Name: _____ Class: _____

NATIONAL JUNIOR COLLEGE
2014 SENIOR HIGH 2 PRELIMINARY EXAMINATION

MATHEMATICS **8864/01**
Higher 1

Additional Materials: Answer Paper **3 hrs**
List of Formulae (MF15)
Cover Sheet

16 September 2014, Tuesday 0815 – 1115 hours

INSTRUCTIONS TO CANDIDATES

Write your name, registration number, subject tutorial group, on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **ALL** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
Up to **2 marks** will be deducted for poor presentation in your work for this paper.

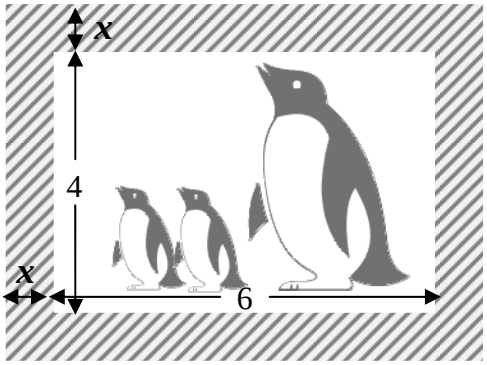
At the end of the examination, fasten all your work securely together. Attach the cover sheet at the front of your answer scripts.
The number of marks is given in the brackets [ ] at the end of each question or part question.

This question paper consists of 6 printed pages, including the cover sheet.

Section A: Pure Mathematics [35 marks]

1 Given that $\frac{2}{e^{2x}} = 3(3e^{2x} - 1)$, use the substitution $u = e^{-2x}$ to find the exact value of x. **[4]**

2 **(a)** The diagram below shows a 4R photo (measuring 4 inch by 6 inch) of penguins.
Qi Er would like to make a frame for the photo.



Given that the width of the frame must be x inches on all sides and the total area of the frame is 15 square inches, show that

$$4x^2 + 20x - 15 = 0 .$$

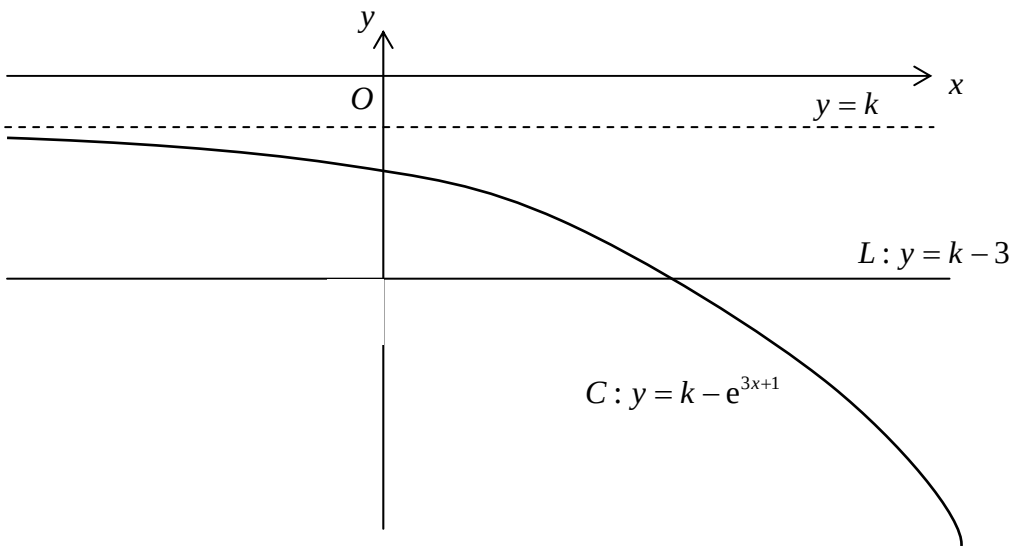
Hence find the exact value of x by completing the square. **[4]**

(b) Find the range of values of k for which the equation $2x^2 + kx = -2$ has real roots. **[2]**

3 **(i)** Differentiate $e^{2x^3 + \ln x}$ with respect to x. **[3]**

(ii) Hence, or otherwise, evaluate $\int \left(\frac{6x^3 + 1}{x} \right) e^{2x^3 + \ln x + 1} + 6x^2 + \frac{1}{x} \, dx$. **[4]**

4 The diagram shows the curve C with equation $y = k - \mathrm{e}^{3x+1}$ and the line L with equation $y = k - 3$, where $k < -1$.



