PDG

Candidate Name ______________________________ (   )

ANDERSON JUNIOR COLLEGE

2014 JC2 Preliminary Examination

ESSENTIALS OF MODERN PHYSICS — 9811/01

Higher 3

Paper 1 — **Thursday 18 September 2014**
3 hours

Additional Materials: Writing Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name and PDG on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, highlighters, glue or correction fluid.

The use of an approved scientific calculator is expected where appropriate.

Section A
Answer **all** questions.
You are advised to spend about 1 hour 50 minutes on Section A.

Section B
Answer any **two** questions.
You are advised to spend about 35 minutes on each question in Section B.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use	
Section A (60 marks)	
1	
2	
3	
4	
5	
Section B (40 marks)	
Significant Figure	
Total (100 marks)	

This document consists of 16 printed pages.

Data

speed of light in free space	$c = 3.00 \times 10^{8}$ m s^{-1}
permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}
	$(1/(36\pi)) \times 10^{-9}$ F m^{-1}
elementary charge	$e = 1.60 \times 10^{-19}$ C
the Planck constant	$h = 6.63 \times 10^{-34}$ J s
unified atomic mass constant	$u = 1.66 \times 10^{-27}$ kg
rest mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
rest mass of proton	$m_p = 1.67 \times 10^{-27}$ kg
molar gas constant	$R = 8.31$ J K^{-1} mol^{-1}
the Avogadro constant	$N_A = 6.02 \times 10^{23}$ mol^{-1}
the Boltzmann constant	$k = 1.38 \times 10^{-23}$ J K^{-1}
gravitational constant	$G = 6.67 \times 10^{-11}$ N m^{2} kg^{-2}
acceleration of free fall	$g = 9.81$ m s^{-2}

Formulae

Lorentz factor	γ	=	$(1-(v/c)^2)^{-1/2}$
length contraction	L	=	L_o/γ
time dilation	T	=	γT_o
Lorentz transformation equations (1 dimension)	x'	=	$\gamma(x-vt)$
	t'	=	$\gamma(t-vx/c^2)$
mass-energy equivalence	E	=	$\gamma m_o c^2$
		=	$\sqrt{(pc)^2+(m_o c^2)^2}$
Wien's displacement law	$\lambda_p T$	=	2.898×10^{-3} m K
Compton shift formula	$\Delta\lambda$	=	$\frac{h}{mc}(1-\cos\theta)$
population distribution of atoms with energy E_x	N_x	=	$N_o\exp(-(E_x-E_o)/kT)$
time-independent Schrödinger equation	$E\Psi$	=	$-\frac{\hbar^2}{2m}\left(\frac{d^2\Psi}{dx^2}\right)+U\Psi$
allowed energy states for a particle in a box	E_n	=	$(n^2h^2)/(8mL^2)$
normalised wave function for a particle in a box	Ψ	=	$(2/L)^{1/2}\sin(n\pi x/L)$
transmission coefficient	T	$\propto$	$\exp(-2kd)$
	where k	=	$\sqrt{\frac{8\pi^2 m(U-E)}{h^2}}$
Drude model of electrical resistivity	ρ	=	$\frac{2m_e<v>}{ne^2\lambda}$
Fermi energy for metals	E_F	=	$\frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3}$
density of energy states for electrons in a metal	$\rho(E)$	=	$\frac{4\pi(2m)^{3/2}}{h^3}\sqrt{E}$
Fermi function	$f(E)$	=	$\frac{1}{1+\exp((E-E_F)/kT)}$
refractive index	n	=	v_1/v_2
phase difference of circularly polarised light	$\frac{\delta}{2\pi}$	=	$\frac{d}{\lambda}\Delta n$
Brewster's angle	$\tan\theta_B$	=	n_2/n_1
attentuation of light intensity	I	=	$I_o\exp(-\mu x)$

Section A

Answer **all** the questions in this Section.

You are advised to spend about 1 hour and 50 minutes on this section.

1 **(a)** The rest mass of particles is often expressed in MeV/c^2 units, where c is the speed of light.

(i) Calculate the rest mass of the electron in MeV/c^2 units. [3]

(ii) Calculate the conversion factor from kg to MeV/c^2 . [2]

(b) A photon of momentum p and energy E makes a head-on collision with a stationary electron of mass m_e .

