

ANDERSON JUNIOR COLLEGE
2014 Preliminary Examination
Pre-University 2

Higher 1 MATHEMATICS

8864/01

Wednesday 17 September (8.00 – 11.00 AM)

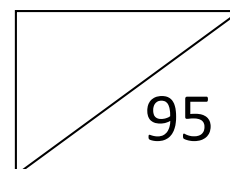
3 hours

READ THESE INSTRUCTIONS FIRST

1. Write your name and PDG in the spaces provided at the bottom of this cover page and on all the scripts you hand in.
2. Answer **all** the questions.
3. Give non-exact numerical answers correct to 3 significant figures, or unless a different level of accuracy is specified in the question.
4. You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
5. You are reminded of the need for clear presentation in your answers.
6. The number of marks is given in brackets [] at the end of each question or part question.
7. At the end of the examination, fasten all your work securely together with this cover page on top using the string provided.
8. You can refer to the guide on resetting of graphic calculator on page 2.

Name		PDG	
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1	2	3	4	5	6
7	8	9	10	11	12



This document consists of 7 printed pages including this page.

[Turn over]

Extract from MF15

STATISTICS

Standard discrete distributions

Distribution of X	$P(X = x)$	Mean	Variance
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$

where $\binom{n}{x} = \frac{n!}{x!(n-x)!}$

Standard discrete distributions

Unbiased variance estimate from a single sample:

$$s^2 = \frac{1}{n-1} \left(\sum x^2 - \frac{(\sum x)^2}{n} \right) = \frac{1}{n-1} \sum (x - \bar{x})^2$$

Regression and correlation

Estimated product moment correlation coefficient:

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\left\{ \sum (x - \bar{x})^2 \right\} \left\{ \sum (y - \bar{y})^2 \right\}}} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\left\{ \sum x^2 - \frac{(\sum x)^2}{n} \right\} \left\{ \sum y^2 - \frac{(\sum y)^2}{n} \right\}}}$$

Estimated regression line of y on x :

$$y - \bar{y} = b(x - \bar{x}), \text{ where } b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

Section A: Pure Mathematics [35 marks]

- 1** Find the range of values of p for which $p(x^2 + 2) < 2x^2 + 6x + 1$ for all real values of x . [4]

- 2** (a) Differentiate $\left(\ln \frac{x}{e^x}\right)^6$. [2]

- (b) Use a non-calculator method to find the exact value of $\int_{-2}^0 \frac{1}{\sqrt{e^{3x-2}}} dx$. [3]

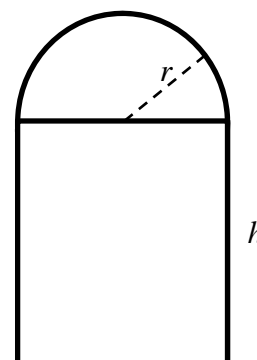
- 3** (a) The equation of a curve is $y = \frac{1}{2-x}$. A point P is moving along the curve so that the y -coordinate is increasing at the constant rate of 0.5 units per second. Find the rate at which the x -coordinate is increasing when $x = -1$. [3]

- (b) The diagram shows a semicircle of radius r m joined to a rectangle of height h m. The perimeter of the shape is 7 m.

- (i) Find the area of the shape in terms of r . [2]

- (ii) Hence, show that the area of the shape is a maximum when

$$h = r = \frac{7}{4 + \pi}. \quad [4]$$



- 4** (i) Solve the equation $\ln(x+5) = 1 + \ln x$ giving your answer in terms of e . [2]
- (ii) Sketch the curve C with equation $y = \ln(x+5) - 1 - \ln x$, stating the exact value of the x -coordinate of its intersection with the x -axis and showing clearly the equation of any asymptotes. [3]
- (iii) Use your graph to solve the inequality $\ln(x+5) > 1 + \ln x$. [1]

- 5 A curve, defined for $x \geq 0$, has equation

$$y = 3x - x^{\frac{3}{2}}$$

- (i) Find the value of $\frac{dy}{dx}$ at the origin O and hence write down an equation of the tangent at O . [3]
- (ii) Find an equation of the tangent at $A(9,0)$. [2]
- (iii) Hence find the coordinates of the point P where the two tangents meet. [1]
- (iv) Find $\int 3x - x^{\frac{3}{2}} dx$ [2]
- (v) Use a non-calculator method to show that the area of the region bounded by the curve and the tangents OP and AP is $16\frac{1}{5}$ units². [3]

Section B: Statistics [60 marks]

- 6 A survey is to be carried out to draw conclusions about the proportion of residents of a town who support the building of a new supermarket. It is proposed to carry out the survey by interviewing a large number of people in the high street of the town, which attracts a large number of tourists.

- (a) Give two different reasons why this proposed method is inappropriate. [2]
- (b) Describe a good method of carrying out the survey. [2]

- 7 In a certain district of a large city, the probability of a household suffering a break-in is a particular year is 0.07 and the probability of its car being stolen is 0.12.

