SERANGOON JUNIOR COLLEGE

2016 JC2 PRELIMINARY EXAMINATION

MATHEMATICS

Higher 2 **9740/2**

21 Sept 2016

3 hours

Additional materials: Writing paper

List of Formulae (MF15)

**TIME : ** 3 hours

READ THESE INSTRUCTIONS FIRST
Write your name and class on the cover page and on all the work you hand in.
Write in blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets **[ ]** at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

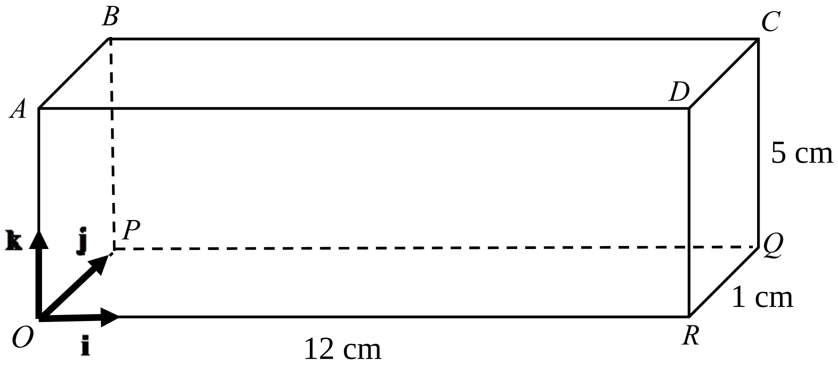
Total marks for this paper is 100 marks

This question paper consists of 7 printed pages (inclusive of this page) and 1 blank page.

[TURN OVER]

Section A: Pure Mathematics [40 marks].

1	**(a)** If $0 < a < b$, solve $\int_0^b x\lvert a-x\rvert\ \mathrm{d}x$, leaving your answers in terms of a and b.	[2]
	(b)(i) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{3-x}{\sqrt{1-x}}\right)$.	[1]
	(ii) Find $\int \frac{3-x}{x^2-3x+2}\mathrm{d}x$.	[2]
	(iii) Hence find $\int \frac{1+x}{(1-x)^{\frac{3}{2}}}\tan^{-1}\sqrt{1-x}\ \mathrm{d}x$.	[3]
	Solution	
	(a) $\int_0^b x\lvert a-x\rvert\ \mathrm{d}x = \int_0^a x(a-x)\ \mathrm{d}x + \int_a^b x(x-a)\ \mathrm{d}x$	
	$= \left[\frac{ax^2}{2}-\frac{x^3}{3}\right]_0^a + \left[\frac{x^3}{3}-\frac{ax^2}{2}\right]_a^b$	
	$= \left(\frac{a^3}{2}-\frac{a^3}{3}\right) + \left(\frac{b^3}{3}-\frac{ab^2}{2}\right) - \left(\frac{a^3}{3}-\frac{a^3}{2}\right)$	
	$= \frac{a^3}{3}+\frac{b^3}{3}-\frac{ab^2}{2}$	
	(bi) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{3-x}{\sqrt{1-x}}\right) = \frac{-\sqrt{1-x}-(3-x)\left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{1-x}}\right)}{1-x}$	
	$= \frac{-2+2x+3-x}{2(1-x)^{3/2}} = \frac{x+1}{2(1-x)^{3/2}}$	
	(ii) $\int \frac{3-x}{x^2-3x+2}\mathrm{d}x = \int \frac{3-x}{(x-2)(x-1)}\mathrm{d}x$	
	$= \int \left[\frac{1}{x-2}-\frac{2}{x-1}\right]\mathrm{d}x$	
	$= \ln\lvert x-2\rvert - 2\ln\lvert x-1\rvert + c$	
	(iii) $\int \frac{1+x}{(1-x)^{\frac{3}{2}}}\tan^{-1}\sqrt{1-x}\ \mathrm{d}x$ $= 2\left(\frac{3-x}{\sqrt{1-x}}\right)\tan^{-1}\sqrt{1-x} - \int 2\left(\frac{3-x}{\sqrt{1-x}}\right)\left(\frac{1}{1+1-x}\right)\left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{1-x}}\right)\ \mathrm{d}x$	
	$= 2\left(\frac{3-x}{\sqrt{1-x}}\right)\tan^{-1}\sqrt{1-x} + \int \frac{3-x}{(2-x)(1-x)}\ \mathrm{d}x$	