(i) Use relativistic mechanics to show that the magnitude of momentum p' of the photon after the interaction is given by

$$p' = \frac{m_e c p}{2p + m_e c}$$ [4]

(ii) In one such interaction, the incident photon has energy 1.25 MeV. Calculate the kinetic energy of the electron after the collision. [3]

2 **(a)** **(i)** Explain how signals are transmitted via a step-index multimode type optical fibre and how information is distorted at the receiving end due to modal dispersion. [3]

(ii) Besides modal dispersion, state one other problem that is commonly associated with optical fibre transmission. [1]

(iii) Explain why the continuous-index multimode type is preferred over the step-index type in long-distance signal transmissions. [2]

(iv) Fig. 2.1 below shows a step-index glass fibre (refractive index n_f) surrounded by a lower optical density cladding (refractive index n_c).

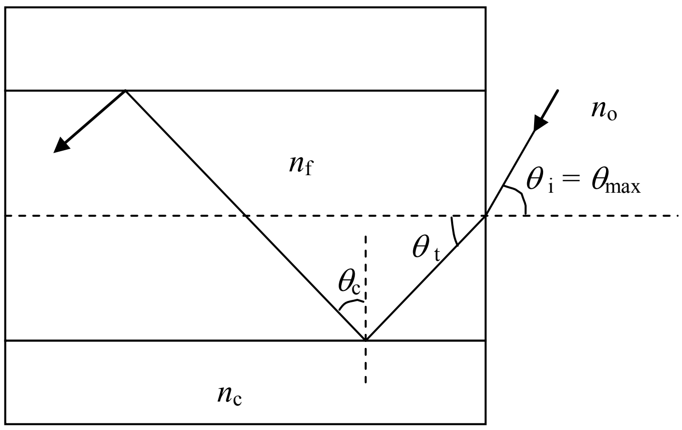

Fig. 2.1

There is a maximum incident angle $\theta_i = \theta_{max}$ such that any ray impinging on the face at $\theta_i > \theta_{max}$ will arrive at an internal wall at an angle less than θ_c, the critical angle, and will not be totally internally reflected. Show that

$$\sin\theta_{max} = \frac{\sqrt{n_f^{\,2} - n_c^{\,2}}}{n_0}$$

where n_o is the refractive index of air. [4]

(b) A portion of a straight glass rod of diameter d and refractive index n is bent into an arc of a circle of mean radius R and a parallel beam of light is shone into it as shown in Fig. 2.2 below.

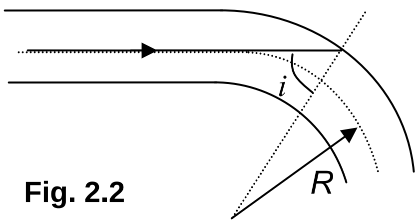

Fig. 2.2

(i) Derive an expression in terms of R and d for the angle of incidence of the central ray on reaching the glass-air interface at the circular arc. [1]

(ii) Show that the smallest value of R which will allow all the light to pass around the arc is given by

$$R = \frac{d}{2}\left(\frac{n+1}{n-1}\right)$$ [4]

3 **(a)** The time-independent Schrodinger equation for the wavefunction Ψ of a particle of mass *m* can be written as

$$E\Psi = -\frac{\hbar^2}{2m}\left(\frac{d^2\Psi}{dx^2}\right) + U\Psi \qquad \text{---- (1)}$$

In the case where *E* > *U*, the expression

$$\Psi = A\ exp\ (ik_1 x) + B\ exp\ (-ik_1 x) \qquad \text{---- (2)}$$

where *A* and *B* are constants and

$$k_1 = \sqrt{\frac{2m(E-U)}{\hbar^2}}$$

is a solution of equation (1).

(i) Distinguish between the energies *E* and *U* in this equation. [1]

(ii) A particle of energy *E*, initially in a region where its potential energy is *U*, approaches a potential energy step located at x = 0 from the left as shown in Fig. 3.1. At the step, the potential energy of the particle decreases from *U* to zero.

