Candidate's Name : ____________________________ CT Group : 14S_____

HWA CHONG INSTITUTION

H3 ESSENTIALS OF MODERN PHYSICS 9811

Preliminary Examinations

C2 | 18 Sep 2015 | Duration: 3 hours

INSTRUCTIONS TO CANDIDATES

Do Not Open This Booklet Until You Are Told To Do So.

Write your name and CT class clearly on the top of this cover page and all answer sheets which you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams, graphs or rough working.

Section A
Answer **all** questions.
You are advised to spend about 1 hr 50 min on Section A.

Section B
Answer any **two** questions.
You are advised to spend about 70 minutes on Section B.

Write your answers on the foolscap paper provided.

Begin each answer on a fresh sheet of paper.

Submit both question paper and answer scripts.

SECTION A	
Q 1	**/ 5**
Q 2	**/ 12**
Q 3	**/ 18**
Q 4	**/ 12**
Q 5	**/ 13**
SECTION B	
Q 6 / 7 / 8 / 9 *	**/ 20**
Q 6 / 7 / 8 / 9 *	**/ 20**
DEDUCTIONS	
Total	**/ 100**

* delete question number as appropriate

Write the calculator brand and model in the box on the right.
If you are using a graphic calculator, you are required to clear the memory of your graphic calculator.

Calculator Model	

This paper consists of **13** printed pages.

Data

speed of light in free space	c	=	3.00×10^{8} m s^{-1}
permeability of free space	μ_0	=	$4\pi \times 10^{-7}$ H m^{-1}
permittivitty of free space	ε_0	=	8.85×10^{-12} F m^{-1}
		=	$(1/(36\pi)) \times 10^{-9}$ F m^{-1}
elementary charge	e	=	1.60×10^{-19} C
the Planck constant	h	=	6.63×10^{-34} J s
unified atomic mass constant	u	=	1.66×10^{-27} kg
rest mass of electron	m_e	=	9.11×10^{-31} kg
rest mass of proton	m_p	=	1.67×10^{-27} kg
molar gas constant	R	=	8.31 J K^{-1} mol^{-1}
the Avogadro constant	N_A	=	6.02×10^{23} mol^{-1}
the Boltzmann constant	k	=	1.38×10^{-23} J K^{-1}
gravitational constant	G	=	6.67×10^{-11} N m^2 kg^{-2}
acceleration of free fall	g	=	9.81 m s^{-2}

Formulae

Lorentz factor	γ	=	$(1 - (v/c)^2)^{-1/2}$
length contraction	L	=	L_0 / γ
time dilation	T	=	γT_0
Lorentz transformation equations (1 dimension)	x'	=	$\gamma(x - vt)$
	t'	=	$\gamma(t - vx/c^2)$
mass-energy equivalence	E	=	$\gamma m_0 c^2$
	E^2	=	$(pc)^2 + (m_0 c^2)^2$
Wien's displacement law	$\lambda_p T$	=	2.898×10^{-3} m K
Compton shift formula	$\Delta\lambda$	=	$\frac{h}{mc}(1 - \cos\theta)$
population distribution of atoms with energy E_x	N_x	=	$N_0 \exp(-(E_x - E_0) / kT)$
time-independent Schrödinger equation	$E\psi$	=	$-\frac{h^2}{2m}\left(\frac{d^2\psi}{dx^2}\right) + U\psi$
allowed energy states for a particle in a box	E_n	=	$(n^2h^2)/(8mL^2)$
normalized wave function for a particle in a box	ψ	=	$(2/L)^{1/2} \sin(n\pi x/L)$
transmission coefficient	T	α	$\exp(-2kd)$ where $k = \sqrt{\frac{8\pi^2 m(U-E)}{h^2}}$
Drude model of electrical resistivity	ρ	=	$\frac{2m\langle v\rangle}{nq^2\lambda}$
Fermi energy for metals	E_F	=	$\frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3}$
density of energy states for electrons in a metal	$\rho(E)$	=	$\frac{4\pi(2m)^{3/2}}{h^3}\sqrt{E}$
Fermi function	$f(E)$	=	$1/(1 + \exp((E-E_F)/kT))$
refractive index	n	=	v_1/v_2
phase difference of circularly polarised light	$\frac{\delta}{2\pi}$	=	$\frac{d}{\lambda}\Delta n$
Brewster's angle	$\tan\theta_B$	=	n_2/n_1
attenuation of light intensity	I	=	$I_0 \exp(-\mu x)$

Section A

Answer **all** questions in this Section.
You are advised to spend about 1 hour and 50 minutes on this section.

