2014 AJC 9811/01 H3 Prelim Solutions
Section A

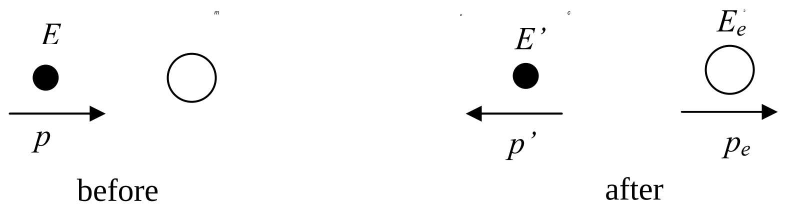
1 (a) (i)

$$E = mc^2 = 9.11\times10^{-31}\times\left(3.00\times10^{8}\right)^2$$
$$= 8.20\times10^{-14}/1.60\times10^{-13}\ \text{MeV}$$
$$m_0 = 0.512\ \text{MeV}/\text{c}^2\ \text{units}$$

(ii)

$$9.11\times10^{-31}kg = 0.512MeV/c^2$$
$$1kg = \frac{0.512}{9.11\times10^{-31}} = 5.62\times10^{29}MeV/c^2$$

(b) (i)



Let p : momentum of photon before collision
p' : momentum of photon after collision
m_e : rest mass of electron
E_e : relativistic energy of electron after collision

COM : $p = p_e - p'$ …… (eqn 1)
COE: $p c + m_e c^2 = p'c + E_e$ ……..(eqn 2)

Also for electron after collision:
$E_e^2 = m_e^2c^4 + p_e^2 c^2$ ……(eqn 3)

Putting eqn (1) and (2) into (3):

$(p c - p' c + m_e c^2)^2 = m_e^2 c^4 + (p + p')^2 c^2$

Expanding all terms and simplifying :
….

$$p' = \frac{m_e cp}{2p + m_e c}$$

(ii)

$$p' = \left[\left(0.51/c^2\right)\times c\times1.25/c\right]/\left[\left(2\times1.25/c\right)+\left(0.51/c^2\right)\times c\right]$$
$$p' = 0.21\ \text{MeV/c}$$
$$E' = p'c = 0.21\ \text{MeV}$$
$$pc + m_e c^2 = p'c + \gamma m_e c^2$$
$$\gamma m_e c^2 - m_e c^2 = pc - p'c$$
$$= 1.25 - 0.21$$
$$= 1.04MeV$$

2 (a) (i) In the step-index multimode type, the core has a relatively large diameter of 50 μm and the refractive index changes abruptly at the cladding. The wide core allows signals to travel by several different paths or modes.
Rays that cross the core more often travel further and therefore take longer to travel down the fibre. The output pulse is spread out in time compared to the input pulse. Dispersion of 30 ns km^{-1} is typical.
In a long fibre, separate pulses may overlap and errors and loss of information will occur at the receiving end.

(ii)
- Spectral/chromatic dispersion: if light source is not monochromatic, a pulse will spread due to different velocities for different wavelengths.
- Bending losses: due to sharp bends of the optical fibre or imperfections at the core-cladding interface.
- Absorption losses: due to property of the fibre, or impurities within.
- Scattering losses: depends on the size of impurity-particles within fibre.
- Coupling losses: due to poor joints.

(iii) In the continuous-index multimode type, the refractive index of the glass varies continuously from a high value at the centre to a low value at the outside so making the boundary between core and cladding indistinct. Signals along longer paths travel faster on average, since the speed of light is inversely proportional to the refractive index.
The arrival times for different modes are then about the same. Dispersion is thereby much reduced to within 1 ns km^{-1}.

