

YISHUN JUNIOR COLLEGE
2014 JC2 PRELIMINARY EXAMINATION

MATHEMATICS
HIGHER 1

8864/1

19 AUGUST 2014
TUESDAY 0800h – 1100h

Additional materials :
Answer paper
List of Formulae (MF15)



TIME 3 hours

READ THESE INSTRUCTIONS FIRST

Write your CTG and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless specifically states otherwise. Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

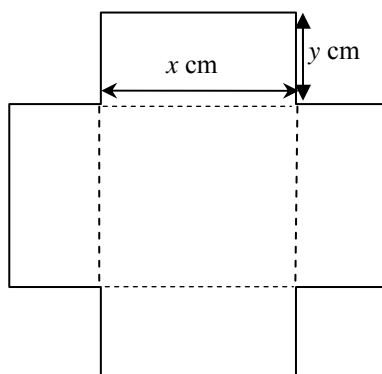
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

Section A: Pure Mathematics [35 marks]

- 1 Find the set of values of k for which the equation $2x^2 + kx + 2 = -4x - 3k$ has two distinct real roots. [3]
- 2 Given that $7e^x = 34 + 5e^{-x}$, find the exact value of x . [3]
- 3 The curve C has equation $y = e^{2x+2} - (3x+4)^2$.
- (i) Sketch C , stating the coordinates of the points of intersection with the axes and the stationary points. [3]
 - (ii) Hence solve the inequality $e^{2x+2} < (3x+4)^2$. [2]
 - (iii) Without the use of a graphing calculator, find the equation of the tangent to C at the point where $x = -1$. [3]
 - (iv) The tangent in (iii) meets C again at point A . Find the coordinates of A , giving your answer correct to 3 decimal places. [1]
- 4 (a) Differentiate
- (i) $\ln(5x^2 - 10)$, [2]
 - (ii) $\frac{2}{(3x-1)^3}$. [2]
- (b) Find $\int \left(2 - \frac{1}{\sqrt{x+3}} \right)^2 dx$. [3]
- 5 A piece of card has the shape of a cross, as shown in the diagram below. The area of the card is 48 cm^2 . It is folded into an open box of height $y \text{ cm}$ with a square base of width $x \text{ cm}$. The volume of the box is $V \text{ cm}^3$.

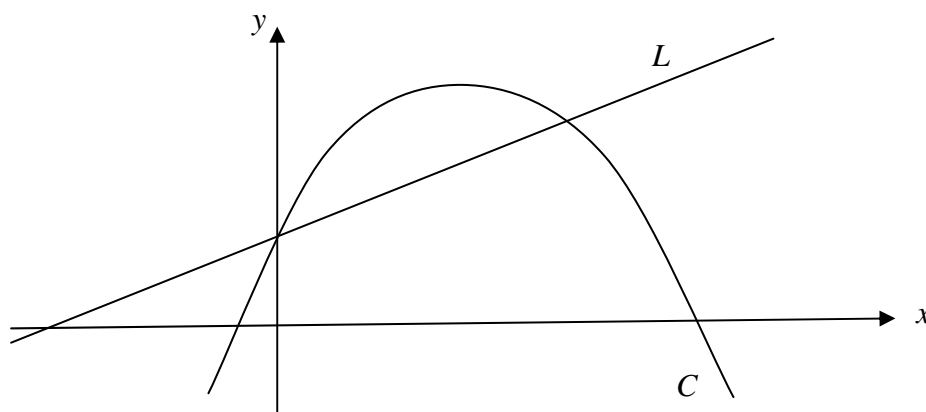


- (i) Show that $V = 12x - \frac{x^3}{4}$. [3]
- (ii) By using differentiation, find the maximum value of V . [4]

- 6 The diagram shows the graphs of

$$C: y = -2x^2 + px + 1 \quad \text{and} \quad L: y = (4 - p)x + 1,$$

where $p > 2$.



- (i) Find, in terms of p , the x -coordinates of the points where C and L intersect. [2]
- (ii) Find the area bounded by C and L , leaving your answer in the form $k(p - r)^3$, where k and r are constants to be determined. [4]

Section B: Statistics [60 marks]

- 7 The student council is conducting a survey on the amount of money the JC 2 students are willing to pay for their Prom night. Nelissa, the chairperson, decides to survey 60 students by standing at the school canteen during a JC2 break time and interviews every 5th student she sees.
- (i) State in this context, one advantage and one disadvantage of this procedure. [2]

The college divided the JC2 students into 4 houses. There are 150 students in Seletar house, 160 students in Rodney house, 200 students in Admiralty house and 190 students in Canberra house.