1 A prism of refractive index n_2 separates media of different refractive indices n_1 and n_3. Light propagates in a plane containing the apex angle Φ of the prism as shown in Fig. 1.

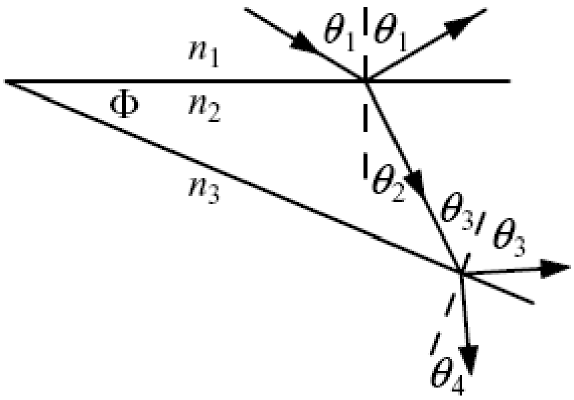


Fig. 1

(i) If light strikes the top surface of the prism at Brewster's angle, show that

$$\tan\theta_2 = \frac{n_1}{n_2}.$$ [2]

(ii) Determine the apex angle Φ, in terms n_1, n_2, and n_3, for which light can fall on both the surfaces at Brewster's angles as it passes through the prism. [3]

2 **(a)** Define proper length. [2]

(b) Using the Lorentz transformation equation $x' = \gamma(x - vt)$, show that the length of an object L measured by an observer O moving at speed v relative to the object is related to the proper length L_0 of the object by $L_0 = \gamma L$. [3]

(c) Two satellites flies past a planet on its equatorial plane at constant speeds where satellite A is moving at twice the speed of satellite B. Given that the ratio of the diameter of the planet as measured by satellite B to that measured by satellite A is given by k, determine the ratio of the diameter of the planet as measured by satellite A to its proper length. [4]

(d) Suggest and explain the difference in the cross-sectional area of the planet as observed by the two satellites. [3]

3 **(a)** **(i)** Discuss qualitatively the failure of the classical theory to explain the radiation from a blackbody at high frequencies. [2]

(ii) Describe qualitatively how the discrepancy between the classical theory and experimental observation was resolved. [2]

(b) **(i)** Sketch a line in a well-labelled graph to show how the intensity of the radiation emitted by an ideal blackbody varies its wavelength. Assume this blackbody has a temperature of 3000 K and label the wavelength axis with appropriate values. [2]

(ii) The temperature of the blackbody in (b)(i) is increased to 6000 K. Sketch another line in the same graph (b)(i) to show how the intensity of the radiation emitted by this ideal blackbody varies with its wavelength. [2]

(c) Describe how the Wien displacement law can be derived from Planck radiation law (you do not have to carry out the actual derivation). [2]