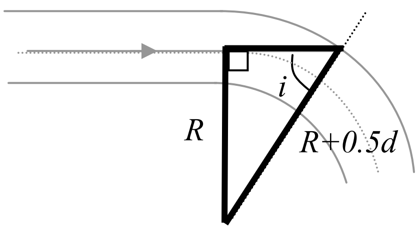
(iv) $\theta_c + \theta_t = 90^0$
$\therefore \sin\theta_t = \cos\theta_c$

By Snell's Law,
$n_o \sin\theta_i = n_f \sin\theta_t$
$= n_f \cos\theta_c$
$= n_f(1 - \sin^2 \theta_c)^{1/2}$

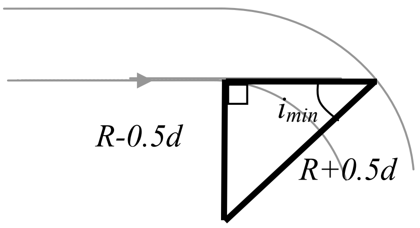
For critical angle condition, (in order to achieve total internal reflection)
$n_f \sin\theta_c = n_c \sin 90^o$
$\sin\theta_c = n_c / n_f$

Hence, $\sin\theta_i = n_f [1 - (n_c / n_f)^2]^{1/2} / n_o$
$\mathbf{\sin\theta_{max} = (n_f^2 - n_c^2)^{1/2} / n_o}$

(b) (i) $\sin i = \dfrac{R}{R + \frac{1}{2}d}$



(ii) angle of incidence is bigger for rays above the central ray



for all light to pass around the arc, need all possible angles of incidence to exceed or equal critical angle, $\sin i_{\min} \geq \sin\theta_c = \dfrac{1}{n}$

$$\frac{R-\frac{1}{2}d}{R+\frac{1}{2}d} \geq \frac{1}{n} \qquad \Rightarrow \quad R \geq \frac{d(n+1)}{2(n-1)}$$

3 (a) (i) U is the <u>potential energy function of a system</u> (due to the particle interacting with its environment), and E is the <u>total energy of the system</u> (the particle and its environment).
E and U are related by the equation $E = U + K$, where K kinetic energy of the particle.

(a) (ii) 1. In the right hand side, U = 0. Hence, TISE becomes

$$E\Psi = -\frac{\hbar^2}{2m}\left(\frac{d^2\Psi}{dx^2}\right)$$

$$\Rightarrow \quad \frac{d^2\Psi}{dx^2} = -\frac{2m}{\hbar^2}E\Psi = -k_2{}^2\Psi \quad \text{where} \quad k_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

Given $\Psi = C\ exp\ (ik_2x)$ where $k_2 = \sqrt{\dfrac{2mE}{\hbar^2}}$

$$\frac{d\Psi}{dx} = Cik_2\exp(\mathrm{i}\,k_2\ x)$$

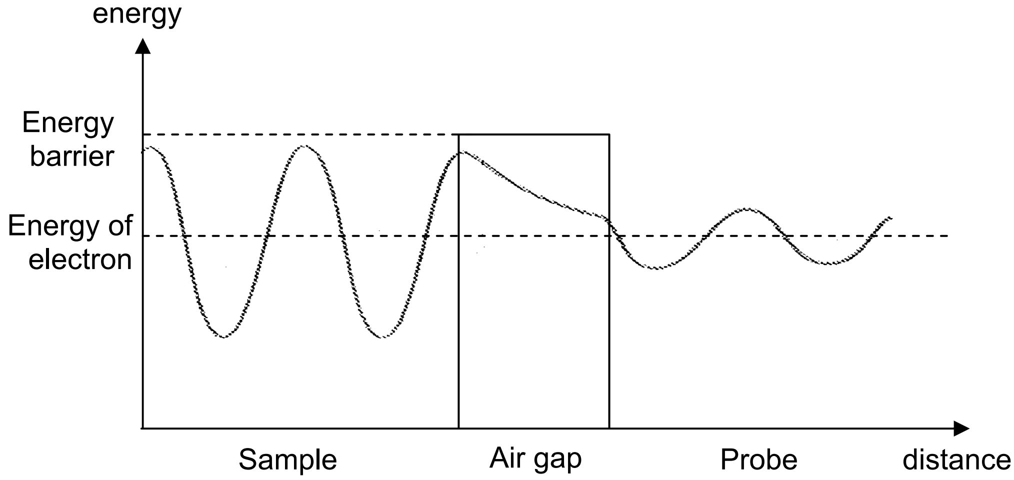
$$\frac{d^2\Psi}{dx^2} = -Ck_2{}^2\exp(\mathrm{i}\,k_2\ x) = -k_2{}^2\Psi$$

Thus, $\Psi = C\ exp\ (ik_2x)$ is a solution of the SE.

