

**Section A: Pure mathematics [35 marks]**

- 1** Use a non-calculator method to find the exact value of  $\int_0^1 \left( \sqrt{x} + \frac{1}{\sqrt{2-x}} \right) dx$ . [3]

- 2** Curves  $C_1$  and  $C_2$  have equations as follows:

$$C_1: y = \ln(x+3), x > -3 \quad \text{and} \quad C_2: y = e^x - 1$$

- (i) Sketch  $C_1$  and  $C_2$  on the same diagram, stating the  $x$ -coordinates of the points of intersection and the equations of asymptotes. [3]
- (ii) Find the area of the region bounded by  $C_1$  and  $C_2$ . [2]
- 3** (a) By means of the substitution  $u = \sqrt{x}$ , or otherwise, find the exact value of  $x$  which satisfies the equation  $\sqrt{x} - \frac{3}{\sqrt{x}} = 2$ . [3]

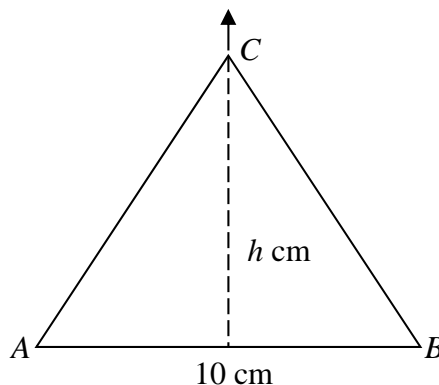
- (b) Find, algebraically, the set of values of  $k$  for which

$$x^2 - 4x + kx + k^2 + 4 > 0$$

for all real values of  $x$ . [3]

- 4** Find the equation of the normal to the curve  $y = \frac{4}{x} - x$  at the point where  $x = 1$ . Hence find the exact coordinates of the point at which this normal meets the curve again. [6]

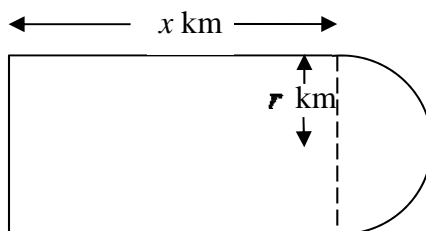
**5**



The diagram above shows an isosceles triangle  $ABC$  with  $AB = 10$  cm and  $AC = BC$ . The height of the triangle is denoted by  $h$  cm. The vertex  $C$  is a variable point and it moves away from the fixed line segment  $AB$  at a constant speed of 2 cm/s.

- (i) Find the length of  $BC$  in terms of  $h$  and hence show that  $h = \sqrt{75}$  when the triangle is equilateral. [2]
- (ii) Calculate the rate of increase of the length of  $BC$  when the triangle is equilateral. [4]

- 6 The diagram below shows the shape of a field which consists of a semi-circle of radius  $r$  km and a rectangle of length  $x$  km.



A low wall measuring 0.5 km long is constructed along the perimeter of the field.

- (i) Let the area of the field be  $A$  km<sup>2</sup>. Show that  $A = \frac{r - r^2(\pi + 4)}{2}$ . [3]
- (ii) Hence use a non-calculator method to find the exact maximum area of the field. [4]
- (iii) It is decided that a field with an area of at least 0.01 km<sup>2</sup> is to be built. Using a graphical method, or otherwise, find the range of values of  $r$  that will allow such a field to be built. [2]

### Section B: Statistics [60 marks]

- 7 An institution has 2000 students. A canteen vendor plans to survey on the types of food the students prefer.

A sample of 400 students is to be selected to take part in the survey. Describe briefly how this sample can be chosen if it is to be

- (i) a simple random sample, [2]
- (ii) a systematic random sample. [2]

- 8 A study of the amount of advertising time on television for a product and the number of sales for the same product in each week was conducted. The results over eight weeks are given below.

