

2014 Preliminary Examination II

Pre-university 2

MATHEMATICS

8864/01

Paper 1

02 September 2014

3 hours

Additional Materials: Answer Paper
 List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, arrange your answers in NUMERICAL ORDER and fasten all your work securely together.

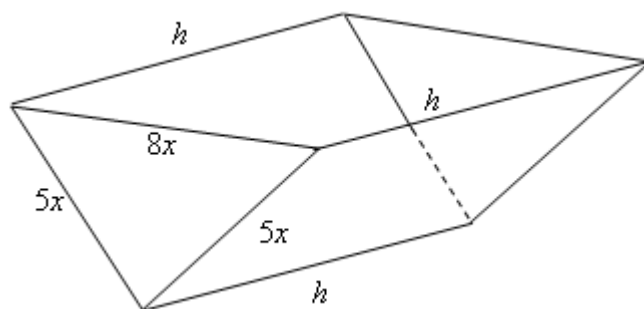
The number of marks is given in brackets [] at the end of each question or part question.

This question paper consists of 6 printed pages.

[Turn over

Section A: Pure Mathematics [35 marks]

- 1 Given $f(x) = e^{2x-1}$,
- (i) Find the exact value of $f(5)$. [1]
 - (ii) Find value of x for which $f(x) = 1000$. Correct your answer to 3 significant figures. [3]
- 2 Sketch the graph A , $x^2 + (y - 2)^2 = 4$. State clearly the coordinates of its centre and intersection(s) with axes. [3]
- On the same diagram, another curve B , $y = x^2 + c$ is drawn.
- (i) State the value of c if the minimum point of B intersects the centre of A . [1]
 - (ii) State the maximum value of c in which there is at least one point of intersection between A and B . [1]
- 3 (a) Find the range of values of k for which $kx^2 - 3x + k > 0$ for all real values of x . [3]
- (b) By considering the discriminant, find the number of intersection point(s) between the curve $y = 2x^2 + 5x + 13$ and the line $y = x + 3$. [3]



The diagram shows an open metal container that has the shape of a prism. The ends are identical isosceles triangles of sides $5x$ cm, $5x$ cm and $8x$ cm, as shown in the diagram above. The sides of the prism are two rectangles of length h cm and width $5x$ cm. The total amount of metal used to make the container is 1000 cm^2 .

- (i) Show that the capacity of the container, $V \text{ cm}^3$, is given by $V = 1200x - 28.8x^3$. [4]
- (ii) Given that x varies, find the value of x when capacity is a maximum and hence find the value of this maximum capacity. [4]

- 5 (i) Sketch the graph of $y = \frac{3x-4}{x-2}$, stating clearly on the diagram any axial intercepts and the equations of any asymptotes. [3]
- (ii) On the same diagram, it is given that the graph intersects the line $4y = x + 12$. Show, at the point(s) of intersection, $x^2 - 2x - 8 = 0$. [1]
- (iii) Hence, solve $\frac{3x-4}{x-2} < \frac{1}{4}x + 3$. [2]
- (iv) Show that $3 + \frac{2}{x-2} = \frac{3x-4}{x-2}$ and integrate $\int \frac{3x-4}{x-2} dx$. [3]

Hence find the **exact** area of the region enclosed by $y = \frac{3x-4}{x-2}$, $4y = x + 12$ and the line $x = 6$. [3]

Section B: Statistics [60 marks]

