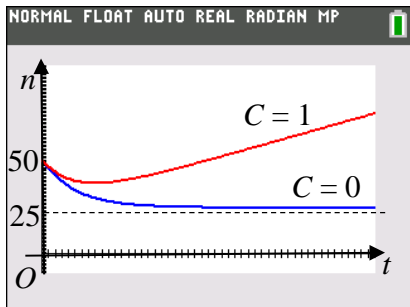




Jurong Junior College
2016 JC2 H2 Mathematics Prelim Paper 2 Solutions

Qn	Solution
1(a)	$\frac{dy}{dx} = \sec^2 y$ $\int \cos^2 y \, dy = \int 1 \, dx$ $\int \frac{\cos 2y + 1}{2} \, dy = \int 1 \, dx$ $\frac{1}{2} \left[\frac{\sin 2y}{2} + y \right] = x + c$ <p>When $y = 0$, $x = 1 \Rightarrow c = -1$</p> $\therefore x = \frac{1}{4} \sin 2y + \frac{1}{2} y + 1$
1(b) (i)	$\frac{d^2 n}{dt^2} = e^{-\frac{t}{5}}$ $\frac{dn}{dt} = \int e^{-\frac{t}{5}} dt = -5e^{-\frac{t}{5}} + C$ $n = 25e^{-\frac{t}{5}} + Ct + D$
1(b) (ii)	$\frac{d^2 n}{dt^2} = e^{-\frac{t}{5}} > 0 \text{ for all values of } t.$ <p>Solution curves are concave upwards.</p>
1(b) (iii)	<p>When $t = 0, n = 50$</p> $50 = 25e^0 + C(0) + D$ $D = 25$ $n = 25e^{-\frac{t}{5}} + Ct + 25$ <p>When $C = 0$, $n = 25e^{-\frac{t}{5}} + 25$. As $t \rightarrow \infty, n \rightarrow 25$</p> <p>When $C = 1$, $n = 25e^{-\frac{t}{5}} + t + 25$. As $t \rightarrow \infty, n \rightarrow \infty$</p> 

Qn	Solution
2(a)	<p>Area of parallelogram</p> $= \mathbf{a} \times (2\mathbf{a} + 3\mathbf{b}) $ $= 2(\mathbf{a} \times \mathbf{a}) + 3(\mathbf{a} \times \mathbf{b}) $ $= 3 \mathbf{a} \times \mathbf{b} $ $= 3 \mathbf{a} \mathbf{b} \sin 30^\circ$ $= 3(4)(5)\frac{1}{2}$ $= 30$
(b) (i)	<p>A vector equation of l is $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$.</p> <p>Let A be the point $(1, 3, 1)$ on l.</p> <p>Perpendicular distance from P to l</p> $= \frac{\left \overrightarrow{AP} \times \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \right }{\left \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \right }$ $= \frac{1}{\sqrt{5}} \left \begin{pmatrix} 1 \\ -4 \\ -3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \right $ $= \frac{1}{\sqrt{5}} \left \begin{pmatrix} 4 \\ -5 \\ 8 \end{pmatrix} \right $ $= \frac{\sqrt{105}}{\sqrt{5}} = \sqrt{21}$
(ii)	<p>Acute angle between l and L</p> $= \cos^{-1} \frac{\left \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right }{\left \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \right \left \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right }$ $= \cos^{-1} \frac{1}{\sqrt{5}}$ $= 63.4^\circ$