(a) (ii) 2. After passing through the potential step, the <u>particle's wave can only move away from the step towards the right, as there is no reflected wave moving to the left</u>. Hence, there is only one term on the right side of the equation representing this wave.
$|C|^2$ represents the intensity of the transmitted wave to the right of the step.
This is in contrast to the wave-function on the left side of the step which consists of an incident wave and a reflected wave, and which therefore requires two terms to represent it.

(b) (i)
- Air space between tip of STM and sample surface acts as potential barrier.
- The tunnelling probability (transmission coefficient) of electrons from the STM tip to sample surface (or sample surface to STM tip) decreases with increasing distance between the tip and surface.
- Hence, as the detected current varies when the STM moves over the sample surface, this indirectly shows the topology of the surface.

(b) (ii)



- Sinusoidal for ingoing and outgoing wave functions
- Decaying exponential in barrier and decreased amplitude in outgoing wave function
- Boundary conditions are met.
- Ingoing and outgoing wavelengths are about the same (N10/I/34 H2QP)

4 **(a)** Directed from side PQ to side RS

(b) Force due to V_H = $eE = e\dfrac{V_H}{w}$

Force due to V_H = force due to magnetic field

$$e\frac{V_H}{w} = Bev$$

$$V_H = Bvw$$

(c) **(i)** I=nAve

$V_H \alpha\ v\ \alpha\ I$

V_H increases

(ii)

$$v = \frac{I}{nAe}$$

$$V_H \alpha \frac{1}{n}$$

Since copper has larger n, V_H decreases

5 **(a)** **(i)** In classical theory, the oscillations of the electron have the same frequency as the incoming and outgoing electromagnetic wave. This gives rise to a single peak of intensity of scattered x-ray wave/radiation at this frequency.

However, experimentally, there is a second peak corresponding to a shorter frequency/longer wavelength when x-rays are scattered, known as the Compton shift.

(a) (ii) The Compton shift can only be explained where the incoming radiation is seen as particles (photons) of discrete quantum energies in collision with the electrons of target material.

The first peak with no change in wavelength is explained in terms of a photon collision with an electron which is tightly bound to an atom. Hence, the photon is essentially interacting with the whole atom.

Since the mass of the atom *M* is much bigger than the mass m_o of the electron, the shift in the wavelength will be given by

$\Delta\lambda = \frac{h}{Mc}(1-\cos\theta)$ which is immeasurably small. This explains the first peak observed without a change in wavelength.

The second peak with a longer wavelength is explained in terms of a photon colliding with a free (stationary) electron. Hence, the incident photon will lose energy to the recoiling electron, resulting in a scattered photon with longer wavelength.

(b) (i) - axes correctly labelled with units
- 15 plots
- 2 peaks near wavelengths 0.0709nm and 0.0749

(b) (ii) 1. Compton shift, $\Delta\lambda \approx 4.0 \times 10^{-12}$ m

(b) (ii) 2. Compton wavelength, λ_c

$$\lambda_c = \frac{\Delta\lambda}{(1-\cos\theta)} = \frac{4.0\times10^{-12}}{1-\cos135^o} = 2.343\times10^{-12} \approx 2.34\times10^{-12}\,m$$

Accept ecf from (bii1).

Do not accept $\lambda_c = \frac{h}{mc} = 2.426\times10^{-12} \approx 2.43\times10^{-12}\,m$ *as the question requires using of plotted graph.*

(c) The change in the wavelength for a given angle is dependent on $\frac{1}{m_{electron}}$.

If a proton is substituted for an electron, the change in the wavelength would be dependent on $\frac{1}{m_{proton}}$.

Since protons have about 2000 times the mass of electons, the change in wavelength is reduced by a factor of about 2000.