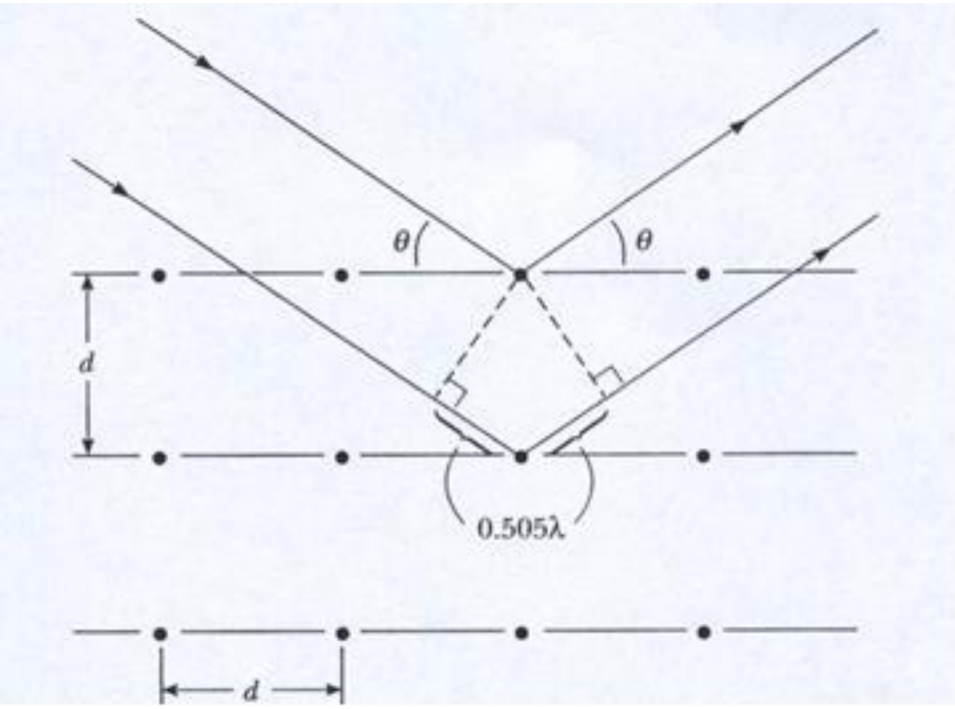
Wien displacement law : $\lambda_{\text{max}} T = 2.898 \times 10^{-3}$ mK

Planck radiation law : $I(\lambda) = \dfrac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)}$

(d) **(i)** Explain what is meant by the wave function Ψ (x,t) of a particle. [2]

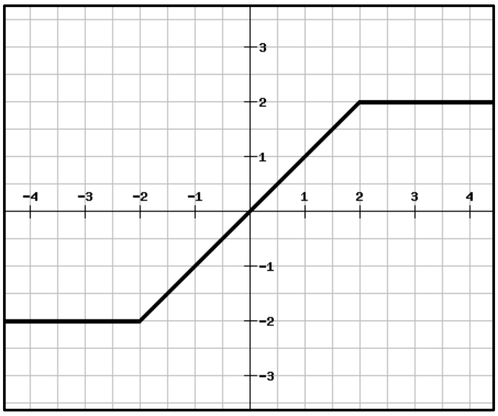
(ii) Give a simple mathematical form of the wave function of a free particle. State the two physical quantities that can be precisely calculated for such a free particle. [3]

(iii) The figure below shows the top three atomic planes of a crystal with its spacing between consecutive planes d . High-energy electrons with de Broglie wavelength λ can penetrate many planes deep into the crystal. A scattered wave from Nth atomic plane inside the crystal can interfere destructively with the scattered waves from the surface of crystal (1st atomic plane). With the aid of the information in the figure below, determine the value of N . [3]

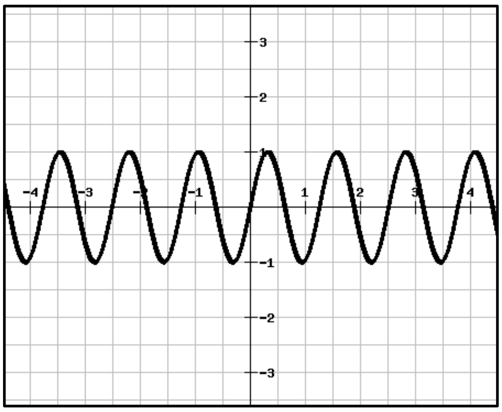


4 (a) The graphs of several functions are shown in Figs. 4.1 to 4.4. For each graph, state whether it can be an acceptable wave function in quantum mechanics. If not, include an explanation of why the function shown is not acceptable.