- 6 It is given that the amount of money donated by individuals during a charity drive is \$ x .
A sample consolidation of the amount collected by 50 individuals finds the following data:

$$\sum x = 4000 \text{ and } \sum (x - 80)^2 = 37874$$

- (i) Find the unbiased estimates of the population mean and variance. [2]
- (ii) A second sample of 60 individuals is selected. Find the probability that the average donation by each individual in this sample is more than \$78. [3]
- 7 In an institute, there are 300 Business, 200 Science and 100 Arts stream students. All the students study Mathematics and they are grouped into 30 separate classes, according to grades and regardless of stream. Each class has an equal number of 20 students. A sample of 60 students is to be chosen to complete a Mathematics Learning Experience survey.
- (i) Describe how systematic sampling can be carried out to choose the 60 students. State a disadvantage of this method in the context of the question. [3]
- (ii) It would be ideal if there is proportionate representation of students from the respective streams to take the survey. Suggest and describe a suitable alternative sampling method. State a disadvantage of this alternative method. [3]
- 8 The weekly sales revenue of a company selling printers has mean \$ μ_0 and standard deviation \$186. To increase its weekly sales, the company implements a sales strategy where a free ink cartridge is given for every printer purchased. To investigate the effectiveness of this strategy, the weekly sales revenue of the company is monitored for 70 weeks and found to have a mean of \$55200.
- (i) Given $\mu_0 = 55165$, the sales strategy is considered effective at $\alpha\%$ level of significance. Find the minimum integer value of α . [4]
- (ii) Find the range of values of μ_0 , to the nearest dollar, for which the sales strategy has been effective at 3% level of significance. [4]

- 9 The events A and B are such that $P(A) = 0.46$, $P(B') = 0.4$ and $P(A | B') = 0.5$.
- (i) Find $P(A \cap B')$. [2]
 - (ii) Find $P(A \cap B)$. [2]
 - (iii) Determine whether A and B are independent events. [2]
 - (iv) For another event C , it is given that A and C are mutually exclusive and $P(B \cap C) = 0.15$. Find the possible range of values of $P(C)$. [3]
- 10 On an orange farm, for quality control, samples of 40 oranges are randomly selected from each harvest batch and the number of spoilt oranges is noted. A batch will be rejected if the number is more than 4 and accepted if less than 4. If the number is exactly 4, then another sample of 20 oranges will be randomly selected from the same harvest batch. The batch will be immediately rejected if the second sample has any spoilt oranges, otherwise it will be accepted. Past records show that 5% of oranges harvested are spoilt.
- (i) Find the probability that a batch is accepted after the first sample. [2]
 - (ii) Find the probability that a batch will be rejected. [3]
 - (iii) For a government health inspection, a random sample of 500 oranges is taken. Determine, using a suitable approximation, the probability that, in the sample, fewer than 20 oranges are spoilt. [4]

- 11 An analysis was done on the relationship between S , the score for a Mathematics test and H , the number of hours spent studying for the test. The maximum possible score for the test is 60. The data is tabulated below.

Study time, H	4	5	3	2	8	6
Score, S	32	39	21	18	48	43

- (i) Give a sketch of the scatter diagram of S against H . [2]
 - (ii) Calculate the product moment correlation coefficient and describe the relationship between H and S . [2]
 - (iii) Find the equations of both regression lines H on S and S on H . [2]
 - (iv) Using the appropriate regression line, estimate the score, to the nearest integer, when the amount of studying time spent is 12 hours. Comment on the reliability of the estimated score. [2]
 - (v) If the scores are moderated with k marks added to each score, state the equation of the regression line S on H after moderation. [1]
 - (vi) Adding more data points may change the value of the product moment correlation coefficient. State a pair of points that, if added, will not cause such a change. [1]
- 12 The running timings of Alex and Bob in 300m races are independent and normally distributed with means 35.8 and 36.1 seconds respectively, and standard deviations 2.7 and 6.2 seconds respectively. The two of them are training to compete in a 300m race event with a track record of 35.2 seconds.
- (i) Determine, with clear working shown, who has the higher chance of breaking the track record. [2]
 - (ii) Find the probability of Bob winning Alex in the race. [3]
 - (iii) Find the timing that Bob can achieve 95% of the time. [2]
 - (iv) Alex went for n number of 300m races. The probability that the total of his running times exceeding $36n$ is 0.35. Find n , correct to the nearest integer. [6]

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