{1}{4(k+2)^2}+\frac{4(k+2)-(k+3)^2}{4(k+1)^2(k+3)^2}$ $=\frac{5}{16}-\frac{1}{4(k+2)^2}+\frac{4k+8-k^2-6k-9}{4(k+1)^2(k+3)^2}$ $=\frac{5}{16}-\frac{1}{4(k+2)^2}+\frac{-k^2-2k-1}{4(k+1)^2(k+3)^2}$ $=\frac{5}{16}-\frac{1}{4(k+2)^2}+\frac{-(k+1)^2}{4(k+1)^2(k+3)^2}$ $=\frac{5}{16}-\frac{1}{4(k+2)^2}-\frac{1}{4(k+3)^2}$ $=\text{RHS of } P_{k+1}$ P_k is true $\Rightarrow P_{k+1}$ is true. Since P_1 is true, and P_k is true $\Rightarrow P_{k+1}$ is true, by mathematical induction, P_n is true for all $n\in\mathbf{Z}^+$.

Qn No.	Solution
11 (b) (i)	$\frac{4n+5}{n(n+1)}=\frac{5}{n}-\frac{1}{n+1}$ $\sum_{n=1}^{N}\frac{4n+5}{n(n+1)}\left(\frac{1}{5^{n+1}}\right)=\sum_{n=1}^{N}\left(\frac{5}{n}-\frac{1}{n+1}\right)\left(\frac{1}{5^{n+1}}\right)$ $=\sum_{n=1}^{N}\left(\frac{1}{n5^{n}}-\frac{1}{(n+1)5^{n+1}}\right)$ $=\frac{1}{1\cdot 5^1}-\cancel{\frac{1}{2\cdot 5^2}}$ $+\cancel{\frac{1}{2\cdot 5^2}}-\cancel{\frac{1}{3\cdot 5^3}}$ $\vdots$ $+\cancel{\frac{1}{(N-1)5^{N-1}}}-\cancel{\frac{1}{N5^{N}}}$ $+\cancel{\frac{1}{N5^{N}}}-\frac{1}{(N+1)5^{N+1}}$ $=\frac{1}{5}-\frac{1}{(N+1)5^{N+1}}$ (shown) $\therefore a=\frac{1}{5},\ b=-1$
11 (b) (ii)	$\sum_{n=1}^{\infty}\frac{4n+5}{n(n+1)}\left(\frac{1}{5^{n+1}}\right)=\lim_{N\to\infty}\left[\frac{1}{5}-\frac{1}{(N+1)5^{N+1}}\right]=\frac{1}{5}$
11 (b) (iii)	**<u>Method 1: Substitution of Indices</u>** $\sum_{n=2}^{N-2}\left[\frac{4n+1}{n(n-1)}\left(\frac{1}{5^{n}}\right)\right]=\sum_{n+1=2}^{n+1=N-2}\left[\frac{4(n+1)+1}{(n+1)((n+1)-1)}\left(\frac{1}{5^{n+1}}\right)\right]$ $=\sum_{n=1}^{N-3}\left[\frac{4n+5}{n(n+1)}\left(\frac{1}{5^{n+1}}\right)\right]$ $=\frac{1}{5}-\frac{1}{(N-2)5^{N-2}}$ **<u>Method 2: Listing of Terms</u>** $\sum_{n=2}^{N-2}\left[\frac{4n+1}{n(n-1)}\left(\frac{1}{5^{n}}\right)\right]$ $=\frac{9}{2\times 1}\left(\frac{1}{5^2}\right)+\frac{13}{3\times 2}\left(\frac{1}{5^3}\right)+\cdots+\frac{4N-7}{(N-2)(N-3)}\left(\frac{1}{5^{N-2}}\right)$ $=\frac{4\cdot 1+5}{1\times 2}\left(\frac{1}{5^2}\right)+\frac{4\cdot 2+5}{2\times 3}\left(\frac{1}{5^3}\right)+\cdots+\frac{4(N-3)+5}{(N-3)(N-2)}\left(\frac{1}{5^{N-2}}\right)$ $=\sum_{n=1}^{N-3}\left[\frac{4n+5}{n(n+1)}\left(\frac{1}{5^{n+1}}\right)\right]$ $=\frac{1}{5}-\frac{1}{(N-2)5^{N-2}}$

Qn No.	Solution
12 (i)	By substituting $(4,1)$ into the equations of C_1 and C_2, $C_1:\ \text{LHS}=(4)^2+16(1-1)^2=16=\text{RHS}$ $C_2:\ \text{LHS}=(4)^2-16(1-1)^2=16=\text{RHS}$ (justified)
12 (ii)	
12 (iii)	Volume of revolution of R about x-axis $=\pi\int_0^4\left[1-\sqrt{\frac{16-x^2}{16}}\right]^2\mathrm{d}x+\pi\int_4^{4\sqrt{2}}\left[1-\sqrt{\frac{x^2-16}{16}}\right]^2\mathrm{d}x$ $=2.17$ units^3 (to 3 s.f.)
12 (iv)	$\int_0^2\sqrt{1-(x-1)^2}\,\mathrm{d}x$ $=\int_\pi^0\left(\sqrt{1-\cos^2\theta}\right)(-\sin\theta)\ \mathrm{d}\theta$ $=\int_0^\pi\left(\sin^2\theta\right)\ \mathrm{d}\theta$ $=\frac{1}{2}\int_0^\pi(1-\cos 2\theta)\ \mathrm{d}\theta$ $=\frac{1}{2}\left[\theta-\frac{\sin 2\theta}{2}\right]_0^\pi$ $=\frac{\pi}{2}$
12 (v)	Area of $S=2\int_0^2 x\ \mathrm{d}y$ $=2\int_0^2\sqrt{16-16(y-1)^2}\,\mathrm{d}y$ $=8\int_0^2\sqrt{1-(y-1)^2}\,\mathrm{d}y$ $=8\left(\frac{\pi}{2}\right)$ $=4\pi$ units^2