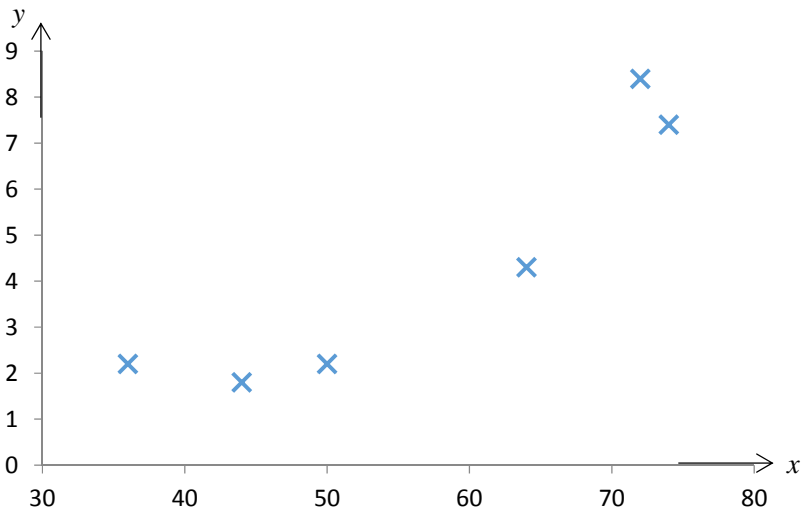
Qn	Solution
3(i)	<p>Area of equilateral $\Delta = \frac{1}{2}x^2 \sin 60^\circ = \frac{\sqrt{3}}{4}x^2$</p> <p>Given that the volume of the box is 250 cm^3</p> $V = \frac{\sqrt{3}}{4}x^2y = 250$ $y = \frac{1000}{\sqrt{3}x^2}$ <p>Surface Area $A = 3xy + 3kxy + 2\left(\frac{\sqrt{3}}{4}x^2\right)$</p> $= 3xy(1+k) + \frac{\sqrt{3}}{2}x^2$ $= 3x(1+k)\frac{1000}{\sqrt{3}x^2} + \frac{\sqrt{3}}{2}x^2$ $= \frac{1000\sqrt{3}(1+k)}{x} + \frac{\sqrt{3}}{2}x^2 \text{ (shown)}$
(ii)	<p>For stationary points, $\frac{dA}{dx} = -\frac{1000\sqrt{3}(1+k)}{x^2} + \sqrt{3}x = 0$</p> $x^3 = 1000(1+k)$ $x = 10(1+k)^{\frac{1}{3}}$ $\frac{d^2A}{dx^2} = \frac{2000\sqrt{3}(1+k)}{x^3} + \sqrt{3} > 0$ <p>Thus, $x = 10(1+k)^{\frac{1}{3}}$ gives a minimum surface area.</p>
(iii)	<p>Since $y = \frac{1000}{\sqrt{3}x^2}$</p> $\frac{y}{x} = \frac{1000}{\sqrt{3}x^3} = \frac{1000}{\sqrt{3}(1000)(1+k)}$ $= \frac{1}{\sqrt{3}(1+k)}$

(iv)	<p>Since $0 < k \leq 1$</p> $1 < 1 + k \leq 2$ $\frac{1}{2} \leq \frac{1}{1+k} < 1$ $\frac{1}{2\sqrt{3}} \leq \frac{1}{\sqrt{3}(1+k)} < \frac{1}{\sqrt{3}}$ <p>i.e. $\frac{1}{2\sqrt{3}} \leq \frac{y}{x} < \frac{1}{\sqrt{3}}$</p>
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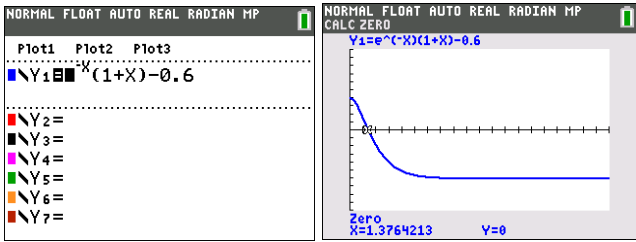
Qn	Solution
4(i)	$a^2b = \frac{1}{2}(1 + \sqrt{3}i)^2(1 - i)$ $= \frac{1}{2}(1 + 2\sqrt{3}i - 3)(1 - i)$ $= (-1 + \sqrt{3}i)(1 - i)$ $= (\sqrt{3} - 1) + (\sqrt{3} + 1)i$
(ii)	$ a^2b = a ^2 b $ $= 2^2 \left(\frac{1}{\sqrt{2}} \right)$ $= 2\sqrt{2}$ $\arg(a^2b) = 2\arg(a) + \arg(b)$ $= 2 \left(-\frac{2\pi}{3} \right) - \frac{\pi}{4}$ $= -\frac{19\pi}{12}$ $\therefore \arg(a^2b) = -\frac{19\pi}{12} + 2\pi = \frac{5\pi}{12}.$
(iii)	<p>Considering the imaginary part of a^2b, we have</p> $2\sqrt{2} \sin \frac{5\pi}{12} = \sqrt{3} + 1$ $\Rightarrow \sin \frac{5\pi}{12} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$

