



RAFFLES INSTITUTION
2014 Year 6 Preliminary Examination

MATHEMATICS

8864/01

Higher 1

16 September 2014

Total Marks: 95

3 hours

Additional Materials : Answer Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and CT group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **9** printed pages.

[Turn over

Section A: Pure Mathematics [35 marks]

- 1** Without using a calculator, solve the inequality

$$2x^2 - 3x - 2 > 0. \quad [2]$$

Hence use a sketch of the graph of $x = e^\theta$ to solve the inequality

$$2e^{2\theta} - 3e^\theta - 2 > 0. \quad [2]$$

- 2** Given that both a and x are real numbers greater than 1, and

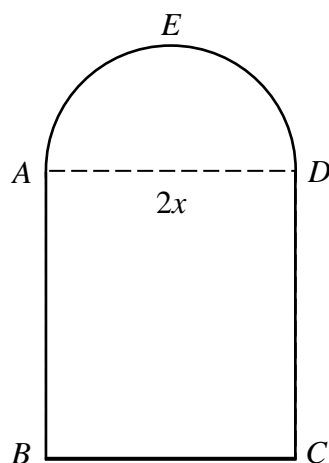
$$\log_a(4 \log_a x) - \log_a(\log_x a) = \log_a 64,$$

find the numerical value(s) of $\log_a x$. [4]

Given further that $a = 3$, write down the exact value(s) of x . [1]

- 3 (a)** Differentiate $\frac{(e^{1-3x})^2}{e^{x+1}}$. [2]

(b) Use a non-calculator method to find the exact value of $\int_{-5}^1 \frac{8}{(7-4x)^2} dx$. [4]



A window in a new building has the shape of a rectangle $ABCD$ joined to a semicircle ADE , as shown in the diagram. It is given that $AD = 2x$ m and the total perimeter $ABCDEA$ of the window is 7 m.

(i) Find the length of AB in terms of x . [2]

(ii) Show that the area of the window, S , is equal to $\left(7x - \frac{\pi+4}{2}x^2\right) \text{ m}^2$. [3]

Use a non-calculator method to find the maximum value of S as x varies. Leave your answer in the form $\frac{P}{2\pi + Q}$, where P and Q are integers to be determined. [4]

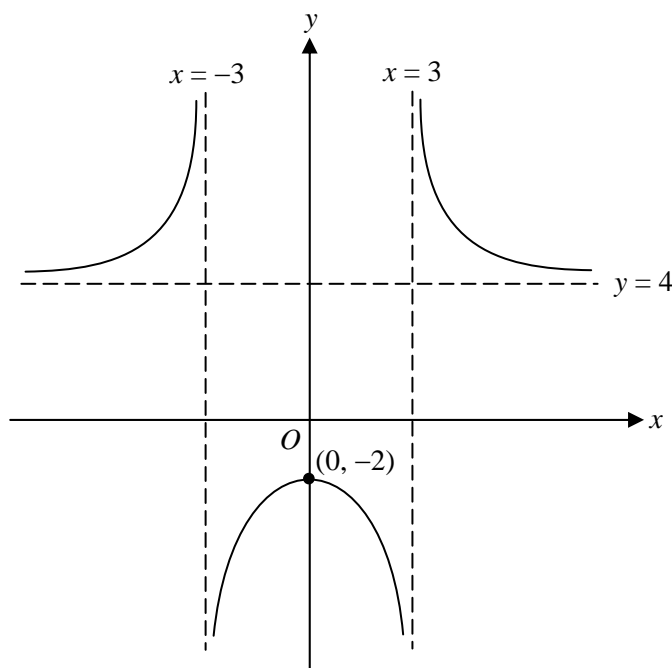
[Turn over

5 Let a, b and c be positive integers.

- (a) Sketch the graph of $y = \frac{ax+b}{x-c}$, labelling clearly, in terms of a, b and c , the asymptotes and the intersections with the axes. [3]

Write down, in terms of a, b and c , an integral that gives the area of the region in the 3rd quadrant bounded by the curve $y = \frac{ax+b}{x-c}$ and the axes. [1]

- (b) The graph of $y = \frac{ax^2+b}{x^2-c}$ is shown below. It passes through $(0, -2)$ and has asymptotes $x = -3, x = 3$ and $y = 4$.



- (i) Find the values of a, b and c . [3]
- (ii) State the range of values of k for which the equation $\frac{ax^2+b}{x^2-c} = k$ has 2 distinct real roots. [2]
- (iii) State the range of values of x for which $\left| \frac{ax^2+b}{x^2-c} \right| = \frac{ax^2+b}{c-x^2}$. [2]

Section B: Statistics [60 marks]