(i) Find the exact value of the x coordinate of the point where C and L intersect **[2]**

(ii) Hence find the area of the finite region which is bounded by the y axis, C and L. Express your answer in the form $\ln p + q + r\mathrm{e}$, where p, q, and r are real constants to be determined. **[6]**

5 The function f is defined by

$$\mathrm{f}(x) = \ln(x+2) + x^2 - 3x \ , \ x \geq 0 \, .$$

(i) Show that the curve $y = \mathrm{f}(x)$ has a minimum point at $x = \dfrac{-1+\sqrt{41}}{4}$. **[4]**

(ii) Given that $\mathrm{f}'(x)$ is positive for $x > \dfrac{-1+\sqrt{41}}{4}$, describe the behaviour of the graph of $y = \mathrm{f}(x)$ for $x > \dfrac{-1+\sqrt{41}}{4}$. **[1]**

(iii) A function g is defined by $\mathrm{g}(x) = 3^x \mathrm{f}(x)$. Find the numerical value of the gradient of the graph of $y = \mathrm{g}(x)$ at the point where $x = 1$. **[1]**

(iv) Sketch the graph of $y = \mathrm{f}(x)$, indicating clearly the axial intercepts. **[2]**

(v) Find the area of the region bounded by $y = \mathrm{f}(x)$, the x-axis and the lines $x = 1$ and $x = 2$, correct to 4 decimal places. **[2]**

Section B: Statistics [60 marks]

6 A particular company, MoonHub, has a 24-hour customer service hotline where calls are recorded for training purposes. In a day, the company receives an estimated total number of 3600 calls, of which an estimated 1960 calls are regarding mobile services, 840 calls are regarding broadband services and the remaining calls are regarding TV services.

Describe how a sample of 50 calls can be selected using systematic random sampling. **[3]**

Suggest another sampling method that is more appropriate for this situation. Justify your answer. **[2]**

7 The table below shows the results of a survey of 100 respondents, in which their favourite creatures and colours were recorded.

	Falcon	**Penguin**	**Lion**
Blue	11	16	8
Red	6	29	10
Green	4	9	7

One respondent is selected at random.

Events *P*, *G* and *R* are defined as follows:

P: the selected respondent's favourite creature is the penguin,

G: the selected respondent's favourite colour is green,

R: the selected respondent's favourite colour is red.

(i) Find $\mathrm{P}(R \cap P')$, **[2]**

(ii) Find the probability that a selected respondent's favourite creature is the lion given that the respondent's favourite colour is blue. **[2]**

(iii) State, with a reason, whether *P* and *G* are mutually exclusive events. **[1]**

8 Events *A* and *B* are such that $\mathrm{P}(A' \cup B) = \frac{2}{5}$, $\mathrm{P}(A \cup B') = \frac{5}{6}$ and $(A \cup B)' = \varnothing$.

(i) Find $\mathrm{P}(A)$ and $\mathrm{P}(B)$. **[4]**

(ii) Determine if *A* and *B* are independent events. **[2]**

9 Maxihard Computers has a service hotline that is in operation every day. Every day, there is a probability of p that there are more than 10 calls to the hotline. The probability Maxihard receives more than 10 calls to the hotline on at least half the days in a year is 0.015.

(Assume that a year has 365 days.)

(i) Show that p is 0.443. **[2]**

(ii) Calculate the probability that Maxihard receives more than 10 calls to the hotline on at most 150 days. **[1]**

(iii) Calculate the probability that Maxihard receives more than 10 calls to the hotline on 180 days given that it receives more than 10 calls to the hotline on more than 150 days. **[3]**

(iv) Using a suitable approximation, find the probability that in a year, the number of days on which Maxihard receives more than 10 calls is between 140 and 190. Leave your answer correct to 5 decimal places. **[3]**

10 SB Coffee Company claims that the average expenditure per customer is \$8.50. It is known that the standard deviation of customer spending is \$0.55.

