

**NANYANG JUNIOR COLLEGE**  
**JC2 PRELIMINARY EXAMINATION**

Higher 1

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**MATHEMATICS**

**8864/01**

Paper 1

**16 September 2014**

**3 hours**

Additional Materials:      Cover sheet  
                                 Answer papers  
                                 Graph paper  
                                 List of Formulae (MF15)

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**READ THESE INSTRUCTIONS FIRST**

Write your name and class on every script you hand in.  
Write in dark blue or black pen on both sides of the paper.  
You may use a soft pencil for any diagram or graph.  
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

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This document consists of 5 printed pages.



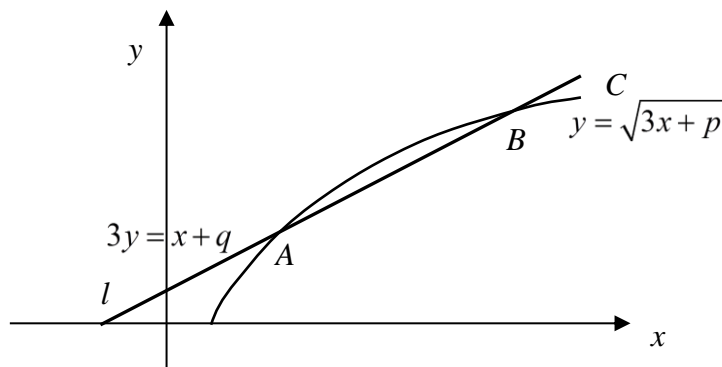
NANYANG JUNIOR COLLEGE  
Internal Examinations

## Section A: Pure Mathematics [35 marks]

- 1 Find the set of values of  $m$  for which  $(5m-3x)^2 > 10-x^2$  for all real values of  $x$ . [4]

- 2 Differentiate  $\ln\left(\frac{e^{x-ax^2}}{\sqrt{1+4x^2}}\right)$  where  $a$  is a constant. [3]

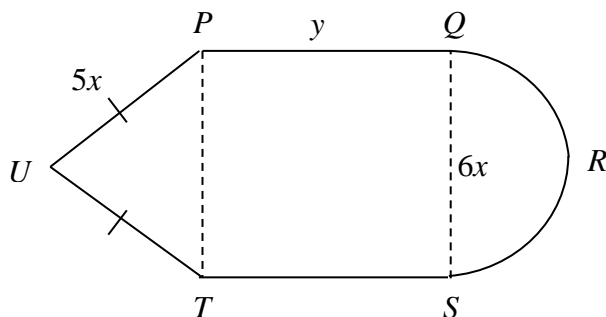
3



The above figure shows the curve  $C$  with equation  $y = \sqrt{3x+p}$  and the line  $l$  with equation  $3y = x + q$ , where  $p$  and  $q$  are constants. The line  $l$  cuts the curve  $C$  at the points  $A$  and  $B$  with  $x$ -coordinates 6 and 9 respectively.

- (i) Find the value of  $q$  and show that  $p = -2$ . [3]
- (ii) Using integration, find the exact area bounded by  $C$  and  $l$ . [3]
- (iii) The point  $R$  on  $C$  is such that the tangent to  $C$  at  $R$  is parallel to  $l$ . Find the point of intersection between the tangent to  $C$  at  $R$  and the line  $y = -x$ . [4]
- 4 (i) Find the exact root of the equation  $3e^{-x} - e^{2-3x} = 0$ . [2]
- (ii) Using differentiation, find the exact coordinates of the stationary point on the curve  $C$ , given by the equation  $y = 3e^{-x} - e^{2-3x}$ . [2]
- (iii) Sketch  $C$ , stating the exact coordinates of the points of intersection with the axes and the stationary point. [2]
- (iv) Use your calculator to find the area of the region bounded by  $C$ , the lines  $y = \frac{2}{e}$ ,  $y = 0$  and  $x = 0$ . [2]

- 5 A piece of wire of length 300 cm is bent to form the perimeter of a closed figure,  $PQRSTUP$ , as shown in the diagram.  $QRS$  is a semicircle with diameter  $6x$  cm,  $PQST$  is a rectangle and  $PTU$  is an isosceles triangle with  $PU = TU = 5x$  cm.



- (i) Given that  $PQ = TS = y$  cm, show that the area enclosed by the figure,  $A \text{ cm}^2$ , is given by  $A = 900x - \frac{9}{2}(4 + \pi)x^2$ . [3]
- (ii) Use differentiation to find the maximum value of  $A$  as  $x$  varies, giving your answer in exact form, explaining briefly how you can tell it is a maximum rather than a minimum. [4]
- (iii) Find the range of values of  $x$ , given that  $3000 \leq A \leq 5000$ . Show clearly how you obtain your answer. [3]