(iv)	\overline{BA} can be obtained by rotating \overline{BC} through 90° in the anticlockwise direction about B . $i(c - b) = a - b$ $\Rightarrow c = -i(a - b) + b$ $= -ia + b(1 + i)$ $= i(1 + \sqrt{3}i) + \frac{1}{2}(2)$ $= (1 - \sqrt{3}) + i$
5	
(i)	${}^6C_2 \times {}^5C_4 = 75$ ways
(ii)	Number of ways if at least one of the sisters are included $=$ number of ways without restriction – number of ways if none of the sisters is included $= {}^{11}C_6 - {}^8C_6$ $= 434$
	<u>Alternative Method</u> ${}^3C_1 \times {}^8C_5 + {}^3C_2 \times {}^8C_4 + {}^3C_3 \times {}^8C_3 = 434$
(iii)	Select a man to be between the 2 sisters and group the 3 of them as one unit and arrange 4 units round a table Number of ways $= {}^3C_1 \times 3! \times 2$ $= 36$
(iv)	First arrange the other 4 persons round the table. There are 4 ways to insert the sisters. Number of ways $= 3! \times 4$ $= 24$

Qn	Solution
6 (i)	$P(C M) = \frac{P(C \cap M)}{P(M)}$ $= \frac{200}{400} = \frac{1}{2}$
(ii)	$P(M \cup C) = P(M) + P(C) - P(M \cap C)$ $= \frac{400}{1000} + \frac{450}{1000} - \frac{200}{1000}$ $= \frac{650}{1000} = \frac{13}{20}$
(iii)	$P(M' \cap B') = \frac{250 + 300}{1000} = \frac{11}{20}$ $P(C) = \frac{9}{20}$ $P(C M) = \frac{1}{2} \neq P(C)$ <p>C and M are not independent.</p>
(iv)	<p>No. of international students in the sample $= 0.2(200 + 250) + 0.3(130 + 300) + 0.05(120) = 225$</p> $P(C \text{international student}) = \frac{P(C \cap \text{international student})}{P(\text{international student})}$ $= \frac{(200 + 250)0.2}{\frac{1000}{225}}$ $= 0.4$
(v)	<p>Number of international students studying Physics $= 0.3(430) = 129$</p> $P(\text{exactly one international student studying Physics}) = \frac{{}^{129}C_1 {}^{871}C_2}{{}^{1000}C_3}$ $= 0.294$ <p>Alternative method</p> $\text{Required Probability} = \frac{129}{1000} \frac{871}{999} \frac{870}{998} \times 3$ $= 0.294$