(d) From the Compton shift formula, the maximum Compton shift is 4.85×10^{-12} m, when θ is 180°.
This is only about 0.001% of the wavelength of visible and thus, is not detectable.

Section B

6 (a) (i) Let *u* be the velocity of the robber's spaceship measured by the stationary frame, i.e., *u* = ¾ *c*.

Let *u'* be the velocity of the robber's spaceship measured by the police.

Also, the police is cruising at a speed of ½ *c* with respect to the stationary frame.

$$u = \frac{u' + v}{1 + u'v/c^2}$$

$$\frac{3c}{4} = \frac{u' + \frac{1}{2}c}{1 + u'/2c}$$

$$u' = \frac{2}{5}c$$

(ii) **1.** According to Galilean/Newtonian physics, velocity of the missile relative to the robber is *c/3 – c/4 = c/12* and it will hit.

2. According to relativity, the robber is traveling at 0.4*c* away from the police, faster than the missile's *c/3*. Hence the missile will not hit.

(iii) Let the time taken be *T*. Within this time, the laser pulse would have moved forward by *cT* and the robber's spaceship 0.4*cT*. Hence,

$$cT = 1.2 \times 10^6 \times 10^3 + 0.4cT$$

$$T = 6.67 \text{ s}$$

(iv) The police observed the robber to be at 1.2×10^9 m away and moving. The distance is a contracted one. Thus, the robber observes the police to be $\frac{1}{\sqrt{1-(2/5)^2}} \times (1.2 \times 10^9) = 1.31 \times 10^9$ m

(v) $T' = \frac{1.31 \times 10^9}{3 \times 10^8} = 4.36$ s

(vi) Consider the event that the pulse is fired. The fastest signal that can reach the robber (to warn him) will travel at the speed of light, which is the same as that of the laser pulse. So the robber will have no time to react at all.

(b) **(i)** $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ **or** $m = \gamma m_0$

(ii) **1.** 260 (±10) MeVc^{-2} ;

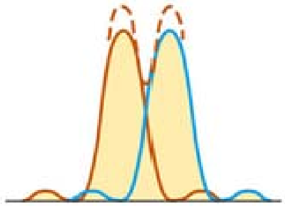
2. 920 (± 20) MeVc^{-2}

(iii) 920 (± 20) MeV

(iv) $E_{total} = Ve + m_0c^2$
V = 920 − 260 = 660 (±30) MV

7 **(a)** The resolving power of an optical instrument is given by the Rayleigh criterion. This states that it is possible for the instrument to separate the images of two point objects if the centre of the central maximum of one image lies on the first dark ring of the other. The criteria is satisfied when the central maxima are separated by an angle given by the relationship:

$$\sin\theta = \frac{1.22\lambda}{D}$$



where D is the aperture diameter.

- The *resolution* of an optical instrument when viewing two point objects is the minimum separation of two objects that can just be distinguished by the instrument

(b) If the angular separation of the dots exceeds Rayleigh's criterion, then you are able to resolve individual dots. Therefore the angular separation of the dots must be smaller than the Rayleigh's criterion,

$$\sin\theta = \frac{1.22\lambda}{D}$$

Use lowest wavelength 400nm to ensure dots blend together for all visible light

Since separation of dots (x) << (y) distance of page to eye

$$\sin\theta \approx \theta \approx \frac{x}{y}$$

$$\frac{x}{y} < \frac{1.22\lambda}{D}$$

$$x < \frac{1.22(400x10^{-9})(0.40)}{0.0025}$$

$$x < 7.81x10^{-5}m$$

$$x < \frac{7.81x10^{-5}m}{0.0254}$$

$$x < 0.00307 \text{ inches}$$

$$\text{dots per inch} = \frac{1}{0.00307} = 330 \text{ dpi}$$

(c) **(i)**

$$eV = \frac{p^2}{2m}$$

Using $p = \frac{h}{\lambda}$

$$eV = \frac{h^2}{2m\lambda^2}$$

$$\lambda = \frac{h}{\sqrt{2me}}$$

$$= \frac{1.23nm}{\sqrt{V}}$$

(ii)

$$hf = h\frac{c}{\lambda} = eV = (1.6x10^{-19})(100x10^{3})$$

$$\lambda = 1.2424x10^{-11}m$$

$$\lambda = \frac{1.23x10^{-9}}{\sqrt{V}}$$

$$V = 9.80kV$$

(iii) Electrons can be easily focused using electric fields, whereas photons are uncharged, and thus they cannot be focused easily.