Assuming these two events are independent of each other, calculate, for a randomly selected household with one car, the following probabilities:

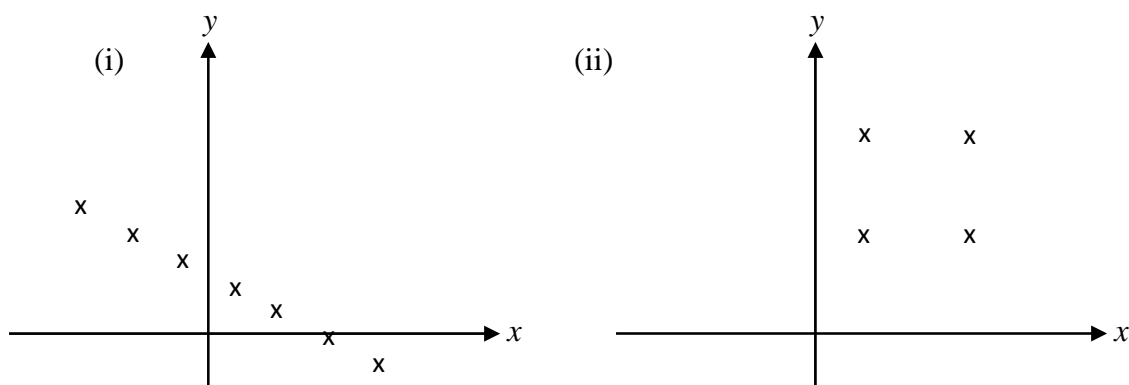
- (i) that the household is a victim of both crimes during that year [1]
- (ii) that the household suffers only one of these misfortunes during that year [2]
- (iii) that the household is a victim of both crimes given that the household suffers at least one of these misfortunes during that year. [3]

- 8 For the events A and B ,

$$P(A \cap B') = 0.32, P(A' \cap B) = 0.11 \text{ and } P(A \cup B) = 0.65.$$

- (a) Find $P(A|B')$. [4]
- (b) Determine whether or not A and B are independent. [3]

- 9 (a) State the value of the product moment correlation coefficient for each of the following scatter diagrams. [2]



- (b) In an agricultural experiment, the relationship between the amount of water supplied, x units, and the yield y units was investigated. Six values of x were chosen and for each value of x the corresponding value of y was measured. The results are shown in the table.

x	1	2	3	4	5	6
y	3	6	8	8	11	10

- (i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]
- (ii) Give a reason why the regression line of x on y is not suitable in this context. [1]
- (iii) Explain why a linear regression model is appropriate, referring both to the scatter diagram and the product moment correlation coefficient. [2]
- (iv) Find the equation of the regression line of y on x , in the form $y = mx + c$, giving the values of m and c correct to 1 decimal place. [3]
- Interpret the meaning of m and c in this context.
- (v) Estimate the increase in yield when the amount of water supplied increases by 5 units. [1]

- 10** A baker bakes cupcakes 7 days a week for sale. The probability that all cupcakes will be sold out on any given day is 0.6, independently of other days.
- Find the probability that all cupcakes will be sold out no less than 4 days of a week. [2]
 - Show that the probability of all cupcakes being sold out on at least 6 consecutive days in a week is 0.0653. [2]
 - If all cupcakes are sold out on at least 6 days of the week, find the probability that this occurs on consecutive days. [2]
 - Using a suitable approximation, estimate the probability that in a year of 52 weeks there will be more than 25 weeks and at most 40 weeks in which all cupcakes will be sold out at least 4 days of a week. [4]

- 11** A supermarket sells durian X . The mass of the durians are normally distributed with mean μ and standard deviation σ in kilograms. This supermarket would discard durians that weigh less than 0.7kg and preserve those that weigh more than 2kg. Based on past data, the supermarket usually discards and preserves 10% and 2% of the durians respectively.

- Show that $\mu = 1.2$ and $\sigma = 0.4$ correct to 1 decimal place. [3]

The supermarket recently imported a new type of durian Y . The mass of the durians are normally distributed with mean 2kg and standard deviation 0.3kg. The price of durian X and Y is \$5 per kg and \$8 per kg respectively. It is assumed that the weight of both types of durian follow a Normal distribution which are independent of each other. [Use $\mu = 1.2$ and $\sigma = 0.4$ for durian X in the questions below.]

- Find the probability that the mass of three durians X is more than twice that of a durian Y selected at random. [3]
- Lucy decides to buy one of each type of durian.
 - Find the probability that the difference in the mass of the durians is no more than 0.5kg. [2]
 - Lucy has only \$20 in her purse. Find the probability that she has enough money for her purchase. [3]

- 12 A company claims that the average lifetime of batteries it produces is 55 hours. A retailer suspects that the claim is inaccurate. To test the claim, the retailer tests a random sample of 100 batteries, and the lifetime x hours are summarized as follows:

$$\sum (x - 55) = -38, \sum (x - 55)^2 = 451.$$

- (i) Find the unbiased estimates of the population mean and variance. [2]
- (ii) Test, at the 5% significance level, whether the company's claim is valid. [4]
- (iii) If the retailer has suspected that the average lifetime of batteries is less than 55 hours, state with a reason whether the conclusion of the test in part (ii) would remain the same. [1]

An improvement is subsequently made to the manufacturing process and the company claims that the average lifetime is now μ_0 hours instead. A test on 8 batteries is conducted and the lifetime in hours, are recorded as follows:

56.3 54.8 54.5 54.4 53.9 55.5 54.6 54.9

It is known that the lifetime of batteries follows a Normal distribution with a standard deviation of 3.9.

- (iv) Using the data from the sample of 8 batteries, find the least value of μ_0 to the nearest hour if there is evidence at 10% level of significance that the company has overstated the average lifetime of batteries. [4]

End of Paper