

**ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT**

**MATHEMATICS
Higher 1**

8864 / 01

Paper 1

26 August 2014

JC 2 PRELIMINARY EXAMINATION

Time allowed: **3 hours**

Additional Materials: List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Index number, Form Class, graphic and/or scientific calculator model/s on the cover page.

Write your Index number and full name on all the work you hand in.

Write in dark blue or black pen on your answer scripts.

You may use a soft pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in the question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

At the end of the examination, fasten all your work securely together.

This document consists of **8** printed pages.



Anglo-Chinese Junior College

[Turn Over]

ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT
JC2 Preliminary Examination 2014

MATHEMATICS 8864
Higher 1
Paper 1

/ 95

Index No:

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Form Class: _____

Name: _____

Calculator model: _____

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

Question no.	Marks
1	/3
2	/5
3	/7
4	/9
5	/11
6	/3
7	/3

Question no.	Marks
8	/4
9	/5
10	/11
11	/11
12	/11
13	/12

Summary of Areas for Improvement

Knowledge (K)	Careless Mistakes (C)	Read/Interpret Qn wrongly (R)	Formula (F)	Presentation (P)

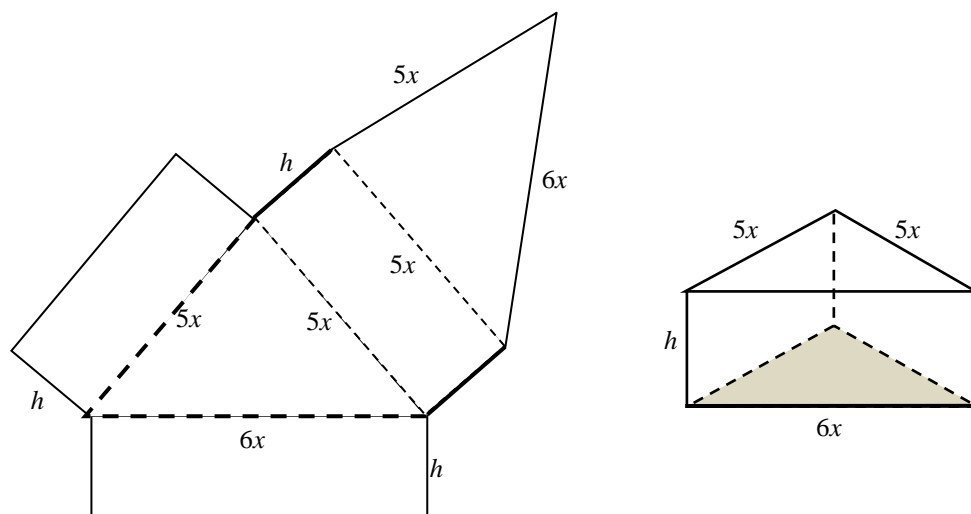
Section A: Pure Mathematics [35 marks]

- 1 The quadratic equation $2x^2 - 2x\sqrt{3} + c = c(x^2 + 2)$, $c \neq 2$ has distinct real roots. Find the range of values of c . [3]

- 2 (i) Show that $\frac{d}{dx} \left[\ln(4x^2 + 1)^2 \right] = \frac{kx}{1 + 4x^2}$, where k is to be determined. [2]

- (ii) Hence, or otherwise, find the exact positive value of a if $\int_0^a \frac{x}{4x^2 + 1} dx = \frac{1}{8} \ln 17$. [3]

3



A piece of cardboard is cut into the shape shown above. The cardboard is then folded along the dotted lines to form a prism of depth h and its cross-section is an isosceles triangle. Given that the volume of the prism is 1200 units^3 .

- (i) Show that $h = \frac{100}{x^2}$. [2]

- (ii) Show that the total surface area, $S \text{ units}^2$, of the prism is given by

$$S = 24x^2 + \frac{1600}{x}. \quad [2]$$

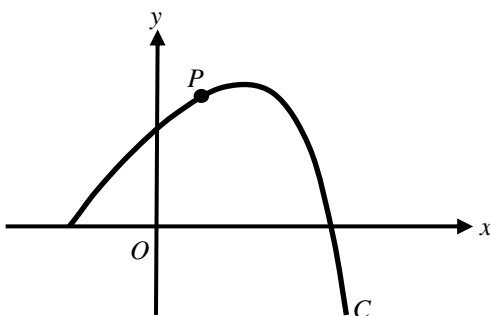
- (iii) Find the value of x for which S has a stationary value. Determine whether S is a maximum or minimum. [3]

