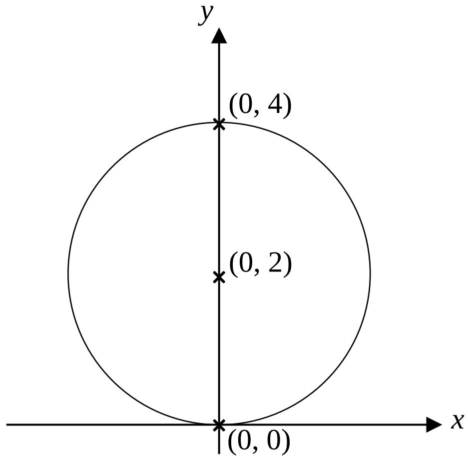
MI PU2 H1 PE II 2014 Suggested Solutions

1(i)	$f(5) = e^{2(5)-1}$ $= e^9$
1(ii)	$e^{2x-1} = 1000$ $2x - 1 = \ln 1000$ $x = 3.95$ (3 sf)
2	 i) $c = 2$ ii) $c = 4$
3(i)	$(-3)^2 - 4(k)(k) < 0$ $(2k+3)(2k-3) > 0$ $k < -\frac{3}{2}$ or $k > \frac{3}{2}$ Since $k > 0$, so $k > \frac{3}{2}$.
3(ii)	$2x^2 + 5x + 13 = x + 3$ $x^2 + 2x + 5 = 0$ Consider $b^2 - 4ac$, $2^2 - 4(1)(5) = -4 < 0$ Since D < 0, there is no point of intersection.
4(i)	$2(0.5 \times 8x \times \sqrt{(5x)^2 - (4x)^2}) + 2(5x \times h) = 1000$ $h = \frac{100 - 2.4x^2}{x}$ Capacity, $V = 0.5(8x)(3x)\left(\frac{100 - 2.4x^2}{x}\right)$ $= 1200x - 28.8x^3$ [Shown]

4(ii)

$$\frac{\mathrm{d}V}{\mathrm{d}x} = 1200 - 86.4x^2$$

At max capacity, $1200 - 86.4x^2 = 0$

$$x^2 = \frac{1200}{86.4}$$

$$x = 3.73 \text{ (3 sf)}$$

Max capacity

= $1200(3.727) - 28.8(3.727)^3 = 2980 \text{ cm}^2$ (3 sf)

Test for max V:

x	3.0	3.73	4.0
$\frac{dy}{dx}$	+ve	0	-ve
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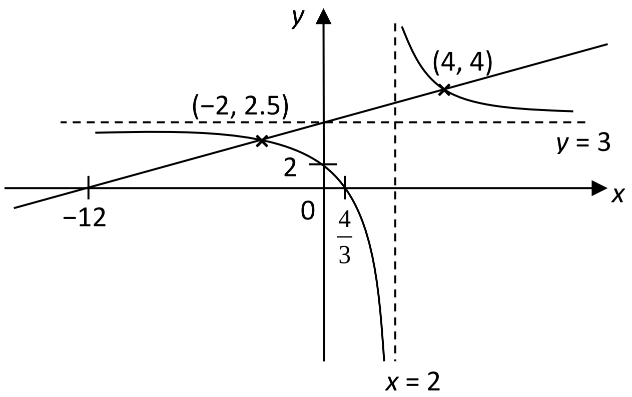
5(i)



5(ii)

$$\left(\frac{1}{4}x + 3\right)(x-2) = 3x - 4$$

$$x^2 + 10x - 24 = 12x - 16$$

$$x^2 - 2x - 8 = 0 \text{ [Shown]}$$

5(iii)

$-2 < x < 2$ or $x > 4$

5(iv)

$$3 + \frac{2}{x-2} = \frac{3(x-2)+2}{x-2} = \frac{3x-4}{x-2}$$

$$3x + 2\ln(x-2) + C$$

Area under line: 0.5(4+4.5)(2) = 8.5 units^2

Area under curve:

$\left[3x + 2\ln(x-2)\right]_4^6 = 6 + 2\ln 2$ units^2

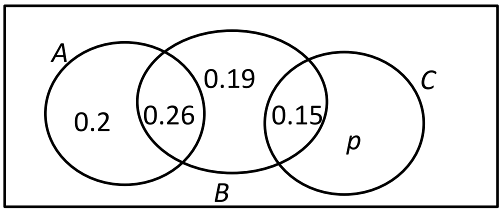
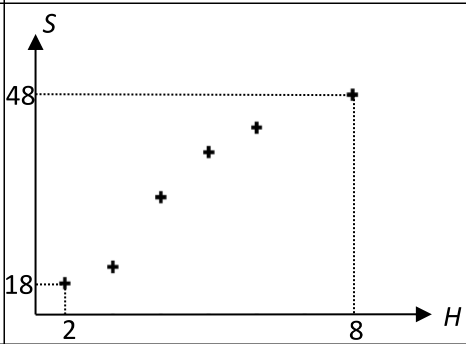
Area enclosed = $2.5 - 2\ln 2$ units^2

6(i)