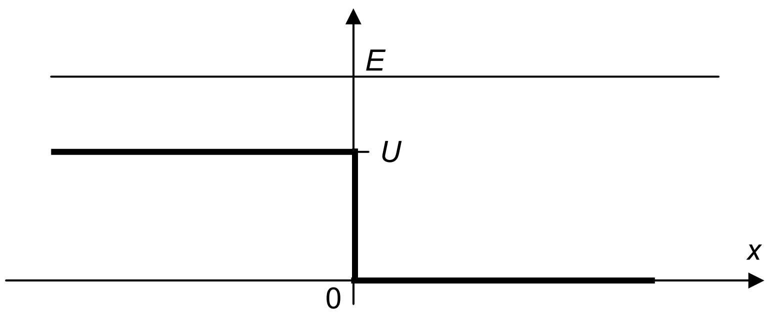

Fig. 3.1

The particle's wavefunction Ψ in the region before the step (x < 0), has the form given by the expression (2). There is a probability that the particle will be reflected and also a probability that it will be transmitted into the region on the right-hand side of the step.

It is suggested that the particle's wavefunction in the region on the right-hand side of the step is

$$\Psi = C\ exp\ (ik_2 x) \qquad \text{---- (3)}$$

where *C* is a constant and

$$k_2 = \sqrt{\frac{2mE}{\hbar^2}}\ .$$

1. Verify that this wavefunction is a solution of the Schrodinger equation.

(Reminder: $\frac{d\left[\,exp\,(ikx)\,\right]}{dx} = ik\ exp\,(ikx)$.
You will need to use the relation $i^2 = -1$.) [2]

2. Suggest a reason for the fact that there is only one term on the right side of the expression (3). [1]

(b) (i) A useful application of tunnelling is the Scanning Tunnelling Microscope (STM). Describe how quantum tunnelling applies to the operation of the STM. [3]

(ii) Fig. 3.2 shows the probe of a STM near the surface of a silicon chip. The corresponding energy diagram is shown in Fig. 3.3.
Copy Fig 3.3 onto your writing paper and sketch the wave function of the electron as it tunnels through the barrier.

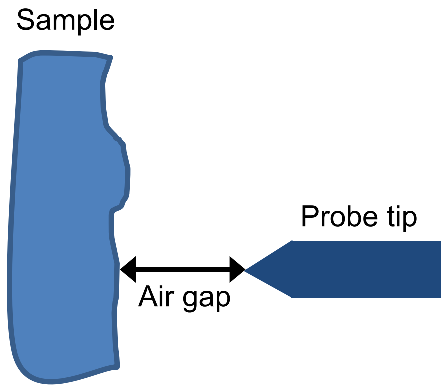

Fig. 3.2

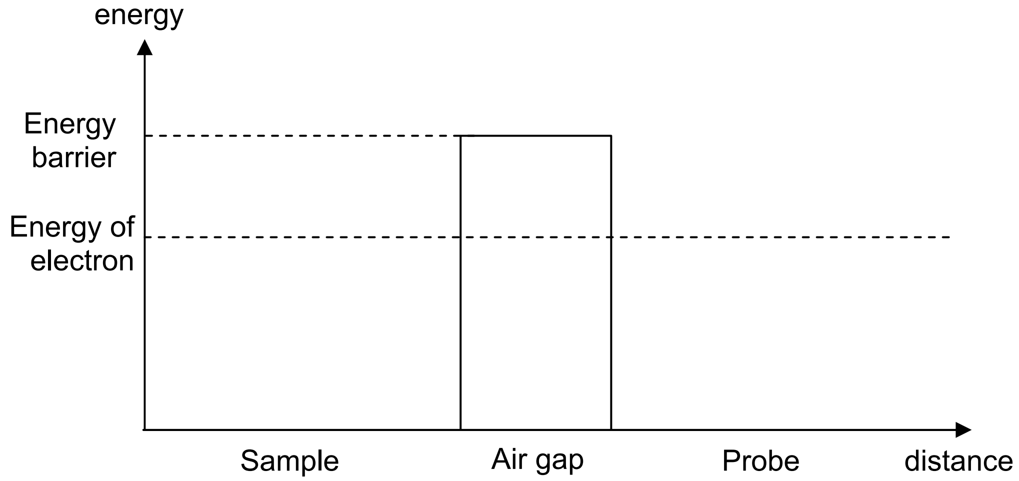

Fig. 3.3

[2]

4 Fig. 4.1 shows a rectangular slice of semiconductor PQRS carrying a current I. The current is due to the free electrons of charge e moving with a speed v. A uniform magnetic field of flux density B acts vertically downwards on the plane of the slice.

