

CATHOLIC JUNIOR COLLEGE
General Certificate of Education Advanced Level
Higher 1
JC2 Preliminary Examination

MATHEMATICS

8864/01

Paper 1

28 August 2014

3 hours

Additional Materials: List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, arrange your answers in **NUMERICAL ORDER**.

Place the cover sheet given in front and fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **5** printed pages, including the cover page.

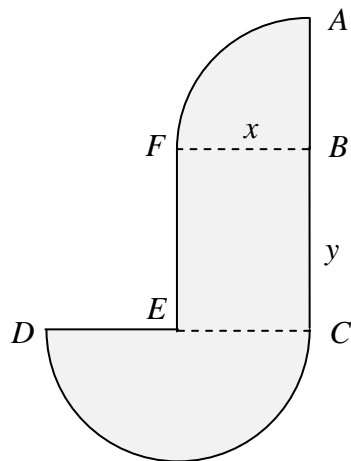


Catholic Junior College

Section A: Pure Mathematics [35 marks]

- 1** The function f is given by $f(x) = kx^2 + \sqrt{98}x + \frac{k}{2}$, where k is a constant.
- (i) Find, algebraically, the set of values of k for which the function $f(x)$ is always positive for all real values of x . [4]
- (ii) It is given that $k = 2$. Find the exact values of m such that $f(\sqrt{2}m) = -9$. [2]

2



The diagram shows a J-shaped piece of cardboard $ABCDEF$. It is made up of a quadrant ABF of radius x cm, a rectangle $BCEF$ of breadth x cm and length y cm, and a semicircle CDE of radius x cm, as shown in the diagram. It is given that the perimeter of the cardboard is 30cm. The area of the cardboard is A cm².

- (i) Find an expression for A in terms of x . [3]
- (ii) Use differentiation to find the value of x which gives a maximum value of A . [3]

3 (a) Differentiate

(i) $2x + \ln(5x^2 + 1)$, [2]

(ii) $\frac{3}{2x+1}$. [2]

- (b) Without using a calculator, find the exact value of $\int_1^3 \frac{4}{(x+1)^4} dx$, simplifying your answer. [3]

4 Two graphs are defined as $C: y = \frac{10x-21}{x-2}$ and $L: y = 2x$.

- (i) Sketch the graphs of C and L on a single diagram, stating clearly the equations of any asymptotes and the exact coordinates of any intersection with the axes. [3]
- (ii) Find the x -coordinates of the points of intersection of the two graphs, C and L , giving your answer correct to 4 decimal places. [2]
- (iii) Write down, as an integral, an expression for the area of the region bounded by the two graphs, C and L . Evaluate this integral. [2]

[Turn Over]

5 The curve C has equation $y = 2^x - x^3$.

- (i) Find the numerical value of the gradient of C at the point P where $x = 2$. [1]
- (ii) Hence find the equation of the normal to C at P , leaving your answer in the form of $y = mx + c$. [4]
- (iii) This normal meets the curve C again at Q , and the y -axis at R . Find the distance QR . [4]

Section B: Statistics [60 marks]

6 On a particular morning, there are 100 students in the college library. A teacher wishes to conduct a survey in the college to find out how students use the library to study. He decides to survey a sample of 10 students in the library on that particular morning.

- (i) Describe how the teacher can obtain a systematic sample of 10 students to take part in the survey on that particular morning. [2]
- (ii) State, in this context, one disadvantage of this procedure. [1]
- (iii) Describe briefly how, in this case, the teacher might choose a more appropriate systematic sample. [1]

7 In a college, each class has 27 students. Over a long period of time, it is found that, in the college, the probability that a student will be late for school on any given day is 0.04, independently of any other student.

For a randomly chosen class, find

- (i) the expected number of students who will be late for school on any given day, [1]
- (ii) the probability that at least two students from the class will be late for school on any given day, [2]
- (iii) the probability that for a week with 5 school days, only Monday is the day that at least two students from the class will be late for school. [2]

It is given that there are 70 classes in the college. Using a suitable approximation, find the probability that at least 10 classes in the college which have at least two students from each class who will be late for school only on Monday, in a given week. [4]

[Turn Over

- 8 Students on a Mathematics Olympiad course were given a Numeracy test and a Mental Arithmetic test. The test marks for the 8 students on the course are given in the table below.