[Volume of prism = base area \times height]

[Turn over]

- 4 (i) Find the exact root of the equation $e^{-x}(2e^{-x} + 1) = 15$. [3]
- (ii) Sketch the curve C with equation $y = e^{-x}(2e^{-x} + 1) - 15$, stating the equation of the horizontal asymptote and the exact coordinates of any points where the curve C crosses the axes. [3]
- (iii) Find the exact area of the region bounded by C , the x -axis and the lines $x = -2$ and $x = -1$. [3]

- 5 (a) The curve C has equation $y = 6 - e^{2x-1} + 5x$ as shown in the diagram below.



O is the origin and $P\left(\frac{1}{2}, \frac{15}{2}\right)$ is a point on C . The tangent to C at P meets the x -axis at R and the normal to C at P meets the x -axis at Q .

Find the coordinates of Q and R . [6]

Hence find the area of triangle PQR . [1]

- (b) On the same diagram, sketch the graphs $y = \frac{x+2}{x-1}$ and $y = 1 + \ln(x-1)$, $x > 1$, including the coordinates of the points where the graphs cross the axes and the equations of any asymptote(s). [2]

Find the x -coordinate(s) of the point(s) of intersection of the curves $y = \frac{x+2}{x-1}$ and $y = 1 + \ln(x-1)$. [1]

Hence solve the inequality $\frac{x+2}{x-1} < 1 + \ln(x-1)$. [1]

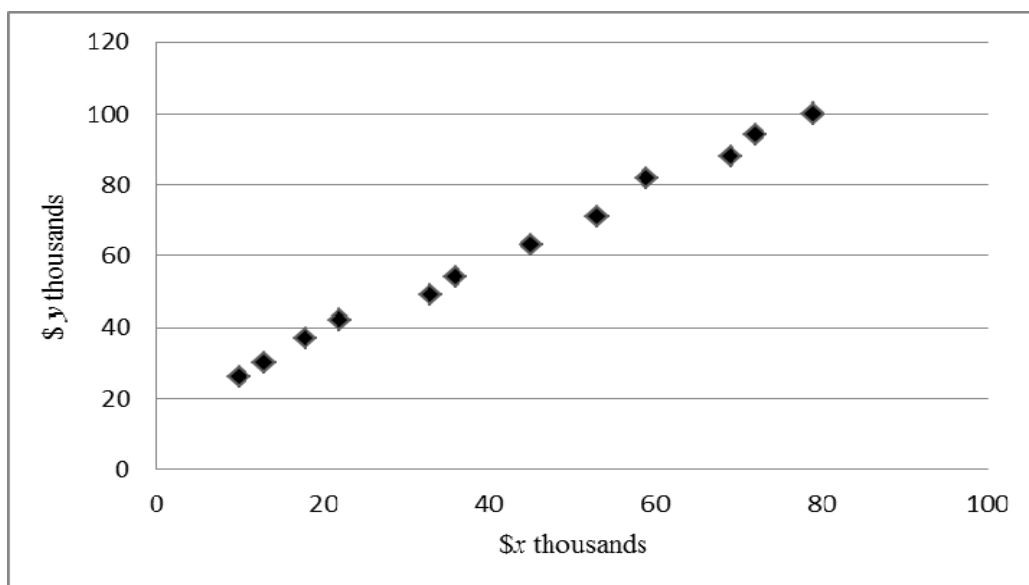
Section B: Statistics [60 marks]

- 6 A hardware store packages washers in bags of eighty. If washers have a mean mass of 12 g and a standard deviation of 5g, what is the probability that a bag will have mass more than 1 kg. [3]
- 7 A company which employs 300 staff wishes to improve its image by introducing new uniforms for the staff to wear. The human resources manager would like to obtain the views of the staff. She has an alphabetical (by surname) list of all the staff members. She decides to survey a sample of 30 staff members. For convenience, she decides to select the 10th, 20th, 30th, 300th name in the list.
- (i) Name the type of sampling the manager is attempting to use. [1]
- (ii) State in this context, one weakness in her method of using this sampling method and describe briefly how she might choose a more appropriate sample using the same sampling method. [2]
- 8 A and B are two events such that $P(A) = \frac{7}{16}$, $P(B) = \frac{2}{3}$ and $P(B|A') = \frac{1}{3}$.
- (i) Find $P(A \cap B)$. [3]
- (ii) State with a reason whether A and B are independent. [1]
- 9 A fair coin and a fair six-sided die are thrown simultaneously. The random variable X is defined as follows:
- If the coin shows a head, then X is the score on the die.
- If the coin shows a tail, then X is twice the score on the die.
- (i) A player throws the coin and the die once.
- (a) Show that the probability of the score being 4 or less is 0.5. [2]
- (b) Find the conditional probability that the coin shows a head, given that the score is more than 4. [2]
- (ii) Another player throws the coin and the die 8 times. Find the probability that all of the 8 scores are 4 or less. [1]

