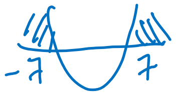
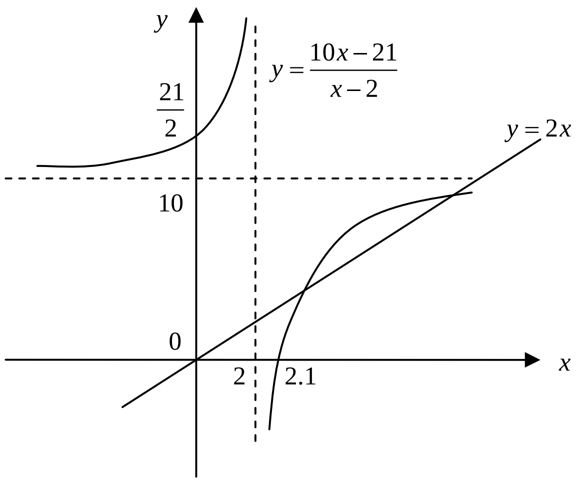
| 1 | | | |
|---|---|---|---|
| | **Solution** | | |
| | **(i)** | $b^2-4ac<0$<br>$\Rightarrow\left(\sqrt{98}\right)^2-4k\left(\dfrac{k}{2}\right)<0$<br>$98-2k^2<0$<br>$49-k^2<0$<br>$\Leftrightarrow k^2-49>0$<br>$(k-7)(k+7)>0$<br>$k<-7 \quad \text{or } k>7$<br><br><br>For the f($x$) to be always positive, $b^2-4ac<0$ and $k>0$<br>$\therefore\ k>7$<br><br>The set of values of $k$ for which the function f($x$) is always positive = $\{k\in\mathbb{R}\ :\ k>7\}$ | |
| | **(ii)** | $\text{f}\left(\sqrt{2}m\right)=-9$ and $k=2$,<br>$\Rightarrow 2\left(\sqrt{2}m\right)^2+\sqrt{98}\left(\sqrt{2}m\right)+\dfrac{2}{2}=-9$<br>$\Rightarrow 4m^2+14m+10=0$<br>$(4m+10)(m+1)=0$<br>$\therefore\ m=-\dfrac{5}{2} \quad \text{or} \quad -1$ | |

| 2 | # | | |
|---|---|---|---|
| | **Solution** | | |
| | **(i)** | $30 = 2x + 2y + \frac{1}{4}2\pi x + \frac{1}{2}2\pi x$<br>$30 = 2x + 2y + \frac{3}{2}\pi x$<br>$60 = 4x + 4y + 3\pi x$<br>$y = \frac{60 - 4x - 3\pi x}{4} = 15 - x - \frac{3}{4}\pi x$<br><br>$A = xy + \frac{1}{2}\pi x^2 + \frac{1}{4}\pi x^2$<br>$= x\left(15 - x - \frac{3}{4}\pi x\right) + \frac{3}{4}\pi x^2$<br>$= 15x - x^2$ | |
| | **(ii)** | $A = 15x - x^2$<br>$\frac{\mathrm{d}A}{\mathrm{d}x} = 15 - 2x$<br>For maximum, $\frac{\mathrm{d}A}{\mathrm{d}x} = 0 \Rightarrow x = \frac{15}{2}$<br><br>Using First Derivative Test, | |

| | $\left(\frac{15}{2}\right)^-$ | $\frac{15}{2}$ | $\left(\frac{15}{2}\right)^+$ |
|---|---|---|---|
| $\frac{\mathrm{d}A}{\mathrm{d}x}$ | $> 0$ | $0$ | $< 0$ |
| | / | – | \ |

Therefore, it is a maximum value at $x = \frac{15}{2}$ .