	$= 2\left(\frac{3-x}{\sqrt{1-x}}\right)\tan^{-1}\sqrt{1-x} + \ln\lvert x-2\rvert - 2\ln\lvert x-1\rvert + c$	
2		
	The cuboid above is formed by the eight vertices *O, A, B, C, D, P, Q* and *R*. Perpendicular unit vectors **i**, **j**, **k** are parallel to *OR*, *OP* and *OA* respectively. The length of *OR*, *OP* and *OA* are 12 cm, 1 cm and 5 cm respectively.	
	(i) Find the Cartesian equation of line *AC*.	[2]
	(ii) Find the acute angle between *CA* and *CR*. Hence, find the shortest distance from *R* to *AC*.	[4]
	(iii) The point *T* is on *AC* produced such that $AT = \lambda AC$ and *M* is the midpoint of *OR*. The unit vector in the direction of *OT* is represented by the vector $\overrightarrow{OV}$. By considering the cross product of relevant vectors, find the ratio of the area of triangle *OVM* to the area of triangle *ORT* in terms of λ.	[3]
	Solution	
	(i) $\overrightarrow{AC} = 12\mathbf{i} + \mathbf{j}$	
	Equation of line *AC*: $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + \alpha\begin{pmatrix} 12 \\ 1 \\ 0 \end{pmatrix},\ \alpha \in \mathbf{R}$	
	Cartesian equation of line is $\frac{x}{12} = y, z = 5.$	
	(ii) Let the acute angle between *AC* and *RC* be *x*. $\cos x = \frac{\left\lvert \begin{pmatrix} 12 \\ 1 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix} \right\rvert}{\sqrt{145}\sqrt{26}}$	
	Therefore, $x = 89.1°$	
	Let the shortest distance required be *y*.	
	$\sin 89.07° = \frac{y}{\sqrt{26}}$	
	$y = 5.10$ cm	

	(iii) $\overrightarrow{OC}=\dfrac{\overrightarrow{OT}+(\lambda-1)\overrightarrow{OA}}{\lambda}$ $\overrightarrow{OT}=\lambda\overrightarrow{OC}-(\lambda-1)\overrightarrow{OA}$ $=\lambda\begin{pmatrix}12\\1\\5\end{pmatrix}-(\lambda-1)\begin{pmatrix}0\\0\\5\end{pmatrix}$ $=\begin{pmatrix}12\lambda\\\lambda\\5\end{pmatrix}$	
	Area of triangle $ORT=\dfrac{1}{2}\left\|\overrightarrow{OT}\times\overrightarrow{OR}\right\|$	
	Area of triangle OVM $=\dfrac{1}{2}\left\|\dfrac{\overrightarrow{OT}}{\left\|\overrightarrow{OT}\right\|}\times\dfrac{\overrightarrow{OR}}{2}\right\|$	
	$=\dfrac{1}{2\sqrt{25+145\lambda^2}}\times\dfrac{1}{2}\left\|\overrightarrow{OT}\times\overrightarrow{OR}\right\|$	
	Therefore the ratio of triangle OVM to area of triangle ORT is $1:2\sqrt{25+145\lambda^2}$.	
3	**(a)** The complex number w is such that $w = a + \mathrm{i}b$, where a and b are non-zero real numbers. The complex conjugate of w is denoted by w*. Given that $\dfrac{(w^*)^2}{w}=3-\mathrm{i}b$, solve for a and b and hence write down the possible values of w.	[3]
	(b) **(i)** Without the use of a graphic calculator, find the roots of the equation $z^2-2z+4=0$, leaving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, $r>0$ and $-\pi<\theta\le\pi$.	[2]
	(ii) Let α and β be the roots found in **(b)(i)**. If $\arg(\alpha)>\arg(\beta)$, find $\left\|\alpha^{10}-\beta^{10}\right\|$ and $\arg\left(\alpha^{10}-\beta^{10}\right)$ in exact form.	[3]
	(c) **(i)** Show that the locus of z where $\arg(z+2\sqrt{3}+\mathrm{i})=-\dfrac{\pi}{6}$ passes through the point $(-\sqrt{3},-2)$.	[1]
	(ii) Find the Cartesian equation of the locus of z in the form $y=mx+c$, stating its domain clearly. Leave your answer in exact form.	[2]
	Solution	
	(a) $\dfrac{(w^*)^2}{w}=3-\mathrm{i}b\Rightarrow\dfrac{(a-\mathrm{i}b)^2}{(a+\mathrm{i}b)}=3-\mathrm{i}b$	

