



H1 Mathematics

8864/01

16 September 2014

Paper 1

3 hours

Additional Materials: Writing paper

List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

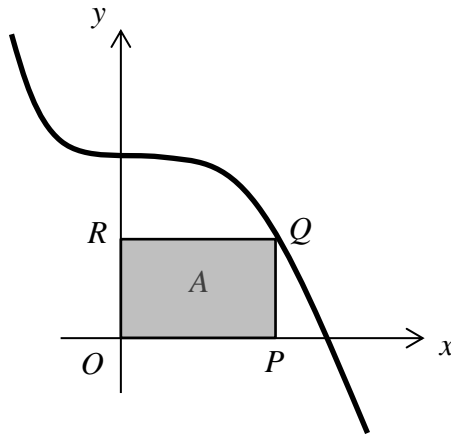
The number of marks is given in brackets [] at the end of each question or part question.

At the end of the examination, fasten all your work securely together.

Section A: Pure Mathematics [35 marks]

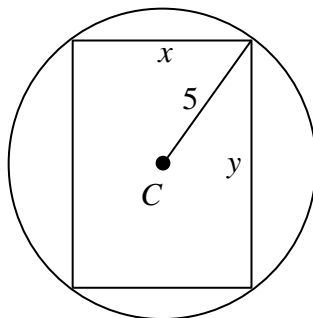
- 1 Find $\frac{d}{dx}\sqrt{7-2x^3}$. Hence find the exact value of $\int_{-1}^1 \frac{x^2}{\sqrt{7-2x^3}} dx$. [5]

2



The diagram shows the curve with equation $y = 6 - 2x^3$ and rectangle $OPQR$ constructed such that the point Q is a point on the curve with coordinates (x, y) . The area of rectangle $OPQR$ is denoted by A .

- (i) Show that $A = 6x - 2x^4$. [1]
- (ii) Use differentiation to find the exact value of x for which A is a maximum. Hence find the maximum value of A . [5]
- 3 A wire of length k units, where $k > 20$, is bent to form a rectangle with breadth x units and length y units. The rectangle is inscribed in a circle with centre C and radius 5 units, as shown in the diagram below.



Show that

(i) $x^2 + y^2 = 100$, [1]

(ii) $8x^2 - 4kx + k^2 - 400 = 0$. [3]

Find the range of values of k for the equation in part (ii) to have real solutions, where $k > 20$. [4]

- 4 The point P lies on the curve $y = \ln(1-x) + c$ where c is a fixed constant. The equation of the normal to the curve at P is $y = x + 2$. Write down the gradient of the tangent to the curve at P and show that the coordinates of P are $(0, 2)$. Hence find the equation of the tangent to the curve at P . [5]

The normal to the curve at P intersects the x -axis at point A and the tangent to the curve at P intersects the x -axis at point B . Find the area of the triangle ABP . [3]

- 5 The curves C_1 and C_2 have equations $y = 5 - e^{-2x}$ and $y = 5 - \frac{1}{6-x}$ respectively.
- (i) Sketch C_1 and C_2 on the same diagram, stating the coordinates of any point(s) of intersection with the axes, and equation(s) of any asymptote(s). Find the coordinates of the point(s) of intersection between C_1 and C_2 . [5]
- (ii) Hence find the area bounded by C_1 , C_2 and the x -axis. [3]

Section B: Statistics [60 marks]

- 6 (a) Explain the meaning of “random” in the phrase “random sample”. [1]
- (b) The National Council on Gaming Addiction wants to assess the level of awareness of gaming addiction among Singaporeans.
- (i) Describe how simple random sampling may be carried out to obtain a sample of size 1000. State one disadvantage of this sampling method. [3]
- (ii) Give a reason why a stratified sample might be preferable in this context. [1]
- 7 The continuous random variable X has the distribution $N(\mu, \sigma^2)$. It is known that $P(X < -2a) = 0.2$ and $P(X > a) = 0.6$, where a is a positive constant. Express σ and μ in terms of a . [5]

[Turn Over]

- 8** An orchard produces honeydew melons and watermelons. The masses of honeydew melons and watermelons are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean mass (kg)	Standard deviation (kg)
Honeydew melons	2.75	0.45
Watermelons	6.00	1.00

- (i) Find the probability that the total mass of two randomly chosen honeydew melons is more than the mass of one randomly chosen watermelon. [3]
- (ii) Find the probability that the mean mass of two randomly chosen honeydew melons is more than the mass of a third randomly chosen honeydew melon by at least 0.1 kg. [3]

The orchard also produces strawberries with an expected mass of 18 g and standard deviation of 1 g.