| 3 | | | |
|---|---|---|---|
| | **Solution** | | |
| **(i)** | **(a)** | $\frac{\mathrm{d}}{\mathrm{d}x}\left[2x+\ln\left(5x^2+1\right)\right]=2+\frac{10x}{5x^2+1}$ | |
| | **(b)** | $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{3}{2x+1}\right)=-3(2x+1)^{-2}(2)$ <br> $=\frac{-6}{\left(2x+1\right)^2}$ | |
| **(ii)** | | $\int_1^3 \frac{4}{(x+1)^4}\mathrm{d}x=\left[\frac{4(x+1)^{-4+1}}{-4+1}\right]_1^3$ <br> $=-\frac{4}{3}\left[(x+1)^{-3}\right]_1^3$ <br> $=-\frac{4}{3}\left[\frac{1}{64}-\frac{1}{8}\right]$ <br> $=\frac{7}{48}$ | |

| 4 | | | |
|---|---|---|---|
| | **Solution** | | |
| | **(i)** |   | |
| | **(ii)** | From GC, $x = 2.1771$ and $x = 4.8229$ (4d.p.) | |

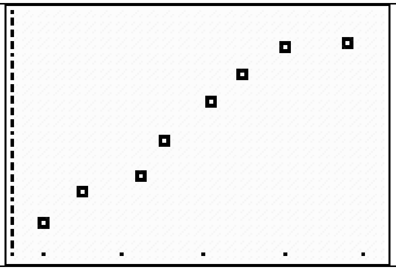
| | | | |
|---|---|---|---|
| | **(iii)** | Area $=\int_{2.1771}^{4.8229}\frac{10x-21}{x-2}-2x\,\mathrm{d}x$<br>$=5.16859454\approx 5.17$ (3 s.f.) | |

| **5** | | | |
|---|---|---|---|
| | **Solution** | | |
| | **(i)** | From GC, $\frac{\mathrm{d}}{\mathrm{d}x}\left(2^x-x^3\right)\Big\vert_{x=2}=-9.2274=-9.23$ (3 SF) | |
| | **(ii)** | When $x = 2$, $y = -4$.<br>Gradient of the normal at $P$ = $-\frac{1}{-9.2274}$ = 0.1083727478<br>= 0.10837<br><br><u>Equation of normal to $C$ at $P$</u><br>$y-(-4)=0.1083727478(x-2)$<br>$y=0.1083727478x-4.216745496$<br>$=0.10837x-4.21675$<br>$=0.108x-4.22$ | |
| | **(iii)** | $y=0.10837x-4.21675$ intersects with $y=2^x-x^3$ at $Q(9.9313264,-3.140492)$<br><br>When $x = 0$, $y = -4.21675$. $R(0,-4.21675)$<br>Distance between $QR$<br>$=\sqrt{(0-9.9313264)^2+(-4.21675+3.140492)^2}$<br>= 9.989473227<br>= 9.99 | |

| 6 | | | |
|---|---|---|---|
| | **Solution** | | |
| # | **(i)** | The teacher stands at the library exit and choose at random starting, say 5.<br><br>Then select the $5^{th}$, $15^{th}$, … students who exit the library until a sample of 10 is selected.<br>Interval: 100÷10 = 10.<br><br>Then choosing every 10th student after the first student is chosen. | |
| | **(ii)** | The sample was biased because it excluded students who visited the library during other times of the day, e.g. afternoon. | |
| | **(iii)** | The teacher should use a list of all the students in the college rather than just those who visited the library in the morning. | |

| 7 | | | |
|---|---|---|---|
| | **Solution** | | |
| # | **(i)** | Let $X$ be r.v. denoting no. of students who will be late for school in a day out of 27 students.<br><br>$X \sim B(27, 0.04)$<br><br>$\mathrm{E}(X) = 27(0.04)$<br><br>$= 1.08$ | |
| | **(ii)** | $\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X \leq 1)$<br>$= 0.29420$<br><br>$= 0.294$ | |
| | **(iii)** | Required prob. = $(0.294199)(1-0.294199)^4$ | |

