YISHUN JUNIOR COLLEGE
2016 JC2 PRELIMINARY EXAMINATION

MATHEMATICS **9740/01**
Higher 2

Paper 1 **18 AUGUST 2016**
THURSDAY 0800h – 1100h

Additional materials :
Answer paper
Graph paper
List of Formulae (MF15)

TIME 3 hours

READ THESE INSTRUCTIONS FIRST

Write your name and CTG in the spaces provided on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, write down the question number of the questions attempted, model of calculator used on the spaces provided on the cover page. Tie your cover page on top of the answer scripts before submission.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 A bakery sells strawberry, blueberry and walnut muffins. During a promotion, a customer purchased all 3 types of muffins and twice as many strawberry muffins as walnut muffins. The promotion price of each strawberry, blueberry and walnut muffin is $1.60, $1.75 and $2.20 respectively. Given that the customer paid $53.40 for 30 muffins, find the number of each type of muffins purchased. [4]

2 Solve the inequality $\dfrac{2}{x+2} \geq \dfrac{x+1}{3}$. [3]

Hence, find the range of values of x for which $\dfrac{2}{x+3} \geq \dfrac{x+2}{3}$. [2]

3 **(i)** Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x\mathrm{e}^{-x}\right)$. [1]

(ii) Hence, find $\displaystyle\int \frac{x-x^2}{\mathrm{e}^x}\,\mathrm{d}x$. [4]

4 It is given that $\mathrm{h}(x) = x\cos x$ for $0 \leq x \leq \dfrac{\pi}{2}$. It is also known that $\mathrm{h}(-x) = \mathrm{h}(x)$ and $\mathrm{h}(\pi + x) = -\mathrm{h}(x)$ for all real values of x.

(i) Sketch the graph of $y = \mathrm{h}(x)$ for $-2\pi \leq x \leq 2\pi$. [3]

(ii) On a separate diagram, sketch the graph of $y^2 = \mathrm{h}(x)$ for $-2\pi \leq x \leq 2\pi$. [2]

5 The planes p_1, p_2 and p_3 have equations $3x + 4y - 7z = 2$, $x - 2y = 4$ and $5x - 4y + a\,z = 3$ respectively, where a is a constant. The point C has position vector $-\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

(i) Given that $a = 2$, find the coordinates of the point of intersection of p_1, p_2 and p_3. [2]

(ii) Find the coordinates of the foot of perpendicular from C to p_2. [3]

(iii) Find the value of a such that p_1, p_2 and p_3 have no common points. [3]

6 **Do not use a calculator in answering this question.**

The complex number z is given by $z = \dfrac{3+\mathrm{i}}{2-\mathrm{i}}$.

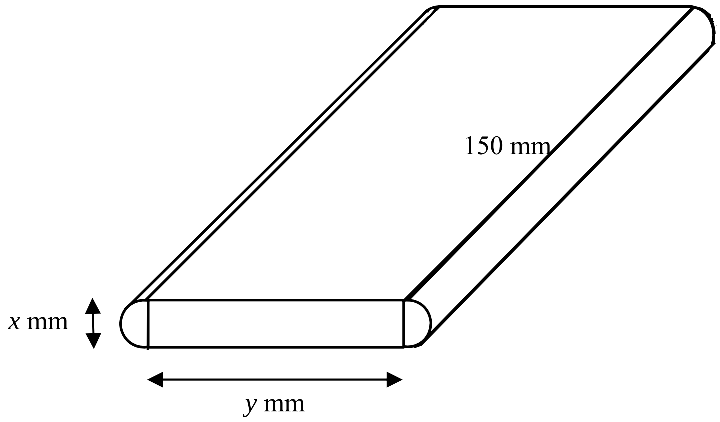
(i) Find $|z|$ and $\arg z$ in exact form. [2]

(ii) Hence, find the exact values of x and y, where $-\pi < y \leq \pi$, such that

$$\mathrm{e}^{x+\mathrm{i}2y} = \frac{3+\mathrm{i}}{2-\mathrm{i}}.$$ [3]

(iii) Find the smallest positive integer n such that $\left(\dfrac{z^2}{z^*}\right)^n$ is purely imaginary. [3]

7 A company manufactures a container of length 150 mm. The container has a uniform cross section made up of a rectangle y mm by x mm and 2 semi-circles of diameter x mm (see diagram).