- (ii) Briefly describe how the student council can obtain a sample using stratified sampling. [2]
- 8 Students in primary 3 have to undergo the Gifted Education Programme (GEP) selection. A student who passes the test at the first round of selection has to sit for a second test. A student who passes both tests will be selected for the GEP. For a randomly chosen student, the probability of passing the test at the first round is 0.3. Among those who sit for the second test, there is a probability of 0.05 of passing it.
- (i) Draw a tree diagram to represent the above situation. [2]
- (ii) Find the probability that a randomly chosen student is not selected for the GEP. [2]
- (iii) Given that a student fails to get into the GEP, find the probability that the student fails the second test. [3]
- (iv) Three primary 3 students are chosen at random. Find the probability that two of them are not selected for the GEP and the other is selected for the GEP. [3]

- 9** A private train company operates train service from Town A to the city centre every day. During the peak hours, there are 25 such train services. It was found that 11% of the train services during the peak hours involve delays. During the peak hours of a randomly chosen day, find the probability that
- (a) (i) there are fewer than 3 train delays , [2]
 (ii) there are between 2 and 9 train delays. [2]
 State two assumptions made in order for you to obtain your answers in part (i) and (ii). [2]
- (b) Days with no more than 2 delays during peak hours are termed “good” days. Commuters are satisfied if there are at least 4 “good” days in a week of 7 days. Find the probability that the commuters are satisfied in a randomly chosen week. [3]
- (c) The train company decides to increase the number of train services during the peak hours to 70 every day so that the percentage of train services during the peak hours that involve delays is reduced to 8%. Using a suitable approximation, find the probability that there are fewer than 8 train delays in a randomly chosen day. [4]
- 10** (a) A fruit stall sells durians and it is found that on average, 1 in 5 durians has a mass greater than 3.5 kg. Assuming that the masses, in kg, of the durians are normally distributed with mean 2.85 kg, find the standard deviation of the distribution. [3]
- (b) The fruit stall also sells mangosteens and mangoes. The masses, in kg, of mangosteens and mangoes have independent normal distributions with means and standard deviations as shown in the following table.
- | | Mean | Standard deviation |
|-------------|------|--------------------|
| Mangosteens | 0.09 | 0.02 |
| Mangoes | 0.40 | 0.04 |
- (i) The fruit seller packs mangosteens in bags of 10. Find the probability that the mass of 1 randomly chosen bag of mangosteens is greater than the total mass of 2 randomly chosen mangoes. [4]
- (ii) Mangosteens cost \$3.00 per kg and mangoes cost \$5.00 per kg. Erica wants to buy 2 bags of mangosteens and 2 mangoes. She has only \$10.00 with her. Find the probability that she has sufficient money to pay for her purchase. [4]
- (c) In order to make more profits, the fruit seller processed the unsold mangoes into jam. The mean mass of a packet of jam is 0.50 kg and the standard deviation is 0.09 kg. Find the probability that the mean mass of 60 randomly chosen packets of jam exceeds 0.52 kg. [3]

- 11** A tea company produces a new detox tea which helps customers to lose weight. The company claims that on average, customers start to lose weight after consuming the tea for 26 consecutive days. The number of days taken for them to start to lose weight is denoted by X . A group of 60 customers consumed the tea every day. It was found that

$$\sum x = 1578 \quad \text{and} \quad \sum (x - \bar{x})^2 = 123.2$$

- (i) Calculate the unbiased estimates of the population mean and variance of X . [2]
 (ii) Test, at the 5% level of significance, whether the company has overrated the product. [4]
 (iii) State, with a reason, whether any assumptions about the population are needed in order for the test to be valid. [1]

The tea company also produces a new bottled rose tea and claims that the mean mass of sugar in a bottle of rose tea is 10 g. The mass of sugar is known to have a normal distribution with standard deviation 0.5 g. A random sample of 25 bottles is selected and a two-tail test is carried out. Find the set of values within which the mean mass of sugar of this sample must lie for the company's claim to be accepted at the 5 % level of significance. [4]

- 12** The table gives the number of reported dengue cases from 25th May to 8th July 2014.

Week (x)	1 (25–31 May)	2 (1–7 Jun)	3 (8–14 Jun)	4 (15–21 Jun)	5 (22–28 Jun)	6 (29Jun– 5 Jul)	7 (6–8 Jul)
Number of dengue cases (y)	456	459	506	552	674	891	282

The above data were extracted from www.dengue.gov.sg on 9 Jul 2014.

- (i) Suggest, in context, a possible reason why the data point (7, 282) appears to be unreasonable. [1]

Remove the data point (7, 282).

- (ii) Give a sketch of the scatter diagram for the data. [2]
 (iii) Calculate the product moment correlation coefficient and comment on its value in the context of the data. [2]
 (iv) Find the equation of a suitable regression line and use it to estimate the number of dengue cases for week 7, giving your answer correct to the nearest whole number. Comment on the reliability of your estimate. [3]

End of Paper