(d) **(i)** The path difference between 2 adjacent scattered electron beams is a$\sin\phi$

For strong scattering, constructive interference between the beams is required. This means the path difference is an integral number of wavelengths

Hence, a $\sin\phi = n\lambda$

(ii) A larger accelerating potential gives the electrons a larger kinetic energy and a larger momentum. With a larger momentum, the

de Broglie wavelength is smaller. (using $\lambda = \frac{1.23}{\sqrt{V}}$)

since wavelength decrease, from a $\sin\phi = n\lambda$,
angle *Φ of the scattered beam decreases*

(e) **(i)** Path difference of electron beam scattered from atom in adjacent rows is b + b$\cos\delta$

For strong scattering, constructive interference between the beams is required. This means the path difference is an integral number of wavelengths
Hence, b + b$\cos\delta = n\lambda$

The resulting diffraction pattern, of less intense maxima, will superpose on the diffraction pattern due to interaction of the electron beam with the atoms on the surface.

(ii) The atoms in the surface of a crystal have fewer neighbouring atoms and will bond differently leading to different arrangement.

Atoms at the surface have more thermal energy and would be spaced further apart.

8 **(a)** **(i)** 2 assumptions of Planck (Serway pg 1156). Either one
Planck assumed that the energy of an oscillator can have only certain discrete values $E_n = nhf$, where n is a positive integer called a quantum number, f is the frequency of the oscillator and h is the Planck constant.
OR
Planck assumed that an oscillator emits or absorbs energy in a single quantum of radiation $E = hf$, where f is the frequency of the oscillator's state and h is the Planck constant, as the oscillator makes a transition from one quantum state to another.

(ii) $E_n = \left(n - \frac{1}{2}\right)\hbar\,\omega$, where n is a positive integer, i.e. n = 1, 2, 3…

(iii) When a quantum particle is in its lowest possible energy state, ie the energy of its ground state, this corresponds to n = 1 from 8(a)(ii). Thus, $E_1 = \frac{1}{2}\hbar\,\omega$ and it is called the zero-point energy.

The zero-point energy is non-zero as all quantum mechanical systems undergo fluctuations even in their ground state, a consequence of their wave-like nature.

(iv) **1.** Given $\Psi = Cxe^{-\alpha x^2}$

$$\frac{d\Psi}{dx} = Ce^{-\alpha x^2} - 2aCx^2e^{-\alpha x^2}$$

$$\frac{d^2\Psi}{dx^2} = -2aCxe^{-\alpha x^2} - \left[4aCxe^{-\alpha x^2} - 4a^2Cx^3e^{-\alpha x^2}\right]$$

$$= -6aCxe^{-\alpha x^2} + 4a^2Cx^3e^{-\alpha x^2}$$

$$= -6a\Psi + 4a^2x^2\Psi$$

Time independent Schrodinger equation

$$E\Psi = -\frac{\hbar^2}{2m}\left(\frac{d^2\Psi}{dx^2}\right) + U\Psi$$

Rearranging, $\dfrac{d^2\Psi}{dx^2} = \dfrac{2m}{\hbar^2}(U - E)\Psi = \dfrac{2m}{\hbar^2}(\frac{1}{2}m\omega^2x^2 - E)\Psi$