Given that the container has a volume of 7200 mm^3, find the exact value of x which gives a container of minimum external surface area. [8]

8 **(a)** A bowl of hot soup is placed in a room where the temperature is a constant 20 °C. As the soup cools down, the rate of decrease of its temperature θ °C after time t minutes is proportional to the difference in temperature between the soup and its surroundings. Initially, the temperature of the soup is 80 °C and the rate of decrease of the temperature is 4 °C per minute. By writing down and solving a differential equation, show that $\theta = 20 + 60\,\mathrm{e}^{-\frac{1}{15}t}$. [6]
Find the time it takes the soup to cool to half of its initial temperature. [2]

(b) The gradient of a curve C is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (x+y)^2.$$

Use the substitution $u = x + y$ to show that the above equation reduces to

$$\frac{\mathrm{d}u}{\mathrm{d}x} = 1 + u^2.$$ [2]

Hence find y in terms of x given that C passes through the origin. [2]

9 **(a)** Given that $y = \ln(\cos x)$, show that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = -1.$$ [2]

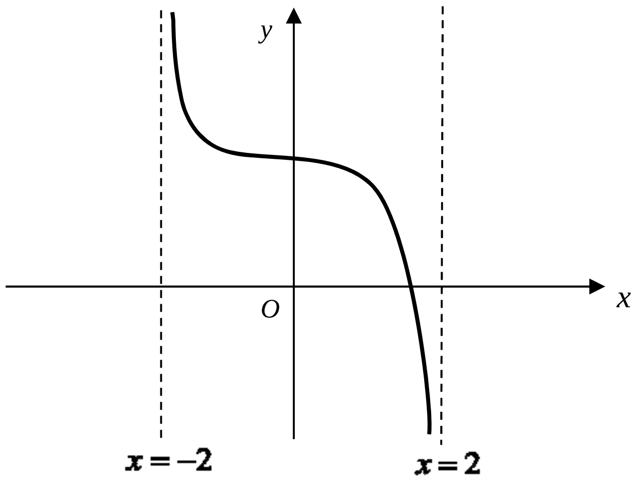
(i) By further differentiation of this result, find the Maclaurin series for y, up to and including the term in x^4. [5]

(ii) Hence, find the Maclaurin series for $\tan x$, up to and including the term in x^3. [2]

(b) Using an appropriate expansion from MF15, find the first three terms of the Maclaurin series for $\ln(k+x)^n$, where n and k are positive constants. [3]

10 **(i)** Find $\displaystyle\int \frac{1}{1+(3-y)^2}\,\mathrm{d}y$. [1]

(ii)



The diagram shows the curve with equation $y = 3 - \dfrac{x}{\sqrt{4-x^2}}$. Find the exact volume of revolution when the region bounded by the curve, the line $y = 1$ and the y-axis is rotated completely about the y-axis. [4]

By using the substitution $x = 2\sin\theta$, find the exact area of the region bounded by the curve, the line $x = 1$ and the axes. [4]

11 In a training session, athletes run from a starting point S towards their coach in a straight line. When they reach the coach, they run back to S along the same straight line. A lap is completed when athletes return to S. At the beginning of the training session, the coach stands at A_1 which is 25 m away from S. After the first lap, the coach moves from A_1 to A_2 and after the second lap, he moves from A_2 to A_3 and so on. The points A_1, A_2, A_3, …, are increasingly further away from S in a straight line where A_iA_{i+1} = 1 m, $i \in \mathbb{Z}^+$. The training session will stop only when the athletes have run more than 1500 m.

An athlete completes his first lap in 20 seconds but the time for each subsequent lap is 15% more than the time for the preceding lap. Given that the athlete must complete each lap he runs and there is no resting time between laps, find the least amount of time to complete the training session, giving your answer correct to the nearest minute. [5]

Assuming that the athlete runs at a constant speed for each lap, find the number of complete laps when he has run for 15 minutes. [2]
Hence, find the distance from S and the direction of travel of the athlete after he has run for exactly 15 minutes. [3]

12 Planes p and q are perpendicular to each other. Plane p has equation $\mathbf{r}\cdot\begin{pmatrix}-2\\3\\1\end{pmatrix}=-4$ and plane q contains the line l with equation $x=0,\ y+1=\dfrac{z-2}{4}$.

(i) Find a cartesian equation of q. [4]

(ii) Find a vector equation of the line m where p and q meet. [2]

(iii) Find the coordinates of the point C at which l intersects m. [3]

(iv) The points A and B have coordinates $(0, -1, 2)$ and $(x, 0, 0)$. If the area of triangle ABC is increasing at a rate of 17 units^2 per second, find the rate of change of x when $x=\sqrt{5}$. [5]

~ End of Paper ~