Centre Number	Index Number	Name	Class
3016			

RAFFLES INSTITUTION
2014 Preliminary Examination

PHYSICS **9811/01**
Higher 3

2 September 2014
Paper 1 **3 hours**

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your index number, name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
Start each question on a new sheet of paper and label the parts clearly.
You may use a soft pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use	
1	/ 17
2	/ 17
3	/ 15
4	/ 17
5	/ 17
6	/ 17
Deduction	
Total	/ 100

This booklet consists of **10** printed pages including the cover page.

Data

speed of light in free space,	c	=	3.00×10^{8} m s^{-1}
permeability of free space,	μ_0	=	$4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space,	ε_0	=	8.85×10^{-12} F m^{-1}
		=	$(1/(36\pi)) \times 10^{-9}$ F m^{-1}
elementary charge,	e	=	1.60×10^{-19} C
the Planck constant,	h	=	6.63×10^{-34} J s
unified atomic mass constant,	u	=	1.66×10^{-27} kg
rest mass of electron,	m_e	=	9.11×10^{-31} kg
rest mass of proton,	m_p	=	1.67×10^{-27} kg
molar gas constant,	R	=	8.31 J K^{-1} mol^{-1}
the Avogadro constant,	N_A	=	6.02×10^{23} mol^{-1}
the Boltzmann constant,	k	=	1.38×10^{-23} J K^{-1}
gravitational constant,	G	=	6.67×10^{-11} N m^{2} kg^{-2}
acceleration of free fall,	g	=	9.81 m s^{-2}

FORMULAE

Lorentz factor	$\gamma = (1 - (v/c)^2)^{-1/2}$
length contraction	$L = L_0/\gamma$
time dilation	$T = \gamma T_0$
Lorentz transformation equations (1 dimension)	$x' = \gamma(x - vt)$
	$t' = \gamma(t - vx/c^2)$
mass-energy equivalence	$E = \gamma m_0 c^2$
	$= \sqrt{(pc)^2 + (m_0 c^2)^2}$
Wien's displacement law	$\lambda_p T = 2.898 \times 10^{-3}$ m K
Compton shift formula	$\Delta\lambda = \frac{h}{mc}(1 - \cos\theta)$
population distribution of atoms with energy E_x	$N_x = N_0 \exp(-(E_x - E_0)/kT)$
time-independent Schrodinger equation	$E\Psi = -\frac{h^2}{2m}\left(\frac{d^2\Psi}{dx^2}\right) + U\Psi$
allowed energy states for a particle in a box	$E_n = (n^2h^2)/(8mL^2)$
normalised wave function for particle in a box	$\Psi = (2/L)^{1/2}\sin(n\pi x/L)$
transmission coefficient	$T = \exp(-2kd)$
	where $k = \sqrt{\frac{8\pi^2 m(U-E)}{h^2}}$
Drude model of electrical resistivity	$\rho = \frac{2m_e\langle v\rangle}{ne^2\lambda}$
Fermi energy for metals	$E_F = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3}$
density of energy states for electrons in a metal	$\rho(E) = \frac{4\pi(2m)^{3/2}}{h^3}\sqrt{E}$
Fermi function	$f(E) = 1/(1 + \exp((E - E_F)/kT))$
refractive index	$n = v_1/v_2$
phase difference of circularly polarised light	$\frac{\delta}{2\pi} = \frac{d}{\lambda}\Delta n$
Brewster's angle	$\tan\theta_B = n_2/n_1$
attenuation of light intensity	$I = I_0\exp(-\mu x)$

1 **(a)** State Einstein's postulates of relativity. [2]

(b) An electrically neutral sub-atomic particle, called a pion, travelling at a very high velocity v, decays into a pair of identical γ-rays as shown in Fig. 1.1. The γ-rays are emitted in opposite directions as shown. There are no other products of the decay.