7 (i)	 <table border="1" data-bbox="263 224 1077 739"> <caption>Data points from the scatter diagram</caption> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr><td>35</td><td>2.2</td></tr> <tr><td>45</td><td>1.8</td></tr> <tr><td>50</td><td>2.2</td></tr> <tr><td>65</td><td>4.2</td></tr> <tr><td>72</td><td>8.5</td></tr> <tr><td>75</td><td>7.5</td></tr> </tbody> </table>	x	y	35	2.2	45	1.8	50	2.2	65	4.2	72	8.5	75	7.5
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(ii)	$r = 0.914099 \approx 0.914$ (to 3 s.f) Though the value of r shows a strong positive linear correlation, from the scatter diagram, it is possible that x and y may have a curvilinear relationship.														
(iii)	For $y = c + dx^2$, $r = 0.93986 \approx 0.940$ Since the value of r for $y = c + dx^2$ is closer to the value of 1, $y = c + dx^2$ is a better model.														
(iv)	$y = -0.88934 + 0.0015441x^2$ When $x = 55$, $y = -0.88934 + 0.0015441(55)^2$ $y = 3.7816 \approx 3.8$ (to 1 d.p)														
	Since $x = 55$ is within the range of data given and $r \approx 0.940$ is close to 1, the estimation is reliable.														

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8 (i)	$P(\text{first person that uses Voyager is the third person selected})$ $= 0.92 \times 0.92 \times 0.08$ $= 0.067712$																																								
(ii)	<ol style="list-style-type: none">Whether a person uses Voyager is independent of another person.The probability that a person uses Voyager is constant for every person in the sample.																																								
(iii)	<p>Let Y be the number of people who use Voyager out of 80 people.</p> $Y \sim B(80, 0.08)$ <p>Since $n = 80 > 50$, $np = 6.4 > 5$, $nq = 73.6 > 5$,</p> $Y \sim N(6.4, 5.888) \text{ approx}$ $P(Y < 10) \xrightarrow{c.c.} P(Y \leq 9.5)$ $= 0.899295$ $= 0.899 \text{ (to 3 s.f.)}$																																								
(iv)	<p>Let V be the number of people who use Voyager out of n people.</p> $V \sim B(n, 0.08)$ $P(V \geq 10) > 0.2$ $1 - P(V \leq 9) > 0.2$ $P(V \leq 9) < 0.8$ <p>Using GC,</p> <table><tr><th colspan="5">NORMAL FLOAT AUTO REAL RADIAN MP</th></tr><tr><th colspan="5">PRESS + FOR Δ to 1</th></tr><tr><th>X</th><th>Y1</th><th></th><th></th><th></th></tr><tr><td>90</td><td>.81786</td><td></td><td></td><td></td></tr><tr><td>91</td><td>.80902</td><td></td><td></td><td></td></tr><tr><td>92</td><td>.79999</td><td></td><td></td><td></td></tr><tr><td>93</td><td>.79078</td><td></td><td></td><td></td></tr><tr><td>94</td><td>.7814</td><td></td><td></td><td></td></tr></table> <p>Least value of $n = 92$</p>	NORMAL FLOAT AUTO REAL RADIAN MP					PRESS + FOR Δ to 1					X	Y1				90	.81786				91	.80902				92	.79999				93	.79078				94	.7814			
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9 (i)	<p>Let C be the number of boxes of chocolate biscuits sold in a day.</p> $C \sim \text{Po}(2.2)$ $P(C = 0) = 0.11080$ $= 0.111 \text{ (to 3 s.f.)}$
(ii)	<p>Let D be the number of days that no boxes of chocolate biscuits were sold out of 7 days.</p> $D \sim B(7, 0.11080)$ $E(D) = 7 \times 0.11080$ $= 0.77562$ $= 0.776$
(iii)	<p>Let S be the number of boxes of strawberry biscuits sold in a day.</p> $S \sim \text{Po}(\lambda)$ $P(S < 2) = 0.6$ $P(S = 0) + P(S = 1) = 0.6$ $e^{-\lambda} \left(\frac{\lambda^0}{0!} \right) + e^{-\lambda} \left(\frac{\lambda^1}{1!} \right) = 0.6$ $e^{-\lambda} (1 + \lambda) = 0.6$
	<p>Using GC,</p> <div data-bbox="247 1323 885 1563">  </div> $\lambda = 1.376$ $= 1.4 \text{ (to 1 d.p.)}$