	$a^2 - b^2 - 2\mathrm{i}ab = (3-\mathrm{i}b)(a+\mathrm{i}b) = 3a + b^2 + \mathrm{i}(-ab+3b)$	
	Equating the real and the imaginary parts:	
	$a^2 - b^2 = 3a + b^2$ **…..**(1) and $-2ab = -ab + 3b$ …(2)	
	From (2) $a = -3$ since $b \neq 0$	
	From (1), $9 - b^2 = -9 + b^2$ $b^2 = 9$ $b = \pm 3$ Possible values of w are $-3 \pm 3\mathrm{i}$	
	(bi) $z^2 - 2z + 4 = 0$ $z = \dfrac{2 \pm \sqrt{4-16}}{2} = 1 \pm \sqrt{3}\mathrm{i}$ $\alpha = 1 + \sqrt{3}\mathrm{i} = 2e^{\mathrm{i}\left(\frac{\pi}{3}\right)}$ and $\beta = 1 - \sqrt{3}\mathrm{i} = 2e^{-\mathrm{i}\left(\frac{\pi}{3}\right)}$	
	(ii) $\alpha^{10} - \beta^{10} = 2^{10}\left(e^{\mathrm{i}\left(\frac{10\pi}{3}\right)} - e^{-\mathrm{i}\left(\frac{10\pi}{3}\right)}\right)$	
	$= 2^{10}\left(2\mathrm{i}\sin\dfrac{10\pi}{3}\right)$	
	$= 2^{10}\left(2\mathrm{i}\sin\left(-\dfrac{2\pi}{3}\right)\right)$	
	$= 2^{10}\left(2\left(-\dfrac{\sqrt{3}}{2}\right)\right)\mathrm{i}$ $= -1024\sqrt{3}\mathrm{i}$	
	$\left\lvert\alpha^{10} - \beta^{10}\right\rvert = 1024\sqrt{3}$	
	So $\arg\left(\alpha^{10} - \beta^{10}\right) = -\dfrac{\pi}{2}$	
	(ci) When $z = -\sqrt{3} - 2\mathrm{i}$, LHS = $\arg(-\sqrt{3} - 2\mathrm{i} + 2\sqrt{3} + \mathrm{i}) = \arg(\sqrt{3} - \mathrm{i})$	
	$= -\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right) = -\dfrac{\pi}{6}$ (Shown)	
	(ii) Gradient of the half line is $\tan(-\dfrac{\pi}{6}) = -\dfrac{1}{\sqrt{3}}$	
	Equation : $y + 2 = -\dfrac{1}{\sqrt{3}}\left(x + \sqrt{3}\right), \quad x > -2\sqrt{3}$ $y = -\dfrac{\sqrt{3}}{3}x - 3$	

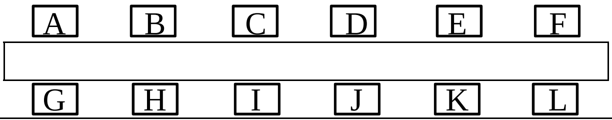
4	A curve C has parametric equations $$x=\sin t,\quad y=\cos t$$ **(i)** Find the equations of tangent and normal to C at the point with parameter t.	[3]
	(ii) Points P and Q on C have parameters p and q respectively, where $0<p<\dfrac{\pi}{2}$ and $0<q<\dfrac{\pi}{2}$. The tangent at P meets the normal at Q at the point R. Show that the x – coordinate of R is $\dfrac{\sin q}{\cos(p-q)}$. Hence, find in similar form the y – coordinate of R in terms of p and q.	[3]
	The tangent at P meets the y-axis at the point A and the normal at Q meets the y-axis at the point B. Taking $q=\dfrac{\pi}{2}-p$,	
	(iii) Show that the area of triangle ARB is $\dfrac{1}{2}\operatorname{cosec}(2p)$.	[3]
	(iv) Find the Cartesian equation of the locus of point R.	[3]
	Solution	
	(i) $x=\sin t \qquad\qquad y=\cos t$ $\dfrac{dx}{dt}=\cos t \qquad\qquad \dfrac{dy}{dt}=-\sin t$ $\dfrac{dy}{dx}=\dfrac{-\sin t}{\cos t}$ $\quad=-\tan t$ Equation of tangent: $y-\cos t=(-\tan t)(x-\sin t)$ $y=(-\tan t)x+(\tan t)(\sin t)+\cos t$ $y=(-\tan t)x+\dfrac{\sin^2 t}{\cos t}+\dfrac{\cos^2 t}{\cos t}$ $y=(-\tan t)x+\sec t$ Equation of normal: $y-\cos t=(\cot t)(x-\sin t)$ $y=(\cot t)x-\cos t+\cos t$ $y=(\cot t)x$	
	(ii) Equation of tangent at P (with parameter p): $y=(-\tan p)x+\sec p$ Equation of normal at Q (with parameter q):	