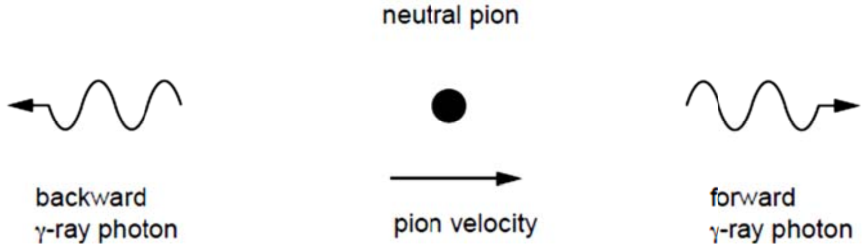

Fig. 1.1

(i) Explain why the pion could not decay to a single γ-ray photon. [2]

(ii) Denoting the rest mass of pion as m, write down an expression for the momentum of the pion in the lab frame in terms of m, v and the Lorentz factor γ. [1]

(iii) The wavelengths of the forward and backward propagating photons are λ_1 and λ_2 respectively as observed in the lab frame.

1. Write down an equation describing the conservation of momentum in the decay process. [2]

2. Write down an equation describing the conservation of mass-energy in the decay process. [2]

(iv) Hence, derive expressions for λ_1 and λ_2 in terms of m, v, c and any other fundamental constants. [3]

(c) The half-life for the decay of a neutral pion at rest in the laboratory is about 18.0 ns. Calculate the expected half-life of the pion measured by an observer in the laboratory if the pion is moving with a velocity of 0.99c. [2]

(d) A scientist in the laboratory observes two pions closely. Pion 1 is stationary while pion 2 is moving at a very high velocity $\boldsymbol{v}$ in the positive x-direction. At time t = 0, both pions are at the origin. Pion 1 decays at time $t = t_1$ while pion 2 decays time $t = \gamma t_1$ as observed by the scientist. γ is the Lorentz factor for pion 2.

(i) The time and space coordinates of the decay of pion 1 is $(t_1, 0)$. Write down the time and space coordinates for the decay of pion 2. [1]

(ii) To another scientist moving alongside pion 2 so that pion 2 appears to be stationary to him, pion 1 is the particle that experiences time dilation and is expected to have a longer lifetime.

Using Lorentz transformations, show that in the rest frame of pion 2, the reversed is indeed observed, i.e., the lifetime of pion 1 is γ times the lifetime of pion 2. [2]

2 **(a)** Both photoelectric effect and Compton scattering can be explained by the photon theory of light. In photoelectric effect, electrons are ejected from a metal surface through the absorption of photons.

(i) Using the Compton-shift formula, explain why it is not possible to eject electrons from a metal surface through Compton scattering of ultraviolet radiation. [2]

(ii) For Compton scattering, describe the feature of the experimental results that could not be explained by the classical wave theory of electromagnetic radiation. [2]

(iii) X-rays having energy of 300 keV undergo Compton scattering from a target. The scattered rays are detected at 37.0° relative to the incident rays.
Calculate

1. the Compton shift at this angle, [2]

2. the energy of the scattered X-ray in keV, [3]

3. the energy of the recoiling electron in keV. [1]

(b) **(i)** Explain what is meant by an *ideal black body*. [1]

(ii) The classical model of black body radiation given by the Rayleigh-Jeans law has major flaws. Identify one such flaw. [1]

(iii) Describe the assumptions made by Planck to address the flaw you identified in **(ii)**. [2]

(iv) The radius of the Sun is 6.96×10^{8} m and its total power output is 3.85×10^{26} W.

1. Assuming the Sun's surface behaves like a black body, calculate the temperature of its surface. [2]

2. Calculate the wavelength at which the energy radiated by the Sun is maximum. [1]

(Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4})

3 **(a)** **(i)** Explain what is meant by the *resolving power* of optical instruments. [1]

(ii) Explain how Rayleigh criterion can be used to determine whether two images are resolved. [2]

(iii) Sketch well labelled diagrams to differentiate the images seen when two point light sources are unresolved, just resolved and fully resolved. [3]

(b) A Newtonian telescope with a large aperture (primary mirror) allows an observer to see dimmer objects by collecting more light than the naked eye can.

In Fig. 3.1, the primary mirror reflects the incoming light onto a secondary mirror, which is then responsible for focusing this light to produce a clear image of the object.