### Section B: Statistics [60 marks]

- 6 A telephone service provider has 2000 subscribed phone numbers in its database. A few months ago, it rolled out a new mobile service. To obtain feedback about this new service, surveyors were engaged to conduct a survey with 80 subscribers from the database.
- (i) Describe how this can be done using systematic random sampling. [2]
- (ii) State one disadvantage of this method in the context of the question. [1]
- 7 In a practice session, a soccer player takes 3 penalty kicks. The probability that he scores in the first kick is 0.7. For each of his second and third kicks, the probability of him scoring
- decreases by 0.2 if he failed to score in the preceding kick,
  - increases by 0.1 if he scored in the preceding kick.
- (i) Draw a tree diagram to represent this situation. [3]
- (ii) Find the probability that the player scores 2 penalty kicks. [2]
- (iii) Given that he scored in 2 out of the 3 penalty kicks, what is the probability that he scores in the third kick? [3]

[Turn Over]

- 8** A school has an enrolment of 1000 students. There are 450 students with a Sports CCA and 500 students with an Aesthetic CCA. Some students have both a Sports CCA and an Aesthetic CCA while 650 have only either a Sports CCA or an Aesthetic CCA but not both. Find the probability that a student chosen at random
- (i) has both a Sports CCA and an Aesthetic CCA; [2]
  - (ii) has a Sports CCA given that he has an Aesthetic CCA. [2]
- 9** In Yummy Orchard, kiwi fruits are harvested monthly for sale. On average, out of every 6 kiwis harvested, 5 are juicy. To maintain the quality of the fruits in the plantation, 80 kiwis are harvested every month.
- (i) Find the expected number of juicy kiwis in a particular month. [1]
  - (ii) Find the probability that more than 70 kiwis harvested in a particular month are juicy. [2]
  - (iii) Three months are chosen at random. Find the probability that one month has more than 70 juicy kiwis harvested while the other two months each has at most 70 juicy kiwis harvested. [3]
  - (iv) The farmer harvested the plantation for 10 consecutive years, with 80 kiwis harvested every month. Using a suitable approximation, find the probability that there are more than 12 months but at most 24 months in which more than 70 juicy kiwis are harvested monthly. [4]
- 10** A study was done to determine whether the heights of 10-year-old boys ( $y$ ), in meters, are related to the heights of their fathers ( $x$ ), in meters. Eight pair of father and sons' heights were measured and their data are shown in the table below.

$x$	1.70	1.72	1.73	1.75	1.76	1.79	1.78	1.80
$y$	1.51	1.53	1.54	1.52	1.57	1.59	1.60	1.62

- (i) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]
  - (ii) It was found that one of the pairs of data was wrongly recorded. With the aid of a scatter diagram, deduce which pair it is, and label it  $P$  on your diagram. [2]
- Omit  $P$  from the data when answering the subsequent parts of the question.
- (iii) Calculate the new product moment correlation coefficient. [1]
  - (iv) Find an extra set of point that can be added without changing the new product moment correlation coefficient. [1]
  - (v) Find the equations of the least squares regression lines of  $y$  on  $x$  and  $x$  on  $y$ . [2]
  - (vi) A 10-year-old boy has height 1.65 m. Use an appropriate line to estimate the height of his father and comment on the reliability of the estimate. [2]

- 11** Studies have shown that the time taken by a person to complete a psychological test  $A$  follows a normal distribution with mean  $\mu$  minutes and standard deviation 4 minutes.

- (i) Find  $\mu$  if 80% of the people who take the test take at least 61.6 minutes to complete the test. [2]
- (ii) 10 people are chosen at random to take the test. Find the probability that less than 7 people take at least 61.6 minutes to complete the test. [2]
- (iii)  $N$  people are chosen at random to take the test. There is a probability of 0.05 that the total time taken by these  $N$  people to complete the test is more than 15 hours. Show that an equation satisfied by  $N$  is given by

$$N + a\sqrt{N} + b = 0,$$

where  $a$  and  $b$  are to be determined. Hence find  $N$ . [4]

In another psychological test  $B$ , the time taken by a person to complete the test is normally distributed with mean 30 minutes and variance 9 minutes<sup>2</sup>.

- (iv) A person is randomly chosen to take both tests  $A$  and  $B$ . Find the probability that the time taken to complete test  $A$  is less than twice the time taken to complete test  $B$ . [3]

- 12** The principal of a school claims that the mean Mathematics score for the National Examination is 78. A random sample of 80 students from the school is taken. Their mathematics scores,  $t$ , are summarised by

$$\sum(t - 78) = -38, \quad \sum(t - 78)^2 = 482.$$

Find unbiased estimates of the population mean and variance. [3]

- (i) Test, at a 5% level of significance, whether the principal has overstated their mean Mathematics score. [4]
- (ii) State, giving a reason, whether it is necessary to assume that the Mathematics score in the school is normally distributed. [1]
- (iii) Explain the meaning of “at the 5% level of significance” in the context of the question. [1]

Another large random sample of  $n$  students gives a mean Mathematics score of 78.6. Given that the population standard deviation of the Mathematics score is now known to be 2.42, find the smallest possible value of  $n$  to conclude that the principal had understated the mean Mathematics score at the 3% level of significance. [5]

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