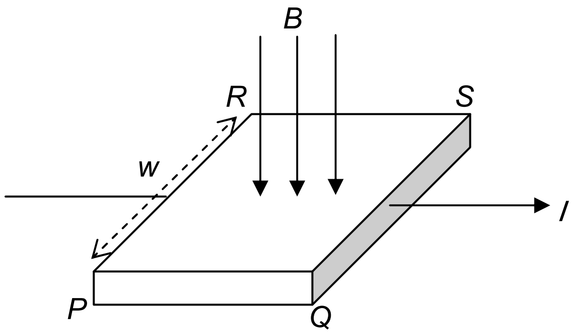

Fig. 4.1

(a) The Hall voltage produced set up an electric field. State the direction of the electric field. [1]

(b) Write down the expression for the force on an electron in the slice due to the Hall voltage V_H and show that

$$V_H = Bvw$$

where w is the width of the slice. [2]

(c) Describe and explain the effect on the Hall voltage if

(i) the current increases, [2]

(ii) the slice of semiconductor is replaced by a piece of copper having the same measurements and carrying the same current. [2]

5 **(a)** Electrons are accelerated from rest in an X-ray tube. X-rays with the minimum wavelength emerge from the tube, strike a target and are Compton-scattered through various angles.

(i) State why a classical picture of the interaction of an electromagnetic wave with an electron **cannot** explain the Compton shift effect. [2]

(ii) Describe how the concept of photons explains for the two peaks observed in the Compton shift effect. [3]

(b) The table in Fig. 5.1 shows experimental data obtained from an experiment exploring the Compton effect. The scattering angle used is 135°.

Wavelength (nm)	Relative Intensity
0.0705	0.333
0.0707	0.706
0.0709	1.06
0.0712	0.764
0.0714	0.423
0.0718	0.179
0.0723	0.107
0.0729	0.036
0.0735	0.153
0.0741	0.564
0.0747	1.115
0.0749	1.265
0.0752	0.846
0.0759	0.453
0.0765	0.264

Fig. 5.1

(i) Plot a graph of Relative Intensity versus Wavelength. [4]

(ii) The Compton shift equation is given by $\Delta\lambda = \lambda_1 - \lambda_o = \lambda_c(1-\cos\theta)$
Using your graph, calculate the following

1. the Compton shift, $\Delta\lambda$ [2]

2. the Compton wavelength, λ_c [2]

(c) For a given angle of scattering, if X-rays were scattered from protons instead of from electrons, explain the effect, if any, on the change in their wavelength. [2]

(d) Explain whether it is possible to detect a change in the wavelength of a visible light photon when the photon is scattered by an electron. [2]

Section B

Answer **two** questions from this Section.

You are advised to spend about 35 minutes on each question.

6 **(a)** In a futuristic planet, a robber escapes in his spaceship which goes at a speed of $\frac{3}{4}c$, the police chases him in their police space cruiser which is cruising at a speed of $\frac{1}{2}c$. The police fire a missile, which is propelled forward at a speed of $\frac{1}{3}c$ as observed by them, at the robber's spaceship.

(i) Calculate the velocity of the robber's spaceship with respect to the police's space cruiser.

(The formula for relativistic velocity addition is $u = \dfrac{u' + v}{1 + u'v/c^2}$. State clearly in your working what values are assigned to *u*, *u'* and *v*.) [3]

(ii) Does the missile reach its target

1. according to Galilean/Newtonian physics?

2. according to Special Relativity? [2]

(iii) The missile missed the robber's spaceship. The police then fires a powerful laser pulse at the robber's spaceship when it is 1.2×10^6 km away from them. Calculate the time *T* it takes the laser pulse to hit the robber's spaceship as observed by the police. [4]

(iv) As observed by the robber, how far is the police from him when the laser gun is fired? [2]

(v) Hence, calculate *T'*, the time taken for the laser pulse to reach the robber as measured by him. [2]

(vi) Explain briefly why it will be difficult for the robber to respond to the incoming laser pulse (such as changing the course of his spaceship when he detects that the pulse is fired). [1]

(b) A muon has a mass m when its speed is v as measured in the laboratory reference frame. The graph in Fig. 6.1 shows the variation with ratio $\frac{v}{c}$ of the mass m. The rest mass of the muon is m_0.