μ = 80

$$\sigma^2 = \frac{37874}{49} = 773 \text{ (3sf)}$$

6(ii)	Since $n = 50$ is large, by Central Limit Theorem, $\bar{x} \sim \mathrm{N}\left(80,\ \frac{773}{60}\right)$ approximately. $\mathrm{P}(\bar{x} > 78) = 0.711$ (3 sf)
7(i)	<u>Method</u> Assign index numbers to each student in each class. Use simple random sampling to choose 2 index numbers from each class and repeat process with same 2 index numbers from all other classes. <u>Disadvantage</u> Students from a stream may be under-represented.
7(ii)	<u>Method</u> Step 1: Divide students into their streams, obtaining 3 strata Step 2: Calculate proportionate number of students to be selected and use random sampling to select the students from each strata. Number of Business students = $\frac{300}{600} \times 60 = 30$ Number of Science students = $\frac{200}{600} \times 60 = 20$ Number of Arts students = $\frac{100}{600} \times 60 = 10$ <u>Disadvantage</u> Unequal number of students from each class to take survey.
8(i)	$\mathrm{H}_0 : \mu = 55165$ $\mathrm{H}_1 : \mu > 55165$ Under H_0, since sample size = 70 is large, by Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(55165,\ \frac{34596}{70}\right)$ approximately. At α% level, we reject H_0 if p-value < $\frac{\alpha}{100}$. Using GC, given $\bar{x} = 55200$, p-value = 0.0577 $0.0577 < \frac{\alpha}{100}$ $\alpha > 5.77$ So minimum $\alpha = 6$.
8(ii)	$\mathrm{H}_0 : \mu = \mu_0$ $\mathrm{H}_1 : \mu > \mu_0$ Under H_0, since sample size = 70 is large, by Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(\mu_0,\ \frac{34596}{70}\right)$ approximately. At 3% level, we reject H_0 if p-value ≤ 0.03. $Z = 1.88079$ Given $\bar{x} = 55200$, in order to consider an effective sales strategy, we must reject H_0, so $\frac{55200 - \mu_0}{\sqrt{\frac{34596}{70}}} > 1.88079$ $\mu_0 < 55158$

9(i)	$P(A \cap B') = 0.5 \times P(B')$ $= 0.2$
9(ii)	$P(A) = P(A \cap B') + P(A \cap B)$ $0.46 = 0.2 + P(A \cap B)$ $P(A \cap B) = 0.26$
9(iii)	$P(A) \times P(B) = 0.276 \neq P(A \cap B)$ A and B are not independent events.
9(iv)	 Max value p: $0.2 + 0.26 + 0.19 + 0.15 + p = 1$ Max $p = 0.2$ For $p = 0.2$, $P(C) = 0.2 + 0.15 = 0.35$ When $C \subset B$, min $P(C) = 0.15$ So $0.15 \leq P(C) \leq 0.35$
10(i)	Let R be the number of spoilt oranges from the first sample. $R \sim B(40, 0.05)$ $P(R \leq 3) = 0.862$
10(ii)	For $P(R > 4)$: Prob = $1 - P(R \leq 4) = 1 - 0.95197 = 0.048$ For $P(R = 4)$: Let S be the number of spoilt oranges from the second sample. $S \sim B(20, 0.05)$ $P(R = 4) \times P(S \geq 1) = 0.0578$ So prob $= 0.048 + 0.0578 = 0.106$ (3 sf)
10(iii)	Let T be the number of spoilt oranges from the sample for the government health inspection. $T \sim B(500, 0.05)$ Since $n = 500$, $np = 25 > 5$, $n(1-p) = 475 > 5$, $T \sim N(25, 23.75)$ approximately $P(T < 20) \xrightarrow{\text{C.C}} P(T < 19.5) = 0.130$
11(i)	
11(ii)	$r = 0.966$ (3 sf) Since r is close to +1, we can conclude that there is a strong positive linear correlation between H and S.
11(iii)	H on S: $H = 0.173S - 1.12$ S on H: $S = 5.4H + 8.3$

11(iv)	Using S on H, S = 5.4(12) + 8.3 = 73 (nearest integer) The score is not valid as the maximum possible mark is 60. OR This is due the input value falls <u>outside the range of data</u> and the result is obtained through <u>extrapolation</u>.
11(v)	$S = 5.4H + 8.3 + k$
11(vi)	The point $\left(\overline{H},\ \overline{S}\right) = \left(4\frac{2}{3},\ 33.5\right)$
12(i)	Let A and B be the training timings of Alex and Bob respectively. $A \sim \mathrm{N}(35.8,\ 2.7^2)$ and $B \sim \mathrm{N}(36.1,\ 6.2^2)$ $\mathrm{P}(A < 35.2) = 0.412$ $\mathrm{P}(B < 35.2) = 0.442$ From results, we observe that Bob has a higher chance.
12(ii)	$\mathrm{P}(A > B) = \mathrm{P}(A - B > 0)$ $A - B \sim \mathrm{N}(-0.3,\ 45.73)$ $\mathrm{P}(A > B) = 0.482$
12(iii)	Let t be the minimum time. $\mathrm{P}(B \le t) = 0.95$ $t = 46.3$s
12(iv)	We cannot assume n is large so, $A_1 + A_2 + ... + A_n \sim \mathrm{N}(35.8n,\ n(2.7^2))$ $\mathrm{P}(A_1 + A_2 + ... + A_n > 36n) = 0.35$ $\mathrm{P}\left(Z > \frac{36n - 35.8n}{2.7\sqrt{n}}\right) = 0.35$ $\frac{0.2n}{2.7\sqrt{n}} = 0.38532$ $n = 27$ (nearest integer)