| | | | |
|---|---|---|---|
| | | $= 0.073008$<br><br>$= 0.0730$ | |
| | | Let $Y$ be r.v. denoting no. of classes with at least 2 students late for school on Monday.<br>$Y \sim B(70, 0.073008)$<br>Since $n = 70$ is large, $np = 5.11056 > 5$, $nq = 64.88944 > 5$,<br>$Y \sim N(5.11056, 4.73745)$ approx.<br>$P(Y \geq 10) = P(Y \geq 9.5)$<br>$= 0.021864$<br>$= 0.0219$ | |

| 8 | | | |
|---|---|---|---|
| | **Solution** | | |
| # | **(i)** | $\bar{x} = \dfrac{6.0+6.5+7.2+7.5+8.1+8.5+9.0+9.8}{8}$<br>$= 7.825$<br>$\bar{y} = \dfrac{153+158+161+197+m+178+182+183}{8}$<br>$= \dfrac{1182+m}{8}$<br>Sub. $\bar{x}$ and $\bar{y}$ into y on x regression line,<br>$\dfrac{1182+m}{8} = 8.734(7.825)+101.031$<br>$m = 172.9964 = 173$ (shown) | |
| | **(ii)** |  | |
| | **(iii)** | $r = 0.97958 = 0.980$ | |

| | | | |
|---|---|---|---|
| | **(iv)** | $x = 0.10987y - 10.78358$<br>$= \quad 0.110y - 10.8$ | |
| | **(v)** | $x = 0.10987(170) - 10.78358$<br><br>= 7.89432<br><br>= 7.89<br>The estimate is **reliable** as ***y* = 170 is within the range of** $153 \le y \le 183$ or **interpolation** is used. In addition, ***r* = 0.980 is close to 1.** | |

| **9** | | | |
|---|---|---|---|
| | **Solution** | | |
| # | **(i)** | $W_1 + W_2 + A_1 + A_2 + A_3 \sim \mathrm{N}(8.2,\ 0.11)$<br>$\mathrm{P}(W_1 + W_2 + A_1 + A_2 + A_3 < 9)$ =0.992 | |
| # | **(ii)** | Let W and A be the random variables denoting the mass of a randomly selected watermelon and papaya in kg, respectively. $W \sim \mathrm{N}(2.3,\ 0.2^2)$ & $A \sim \mathrm{N}(1.2,\ 0.1^2)$<br>$W - 2A \sim \mathrm{N}(-0.1,\ 0.08)$<br>$\mathrm{P}(\lvert W - 2A \rvert > 0.1) = \mathrm{P}(W - 2A > 0.1) + \mathrm{P}(W - 2A < -0.1)$<br>=0.740 | |
| | **(iii)** | $4W + 6A \sim \mathrm{N}(16.4,\ 1)$<br>$\mathrm{P}(4W + 6A > 16) = 0.655$ | |
| | **(iv)** | Let $X$ be the random variables denoting the mass of a randomly selected hamper in kg.<br>$X \sim \mathrm{N}(\mu,\ \sigma^2)$<br>$\mathrm{P}(X < 6) = 0.05$ $\quad$ $\mathrm{P}(X > 8) = 0.03$<br>$\mathrm{P}\left(Z < \frac{6-\mu}{\sigma}\right) = 0.05$ $\quad$ $\mathrm{P}\left(Z > \frac{8-\mu}{\sigma}\right) = 0.03$ | |

| | | | |
|---|---|---|---|
| | | $\mathrm{P}\left(Z<\frac{8-\mu}{\sigma}\right)=0.97$<br>$\frac{6-\mu}{\sigma}=-1.644853626---(1)$<br>$\frac{8-\mu}{\sigma}=1.88079361---(2)$<br>By GC, $\mu=6.93$ $\sigma^2=0.322$ | |