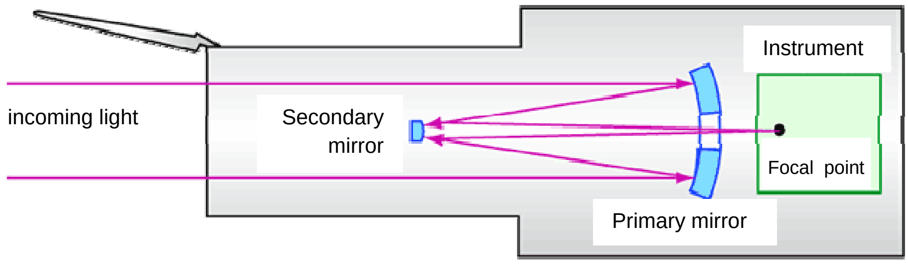

Fig. 3.1

(i) The primary mirror of the telescope has a diameter of 0.25 m. Estimate the angle subtended by two stars that are just resolved by the telescope. [2]

(ii) If those two stars are known to be 1.6×10^{13} m apart, how far away are they from Earth? [1]

(c) **(i)** Explain how transmission electron microscopes (TEM) can achieve higher resolutions than normal optical microscopes. [2]

(ii) Explain how the resolution of TEM can be increased. [2]

(iii) The typical accelerating voltage in a TEM is 100 kV. Calculate the de Broglie's wavelength of the electron. [2]

4 (a) The time-independent Schrodinger's Equation for a quantum particle of mass m is

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)\right]\psi(x)=E\psi(x),$$

where $\psi(x)$ is the wave function, $V(x)$ is the potential energy function and E is the total energy of the quantum particle.

Explain the term *wave function*. [1]

(b) A particle moving along the x–axis is trapped by two hard walls located at $x=-L$ and $x=L$ as shown in Fig. 4.1.

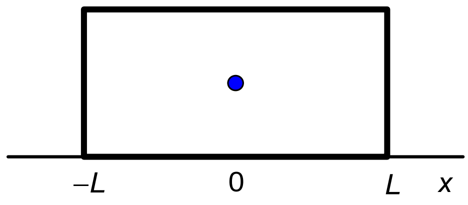

Fig. 4.1

Mathematically, the situation can be described with the following potential energy:

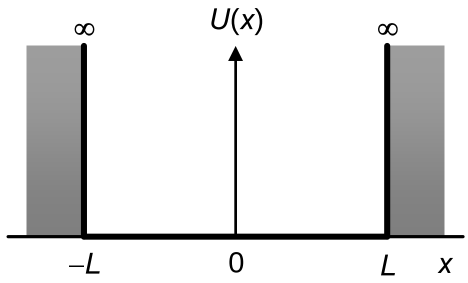

Fig. 4.2

(i) Given that

$$\psi_1(x)=R\sin\left[\frac{\sqrt{2mE}}{\hbar}x+\phi\right],$$

where R and ϕ are constants, show that $\psi_1(x)$ is a solution of the Schrodinger's Equation in the interval $-L\le x\le L$. [3]

(ii) Write down the wave function, $\psi_2(x)$, in the region $x<-L$ and $x>L$ [1]

(iii) Deduce the value of R. [4]

(iv) By considering the boundary condition at $x=-L$ and $x=L$, derive the expression for the energy of the particle. [4]

(c) A particle undergoes simple harmonic oscillation with angular frequency ω. The energy of the n^{th}-stationary state is E_n.

(i) State the zero-point energy E_1 of this particle. [1]

(ii) Determine the quantum number n such that the difference in energy between this level and the next higher level is less than 1 % of E_n. [3]

5 **(a)** The Drude model of electrical conduction has a bearing on Ohm's Law and shows that resistivity can be related to the motion of free electrons in metals.