	$y=(\cot q)x$ Equating the equation of tangent at P and the equation of normal at Q, we have $(\cot q)x=(-\tan p)x+\sec p$ $(\cot q+\tan p)x=\sec p$ $\left(\dfrac{\cos q}{\sin q}+\dfrac{\sin p}{\cos p}\right)x=\dfrac{1}{\cos p}$ $\left(\dfrac{\cos p\cos q+\sin p\sin q}{\cos p\sin q}\right)x=\dfrac{1}{\cos p}$ $\left(\dfrac{\cos(p-q)}{\cos p\sin q}\right)x=\dfrac{1}{\cos p}$ $x=\dfrac{\sin q}{\cos(p-q)}$ (Shown) Substitute $x=\dfrac{\sin q}{\cos(p-q)}$ into the equation of normal at Q, $y=(\cot q)\left(\dfrac{\sin q}{\cos(p-q)}\right)$ $y=\dfrac{\cos q}{\cos(p-q)}$	
	(iii) Coordinates of A: When $x=0$, $y=\sec p$ A is $(0,\sec p)$ or $(0,\sin p\tan p+\cos p)$ Coordinates of B: When $x=0$, $y=0$ B is $(0,0)$. Since R is $\left(\dfrac{\sin q}{\cos(p-q)},\dfrac{\cos q}{\cos(p-q)}\right)$, Area of Triangle ARB $=\dfrac{1}{2}(\sec p)\left(\dfrac{\sin q}{\cos(p-q)}\right)$ $=\dfrac{1}{2}\left(\dfrac{1}{\cos p}\right)\left(\dfrac{1}{2\sin p}\right)$ $=\dfrac{1}{2}\left(\dfrac{1}{2\sin p\cos p}\right)$ $=\dfrac{1}{2}\operatorname{cosec}(2p)$ (Shown)	

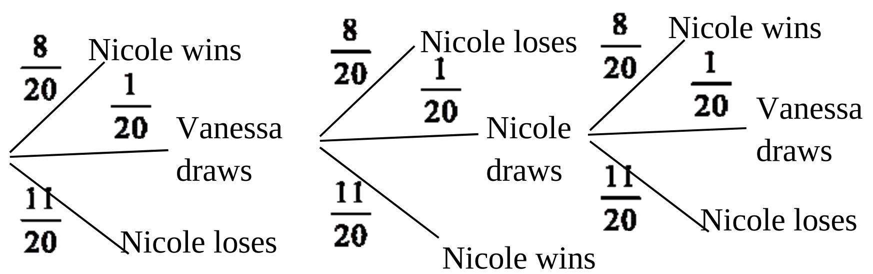
(iv)

When $q=\frac{\pi}{2}-p$, $x=\dfrac{\sin\left(\frac{\pi}{2}-p\right)}{\cos\left(2p-\frac{\pi}{2}\right)}$ $=\dfrac{\cos p}{\sin 2p}$ $=\dfrac{\cos p}{2\sin p\cos p}$ $=\dfrac{1}{2\sin p}$	**Alternatively,** Since $p=\frac{\pi}{2}-q$, $x=\dfrac{\sin q}{\cos\left(\frac{\pi}{2}-2q\right)}$ $=\dfrac{\sin q}{\sin 2q}$ $=\dfrac{\sin q}{2\sin q\cos q}$ $=\dfrac{1}{2\cos q}$
$y=\dfrac{\cos\left(\frac{\pi}{2}-p\right)}{\cos\left(2p-\frac{\pi}{2}\right)}$ $=\dfrac{\sin p}{\sin 2p}$ $=\dfrac{\sin p}{2\sin p\cos p}$ $=\dfrac{1}{2\cos p}$	**Alternatively,** $y=\dfrac{\cos q}{\cos\left(\frac{\pi}{2}-2q\right)}$ $=\dfrac{\cos q}{\sin 2q}$ $=\dfrac{\cos q}{2\sin q\cos q}$ $=\dfrac{1}{2\sin q}$
Using $\sin^2 p+\cos^2 p=1$, $\left(\dfrac{1}{2x}\right)^2+\left(\dfrac{1}{2y}\right)^2=1$ $\dfrac{1}{4x^2}+\dfrac{1}{4y^2}=1, \quad x>0, y>0$	**Alternatively,** Using $\sin^2 q+\cos^2 q=1$, $\left(\dfrac{1}{2y}\right)^2+\left(\dfrac{1}{2x}\right)^2=1$ $\dfrac{1}{4x^2}+\dfrac{1}{4y^2}=1, \quad x>0, y>0$

Section B: Statistics [60 marks]