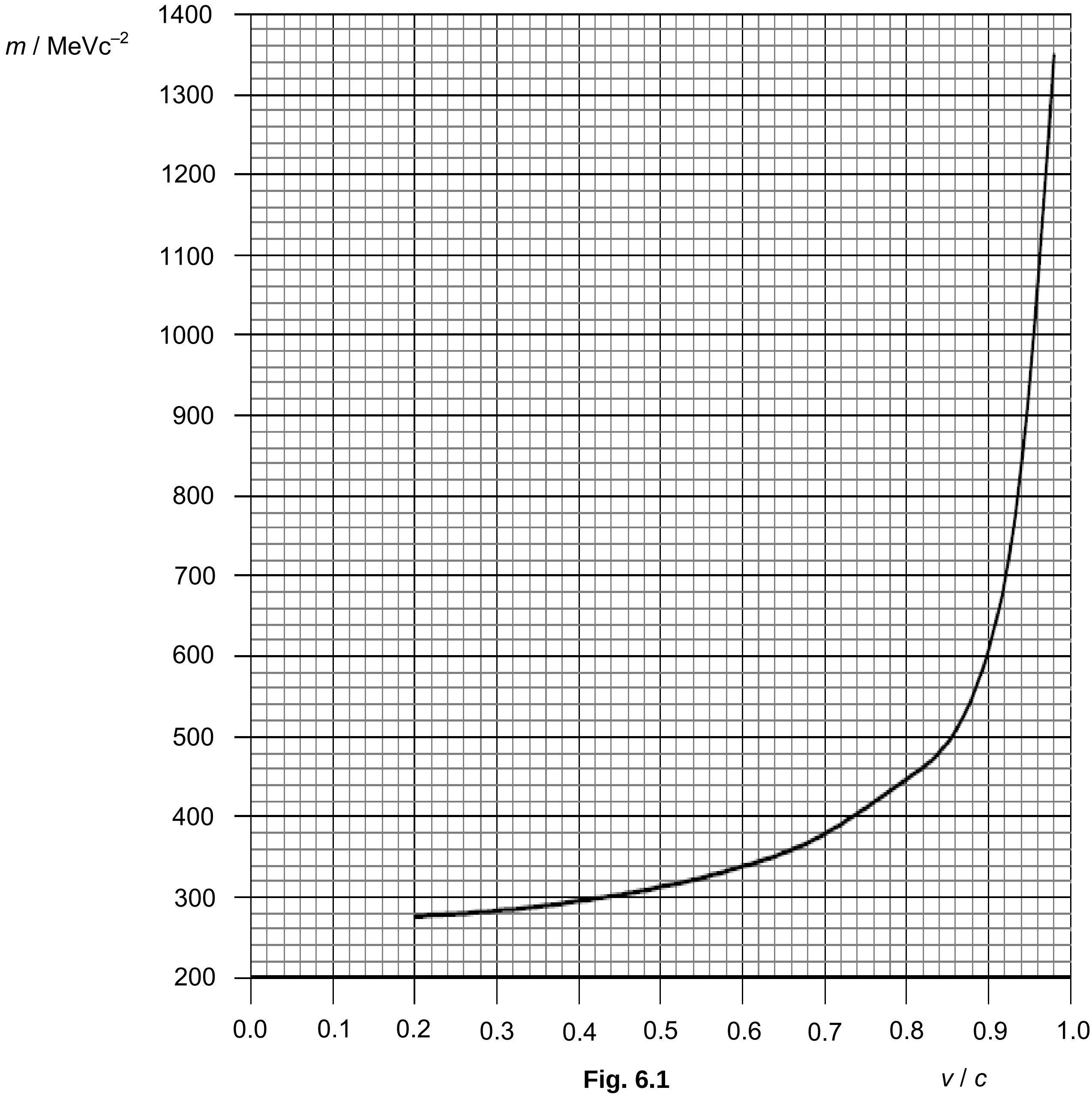

Fig. 6.1

(i) Write down an equation for the above curve. [1]