(i) The current density J in a conductor is the current per unit cross-sectional area. Show that

$$J = nev,$$

where n is the number density of free electrons, v is the drift velocity and e is the elementary charge. [3]

(ii) If a potential difference V is applied across a metal wire of length L and cross-sectional area A, show that an alternative representation of current density is

$$J = \frac{V}{\rho L},$$

where ρ is the resistivity of the conductor. [2]

(b) The resistivity ρ of zinc varies with temperature T according to

$$\rho = \rho_0[1 + \alpha(T - T_0)],$$

where ρ_0 is the resistivity of zinc at 20 °C and α is the temperature coefficient of zinc.

molar mass of zinc = 65.37 g mol^{-1}
density of zinc = 7.133 g cm^{-3}
resistivity of zinc at 20°C = 5.920 x 10^{-8} Ω m
temperature coefficient of zinc = 3.70 x 10^{-3} °C^{-1}

(i) A potential difference of 0.0320 V is applied across the ends of a zinc wire of length 10.0 cm. The temperature of the wire is 55.0 °C. Each zinc atom contributes two free electrons for conduction. [1]

Calculate the resistivity of zinc at 55.0 °C.

(ii) Calculate the drift velocity of the free electrons in the zinc wire at 55.0 °C. [3]

(c) Use the Drude model to show that the drift velocity v of a free electron in a cylindrical wire along which an electric field E has been applied is given by

$$v = \frac{Ee\lambda}{\sqrt{12kTm_e}},$$

where e is the elementary charge, λ is the mean free path of the free electrons, k is the Boltzmann constant, T is the thermodynamic temperature of the wire and m_e is the mass of electron. [5]

(d) Aluminium, which is trivalent, has a density of 2700 kg m^{-3}. One mole of aluminium has a mass of 0.027 kg.

Calculate the Fermi energy of aluminium at a temperature of absolute zero. Leave your answer in electron-volt (eV). [3]

6 **(a)** **(i)** State Malus' law and explain the meaning of the symbols used. [2]

(ii) Explain why, in reality, the intensity of the transmitted light is lower than predicted by Malus' law. [1]

(iii) An unpolarised light of intensity I_0 is incident on an ideal polariser. Deduce the intensity of the transmitted light in terms of I_0. [2]

(b) A horizontal beam of light has an unpolarised component of intensity I_0 and a polarised component of intensity I_p. The plane of polarisation of the polarised component is oriented at an angle of θ with respect to the vertical. The data in Fig. 6.1 give the intensity $I_{transmitted}$ measured through a polariser with an orientation ϕ with respect to the vertical.

ϕ / °	$I_{transmitted}$/ W m^{-2}
0	18.4
10	21.4
20	23.7
30	24.8
40	24.8
50	23.7
60	21.4
70	18.4
80	15.0
90	11.6

ϕ / °	$I_{transmitted}$/ W m^{-2}
100	8.6
110	6.3
120	5.2
130	5.2
140	6.3
150	8.6
160	11.6
170	15.0
180	18.4

Fig. 6.1

(i) Plot the graph of $I_{transmitted}$ against ϕ on a graph paper. [2]

(ii) From the graph, determine

1. the angle θ of the polarised light, [1]

2. the intensity I_0 of the unpolarised component, and [2]

3. the intensity I_p of the polarised component? [2]

(c) A prism of refractive index n_2 separates media of different refractive indices n_1 and n_3. Light propagates in a plane containing the apex angle Φ of the prism as shown in Fig. 6.2.

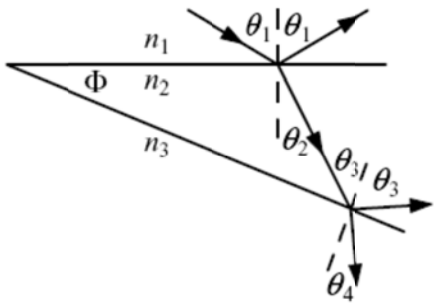

Fig. 6.2

(i) If light strikes the top surface of the prism at Brewster's angle, show that

$$tan\,\theta_2 = \frac{n_1}{n_2}.$$ [2]

(ii) Determine the apex angle Φ, in terms n_1, n_2, and n_3, for which light can fall on both the surfaces at Brewster's angles as it passes through the prism. [3]

End of Paper