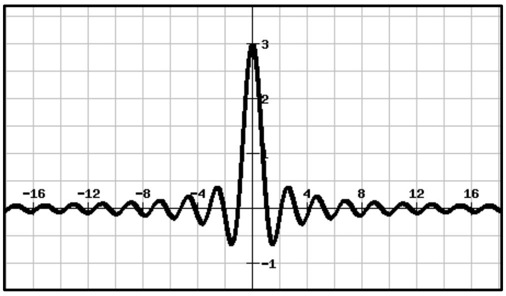
(i) [1]



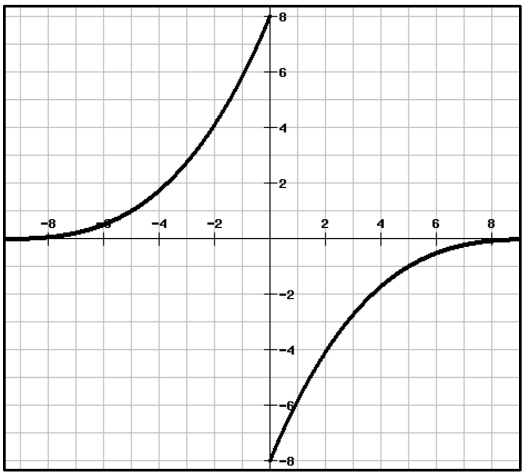
(ii) [1]



(iii) [1]

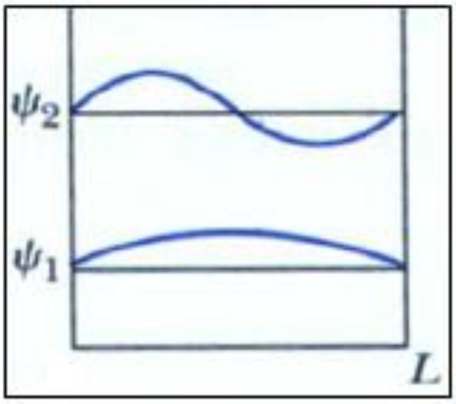


(iv) [1]



(b) **(i)** Use the properties of the wave function to explain why a particle in a box can only have certain energies, but not others. [2]

(ii) The wave functions for the particle in a box in two different states (n = 1 and n = 2) are sketched in Fig. 4.5. The width of the box is L.



1. State for which of the two values of n the particle has the most energy. [1]

2. State, with a reason, whether you are more likely to find a particle at x = ½ L for n = 1 or for n = 2. [2]

3. Sketch the probability distribution for a particle in a box for n = 4. [1]

(iii) State whether an atomic particle can be at rest at the centre of a microscopic box.
If so, describe a procedure by which you could arrange this. [2]
If not, provide a reason for why this cannot be the case.

5 An intrinsic silicon semiconductor has a band gap energy E_g of 1.1 eV.

(a) **(i)** Using your knowledge of an energy band diagram,

1. sketch two dotted lines and label them T and C, representing the top and bottom of its conduction band respectively,
2. sketch two dotted lines and label them V and B, representing the top and bottom of its valence band respectively,
3. draw an axis and label it E, representing the direction of increasing energy level for mobile electrons. [1]

(ii) Sketch in the same diagram in (a)(i) and label P the line representing the Fermi distribution at room temperature. Indicate with arrows and values in your sketch the two parts of your line which corresponds to 50 % and 100 % probabilities of occupancy by mobile electrons. [1]

(iii) Sketch in the same diagram in (a)(i) and label F the line representing the Fermi level. Indicate in your sketch the difference in energy in eV between the bottom of the conduction band and the Fermi level. [1]

(b) The mentioned semiconductor is doped and is transformed into a n-type semiconductor.

