Solns:

1 (i) $a = -1 < 0$

$b^2 - 4ac = (1)^2 - 4(-1)(-1)$
$= -3$

$b^2 - 4ac < 0$
Therefore $-x^2 + x - 1$ is always negative for all real values of x

(ii) Since $-x^2 + x - 1 < 0 \Rightarrow x^2 - x + 1 > 0 \; for \; all \; x$

$2x^2 + 5x - 3 > 0$

$(2x-1)(x+3) > 0$

$x < -3 \quad or \quad x > \frac{1}{2}$

2 (i)

$$y = \frac{e^{2x}+1}{e^{2x}-1}$$

$ye^{2x} - y = e^{2x} + 1$

$e^{2x}(y-1) = y+1$

$e^{2x} = \frac{y+1}{y-1}$ (Shown)

(ii)

$$e^{2x} = \frac{-\frac{5}{4}+1}{-\frac{5}{4}-1} = \frac{-\frac{1}{4}}{-\frac{9}{4}}$$

$e^{2x} = \frac{1}{9}$

$2x = \ln\frac{1}{9}$

$x = -1.10 \; (3sig)$

3 perimeter $= 30 = AB + BF + FC + CD + DE + EA$

$30 = y + \sqrt{(3x)^2 + (4x)^2} + 4x + y + 4x + \sqrt{(3x)^2 + (4x)^2}$

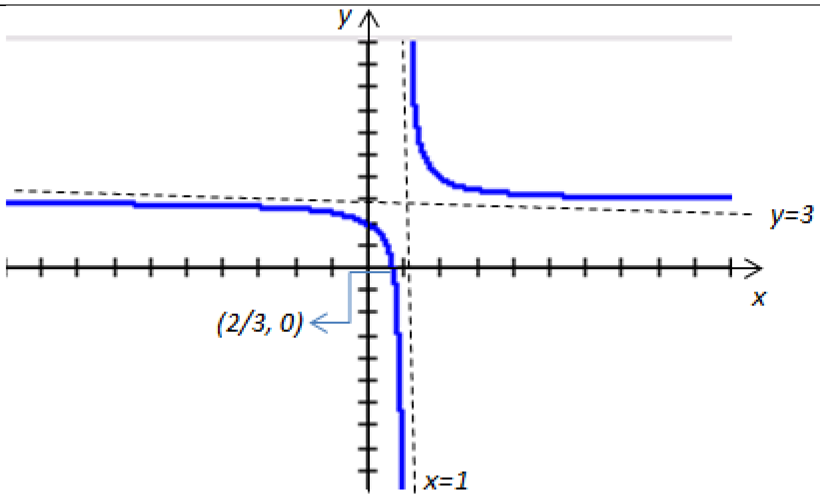
$30 = 2y + 8x + 10x$

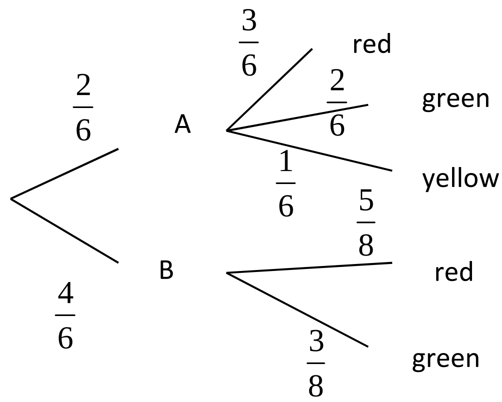
$30 = 2y + 18x$

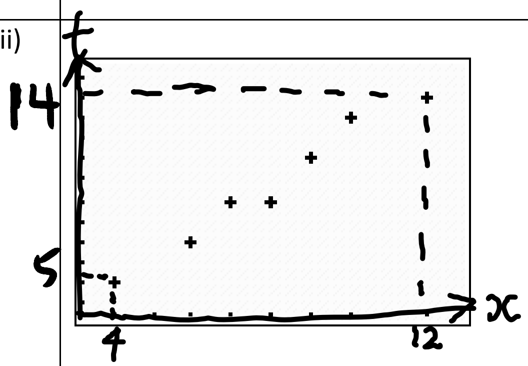
$y = 15 - 9x$

	$A = 3x \times y + 3x \times 4x$ $= 12x^2 + 3xy$ $= 12x^2 + 3x(15 - 9x)$ $= 45x - 15x^2$
	$\frac{dA}{dx} = 45 - 30x = 0$ $x = \frac{3}{2}$ $\frac{d^2A}{dx^2} = -30 < 0$ A is a maximum. $A = 45\left(\frac{3}{2}\right) - 15\left(\frac{3}{2}\right)^2 = \frac{135}{4}$ Maximum A is $\frac{135}{4}$
4(a)	$\int \frac{\left(2+\sqrt{x}\right)^2}{\sqrt{x}}\,\mathrm{d}x$ $= \int \frac{4+4\sqrt{x}+x}{\sqrt{x}}\,\mathrm{d}x$ $= \int \left(\frac{4}{\sqrt{x}} + 4 + \sqrt{x}\right)\mathrm{d}x$ $= 8x^{\frac{1}{2}} + 4x + \frac{2}{3}x^{\frac{3}{2}} + C$
(b)	$\int_0^2 \left(\frac{1}{(8-5x)^3} - \mathrm{e}^{7-3x}\right)\mathrm{d}x$ $= \left[\frac{1}{(-2)(-5)(8-5x)^2} - \frac{\mathrm{e}^{7-3x}}{(-3)}\right]_0^2$ $= \frac{1}{40} + \frac{\mathrm{e}}{3} - \frac{1}{640} - \frac{\mathrm{e}^7}{3} = \frac{3}{128} + \frac{\mathrm{e}}{3} - \frac{\mathrm{e}^7}{3}$
5(i)	Let V be volume of water in the inverted cone, h be depth of water and r be radius of the surface of water. Using similar triangles, $\frac{r}{h} = \frac{5}{20} = \frac{1}{4}$ $r = \frac{1}{4}h$

	$V=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(\frac{1}{4}h\right)^2 h$ $=\frac{1}{48}\pi h^3$ $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{1}{16}\pi h^2$ $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{dh}}\times\frac{\mathrm{dh}}{\mathrm{d}t}$ $5.5=\frac{1}{16}\pi h^2\times\frac{\mathrm{dh}}{\mathrm{d}t}$ $\frac{\mathrm{dh}}{\mathrm{d}t}=\frac{16}{\pi(4)^2}\times 5.5$ $=\frac{11}{2\pi}\mathrm{cms}^{-1}$
(ii)	Change in vol $=\frac{\pi\left(10^3-4^3\right)}{48}=\frac{39\pi}{2}$ Time required $=\frac{\Delta V}{\frac{dV}{dt}}=\frac{39\pi}{2}\div 5.5=\frac{39\pi}{11}s$