5	Nicole decides to celebrate her birthday with 9 boys and 2 girls whose names are Vanessa and Sally.	
	(a) (i) They have a dinner at a restaurant that can only offer them a rectangular table as shown in the following diagram, with seats labelled A to L as shown. 	
	Find the number of ways in which at least one girl must be seated at the seats A, F, G and L.	[2]
	(ii) Find the number of ways in which they can sit if instead, the restaurant offers them 2 indistinguishable round tables of 6.	[2]
	(iii) After the dinner, they went for a movie together. They bought tickets for seats in a row. Find the number of ways where Nicole and Vanessa must be seated together but not Sally.	[2]
	(b) After the celebration, Nicole plays a card game with Vanessa. The pack of 20 cards are numbered 1 to 20. The two friends take turns to draw a card from the pack. If a prime number is drawn, the player wins the game. If a composite number (4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20) is drawn, the player loses the game and the other player wins. If the number ‘1’ is drawn, the card is returned and the other player draws the next card. Nicole draws the first card. Find the probability of her winning the game.	[3]
	Solution	
	(i) Number of ways $=12!-\binom{9}{4}4!8!$ $-\binom{9}{4}4!$ (for the selection and the arrangement of the 4 guys to be seated at A, F, G and L.)	
	$=357073920$	
	OR Number of ways $=12!-\binom{9}{5}8!4!$ $-\binom{9}{5}8!$ (for the selection of the 5 guys that is to be seated at B, C, D, E, H, I, J, K with the 3 girls)	
	$=357073920$	
	OR Number of ways $=12!-\binom{8}{3}9!3!$ $-\binom{8}{3}3!$ (for the selection of the 3 seat in the slot B, C, D, E, H, I, J, K)	
	$=357073920$	

	(ii) Number of ways $=\dfrac{\binom{12}{6}5!5!}{2!}$ – For $\binom{12}{6}5!5!$	
	$= 6652800$	
	(iii) Number of ways $=9!\binom{10}{2}2!2!$ **Alternative solution** $10!\times 9\times 2=65318400$ – For $\binom{10}{2}2!$ (the selection of the slots to separate Nicole and Vanessa with Sally)	
	$=65318400$	
	(b)	



	Probability of Nicole winning $=\frac{8}{20}\left[1+\left(\frac{1}{20}\right)^2+\left(\frac{1}{20}\right)^4+......\right]+\frac{11}{20}\left[\frac{1}{20}+\left(\frac{1}{20}\right)^3+\left(\frac{1}{20}\right)^5+...\right]$ First – 1st infinite series, Second – 2nd infinite series	
	$=\frac{8}{20}\cdot\frac{1}{1-\left(\frac{1}{20}\right)^2}+\frac{11}{20}\cdot\frac{\frac{1}{20}}{1-\left(\frac{1}{20}\right)^2}=\frac{3}{7}$	
6	In a telephone enquiry service, 92% of calls to it are successfully connected. The probability of any call being successfully connected is constant. A random sample of 60 calls is taken each day.	
	(i) State, in context, an assumption needed for it to be well modelled by a binomial distribution.	[1]
	(ii) On a given day, it is found that at most 55 calls went through successfully. Find the probability that there are at least 50 successful calls in the sample of 60.	[2]
	(iii) Estimate the probability that the number of successful calls on any day is less than 55 in a sample of 60.	[4]
	(iv) The number of successful calls is recorded daily for 70 consecutive days. Find the approximate probability that the average number of successful calls in a day is not more than 55.	[2]

	Solution	
	(i) The event that a call is successfully connected is independent from the event of other calls being successfully connected.	
	(ii) Let X be the random variable denoting the number of successful calls, out of a sample of 60 calls. $X \sim \mathrm{B}(60, 0.92)$	
	$\mathrm{P}(X \geq 50 \mid X \leq 55)$ $= \dfrac{\mathrm{P}(50 \leq X \leq 55)}{\mathrm{P}(X \leq 55)}$	
	$= \dfrac{\mathrm{P}(X \leq 55) - \mathrm{P}(X \leq 49)}{\mathrm{P}(X \leq 55)}$	
	$= \dfrac{0.52982 - 0.074926}{0.52982}$	
	= 0.986 (3s.f.)	
	(iii) Let Y be the random variable denoting the number of calls that are not successfully connected, out of a sample of 60 calls. $Y \sim \mathrm{B}(60, 0.08)$	
	Since n = 60 is large, $np = 60(0.08) = 4.8(< 5)$ $\therefore Y \sim \mathrm{Po}(4.8)$ approx	
	(table below)	
	$\mathrm{P}(\text{less than 55 successful calls})$ $= \mathrm{P}(\text{at least 6 calls that are not successfully connected})$	
	$= \mathrm{P}(Y \geq 6)$ $= 1 - \mathrm{P}(Y \leq 5)$	
	= 0.349 (3s.f.)	
	(iv) $X \sim \mathrm{B}(60, 0.92)$ Since n = 70 is large, by Central Limit Theorem, $\overline{X} \sim \mathrm{N}\left(55.2, \dfrac{4.416}{70}\right)$ approx	
	$\mathrm{P}\left(\overline{X} \leq 55\right) = 0.213(\text{to 3 s.f})$	
	Alternative solution: Let W be the random variable denoting the number of successful calls out of a sample of 4200. $W \sim \mathrm{B}(4200, 0.92)$ Since n is large, np = 3864>5, nq = 336>5 $W \sim \mathrm{N}(3864, 309.12)$ approx $\mathrm{P}(W \leq 55 \times 70) \xrightarrow{CC} \mathrm{P}(W < 3850.5) = 0.221(\text{to 3 s.f})$	