- (iii) Find the probability that the total mass of 50 randomly chosen strawberries exceeds 800 g. [3]

- 9** John plays a game with at most 3 sets. The probability that he wins the first set is 0.7. For the second set,

- the probability that he wins the set given that he wins the first set is 0.7.
- the probability that he wins the set given that he did not win the first set is 0.4.

He wins the games if he wins two sets. The game stops when he wins two sets or loses two sets. If he wins one set and loses one set, he gets to play a third set. The probability that he wins the third set is 0.7, independent of the outcome of the first two sets. In this case, he wins the game if he wins the third set.

- (i) Represent the above information using a tree diagram. [3]
- (ii) Calculate the probability that
- (a) he plays all 3 sets. [2]
 - (b) he wins the game. [2]
 - (c) he wins the game given that he did not win the first set. [2]

- 10** A company manufactures a large number of calculators. It is known that 5% of calculators manufactured are found to be defective. From a batch of calculators manufactured, a random sample of 20 calculators is drawn for inspection.
- (i) State, in the context, two assumptions needed for the number of defective calculators in the sample to be well modelled by a binomial distribution. [2]
 - (ii) A batch is labelled as acceptable if the sample of 20 calculators drawn has fewer than 3 defective calculators. Eighty batches are produced daily. Using a suitable approximation, find the probability that in a day, at least 90% of the batches are acceptable. [5]
 - (iii) A quality control inspector takes another new sample of size k for inspection. He will reject the batch if this sample contains at least 3 defective calculators. The probability that he rejects the batch is more than 0.8. Find the least value of k . [3]
- 11** Consumers of ABC Juice company complained that the actual volume in a packet of apple juice is less than the printed value of 300 ml on the packet. The manager took a random sample of 100 packets and measured the volume, x ml of apple juice in each packet. The results are summarised by

$$\sum x = 29800, \sum x^2 = 8908058 .$$

The population mean volume of the packets is denoted by μ ml.

- (i) Calculate unbiased estimates of the population mean and variance. [2]
 - (ii) The null hypothesis $\mu = 300$ is to be tested against the alternative hypothesis $\mu < 300$. Find the p -value of the test and state the meaning of this p -value in the context of the question. [You are not required to conduct the test.] [2]
- Hence find the set of values of α , where $\alpha\%$ is the level of significance, such that the consumers' complaint is justified. [2]
- (iii) Using the above sample, another test is conducted at 12% level of significance. The hypotheses are:

Null hypothesis: the population mean volume is equal to μ_0 ml

Alternative hypothesis: the population mean volume is not equal to μ_0 ml

Suppose that the population standard deviation is known to be 11 ml, find the possible values of μ_0 such that the null hypothesis is not rejected. [4]

- (iv) State whether it is necessary to assume that the volume of apple juice in a packet follows a normal distribution for your calculations in parts (ii) and (iii). [1]

[Turn Over

- 12** A teacher records the number of hours her students spent revising Mathematics and the scores they obtained for their Mathematics paper. The following table shows the results:

Student	A	B	C	D	E	F	G	H	I	J
No of hours (x)	15	22	25	28	31	33	36	39	42	48
Scores (y)	49	50	55	70	57	65	60	72	28	75

- (i) Draw a scatter diagram for the above data. [2]
- (ii) Identify and circle the outlier in the scatter diagram. [1]
- (iii) Remove the outlier and find the equation of the regression line of y on x . Interpret the slope of this line in the context of the question. [2]

The teacher realizes that the outlier is a result of her recording a wrong score for a particular student. She rectifies the error and enters the appropriate score for that student. The equation of the regression line y on x with the rectified data point is recalculated and its new equation is $y = 36.2 + 0.817x$.

- (iv) Show that the rectified score is 70, to the nearest integer. [3]
- (v) Calculate the product moment correlation coefficient for the new data. [1]
- (vi) Estimate the score of a student, to the nearest integer, if he has spent 60 hours revising Mathematics. Comment on the validity of your answer. [2]

END OF PAPER