6	$y=\frac{px+q}{x-1}=p+\frac{p+q}{x-1}$ Since $y=3$ is a asymptote of C, so $p=3$ Sub $p=3$ and $A(2, 4)$ into the equation: $4=\frac{3(2)+q}{2-1}\Rightarrow 4=6+q$ $\therefore q=-2$

	
	$y=\ln(e^x-x)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{e^x-x}(e^x-1)=\frac{e^x-1}{e^x-x}.$ Area of shaded region $=\int_0^1 3-x-\frac{2e^x-2}{e^x-x}dx=\int_0^1(3-x)dx-2\int_0^1\frac{e^x-1}{e^x-x}dx$ $=\left[3x-\frac{x^2}{2}\right]_0^1-2\left[\ln(e^x-x)\right]_0^1$ from part (i) $=\frac{5}{2}-2\ln(e-1)$
7(i)	$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$ $=\frac{1}{3}+\frac{1}{4}-0$ $=\frac{7}{12}$
(ii)	$\mathrm{P}(A\mid B)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}=\frac{\mathrm{P}(A)\times\mathrm{P}(B)}{\mathrm{P}(B)}=\frac{1}{3}$
(iii)	$\mathrm{P}(A'\cup B)$ $=1-\mathrm{P}\left[(A)-\mathrm{P}(A\cap B)\right]$ $=1-\left[\frac{1}{3}-\frac{1}{6}\right]$ $=\frac{5}{6}$
8	(i) Simple random sampling, Certain branches may not be represented in the sample. (ii) The names of all members are to be listed and numbered from 1 to 1450. To obtain a

	sample of 60 members from a population of 1450 members, we may make k = 1450/60= 24. In this case, we start with randomly chosen number from amongst the first 24, say number 10 and then select every 24th member, say 34, 58, 82…….. until 60 numbers are selected. Members corresponding to these numbers form systematic sample.
9(i)	
(ii)	$\mathrm{P}(\text{green}) = \frac{2}{6}\times\frac{2}{6}+\frac{4}{6}\times\frac{3}{8}=\frac{13}{36}$
(iii)	$\mathrm{P}(\text{box B given red ball is chosen}) = \dfrac{\mathrm{P}(\text{box B and red})}{\mathrm{P}(\text{red})}$ $= \dfrac{\frac{4}{6}\times\frac{5}{8}}{\frac{2}{6}\times\frac{3}{6}+\frac{4}{6}\times\frac{5}{8}}$ $= \dfrac{\frac{5}{12}}{\frac{7}{12}} = \frac{5}{7}$
10(i)	Let X be weight in grams of a randomly selected loaf . Given $X \approx N(\mu, 24^2)$ $H_0 : \mu = 1000$ $H_1 : \mu \neq 1000$ Using GC, $\bar{x} = 970.11$ Under H_0, $\bar{X} \sim N(1000, \frac{24^2}{9})$ Using two tailed Z-test, p-value $\approx 0.000187 < 0.05$ Reject H_0 and conclude that there is sufficient evidence at 5% level of significance to conclude that mean weight of a loaf is not equal to 1000 grams
(ii)	5% level of significance means that the probability that mean weight of a loaf either less than 1000 grams or greater than 1000 grams when in fact the mean weight is 1000 grams is 0.05.
(iii)	Using GC, $\bar{x} = 970.11$ Under H_0, $\bar{X} \sim N(1000, \frac{24^2}{9})$