Advertising time (minutes) $x$	10	12	15	14	17	16	22	20
Sales (thousands) $y$	2.3	2.8	3.0	3.1	3.2	2.9	5.0	4.0

- (i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]
- (ii) Find the product moment correlation coefficient and comment on its value in the context of the question. [2]
- (iii) Find the equation of the regression line of  $y$  on  $x$ , in the form  $y = mx + c$ , giving the values of  $m$  and  $c$  correct to 3 decimal places. Sketch this line on your scatter diagram. Interpret the value of  $m$  in terms of the amount of advertising time on television and the number of sales. [3]
- (iv) Use the equation of your regression line to calculate an estimate of the sales for the product in a week in which the advertising time is 19 minutes. Comment briefly on the reliability of the estimate. [2]

- 9 (a)** A factory produces plastic cups and they are checked for cracks. It is known that 10% of the cups have cracks.
- (i) Find the probability that in a random check of 10 cups, at least 8 cups have no cracks. [3]
- (ii) A random check is conducted on 250 cups. By using a suitable approximation, find the probability that between 5 and 15 cups inclusive have cracks. [5]
- (b)** In a quality control process, a random sample of 15 cups is tested. The batch is rejected if more than four cups have cracks, and accepted if three or fewer have cracks. If exactly four cups have cracks, another random sample of 10 cups will be tested. The batch is accepted if none of the cups in the second sample have cracks and rejected otherwise. Find the probability that the batch of cups is rejected. [3]
- 10 (a)** The random variable  $X$  has mean 11 and standard deviation 3. The mean of a random sample of 50 observations of  $X$  is denoted by  $\bar{X}$ . Find
- (i)  $P(9.2 < \bar{X} < 12.2)$ , [2]
- (ii) the value of  $c$  for which  $P(\bar{X} < c) = 0.03$ . [1]
- (b)** A soft drink manufacturer produces small and large bottles of drink. The volumes of both sizes of drink are assumed to follow independent normal distributions, with means and standard deviations as shown in the following table.
- |                       | Mean (ml) | Standard deviation (ml) |
|-----------------------|-----------|-------------------------|
| Small bottle of drink | 338       | 3                       |
| Large bottle of drink | 1010      | 12                      |
- (i) Find the probability that one large bottle of drink selected at random has a volume greater than the combined volume of three small bottles of drink selected at random. [3]
- (ii) Find the probability that one large bottle of drink selected at random has a volume larger than three times that of one small bottle of drink selected at random. [3]
- (iii) Small bottles of drink are advertised to have a volume of 340ml. Find the probability that a randomly selected pack of a dozen small bottles of drink has an average volume less than the advertised amount. [2]

- 11 (a)** Given that  $P(A \cap B) = 0.3$ ,  $P(A | B') = 0.2$  and  $P(B) = 0.4$ , find  $P(A)$ . [4]

- (b)** In a multi-storey carpark, 60% of the car owners are men and the rest are women. 90% of the cars belonging to the men are continental cars as are 70% of the cars belonging to the women. Turbochargers are installed to 5% of the non-continental cars and 1% of the continental cars irrespective of whether the car is owned by a man or woman.

If a car is chosen at random, find the probability that it is

- (i)** a continental car with turbocharger and is owned by a man, [2]  
**(ii)** a continental car without a turbocharger. [3]

Another car is chosen at random. Given that it is not a continental car, calculate the probability that the car is owned by a woman. [3]

- 12** Company A claims that the mean monthly salary of a fresh mathematics graduate is \$2300. However, Company B suspects that this is not the correct mean monthly salary and a random sample of 50 fresh mathematics graduates is surveyed. The salaries, \$ $x$ , of these 50 fresh mathematics graduates are summarised by

$$\sum (x - 2300) = 635.78 \qquad \sum (x - 2300)^2 = 130584.32$$

- (i)** Calculate the unbiased estimates of the mean and variance of the monthly salary of a fresh mathematics graduate. [3]  
**(ii)** Test at 10% significance level whether Company B's suspicion is supported. [4]

Company B decides to further its investigation by surveying another random sample of 50 fresh mathematics graduates and the mean of this sample is denoted by \$ $k$ . A test is carried out, at the 5% significance level, to determine whether the mean salary is more than \$2300.

- (a)** State the appropriate hypotheses for the test. [2]  
**(b)** It is given that the null hypothesis is rejected. By using the unbiased estimate of the population variance found in part (i), find the least value of  $k$ . [4]