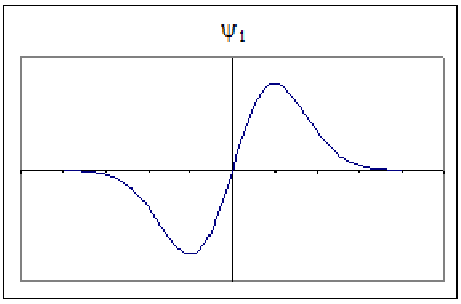
Comparing coefficients of x^2,

$$\frac{2m}{\hbar^2}(\frac{1}{2}m\omega^2) = 4a^2 \quad \Rightarrow \quad a = \frac{m\omega}{2\hbar}$$

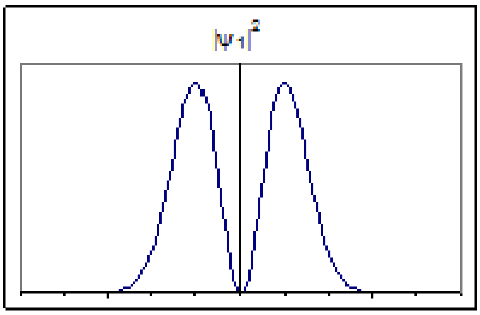
Comparing constants,

$$-\frac{2m}{\hbar^2}E = -6a \quad \Rightarrow \quad E = 3\frac{\hbar^2}{m}a \quad \Rightarrow \quad E = \frac{3}{2}\hbar\omega \quad \text{(shown)}$$

(a) (iv) 2.



(a) (iv) 3.



(a) (v) 1. Total energy of oscillator = max potential energy
= ½ $m\omega^2x_o^2$ = ½ $k\,x_o^2$ = ½ (25)(0.40)^2 = 2.0 J.

2. From **(a)(ii)**, total energy of quantum harmonic oscillator
$= E_n = \left(n - \frac{1}{2}\right)\hbar\,\omega = E_n = \left(n - \frac{1}{2}\right)hf$

Frequency $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{25}{2.5}} = 0.50329Hz$

$\Rightarrow$ n – ½ = 2.0 / (6.63 x 10^{-34} x 0.50329) = 5.99 x 10^{33}
$\Rightarrow$ n = 5.99 x 10^{33} + ½ $\approx$ 6.0 x 10^{33}

3. Fractional decrease

$$= \frac{1hf}{\left(n - \frac{1}{2}\right) hf} = \frac{1hf}{5.99 \times 10^{33}\ hf} = 1.668 \times 10^{-34} \approx 1.7 \times 10^{-34}$$

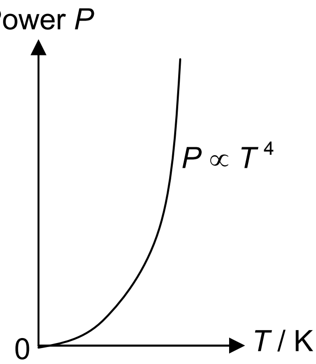
The fractional change is very small. This means that the change in the energies by discrete amounts is very small for a macroscopic oscillator like a spring-mass system. This would be perceived as a continuous change for a macroscopic oscillator.

Hence quantum effects are not significant when applied to macroscopic systems.

(b) (i) A black body is a theoretical object that absorbs all wavelengths of electromagnetic radiation/all light falling on it. It does not reflect any radiation and hence appears completely dark.

(b) (ii) A hollow enclosure, such as a cylinder, with its interior walls blackened with soot and a small hole/opening.

(b) (iii)



Note: Stefan's law states that, for a black body emitter, the radiated power per unit area P is directly proportional to the fourth power of the black body's temperature T, in kelvin, as given by $P = \sigma T^4$ where the Stefan-Boltzmann constant σ = 5.67 x 10^{-8} W m^{-2} K^{-4}.

(b) (iv) Classical Physics predicted that the oscillating electric charges of molecules radiate/absorb electromagnetic waves continuously. At long wavelengths, classical theory agrees with experimental results.
At short wavelengths, classical theory predicts that the intensity of radiation will tend to infinity (ultra violet catastrophe) which contradicts the experimental observations where intensity approaches to a zero value.

9 (a) $U = -\frac{ke^2}{r}$

$(5.76)(1.6x10^{-19}) = \frac{1}{4\pi\varepsilon_0} x \frac{(1.6x10^{-19})^2}{r}$

r_m = 0.25 nm

(b) (i) To account for the potential energy due to the overall Coulomb attraction of neighbouring Na^+ and Cl^- in the entire lattice.