	Since H_0 is rejected in favour of H_1, p value < 0.05 $P(\bar{X} < 970.11) < 0.05$ $P\left(\frac{\bar{X}-\mu_0}{24/3} < \frac{970.11-\mu_0}{8}\right) < 0.05$ $P\left(Z < \frac{970.11-\mu_0}{8}\right) < 0.05$, where $Z \approx N(0,1)$ Using GC, $\frac{970.11-\mu_0}{8} < -1.644853626$ $\mu_0 > 983\ (3 sigfig)$
11(i)	$t = 1.19x + 0.19$ From GC, $\bar{x} = 8$. $\bar{t} = \frac{55+k}{7}$ Subt. $(\bar{x}, \bar{t})$ into linear regression line of t on x: $\frac{55+k}{7} = 1.19(8) + 0.19$ $k = 12.97 \approx 13$ (nearest integer)
(ii)	 From GC, r between x and t = 0.984
(iii)	Since x is the independent variable, we should use the linear regression line of t on x. $t = 0.19 + 1.19x$ (a) When $x = 14$, t = 0.19 + 1.19(14) = 16.85 mins The estimate may be unreliable as it is obtained by extrapolation as $x = 14$ lies outside the given data range of x. (b) When $t = 8$, $x = (8 - 0.19)/1.19 = 6.56 \approx 7$ customers The estimate may be reliable as it is obtained by interpolation as $t = 8$ lies within the given data range of x and the linear model is suitable for the given data as r is close to 1.
12	Let A be the r.v "Number of students getting an A in a class of 20".

	$A \sim \mathrm{B}(20, 0.3)$
(i)a)	$P(A=5)=0.17886 \approx 0.179$
b)	$P(A>7)=1-P(A\le 7)$ $=1-0.77227$ $=0.22773$ ≈ 0.228
(ii)	Let X be the r.v "Number of classes with less than 8 students getting an A out of 40 classes". $X \sim \mathrm{B}(40, P(A<8))$ $X \sim \mathrm{B}(40, 0.77227)$ $P(X=25)=0.014455 \approx 0.0145$
(iii)	Let Y be the r.v "Number of students getting an A out of 800 students". $Y \sim \mathrm{B}(800, 0.3)$ Since n is large and $np=240>5$ $nq=560>5$, $Y \sim \mathrm{N}(240,168)$ approximately $P(Y\ge 250) \xrightarrow{c.c} P(Y \ge 249.5)=0.23180$
13a)	$\mathrm{P}(12<X<15)=\mathrm{P}(X>15)=0.4$ $\Rightarrow \mathrm{P}(X<12)=0.2$ $\mathrm{P}(Z<\frac{12-\mu}{\sigma})=0.2$ $\frac{12-\mu}{\sigma}=-0.84162$ $12+0.84162\sigma=\mu$ ---(1) $\mathrm{P}(X<15)=0.6$ $\mathrm{P}(Z<\frac{15-\mu}{\sigma})=0.6$ $\frac{15-\mu}{\sigma}=0.25335$ $15-0.25335\sigma=\mu$ ---(2) Solving (1) and (2), $12+0.84162\sigma=15-0.25335\sigma$ $1.09497\sigma=3$ $\sigma=2.7398\approx 2.74$ $\mu=14.3$
(b)i)	Let F be the weight of a fruity muffin in grams. $F \sim \mathrm{N}(220, 25^2)$

	$\overline{F} = \frac{F_1 + F_2 + ... + F_n}{n} \sim N\left(220, \frac{25^2}{n}\right)$ $\mathrm{P}\left(\overline{F} < 210\right) = 0.164$ $\mathrm{P}\left(Z < \frac{210-220}{25/\sqrt{n}}\right) = 0.164$ $\frac{210-220}{25/\sqrt{n}} = -0.97815$ $n \approx 5.98 \approx 6\left(\text{ nearest integer}\right)$
(ii)	F~N(220, 25^2) Let C be the weight of a chocolaty muffin in grams. C~N(112, 15^2) $C_1 + C_2 + C_3 - 2F \sim$ $N(3\times112 - 2\times220, 3\times15^2 + 2^2\times25^2)$ $= N(-104, 3175)$ $\mathrm{P}\left(\lvert C_1 + C_2 + C_3 - 2F\rvert < 50\right)$ $= \mathrm{P}\left(-50 < C_1 + C_2 + C_3 - 2F < 50\right)$ $= 0.166$ Assumption : The weights of fruity muffins and chocolaty muffins are independent of one another.