(i) Sketch in the same diagram in (a)(i) and label Q the line representing the Fermi distribution at room temperature. Indicate with arrows and values in your sketch the two parts of your line which corresponds to 50 % and 100 % probabilities of occupancy by mobile electrons. [1]

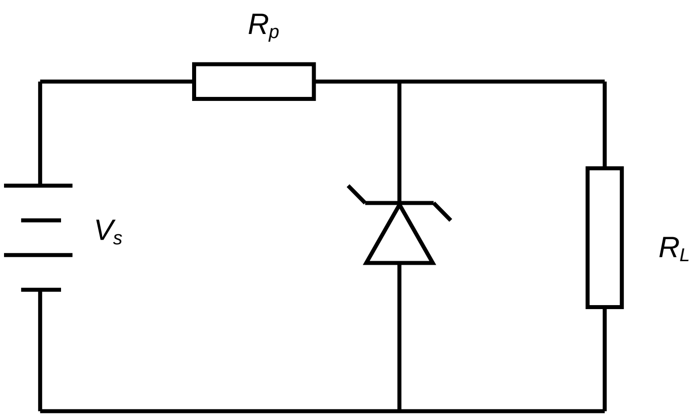
(ii) Sketch in the same diagram in (a)(i) and label G the line representing the Fermi level. Hint: use the knowledge of the position of the highest occupied energy level by mobile electrons when this n-type semiconductor is at zero Kelvin. [1]

(iii) Assume the mobile electrons in n-type semiconductor behave like ideal gas molecules.

1. Calculate the average kinetic energy in eV of a mobile electron at 23 ^{o}C . [1]

2. Estimate the energy gap in eV between the bottom of the conduction band and the Fermi level for this n-type semiconductor at 23 ^{o}C .And explain briefly how you arrive at your own deduction in relation to the donor atoms. [1]

(c) An electrical circuit is setup as shown below.



The supply voltage V_s is fluctuating with a voltage of 11.0 ± 1.0 V. The zener diode has a breakdown voltage of 6.0 V. An electrical load which has a fixed resistance R_L of 100 Ω , should be maintained at 6.0 V at all times and is connected in parallel to the zener diode.

Determine the minimum power rating of the zener diode. [6]

Section B

Answer **two** questions from this Section.
You are advised to spend about 35 minutes on each question.

6 **(a)** **(i)** What do you understand by 'rest-mass energy'? [2]

(ii) What do you understand by 'total energy of a free particle in motion'? [2]

(b) **(i)** A proton and an antiproton, both of which have the same mass of 938 MeV/c^2, are both traveling at a speed of 0.981c towards each other. They collide to form a new particle. Calculate the mass of this new particle in units of MeV/c^2. [3]

(ii) Alternatively, we can also produce the same particle by firing a proton at a stationary antiproton. Show that in this situation, the proton will need to be moving at a speed of $0.999816c$. [6]

(iii) Calculate the ratio of the total kinetic energies that needs to be provided to the protons and antiprotons for the second method to that of the first method. [4]

(iv) Observer O performed the experiment via method described in b(i). At the same time, Observer O' who is flying past the laboratory at the same speed as the anti-proton observes the experiment being done via the method described in b(ii). Using the equation $v' = \frac{v-u}{1-\frac{uv}{c^2}}$, calculate the speed of the proton as viewed by observer O'. Hence deduce whether kinetic energy is an invariant quantity or not. [3]

7 **(a)** A free electron has a wave function of

$$\psi(x) = A\sin\left(5\times10^{10}x\right)$$

where x is measured in meters.

Find the electron's

(i) de Broglie wavelength, [1]

(ii) momentum and [1]

(iii) energy (in electron volt). [1]

(b) **(i)** Explain how the Compton Effect provides an evidence for a particulate nature of electromagnetic radiation. [3]

(ii) In a Compton shift experiment, an x-ray photon collided with a stationary electron. The photon recoils back in the direction it came from after the collision. By taking into consideration of the conservation of energy and momentum, deduce the change of the wavelength of the photon in this process. [4]

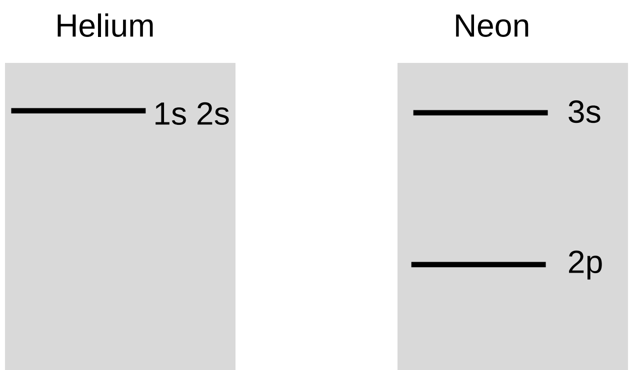
(c) Population inversion and metastable level are two important terms in the lasing process.
Explain what is meant by