Number of successful calls	Number of calls that are not successfully connected
54	6
53	7
52	8

7	**(a)** Tickets are sold for the closing ceremony of an international swimming competition. It is desired to sample 1% of the spectators to find their opinions of the goodie bags received during the closing ceremony.	
	(i) Give a reason why it would be difficult to use a stratified sample.	[1]
	(ii) Explain how a systematic sample could be carried out.	[2]
	(b) The random variable X has the distribution $\mathrm{N}(18, 3^2)$ and the random variable Y has the distribution $\mathrm{N}(\mu, \sigma^2)$. The random variable T is related to X and Y by the formula $T = \dfrac{X_1 + X_2 + 3Y}{4}$, where X_1 and X_2 are two independent observations of X. Given that $\mathrm{P}(T < 10) = \mathrm{P}(T > 30) = 0.0668$, find the value of σ and the exact value of μ.	[5]
	(c) A survey done on students in a particular college found that the amount of time a student spends on social media in a week is normally distributed with mean 7 hours and variance 4 hours^2. Five students are randomly chosen. Find the probability that the fifth student is the second student who spends more than 10 hours a week on social media.	[2]
	Solution	
	(ai) Though the tickets issued might have a serial number indicated, but some people who have purchased the tickets, may not turn up for the closing ceremony and so it is difficult to obtain the actual sampling frame.	
	(ii) To have a sample consisting of 1% of the spectators present, the sampling interval will be 100. Randomly select a number between 1 to 100 say *r*. So at the entrance point, every *r*th person for each interval of 100 will be selected for the survey until the sample is obtained.	
	(b) $\mathrm{E}(T) = 20$	
	$\frac{1}{4}\left(2(18) + 3\mu\right) = 20$	
	$\mu = \frac{44}{3}$	
	$\mathrm{Var}(T) = \mathrm{Var}\left(\frac{X_1 + X_2 + 3Y}{4}\right)$	
	$= \frac{1}{4^2}\left(2\mathrm{Var}(X) + 3^2\,\mathrm{Var}(Y)\right)$	
	$= \frac{1}{4^2}\left(2(3^2) + 9\sigma^2\right) = \frac{9}{8} + \frac{9}{16}\sigma^2$	
	$\mathrm{P}(T < 10) = 0.0668$	
	$\mathrm{P}\left(Z < \frac{10 - 20}{\sqrt{\frac{9}{8} + \frac{9}{16}\sigma^2}}\right) = 0.0668$	

	$\dfrac{10-20}{\sqrt{\dfrac{9}{8}+\dfrac{9}{16}\sigma^2}}=-1.500$	
	$\left(\dfrac{10-20}{-1.500}\right)^2=\dfrac{9}{8}+\dfrac{9}{16}\sigma^2$	
	$\sigma=8.77533=8.78\,(3\text{sf})$	
	(c) Let X be the random variable "time taken by a randomly chosen student on social media".	
	$X \sim \mathrm{N}(7,2^2)$	
	Required probability $=4\left[\mathrm{P}\left(X>10\right)\right]^2\left[\mathrm{P}\left(X\le 10\right)\right]^3$	
	$=0.014508$	
	$=0.0145\,(3\text{ s.f})$	
8	An advertising display contains a large number of light bulbs which are continually being switched on and off every day in a week. The light bulbs fail independently at random times. Each day the display is inspected and any failed light bulbs are replaced. The number of light bulbs that fail in any one-day period has a Poisson distribution with mean 1.6.	
	(i) State, in the context of the question, one assumption that needs to be made for the number of light bulbs that fail per day to be well modelled by a Poisson distribution.	[1]
	(ii) Estimate the probability that there are fewer than 17 light bulbs that needs to be replaced in a period of 20 days.	[2]
	(iii) Using a suitable approximation, find the probability that there will be not fewer than 20 days with more than two light bulbs that will need to be replaced per day in a period of 8 weeks.	[4]
	(iv) The probability of at least three light bulbs having to be replaced over a period of n consecutive days exceeds 0.999. Write down an inequality in terms of n to express this information, and hence find the least value of n.	[4]
	Solution	
	(i) The average number of light bulbs that fail in a given time interval is proportional to the length of the time interval.	
	(ii) Let V be the random variable denoting "the number of light bulbs that needs to be replaced in 20 days." $V \sim \mathrm{Po}(20\times 1.6)$ $V \sim \mathrm{Po}(32)$ Since λ = 32 > 10, $V \sim$ N(32, 32) approximately	