(a) A test at 10% level of significance is carried out based on a sample of 50 randomly chosen customers. The company's claim was not rejected. If the sample mean expenditure is \$$C$, find the range of possible values for C. **[4]**

(b) An economic recession hit. The company then claimed that the average expenditure per customer is reduced. A random sample of 60 customers was taken to test this hypothesis. The expenditure data for customers in this sample is summarised below,

$$\sum(x-8.5)=-3; \qquad \sum(x-8.5)^2=18.46$$

(i) Based on the sample data above, calculate the unbiased estimates of the population mean and variance. **[2]**

(ii) Assuming that the standard deviation of expenditure per customer did not change, test the company's claim about the reduced expenditure at a 5% level of significance. **[4]**

(iii) For **(ii)**, explain, in the context of the question, what "5% level of significance" means. **[1]**

11 A vegetable stall sells carrots and tomatoes. The mass of a randomly selected carrot and tomato are normally distributed with means 85 g and 112 g as well as standard deviations σ g and 9 g respectively.

(i) There is a 90% chance that a randomly selected carrot has mass greater than 80g. Show that the value of σ is 3.9, correct to 1 decimal place. **[2]**

(ii) The shopkeeper selects two tomatoes randomly. Find the probability that one tomato weighs more than 116 g while the other tomato weighs between 105 g and 110 g. **[2]**

(iii) Find the probability that thrice the mass of a randomly chosen carrot differs from the total mass of two randomly selected tomatoes by more than 10 g. **[4]**

(iv) State an assumption needed for your calculations in **(iii)**. **[1]**

(v) A sample of sixty carrots is chosen. Find the probability that the mean mass of carrots in this sample is at most 85.6 g. **[3]**

12 Amethyst College wishes to study the relationship between students' average number of hours of sleep per day *t*, and students' Grade Point Average (GPA) *p*, out of a maximum of 4. Ten students were randomly selected, and the results are summarised in the table below.

Student	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**J**	**K**
***t* (hours)**	6.9	8.0	5.2	3.7	*x*	7.2	6.6	4.4	4.1	5.8
p	3.51	3.92	2.46	2.11	1.74	3.38	3.80	2.75	1.99	3.04

(i) Given that $(\bar{t}, \bar{p})$ is $(5.54, 2.87)$, show that $x = 3.5$. **[1]**

(ii) Give a sketch of the scatter diagram for the data, labelling also the point $(\bar{t}, \bar{p})$. **[2]**

(iii) Calculate the product moment correlation coefficient and comment on its value in the context of the question. **[2]**

(iv) Find the equation of the regression line of *p* on *t*. Sketch this line on your diagram in **(ii)**. **[2]**

(v) Calculate an estimate of a student's GPA if this student sleeps an average of 7.6 hours a day. Comment on the reliability of this estimate. **[2]**

(vi) Explain, in context, why it is unsuitable to use the equation in **(iv)** to estimate the average number of hours of sleep of a student with a GPA of 3.14. **[1]**

(vii) The college decided to increase all students' GPA by 0.02. Without any further calculations, state the new equation of the regression line of *p* on *t*. **[2]**

—— End of Paper ——

NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 1

MATHEMATICS **8864/01**
Higher 1
16 September 2014, Tuesday 0815 – 1115 hrs

Candidate Name: _____ Registration No.:_____

Subject Class: _____ Subject Tutors: _____

Cover Sheet

INSTRUCTIONS TO CANDIDATES

Write your name, registration number, subject tutorial group, subject tutor's name and calculator model in the spaces provided on the cover sheet and attached it on top of your answer paper.
Circle the questions you have attempted and arrange your answers in **NUMERICAL ORDER**.

Write your calculator's model number(s) in the box below.

Scientific Calculator Model:

Graphic Calculator Model:

For official use

Question No.	Marks Obtained	TOTAL MARKS
1		*4*
2		*6*
3		*7*
4		*8*
5		*10*
6		*5*
7		*5*
8		*6*
9		*9*
10		*11*
11		*12*
12		*12*
Presentation	**– 1 / –2**	
TOTAL		**95**
GRADE		