(i) population inversion [2]

(ii) metastable level [2]

(d) A hypothetical atom has two energy states, with a transition wavelength of 580 nm between them. In a particular sample at 300 K, 4.0 x 10^{20} such atoms are in the lower energy state. How many atoms are in the upper energy state, assuming they are in thermal equilibrium with the surroundings? [2]

(e) Explain, with the aid of the energy levels diagram below, how the helium-neon laser achieves *population inversion*. Identify the *metastable levels* in the diagram.



[4]

8 (a) An electron is trapped in a one-dimensional box, with sides of 5.0 x 10^{-10} m (about the diameter of an atom).
Calculate the energy required, in electron volt, to excite the electron from the first to the second energy level. [3]

(b) (i) The time-independent Schrödinger equation for the wave function ψ of a particle of mass m can be written as

$$E\psi = -\frac{\hbar^2}{2m}\left(\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}\right) + U\psi \text{ .}$$

Show that the expression

$$\psi = Ax + B \text{ ,}$$

where A and B are constants, is a solution of the time-independent Schrödinger equation for the (E = 0) energy level of a particle in a box. [2]

(ii) 1. The particle is confined to the region 0 ≤ x ≤ L.

Explain what constraints the boundary conditions at x = 0 and x = L place on the constants A and B. [2]

2. Comment on the result obtained in part **1.** [1]

(c) A potential well is a potential-energy function $U(x)$ that has a minimum. Parts **(a)** and **(b)** dealt with the particle in a box, which is a rudimentary potential well with a function $U(x)$ that is zero within a certain interval and infinite everywhere else. A better approximation to several actual physical situations is a finite well, which is a potential well with straight sides but *finite* height. Fig. 8.1 shows a potential-energy function that is zero in the interval $0 \le x \le L$ and has the value U_0 outside this interval. This function is often called a square-well potential.

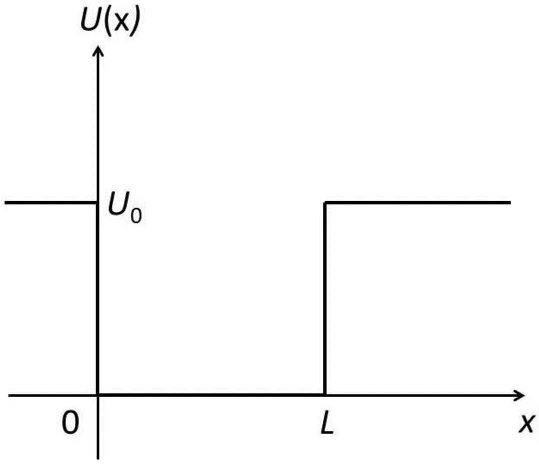


Figure 8.1

(i) Show that the expression

$$\psi = C\exp(\kappa x) + D\exp(-\kappa x),$$

where $\kappa = [2m(U_0 - E)]^{1/2}/\hbar$ is a positive number, is a solution of the time-independent Schrödinger equation outside a finite well of height U_0.
(Reminder: $\frac{\mathrm{d}[\exp(au)]}{\mathrm{d}u} = a\exp(au)$ and $\frac{\mathrm{d}^2[\exp(au)]}{du^2} = a^2\exp(au)$.) [3]

(ii) Given that $D = 0$ for $x < 0$ and $C = 0$ for $x > L$, determine what will happen to $\psi(x)$ in the limit $U_0 \to \infty$. [3]

(d) We will now compare the finite-depth potential well with the infinitely deep well. First, because the wave functions for the finite well do not go to zero at $x = 0$ and $x = L$, the wavelength of the sinusoidal part of each wave function is *longer* than it would be with an infinite well.
Second, a well with finite depth U_0 has only a *finite* number of bound states and corresponding energy levels, compared to the *infinite* number for an infinitely deep well. The number of levels there are depends on the magnitude of U_0 in comparison with the ground-level energy for the infinitely deep well.