$P(V < 17) \xrightarrow{\text{c.c}} P(V < 16.5)$
$= 0.003071651$
$= 0.00307$

(iii) $P(X > 2) = 1 - P(X \le 2) = 0.21664$

Let Y be the random variable denoting "the number of days in which at least three light bulbs will need to be replaced out of 56 days"

$$Y \sim B(56, 0.21664)$$

Since $n = 56$ is large, $np = 12.132$ (>5), $nq = 43.868$

$$Y \sim N(12.132, 9.5036) \text{ approximately}$$

$P(Y \ge 20) \xrightarrow{\text{c.c}} P(Y > 19.5)$
$= 0.0084228$
$= 0.00842$

(iv) Let W be the random variable denoting "the number of light bulbs that need to be replaced in n consecutive days."

$$W \sim Po(1.6n)$$

$P(W \ge 3) > 0.999$

$1 - P(W \le 2) > 0.999$

$1 - 0.999 > P(W \le 2)$

$$0.001 > \frac{e^{-(1.6n)}(1.6n)^0}{0!} + \frac{e^{-(1.6n)}(1.6n)^1}{1!} + \frac{e^{-(1.6n)}(1.6n)^2}{2!}$$

$$e^{-(1.6n)} + e^{-(1.6n)}(1.6n) + e^{-(1.6n)}(1.28n^2) < 0.001$$

$n = 7$, $P(W \le 2) = 0.00102$

$n = 8$, $P(W \le 2) = 0.000264$

$n = 9$, $P(W \le 2) = 0.0000664$

Least value of n is 8.

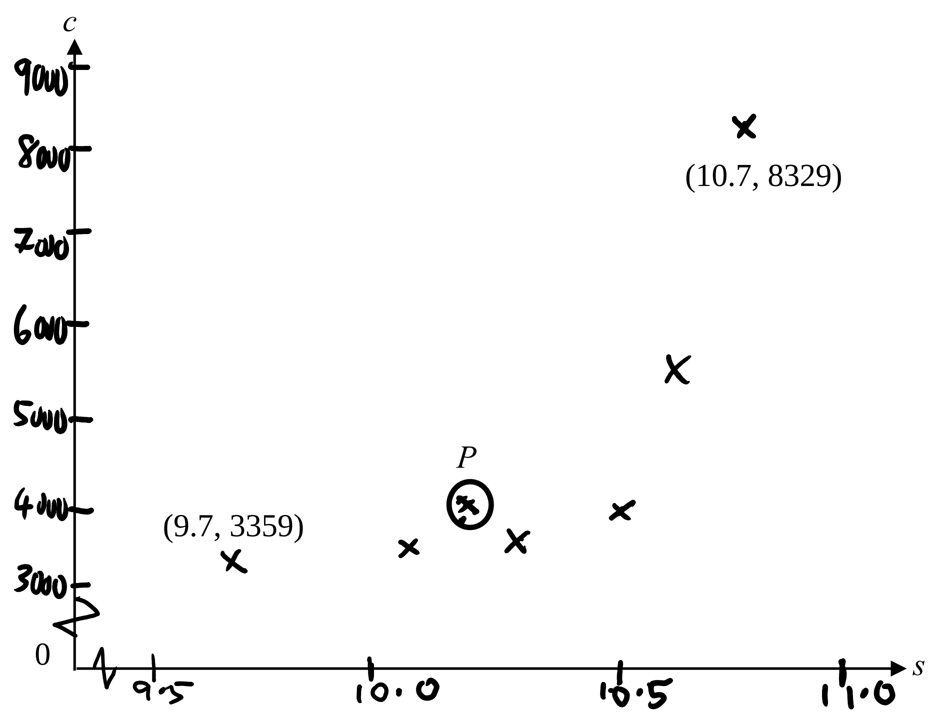
9 **(a)** Observations of 10 pairs of values (x,y) are shown in the table below.

x	1	2	3	4	5	6	7	8	9	10
y	0.5	0.6	0.8	0.95	a	1.21	1.36	1.55	1.87	2.11

It is known that the equation of the linear regression line of y on x is $y = 0.17321x + 0.24133$. Find a, correct to 2 decimal places. [2]

(b) A student wanted to study the relationship between the number of commercial crimes (c) and the mean years of schooling (s) of the offenders. The following set of data was obtained.