(ii) Use the graph in Fig. 6.1 to determine

1. the rest mass of muon. [1]

2. the mass of a muon when it is moving with speed of 0.95 c. [1]

(iii) State the total energy in MeV of a muon when it has a speed of 0.95 c. [1]

(iv) The charge on a muon is -1.6×10^{-19} C. Calculate the potential difference through which the muon must be accelerated in order to attain a speed of 0.95 c. [2]

7 **(a)** State what is meant by the *resolving power* of an optical instrument such as a microscope. [1]

(b) A laser printer puts tiny dots of ink on a page. The dots should be sufficiently close together such that we would see the letters or graphics instead of the individual dots. Determine the minimum number of dots per inch (dpi) to ensure that individual dots cannot be resolved when viewing a page 0.40m from the eye in bright light. You may take the diameter of a pupil as 2.5 mm and the range of wavelength for visible light as 400 nm to 700 nm. (1 inch is equal to 2.54 cm) [3]

(c) A beam of electrons is accelerated from rest through a potential difference V in an electron microscope.

(i) Show that the wavelength λ (measured in nm) of an electron in the beam is given by

$$\lambda = \frac{1.23}{\sqrt{V}}$$ [2]

(ii) Hence determine the accelerating voltage required for the electrons of an electron microscope if the microscope is to have the same resolving power as could be obtained using 100 keV gamma rays. [2]

(iii) Suggest an advantage of using electrons rather than photons of the same wavelength for the microscope. [1]

(d) Experiments using low energy electron diffraction give information about the arrangement of atoms very near the surface of a crystal. Fig. 7.1 illustrates the principle of such an experiment.

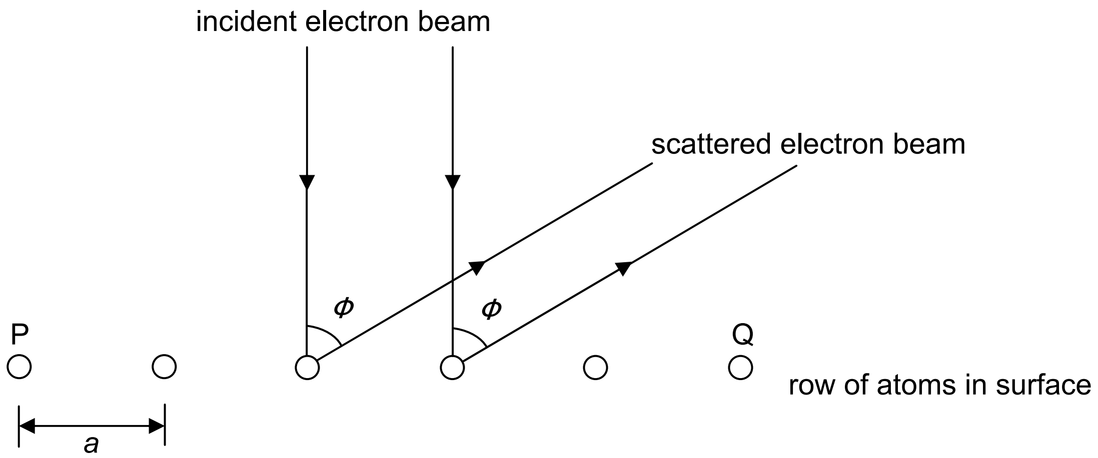

Fig. 7.1

An electron gun, in which electrons are accelerated from rest through a potential difference V, directs a beam of electrons normally towards the surface of the crystal. The wavelength associated with the electrons in the beam is λ. The beam is scattered by the atoms in the surface plane of the crystal. One row of atoms, of regular spacing a, is shown.

(i) Strong scattering of the electron beam is observed at an angle Φ to the normal to the surface. Derive, in terms of λ, a and Φ, the condition for this to take place. [3]

(ii) Describe and explain what happens to the direction of the strongly-scattered beam as the accelerating potential V is increased. [3]

(e) The electron beam in the experiment in **(d)** may penetrate the crystal so that it interacts with atoms below the surface. Fig. 7.2 illustrates a row of atoms RS in the first plane below the surface. Each atom is a distance *b* below the corresponding atom in the surface row PQ.