- 6** Consider events A and B , where $P(A) = \frac{3}{8}$, $P(B') = \frac{1}{4}$ and $P(A \cup B) = \frac{7}{8}$.
- (i) Write down $P(B)$ and find $P(A \cap B)$. [2]
 - (ii) Determine whether events A and B are independent. [1]
 - (iii) Find $P(A' | B')$. [2]
- 7** In a particular country, public transport is organised through five transportation hubs. To regulate the transportation cost for commuters aged 21 to 60 years old, a study is to be conducted on the average amount of money spent per month on public transport. For the purpose of this study, 60 commuters from each of the 4 groups: Group I (aged 21 to 30 years old), Group II (aged 31 to 40 years old), Group III (aged 41 to 50 years old) and Group IV (aged 51 to 60 years old) were identified at a randomly chosen transportation hub, and were asked their expenditure per month on public transport.
- (i) State the name given to this method of sampling. [1]
 - (ii) Explain, in the context of the question, an advantage and a disadvantage for this method of sampling. [2]
 - (iii) Suggest how you would improve the selection procedure in the context of this question. [1]
- Based on the responses given by the commuters surveyed, the average expenditure for Group I and Group III are both \$100, and the average expenditure for Group II and Group IV are both \$50.
- (iv) Suggest a value for the product moment correlation coefficient between the average amount of money spent and the age of the commuter. Give a reason for the choice of your value. [2]

[Turn over]

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- 8** Two fair dice are thrown. Write down the probability that the total score obtained is more than 10. [1]

Box *A* contains 2 green balls and 6 blue balls. Box *B* contains 3 green balls and 1 blue ball. Two fair dice are thrown, and Box *A* is selected if the total score obtained is more than 10 or less than 4. Otherwise, Box *B* is selected. One ball is then chosen from the selected box and its colour noted.

Draw a tree diagram to represent this situation. [3]

Find the probability that

- (i) the ball chosen is green, [2]
- (ii) the ball chosen is from Box *A*, given that its colour is blue. [2]
- 9** The test scores for Mathematics Paper 1 (x) and Paper 2 (y) for a certain class of students are given in the table below:

x	22	23	24	25	25	26	28	30	31	32	35	36	38	40	42	43
y	28	24	26	21	28	28	20	25	28	30	37	38	40	50	43	37

The full mark for each paper is 50.

- (i) Draw a scatter diagram for these values. [2]
- (ii) Calculate the product moment correlation coefficient between x and y . [1]

The regression line of y on x is given by $y = -0.120 + 1.010x$ (corrected to 3 decimal places) and the regression line of x on y is given by $x = a + by$.

- (iii) Find the values of a and b , giving your answer to 3 decimal places. [2]
- (iv) One student from the class is absent for Mathematics Paper 2. Her Paper 1 test score is 50. Use an appropriate equation to estimate her Paper 2 test score. Comment on your result. [2]
- (v) Another student from the class is absent for Mathematics Paper 1. His Paper 2 test score is 26. Use an appropriate equation to estimate his Paper 1 test score. Comment on your result. [2]

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- 10** The masses, in g, of small and large packets of paperclips are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean mass	Standard deviation
Small packets	50	2
Large packets	100	$\sqrt{10}$

One small packet and one large packet of paperclips are chosen at random. Find the probability that

- (i) the total mass of the two packets is at most 154 g, [2]
- (ii) the mass of the small packet is at most 52 g and the mass of the large packet is at most 102 g, [2]
- (iii) Explain briefly why the answer to (ii) is smaller than the answer to (i). [1]

Let p be the probability that the mass of the large packet exceeds twice the mass of the small packet by at least 5 g.

- (iv) Find p , correct to 4 decimal places. [3]
- (v) Write down, in terms of p , the probability that the mass of the large packet and twice the mass of the small packet differ by less than 5 g. [2]

[Turn over]

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- 11** The traffic police department from a small town is proposing to pass a new law on speeding. To justify the passing of the law, they claim that the number of speeding offences has increased significantly over the last few years.

The number of speeding offences reported per day, x , for the period June - August 2014, is as follows:

x	3	4	5	6	7	8	9	10
Frequency, f	1	6	24	38	13	6	2	2

- (i) Show that the mean for this data is 6 and calculate its variance, giving your answer as a fraction in its lowest term. [2]

Long-term records showed that the number of speeding offences reported per day has mean 5.7 and variance 1.5.

- (ii) Test, at the 1 % level of significance, whether the mean number of offences reported per day has increased. State appropriate hypotheses for the test, and define any symbols used. [4]
- (iii) Explain, in the context of the question, what is meant by "1 % level of significance". [1]

Speeding at least 40 km/h above the stipulated speed limit is considered a serious offence. To further support the passing of the law, the traffic police department also studied the number of serious offences, y , amongst those reported daily. The figure for the same period June - August 2014 is summarised as follow:

$$\Sigma(y - 3.2) = 27.6, \quad \Sigma(y - 3.2)^2 = 123.28.$$

Long-term records showed that the number of serious offences reported per day follows a normal distribution with a mean of 3.2.

- (iv) Find, to 1 decimal place, the minimum level of significance (in %) that would provide further evidence to support the passing of the law. [3]

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- 12** A factory produces 2000 writing pads daily. According to the records of the factory, the probability that a writing pad produced is substandard is 0.013.

- (i) Find the expected number of substandard writing pads produced in a day. [1]
- (ii) Use a suitable approximation to find the probability that there are no more than 31 substandard writing pads produced in a day. [4]

The writing pads are packed into boxes of 10 for retailing purpose.

- (iii) Show that the probability that a randomly chosen box contains substandard writing pads is 0.1227, correct to 4 decimal places. [2]

On a particular day, the factory received an order of 52 boxes to be delivered to a local book store.

- (iv) Find the probability that at most 3 of the boxes delivered contain substandard writing pads. [2]
- (v) Find the probability that the mean number of substandard writing pads delivered per box is less than 0.1. [3]

— End of Paper —