[Turn over]

- 10** A firm wishes to assess whether there is a linear relationship between the annual amount spent on advertising, x thousand dollars, and the annual profit, y thousand dollars. The following table and scatter diagram show the data for 12 years.

Year	1	2	3	4	5	6	7	8	9	10	11	12
x	18	36	45	22	69	72	13	33	59	79	10	53
y	37	54	63	42	84	91	33	49	79	98	32	71



- (i) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]
- (ii) A manager claims that this result shows that spending more money on advertising in the future will result in greater profits. Make two criticisms of this claim. [2]
- (iii) Find the equation of the regression line of y on x , in the form $y = a + bx$, giving the values of a and b correct to 3 significant figures. [1]
- (iv) Give a practical interpretation of the values of a and b . [2]
- (v) Use your equation to estimate the profit when the amount spent on advertisement is \$50,000. Comment on the reliability of your estimate. [2]
- (vi) State whether you should use the regression line of y on x or the regression line of x on y in order to give the best estimate of the amount spent on advertisement when the profit is \$80,000. Give a reason for your answer. [2]

- 11** The marks of students taking a Mathematics examination were normally distributed with mean m . The marks x , of 400 randomly chosen students whose scripts were marked by a particular examiner were recorded and are summarised as follows.

$$\sum (x - 50) = 3000 \qquad \sum (x - 50)^2 = 233571$$

- (i) Find the unbiased estimates of the population mean and variance. [2]
 - (ii) If $m = 60$, test at the 5% significance level whether the examiner was marking too strictly is justified. [4]
 - (iii) If $m = 60$ and a test is carried out at the $\alpha\%$ level of significance, find the set of values of α in order to justify that the marker was not marking too strictly. [1]
 - (iv) Find the set of possible values of m , such that the claim that the marker is marking too strictly at 5% significance level is justified. [4]
- 12** Electrical components are packed independently in boxes of 48. The number of defective electrical components in a box is denoted by D . $p\%$ of the components in a box are defective. The probability that a randomly selected box contains fewer than 1 defective component is 0.30065.
- (i) State, in context, an assumption needed for D to be well modelled by a binomial distribution. [1]
 - (ii) Write down an equation for the value of p , and find this value numerically. [3]
 - (iii) Use a suitable approximation to find the probability that, in 100 randomly selected boxes, less than 36 boxes contain fewer than 1 defective component in each box. [4]
 - (iv) 100 boxes are selected at random. Find the probability that the average number of defective components in a box exceeds 1. [3]
- 13 (a)** The random variable X has a normal distribution with mean 50 and standard deviation 8.
- (i) Find the positive value of a such that $P(50 - a < X < 50 + a) = 0.95$. [3]
 - (ii) The random variable Y is related to X by the formula $Y = bX - 40$, where b is a positive constant. Find the expressions of $E(Y)$ and $\text{Var}(Y)$ in terms of b . Given that $P(Y < 74) = 0.0668$, find the value of b . [4]

[Turn over]

- (b) The amount of time spent reading emails per day by a teenager follows a normal distribution with mean 2.3 hours and standard deviation $\sqrt{45}$ minutes. The amount of time spent reading emails per day by an adult follows a normal distribution with mean 2.8 hours and standard deviation $\sqrt{50}$ minutes.
- (i) An adult and a teenager are chosen at random. Find the probability that the adult and teenager each spends less than 2.5 hours reading emails per day. [2]
- (ii) Find the probability that the total time spent reading emails by three randomly chosen teenagers exceeds twice the amount of time spent reading emails by a randomly chosen adult by more than 2 hours. [3]

~ End of Paper ~