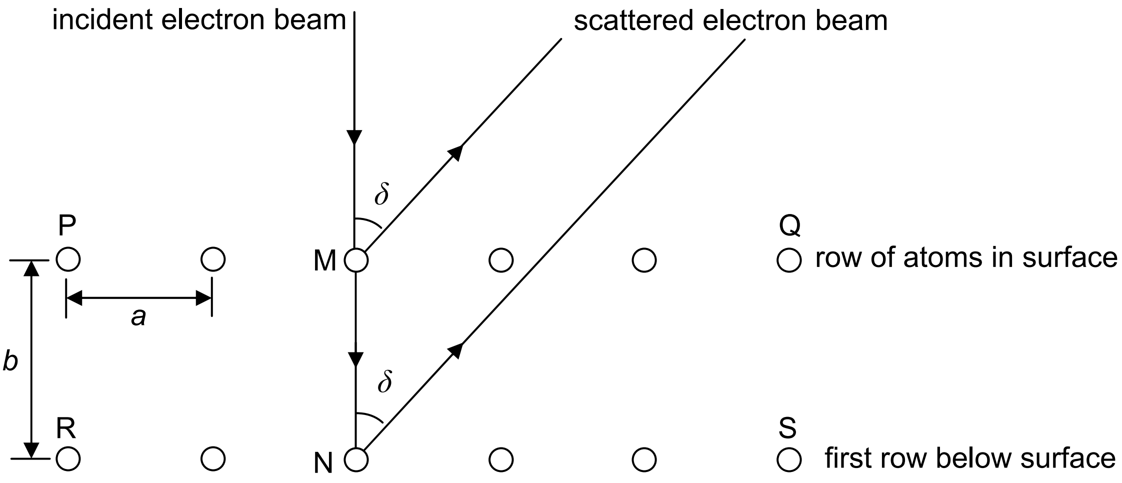

Fig. 7.2

(i) Consider the interaction of the electron beam with atoms M and N, and the condition for strong scattering to be observed in a direction at an angle δ to the normal to the surface. Suggest how this interaction might modify the diffraction pattern obtained due to the interaction of the electron beam with the atoms in the surface row PQ. [3]

(ii) The arrangement and spacing of atoms in the surface of a crystal is often different from that inside the crystal. Suggest a reason for this. [2]

8 **(a)** A particle of mass m moves along a straight line. The motion is simple harmonic with angular frequency ω. The potential energy $U(x)$ of the particle is given by

$$U(x) = \frac{1}{2}m\omega^2 x^2$$

where x is the displacement of the particle from its equilibrium position.

(i) State the assumption of Planck's quantum concept about the energy of the harmonic oscillator. [1]

(ii) Write down a general expression for the total energy E_n of the harmonic oscillator. [1]

(iii) Use your expression in **8(a)(ii)** to explain what is meant by zero-point energy. [2]

(iv) The function $\Psi = Cxe^{-\alpha x^2}$ is a solution to the Schrodinger Equation for the quantum harmonic oscillator, where C and α are constants

1. Show that the energy level of this solution is

$$E = \frac{3}{2}\hbar\omega$$

(Hint: $\frac{d\,(e^u)}{dx} = e^u \frac{du}{dx}$; $\frac{d\,(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$, where u and v are functions of x.) [3]

2. Sketch the wave-function Ψ as a function of displacement x for both positive and negative values of x. [1]

3. Sketch the probability density function of the harmonic oscillator as a function of displacement x, when it is in the energy state in **8(a)(iv)** part **1**. [1]

(v) A 2.5 kg mass is attached to a spring of negligible mass and with a spring constant equal to 25 N m^{-1}. The spring is stretched 0.40 m from its equilibrium position and released.
Calculate the following data for the oscillator:

1. the total energy, [1]

2. the corresponding quantum number n and [2]

3. the fractional decrease in the energy of the oscillator if its energy quantum number decreases by one. Hence, explain whether the quantum concept with respect to the macroscopic oscillator described in **8(a)(v)** is significant. [3]

(b) **(i)** State what is meant by an ideal black body. [1]

(ii) Give a brief description of a practical source of black body radiation. [1]

(iii) Sketch a graph to show the variation of the total power radiated by a black body with the temperature of the body. [1]

(iv) Describe how classical physics failed to explain the black body radiation. [2]

9 **(a)** A molecule of sodium chloride, consisting of a sodium ion of charge +*e* and a chlorine ion of charge –*e*, can exist in the gaseous phase.