(i) State, with a reason, whether the kinetic energy for a given level (n = 1, 2, 3, …) in a finite well is larger, smaller or the same as the kinetic energy for the equivalent level in an infinitely deep well. [2]

(ii) In the Fig. 8.2, sketch the shape of the wave functions for the three lowest bound states for a particle in a finite potential well. [2]

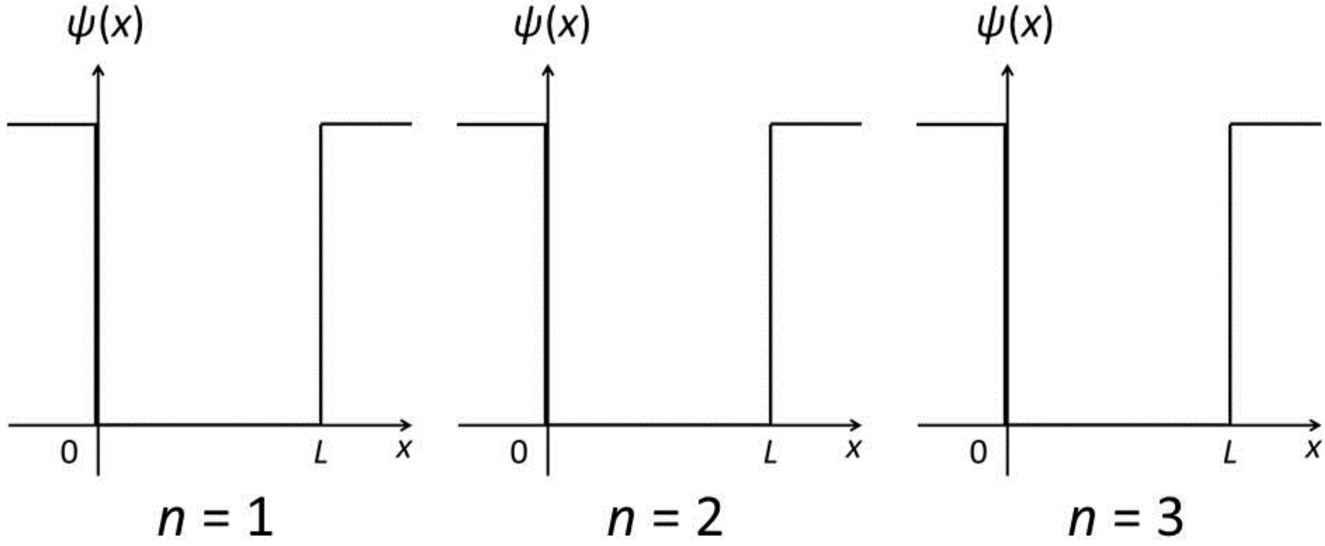


Fig. 8.2

(iii) In the Fig. 8.3, sketch the shape of the probability-distribution functions for the square-well wave functions in part **(ii)**. [2]

