

INNOVA JUNIOR COLLEGE  
JC 2 PRELIMINARY EXAMINATION 2  
in preparation for General Certificate of Education Advanced Level  
**Higher 1**

CANDIDATE  
NAME

CLASS

INDEX NUMBER

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**MATHEMATICS**

**8864/01**

**16 September 2014**

**3 hours**

Additional Materials:      Answer Paper  
                                    Cover Page  
                                    List of Formulae (MF 15)

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**READ THESE INSTRUCTIONS FIRST**

**Do not open this booklet until you are told to do so.**

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper. You may use a soft pencil for any diagrams or graphs.

**Do not use staples, paper clips, highlighters, glue or correction fluid.**

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

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This document consists of **6** printed pages.



## Section A: Pure Mathematics [35 marks]

1 (a) Find  $\int \left( \frac{2}{x} - 1 \right)^2 dx$ . [2]

(b) Find the exact value of  $a$  such that

$$\int_{\frac{2}{3}}^a \frac{1}{3x-1} dx = \int_0^4 \frac{1}{\sqrt{x}} dx. \quad [3]$$

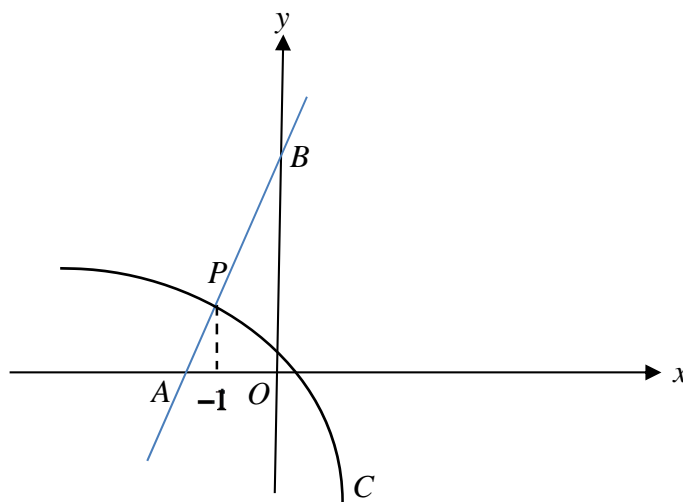
2 The temperature,  $T$  °C, of a turkey removed from a freezer is given by the formula  $T = 24 - 29e^{-0.3t}$ , where  $t$  is the time in hours since the turkey was removed from the freezer.

(i) Find the value of  $t$  when  $T = 18$ . [2]

(ii) Find the rate of change of  $T$  when  $t = 10$ . [2]

(iii) Explain why the temperature of the turkey can never reach 24 °C. [1]

3

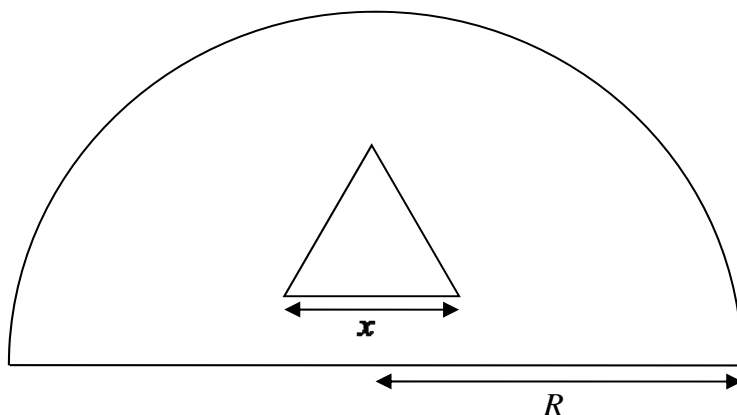


The curve  $C$  has equation  $y = \ln(x^2 - 3x + 2)$ . Part of  $C$  is shown in the diagram above.  $O$  is the origin and  $P$  is the point on  $C$  for which  $x = -1$ . The normal to  $C$  at the point  $P$  meets the  $x$ -axis at  $A$  and the  $y$ -axis at  $B$ . Without using a calculator,

(i) find the equation of the normal at  $P$ . Leave your answer in the form  $y = ax + b + \ln c$ , where  $a$ ,  $b$  and  $c$  are constants to be determined. [4]

(ii) find the exact coordinates of  $A$  and  $B$  and hence show that the exact area of triangle  $OAB$  is  $\frac{1}{60}(p + q \ln 6)^2$ , where  $p$  and  $q$  are integers to be found. [4]

- 4 The diagram shows a semicircular garden of radius  $R$  m. An equilateral triangle of side  $x$  m is set aside in the garden to build a fountain. The remaining part of the garden is then covered with flowers.



- (i) Show that the area of the fountain is  $\frac{\sqrt{3}}{4}x^2$  m<sup>2</sup>. [2]

The difference between the diameter of the garden and the side of the fountain is 20 m. The area covered with flowers is  $A$  m<sup>2</sup>.

- (ii) Show that  $A = 50\pi + 5\pi x + \left(\frac{\pi}{8} - \frac{\sqrt{3}}{4}\right)x^2$ . [3]
- (iii) Hence, find the value of  $x$  for which  $A$  is a maximum. [4]

- 5 (i) Sketch, on a single diagram, the graphs of

$$C_1 : y = \frac{3}{x+3} \quad \text{and} \quad C_2 : y = 4e^{-x^2},$$

stating clearly the axial intercepts and the equations of the asymptotes of the graphs. [3]

- (ii) State the  $x$ -coordinates of the points of intersection of  $C_1$  and  $C_2$ . Hence, or otherwise, find the range of values of  $x$  for which  $\frac{e^{x^2}}{x+3} > \frac{4}{3}$ . [3]
- (iii) Find the numerical value of the area of the region bounded by  $C_1$  and  $C_2$ . [2]

**Section B: Statistics [60 marks]**

- 6** Ace Junior College has 1000 students of which 60% are in JC1 level and the rest are in JC2 level. A survey is to be conducted to find out the students' opinions on the current academic programmes of the college. A sample of 100 students is taken.