| 10 | | | |
|---|---|---|---|
| | **Solution** | | |
| # | **(i)** | Unbiased est. of population mean,<br>$\hat{\mu}=\bar{x}=\frac{\sum(x-500)}{80}+500=\frac{50}{80}+500=\frac{4005}{8}=500.625\approx 501$<br><br>Unbiased est. of population variance,<br>$\hat{\sigma}^2=\frac{1}{80-1}\left[1150-\frac{(50)^2}{80}\right]$<br>$\hat{\sigma}^2=14.16139241\approx 14.2$ | |
| | **(ii)** | Let $X$ be the random variable denoting the mass of a randomly chosen vase in grams.<br>Since $n$ is large, by C.L.T.,<br>Assuming $H_0$ is true,<br>$\overline{X}\sim N\left(500,\frac{14.16139241}{80}\right)$ approximately<br>$H_0:\mu=500$<br>$H_1:\mu\neq 500$<br>Value of test statistic, $z=\frac{500.625-500}{\sqrt{\frac{14.16139241}{80}}}=1.485497866$<br>Use GC, $p$-value = 0.1374120513 = 0.137<br><br>Since $p$-value = 0.137 > 0.1, we do not reject $H_0$.<br>Hence, there is insufficient evidence at 10% significance | |

| | | | |
|---|---|---|---|
| | | level to indicate the company's claim is not valid. | |
| | **(iii)** | Given the company's claim is valid, we do not reject $H_0$.<br>$z < 1.644853626$<br>$\dfrac{501-500}{\left(\dfrac{9}{\sqrt{n}}\right)} < 1.644853626$<br>$n < 219.1490195$<br><br>The largest sample size is 219.<br><br>Assume that sample size is sufficiently large and Central Limit Theorem applies since population distribution is unknown. | |

| **11** | | | |
|---|---|---|---|
| | **Solution** | | |
| **(a)** | **(i)** | P(Western Wild on 2nd day) = P(Western Wild on 1st day and 2nd day) + P(Asian Delights on 1st day and Western Wild on 2nd day)<br>$= \frac{1}{3}\cdot\frac{1}{2}\left(\frac{1}{3}\right)+\frac{2}{3}\left(\frac{4}{6}\right)$<br>$=\frac{1}{2}$ or 0.5 | |
| | **(ii)** | P(Asian Delights on at least 2 of the 3 days)<br>= P(Asian Delights on 2 of the 3 days) + P(Asian Delights on all 3 days)<br>$= \frac{2}{3}\cdot\frac{2}{6}\cdot\frac{10}{12}+\frac{2}{3}\cdot\frac{4}{6}\cdot\frac{8}{12}+\frac{1}{3}\cdot\frac{5}{6}\cdot\frac{5}{12}+\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{6}$<br>$=\frac{137}{216}$ or 0.634 | |
| | **(iii)** | (i) P(Asian delights on 3rd day given Western Wild on 2nd day)<br>= P(Asian delights on 3rd day and Western Wild on 2nd day)/ P(Western Wild on 2nd day) | |

| | | | |
|---|---|---|---|
| | | $= \dfrac{\frac{1}{3}\cdot\frac{1}{6}\cdot\frac{11}{12}+\frac{2}{3}\cdot\frac{4}{6}\cdot\frac{8}{12}}{\frac{1}{2}}$<br>$= \dfrac{25}{36}$ or 0.694 | |
| (b) | (i) | $P(A\cap B') = 0.6(1-0.3)$<br>$= 0.42$ | |
| | (ii) | $P(A'\cap B') = 1 - P(A\cup B)$<br>$= 1-[P(A\cap B')+P(B)]$<br>$= 1 - (0.42 + 0.3)$<br>$= 0.28$ | |
| | (iii) | Using $\dfrac{P(B\cap A')}{P(A')} = 0.2$,<br>$\dfrac{P(B)-P(A\cap B)}{1-[P(A\cap B')+P(A\cap B)]} = 0.2$<br>$\dfrac{0.3-P(A\cap B)}{1-[0.42+P(A\cap B)]} = 0.2$<br>Solving, $P(A\cap B) = 0.230$ | |