Year	2009	2010	2011	2012	2013	2014	2015
Mean years of schooling (s)	9.7	10.1	10.2	10.3	10.5	10.6	10.7
No. of commercial crimes (c)	3359	3504	4080	3507	3947	5687	8329

	(i) Draw a scatter diagram for these values.	[2]
	(ii) One of the values of c appears to be incorrect. Circle this point on your diagram and label it P.	[1]
	It is thought that the number of commercial crimes (c) can be modelled by one of the formulae after removing the point P. (A) $c = a + b\left(100^s\right)$ (B) $c = a + bs$ (C) $c = a + b\ln s$ where a and b are non-zero constants.	
	(iii) With reference to the scatter diagram, explain clearly which model is the best model for this set of data. For the case identified, find the equation of a suitable regression line.	[2]
	(iv) Using the regression line found in **(iii)**, estimate the number of commercial crimes (to the nearest whole number) when the mean years of schooling reaches 11.	[2]
	(v) Comment on the reliability of your answer in part **(iv)**.	[1]
	Solution	
	(a) Using GC, $\bar{x} = 5.5$	
	$\bar{y} = \dfrac{10.95 + a}{10}$	
	Since $\left(\bar{x}, \bar{y}\right)$ lies on the regression line,	
	$\dfrac{10.95 + a}{10} = 0.17321\left(5.5\right) + 0.24133$	
	$a = 0.98985 \approx 0.99$ (correct to 2 decimal places)	
	(b) (i) and (ii) 	

	(iii) From the scatter diagram (after removing the outlier), as s increases, c increases at an increasing rate. Hence model (A) is the best model.	
	From GC, $c = 2862.048513 + \left(1.965434 \times 10^{-18}\right)\left(100^{s}\right)$	
	(iv) When $s = 11$,	
	$c = 2862.048513 + \left(1.965434 \times 10^{-18}\right)\left(100^{11}\right)$ ≈ 22516	
	(v) The estimate is unreliable because the data substituted is outside the data range and so the linear relationship between c and 100^{s} may not hold.	
10	In the latest Pokkinon Roll game, players go to a battle arena to use their Pokkinon character to battle against each other. Alvin and Billy are interested to know how long it takes before someone wins a battle. The time taken by a randomly chosen player to win a game follows a normal distribution.	
	(a) Alvin claims that on average, it will take at most 190.0 seconds to win a battle. To verify his belief, he surveyed a randomly chosen sample of 45 Pokkinon Roll gamers and found out that the mean is 195.0 seconds with a variance of 206.0 seconds^2.	
	Carry out an appropriate test at 1% level of significance whether there is any evidence to doubt Alvin's claim. State an assumption needed for the calculation.	[5]
	(b) Billy also obtained his own data by recording the time taken, in seconds, by 5 randomly chosen gamers as shown below.	
	188.0 190.0 k 186.0 187.0	
	However, he believes that it will take 190.0 seconds on average to win a battle. When he conducted the test at 4.742% level of significance, his conclusion is one where the null hypothesis is not rejected. The sample mean time taken is denoted by $\bar{x}$.	
	Given that s^2 is the unbiased estimate of the population variance and that the maximum range of values of $\bar{x}$ is $188 \le \bar{x} \le a$, write down an equation involving s and a.	[1]
	Hence or otherwise find the values of a and k, leaving your answers to the nearest integer.	[5]
	Solution	
	(a) Assume that the time taken to win any battle is independent of other battles.	
	Let Y denote the time taken to win a randomly chosen battle	
	$s^2 = \frac{45}{44}(206) = 210.68$	
	$H_0 : \mu = 190$ $H_1 : \mu > 190$	
	Under H_0 $\bar{Y} \sim N\left(190, \frac{210.68}{45}\right)$	
	$\mu = 190$, $\bar{y} = 195$, $n = 45$, $s = \sqrt{210.68}$	
	Using G.C, p-value is 0.0104	
	Since p value > 0.01, we do not reject H_0 and conclude that there is insufficient evidence to doubt Alvin's claim at 1% level of significance.	

	(b) $H_0 : \mu = 190$ $H_1 : \mu \neq 190$	
	2-tailed T-test at 4.742% level of significance	
	$T \sim t(4)$	
	$a = \dfrac{2.82844s}{\sqrt{5}} + 190$	
	$a = 1.26s + 190$	
	$188 \leq \bar{x} \leq a$	
	$\dfrac{-2\sqrt{5}}{s} \leq \dfrac{\bar{x} - 190}{\frac{s}{\sqrt{5}}} \leq \dfrac{\sqrt{5}(a - 190)}{s}$	
	Since $\dfrac{-2\sqrt{5}}{s} = -2.82844$	
	$s = 1.5811$	
	$s^2 = 2.5$	
	So $a = \dfrac{2.82844(1.5811)}{\sqrt{5}} + 190 = 192$	
	OR by symmetry of curve, a = 192	
	$\sum x = 751 + k$	
	$\sum x^2 = 141009 + k^2$	
	$s^2 = \dfrac{1}{4}\left[141009 + k^2 - \dfrac{(751 + k)^2}{5}\right]$	
	So $2.5 = \dfrac{1}{4}\left[141009 + k^2 - \dfrac{(751 + k)^2}{5}\right]$	
	k =189 or k = 186.5 (rejected since $188 \leq \bar{x} \leq a$)	

THE END