(iv)	<p>Let T be the total number of boxes of chocolate and strawberry biscuits sold in 7 days.</p> $T \sim \text{Po}(7 \times 2.2 + 7 \times 1.376) = \text{Po}(25.032)$ $P(T > 25) = 1 - P(T \leq 25)$ $= 0.44962$ $= 0.450 \text{ (to 3 s.f.)}$
(v)	<p>Let X be number of boxes of chocolate biscuits sold in 30 days.</p> $X \sim \text{Po}(30 \times 2.2) = \text{Po}(66)$ <p>Since $\lambda = 66 > 10$, $X \sim N(66, 66)$ approx</p> <p>Let Y be number of boxes of strawberry biscuits sold in 30 days.</p> $Y \sim \text{Po}(30 \times 1.376) = \text{Po}(41.28)$ <p>Since $\lambda = 41.28 > 10$, $Y \sim N(41.28, 41.28)$ approx</p> $X - Y \sim N(24.72, 107.28) \text{ approx}$ $P(X - Y > 0) \xrightarrow{c.c.} P(X - Y > 0.5)$ $= 0.99032$ $= 0.990 \text{ (to 3 s.f.)}$

<p>10 (i)</p>	<p>Choose a plant randomly from the first 10 plants, say the 5th plant.</p> <p>Choose every 10th plant thereafter until 8 plants are selected i.e. 5th, 15th, 25th, ...</p> <p>The 8 plants selected will be <i>evenly spread out</i> across the row of 80 plants.</p>
<p>(ii)</p>	<p>Unbiased estimate of the population mean, $\hat{\mu}$</p> $= \frac{\sum (x - 150)}{80} + 150$ $= -\frac{160}{80} + 150$ $= 148$ <p>Unbiased estimate of the population variance, s^2</p> $= \frac{1}{80-1} \left[\sum (x-150)^2 - \frac{(\sum (x-150))^2}{80} \right]$ $= \frac{1}{79} \left[5520 - \frac{(-160)^2}{80} \right]$ $= \frac{5200}{79}$
<p>(iii)</p>	<p>$H_0 : \mu = 150$ $H_1 : \mu < 150$</p> <p>Under H_0, since $n = 80 > 50$, by Central Limit Theorem,</p> $\bar{X} \sim N\left(150, \frac{5200}{79(80)}\right) \text{ approx.}$ <p>Test statistic $Z = \frac{\bar{X} - 150}{\sqrt{\frac{5200}{79(80)}}} \sim N(0,1) \text{ approx.}$</p> <p>From GC, $p\text{-value} = 0.013731$ $= 0.0137 \text{ (to 3 s.f.)}$</p> <p>$\alpha = 0.01$</p> <p>Since $p\text{-value} = 0.0137 > \alpha = 0.01$, we do not reject H_0 at 1% level of significance and conclude that there is insufficient evidence that the researcher's claim is invalid.</p>

(iv)	It means that there is a probability of 0.01 of concluding that the population mean mass of a new variety of potato is less than 150g given that the population mean mass of a new variety of potato is in fact 150g.
	<p>Unbiased estimate of the population variance $= \frac{8}{7}k^2$</p> <p>$H_0 : \mu = 150$ $H_1 : \mu < 150$</p> <p>Under H_0, test statistic $T = \frac{\bar{X} - 150}{\sqrt{\frac{S^2}{8}}} \sim t(7)$</p> <p>$\alpha = 0.01$</p> <p>Researcher's claim is invalid at 1% level of significance</p> <p>$\Rightarrow H_0$ is rejected at 1% level of significance</p> <p>$\Rightarrow t \leq -2.9980$</p> <p>$\Rightarrow \frac{148.5 - 150}{\sqrt{\frac{k^2}{7}}} \leq -2.9980$</p> <p>$\Rightarrow k \leq 1.3238$</p> <p>$\therefore k \leq 1.32$ (to 3 s.f)</p>