(i) Describe how systematic sampling could be used to obtain the sample. [2]

The college Administrative Manager suggests interviewing the first 60 JC1 students and the first 40 JC2 students that come to college on a particular day.

(ii) Name this sampling method and state one disadvantage of this method in this context. [2]

- 7** Over a long period of time, it is found that 73% of the population in country A can roll their tongues. A group of 16 people is randomly selected from the population. The number of people in this group who can roll their tongues is the random variable  $R$ .

(i) State, in context, an assumption needed for  $R$  to be well modelled by a binomial distribution. [1]

Assume now that  $R$  has a binomial distribution. Find the probability that, in this group,

(ii) exactly 12 can roll their tongues, [2]

(iii) at least 10 can roll their tongues. [2]

A group of 50 people is chosen at random. Use a suitable approximation to estimate the probability that more than 38 people can roll their tongues. State the mean and variance of the distribution that you use. [4]

- 8** (i) A company produces brand A cell phones. Given that 36.9% of brand A cell phones has battery life time of less than 6.8 hours and 52.2% of them has battery life time of more than 7.8 hours. Assuming a normal distribution, find the mean lifetime of a randomly chosen brand A cell phone. [4]
- (ii) Another company produces brand B cell phones with mean life time 8.5 hours and standard deviation 3.61 hours. A random sample of 60 brand B cell phones is taken. Calculate the probability that the mean battery life time of this sample lies between 7.8 and 9.1 hours. [3]

- 9** A computer shop in Sim Nan Centre sells thumb drives. The number of thumb drives,  $x$ , sold in each week together with the profits,  $y$  dollars, from the sale of the thumb drives are given in the following table.

$x$	11	12	14	15	16	18	20	21
$y$	29	34	41	40	43	46	51	54

- (i) Give a sketch of the scatter diagram for the data as shown on your calculator. [2]
  - (ii) Calculate the product moment correlation coefficient and comment on its value in relation to your scatter diagram. [2]
  - (iii) Find the equation of the regression line of  $y$  on  $x$ , expressing your answer in the form  $y = mx + c$ . Hence, or otherwise, explain the meaning of  $m$  in the context of the data. [2]
  - (iv) Using an appropriate regression line, estimate the number of thumb drives that needs to be sold in a week if a profit of 40 dollars is to be made. Give your answer to the nearest whole number. [2]
  - (v) The owner of the shop uses the regression line in part (iii) to estimate the profit when 25 thumb drives were sold in a week. Comment on the reliability of this estimation. [1]
- 10** In a lucky draw, there are 20 prizes allocated at random to 20 prize-winners. Ashley, Bernie and Charles are three of the prize-winners. Of the prizes, 5 are laptops, 7 are cameras and 8 are watches.
- (i) Show that the probability that Ashley gets a laptop and Bernie gets either a camera or a watch is  $\frac{15}{76}$ . [1]
  - (ii) Find the probability that either Ashley or Charles (or both) gets a laptop. [2]
  - (iii) Find the probability that Bernie gets a camera given that Charles gets either a camera or a watch. [3]

- 11** A factory produces fruity bars which the manufacturer claims contain 30 grams each, on average. A random sample of 60 fruity bars is taken and their masses,  $x$  grams, are recorded. The results are summarized by

$$\sum (x - 30) = -15 \quad \text{and} \quad \sum (x - 30)^2 = 160.$$

- (i) Find the unbiased estimates of the population mean and variance. [3]
- (ii) Test, at the 5% significance level, whether the manufacturer is overstating the average mass of a fruity bar. [5]
- (iii) State, giving a reason, whether it is necessary to assume a normal distribution for the test to be valid. [1]

The manufacturer purchases new machinery to produce the fruity bars. The mass of the fruity bars are normally distributed with standard deviation 1.6 grams. The manufacturer claims the mean mass is now 35 grams. A new random sample of 60 fruity bars is taken.

- (iv) Find the range of values within which the mean mass of this sample must lie for the manufacturer's new claim to be rejected at the 5% level of significance. [4]

- 12** The time taken, in seconds, for boys and girls in a Junior College to run 100 metres have independent normal distributions. The means and standard deviations of these distributions are shown in the following table.

	Mean (in second)	Standard deviation (in second)
Boy	19.6	2.3
Girl	23.1	3.2

- (i) Find the probability that the time taken by a randomly chosen boy to run 100 m is between 15 and 18 s. [1]
- (ii) Find the probability that the total time taken by two randomly chosen girls to run 100 m is more than twice the time taken by a randomly chosen boy to run 100 m, stating clearly the mean and variance of the distribution that you use. [5]

During the college sports meet, a team of 2 randomly chosen boys and 2 randomly chosen girls is required to participate in a 4×100 m novelty relay race.

Stating clearly the mean and variance of all distributions that you use, find the probability that

- (iii) the total time taken by the team to complete the 400 m is between 55 and 80 s. [3]
- (iv) the total time taken by the team to complete the 400 m is less than 75 s, given that the total time taken by the team is between 55 and 80 s. [3]