α = 1.75 (between 1.7 to 2.1)

(for the ions shown in fig 9.2, $\alpha = 6 - \frac{12}{\sqrt{2}} + \frac{8}{\sqrt{3}} \approx 2.13$.

Note this value is not the actual α *which is due to the whole lattice)*

(ii) The repulsive potential energy is due to the overlap of the electron wave functions of neighbouring ions in the crystal lattice
According to Pauli exclusion principle, no two electrons can occupy the same state and this gives rise to a repulsive force which prevents the ions from getting closer.

(iii)

$$U_c = -\frac{\alpha k e^2}{r} + \lambda e^{-\frac{r}{\rho}}$$

$$\frac{dU_c}{dr} = \frac{\alpha k e^2}{r^2} - \frac{\lambda}{\rho} e^{-\left(\frac{r}{\rho}\right)}$$

At equilibrium position at $r = r_0$, $\frac{dU_c}{dr} = 0$

Hence $\frac{\alpha k e^2}{r_0^{\;2}} - \frac{\lambda}{\rho} e^{-\left(\frac{r_0}{\rho}\right)} = 0$

$$\lambda e^{-\left(\frac{r_0}{\rho}\right)} = \frac{\rho \alpha k e^2}{r_0^{\;2}} \quad .....(1)$$

At equilibrium position at $r = r_0$,

$$U_c = -\frac{\alpha k e^2}{r_0} + \lambda e^{-\frac{r_0}{\rho}}$$

Subst (1), we have $U_c = -\frac{\alpha k e^2}{r_0} + \frac{\rho \alpha k e^2}{r_0^{\;2}}$

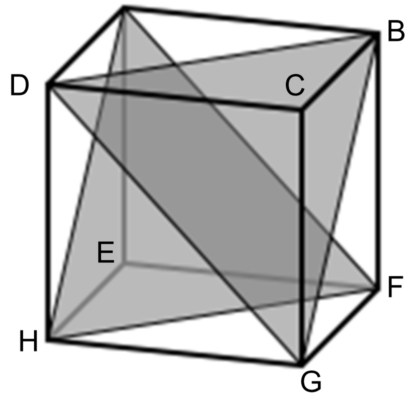
$$U_o = \frac{\alpha k e^2}{r_o}\left(\frac{\rho}{r_o} - 1\right)$$

(iv) r_0 is larger than r_m
because in there are more overlapping of the electron wave functions as there are more neighbouring ions in a crystal lattice than in a molecule.

(v) **1.** Using Bragg's equation,
$n\lambda = 2\,d'\sin 13.9$
$(1)(78.2 \times 10^{-12}) = 2\,d'\sin 13.90$
$d' = 0.163$ nm

2.

A



B

AHF and BDG are adjacent planes.

Distance EC = $\sqrt{3}\,d$

distance between planes AHF and BDG (d') is one-third of distance EC

$$d' = \frac{\sqrt{3}\,d}{3} = \frac{d}{\sqrt{3}}$$

$$d = \sqrt{3}d'$$
$$= \sqrt{3}(0.163nm)$$
$$= 0.282nm$$

(vi) Mass of one "molecule" of NaCl is

$$M = \frac{M_m}{N_A} = \frac{(23+35.5)10^{-3}}{6.02\text{x}10^{23}} = 9.718 \text{ x } 10^{-26} \text{ kg}$$

1 unit cell contains ½ NaCl molecule

$\Rightarrow$ mass one unit cell = 0.5(9.718x10^{-26}) = 4.859 x 10^{-26} kg

$$\text{Density of NaCl} = \frac{\text{mass of unit cell}}{\text{volume of unit cell}}$$
$$= \frac{\text{mass of 0.5 NaCl}}{d^3}$$
$$= \frac{4.859x10^{-26}}{(0.282x10^{-9})^3}$$
$$= 2.17x10^3\,kgm^{-3}$$