Student	A	B	C	D	E	F	G	H
Numeracy test marks, x	6.0	6.5	7.2	7.5	8.1	8.5	9.0	9.8
Mental Arithmetic test marks, y	153	158	161	167	m	178	182	183

- (i) Given that the regression line of y on x is $y = 8.734x + 101.031$, show that the value of m is 173 correct to 3 significant figures. [3]

For the remainder of this question, use the value of $m = 173$.

- (ii) Give a sketch of the scatter diagram for the data. [2]
 (iii) State the value of the product moment correlation coefficient. [1]
 (iv) Find the regression line of x on y . [1]
 (iv) A 9th student, Clare scored 170 marks in the Mental Arithmetic test.

Use a suitable regression line to estimate the Mental Arithmetic test mark for Clare, correct to 2 decimal places. Comment on the reliability of the estimate obtained. [3]

- 9 A fruit stall sells watermelons and papayas. The masses, in kilograms, of the fruits have independent normal distributions. The means and standard deviations of these distributions, and the selling prices, in \$ per kilogram, are shown in the following table.

	Mean (kg)	Standard deviation (kg)	Selling price (\$ per kg)
Watermelon	2.3	0.2	4
Papaya	1.2	0.1	6

State clearly the mean and variance of all distributions that you use.

- (i) Two watermelons and three papayas are chosen at random. Find the probability that the total mass is less than 9kg. [2]
 (ii) Find the probability that the mass of a randomly chosen watermelon differs from twice the mass of a randomly chosen papaya by at least 0.1kg. [3]
 (iii) Find the probability that the total selling price of a randomly chosen watermelon and a randomly chosen papaya is more than \$16. [2]

Another fruit stall sells fruits in hampers. The masses of these hampers have a normal distribution. Over a long period of time, it is found that 5% of the hampers have a mass less than 6 kg and 3% have a mass more than 8 kg.

- (iv) Find the mean and variance of this distribution. [5]

[Turn Over]

- 10** A company producing ceramic vases claims that a vase has a mean mass of 500 grams. To test this claim, a random sample of 80 vases is checked and the masses, x grams, of the vases are summarised by $\sum(x - 500) = 50$ and $\sum(x - 500)^2 = 1150$.

- (i) Find unbiased estimates of the population mean and variance. [3]
- (ii) Test, at the 10% significance level, whether the company's claim is valid. [4]
- (iii) The manufacturing process is reviewed and the new population standard deviation is known to be 9 grams. A new random sample of n ceramic vases is chosen and the mean of this sample is 501 grams. At 10% level of significance, what should be the largest sample size in order for the company's claim to be valid? You should state any assumptions that you need to make. [5]

- 11 (a)** A 3-day resort stay offers guests two choices for breakfast: a Western Wild breakfast or an Asian Delights breakfast. The probability of a guest choosing a Western Wild breakfast buffet on the first day is $\frac{1}{3}$. The probability that a guest chooses the same type of breakfast as the one he chose the day before is half the probability with which he chose that type of breakfast buffet the day before. A random guest signed up for the 3-day resort stay and had breakfast daily.

Find the probability that he had

- (i) Western Wild breakfast on the second day, [2]
 - (ii) Asian Delights breakfast on at least 2 of the 3 days, [3]
 - (iii) Asian Delights breakfast on the third day given that he had Western Wild breakfast on the second day. [3]
- (b)** For events A and B , it is given that $P(B) = 0.3$, $P(B | A') = 0.2$ and $P(A | B') = 0.6$.
- (i) Show that $P(A \cap B') = 0.42$. [1]
 - (ii) Find $P(A' \cap B')$. [2]
 - (iii) Find $P(A \cap B)$. [2]

END OF PAPER