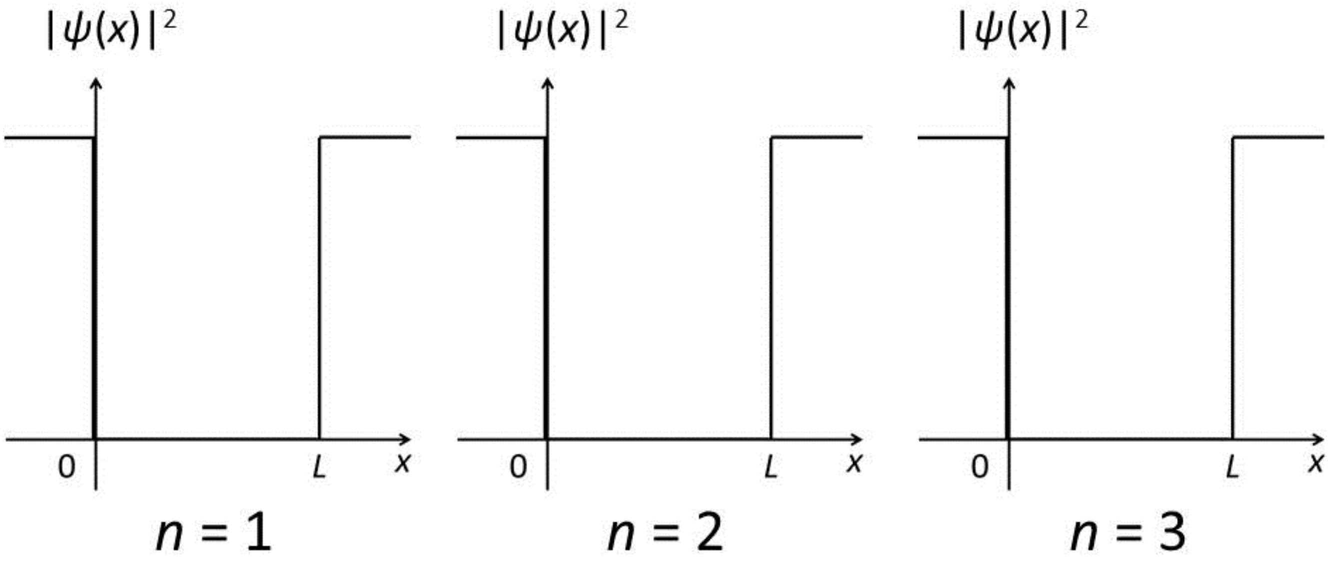


Fig. 8.3

9 **(a)** The resistivity ρ of a zinc metal varies experimentally with its temperature T and obeys the following equation;

$$\rho = \rho_0[1 + \alpha(T - T_0)],$$

where ρ_0 is the resistivity of zinc at a temperature T_o ,
α is the temperature coefficient of zinc.

The followings are the information related to the zinc metal:

molar mass of zinc = 65.37 g mol^{-1}
density of zinc = 7.133 g cm^{-3}
resistivity of zinc at 20°C = 5.920 x 10^{-8} Ω m
temperature coefficient of zinc = 3.70 x 10^{-3} °C^{-1}

(i) If a potential difference V is applied across a metal wire of length L and cross-sectional area A, show that an alternative representation of current density (current per unit perpendicular cross-sectional) is given by

$$J = V / (\rho L)$$

where ρ is the resistivity of the metal. [2]

(ii) A potential difference of 0.0320 V is applied across the ends of a zinc wire with a length of 10.0 cm. The temperature of the wire is 55.0 °C. Assume each zinc atom contributes two free electrons for electrical conduction, calculate the resistivity of this wire at 55.0 °C. [2]

(iii) Calculate the drift velocity of the free electrons in this wire at 55.0 °C. [3]

(b) The Drude's free-electron model is one where each atom in a metal loses its valency electrons and becomes a positively charged ion. These positively charged ions form a fairly rigid lattice. The free electrons lost by the ions are free to wander within the metal. The free electrons are treated like ideal gas molecules confined in a container.

(i) What are the two assumptions for this model? [2]

(ii) Use the Drude model to show that the drift velocity v of a free electron in a cylindrical wire along which an electric field E has been applied is given by

$$v = \frac{Ee\lambda}{\sqrt{12kTm}},$$

where e is the elementary charge,
λ is the mean free path of the free electrons,
k is the Boltzmann constant,
T is the thermodynamic temperature of the wire and
m is the mass of electron. [5]

(iii) Describe the variations of the resistivity of a metal in (a) and (b) with its temperature (i.e. how the resistivity of a metal changes with the change in its temperature). Suggest a reason if there is a difference between the two mentioned variations. [3]

(c) Aluminium metal, which is trivalent, has a density of 2700 kg m^{-3}. One mole of aluminium has a mass of 0.027 kg. Calculate the Fermi energy of aluminium at a temperature of absolute zero. Leave your answer in electron-volt (eV). [3]

End of Paper