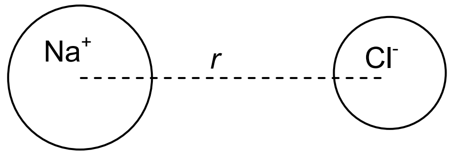

Fig. 9.1

Assuming a hard-sphere model of the molecule as shown in Fig. 9.1, the electrostatic potential energy *U* of the molecule is given by

$$U = -\frac{ke^2}{r}$$

where *r* is the distance between the centres of the ions and $k = \frac{1}{4\pi\varepsilon_0}$.

Determine the equilibrium separation r_m of the ions if the dissociation energy of the sodium chloride molecule is 5.76 eV. [2]

(b) Fig. 9.2 shows the arrangement of sodium and chloride ions in rock salt based on the simple cubic structure. A unit cell of this arrangement is shown in Fig. 9.3.

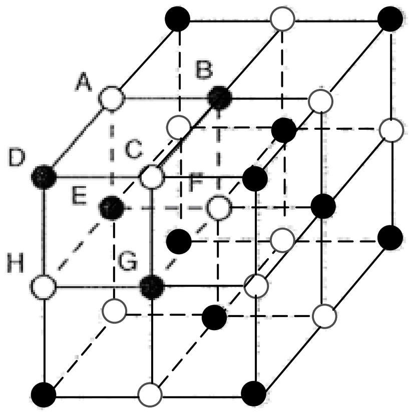

Fig. 9.2

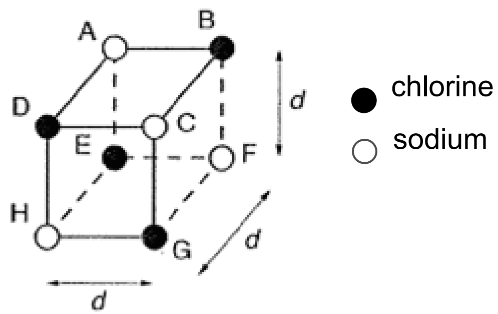

Fig. 9.3

The equation in **(a)**, which relates the potential energy of a sodium chloride molecule, must be modified when a simple cubic crystal structure is considered. The equation can be written as

$$U_c = -\frac{\alpha ke^2}{r} + \lambda e^{-\frac{r}{\rho}}$$

In this equation, *r* is the separation between nearest-neighbouring ions in the crystal and α, λ and ρ are constants.

(i) State why it is necessary to introduce the constant α. Suggest the order of magnitude of α. [2]

(ii) It is reasonable to include the potential energy term $\lambda e^{-\frac{r}{\rho}}$ which is associated with a repulsive force. Suggest the origin of this repulsive potential energy. [2]

(iii) If the equilibrium separation of ions in the crystal is r_o, show that the expression for the potential energy U_0 at equilibrium separation is

$$U_o = \frac{\alpha k e^2}{r_o}\left(\frac{\rho}{r_o} - 1\right)$$ [4]

(iv) Suggest whether r_o is greater than, equal to or less than the value of r_m found in **(a)**. Give a reason for your answer. [2]

(v) X-ray diffraction can be used to study the arrangement of ions and the size of the unit cell of sodium chloride salt crystals as shown in Fig. 9.2 and Fig. 9.3.

1. For X-rays of wavelength 78.2 pm, the first-order diffraction angle corresponding to X-ray diffraction from the crystal planes of type AHF is found to be 13.90°. Determine the spacing for this set of crystal planes. [2]

2. Determine the value of the side of the unit cell *d*. [3]

(vi) The relative atomic masses of Na and Cl are respectively 23 and 35.5. Using the value of *d* in **(b)(v)2.**, calculate the density of the sodium chloride salt crystals. [3]