Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 1)

8864

22 September 2014

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

At the end of the examination, **attach the question paper to the front of your answer script.**

The total number of marks for this paper is **95**.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Score												
Max Score	**6**	**9**	**10**	**10**	**3**	**4**	**7**	**9**	**11**	**12**	**14**	**95**

Section A: Pure Mathematics [35 marks]

1 **(i)** Using an algebraic method, solve $\ln\left(\dfrac{4x^3}{3x+2}\right) - 2\ln x = \ln(x-4)$. [4]

(ii) Hence solve $\ln\left(\dfrac{4e^{6x}}{3e^{2x}+2}\right) - 4x = \ln\left(e^{2x}-4\right)$. [2]

2 Differentiate $\dfrac{x}{2x+3}$. [3]

Hence, or otherwise,

(a) find the derivative of $f(x) = \dfrac{\ln x}{\ln x^2 + 3}$, and determine the set of values of x where the derivative exists. [4]

(b) evaluate, without using a calculator, the integral $\displaystyle\int_{-1}^{0} \frac{1}{(4x+6)^2}\,dx$. [2]

3 The curve C has equation $y = 3^x + x^2 - \dfrac{1}{3}x$.

(i) Sketch C, stating the coordinates of any points of intersection with the axes and the coordinates of any stationary points. [3]

(ii) Find the numerical value of the gradient of C at the point where $x = -3$. Give your answer correct to 4 decimal places. [1]

(iii) Hence find the equation of the normal to C at the point where $x = -3$. Give your answer in the form $y = ax + b$, with a and b correct to 4 decimal places. [3]

(iv) Given that the normal to C in part **(iii)** meets the curve again at a point where $x > 0$, state the x-coordinate of this point of intersection. Hence write down an integral that gives the area of the finite region between C and the normal, and evaluate this integral numerically. [3]

4 The curve C has equation $y = 4x^3 + px^2 + 3x + q$, where p and q are real constants.

(i) By using differentiation, find the set of values of p for which C has two stationary points. [4]

(ii) It is given that C has a stationary point at $\left(-\frac{1}{2}, -3\right)$.

(a) Show that $p = 6$ and find the value of q. Hence determine the nature of the stationary point at $\left(-\frac{1}{2}, -3\right)$. [5]

(b) Explain whether the nature of this stationary point can be deduced from the values of p found in the earlier parts. [1]

Section B: Statistics [60 marks]

5 Independent events A and B are such that P(A) = p and P(B) = q, where $p > q$. Given that $\mathrm{P}(A \cup B) = 0.58$ and $\mathrm{P}(A \cap B) = 0.12$, find a quadratic equation satisfied by p, and hence find $\mathrm{P}(A \cup B')$. [3]

6 The organiser of an overseas education fair would like to survey 1% of the people attending the one-day event on their opinions about the fair.

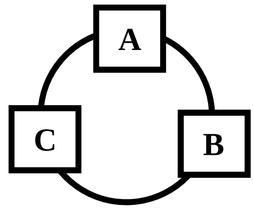
(i) The organiser has identified that the people at the event comprise students and working adults and has decided to target these two groups for his survey.

Give one advantage of using a stratified sample in comparison to a sample obtained by quota sampling. Suggest why it would be difficult for the organiser to obtain a stratified sample. [2]

(ii) Explain how the event organiser can conduct his survey using a systematic sample. [2]

7 For a particular activity, three boxes A, B and C are placed in a circular arrangement.

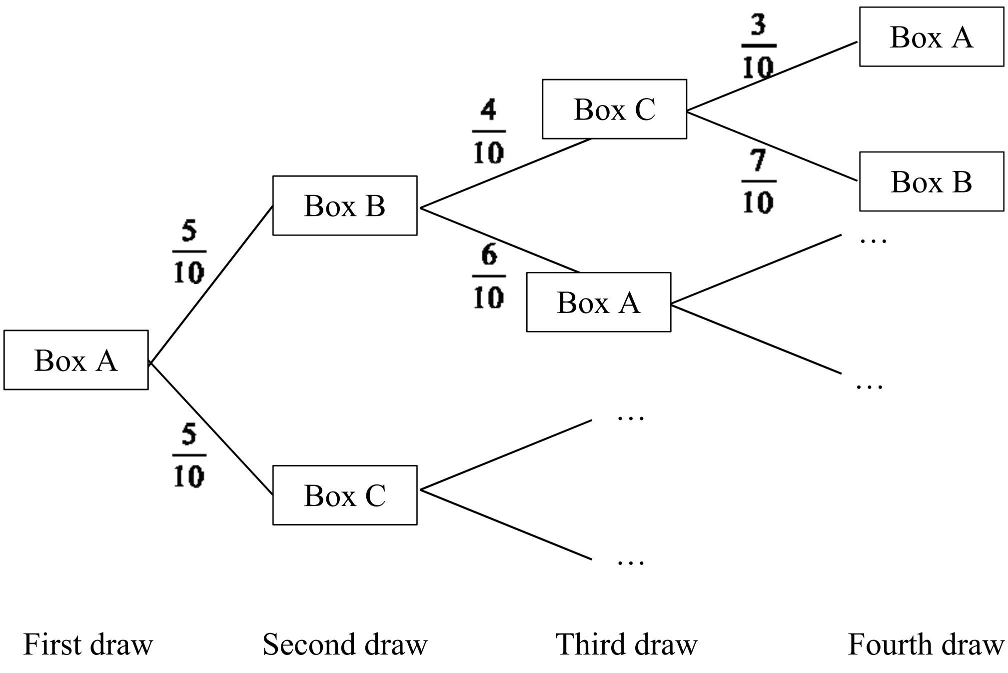
Box A contains 5 red and 5 white balls.
Box B contains 4 red and 6 white balls.
Box C contains 3 red and 7 white balls.



For each round, a participant of this activity will draw a ball randomly from a box at any one time **without** replacement. The participant begins his first draw from box A, and the subsequent draws are based on the following rules:

- If a red ball is drawn, the next draw is from the box in the clockwise direction.
- If a white ball is drawn, the next draw is from the box in the anti-clockwise direction.

A tree diagram is used to illustrate all the possible outcomes from the first draw to the fourth draw.



(i) Complete the above tree diagram. [2]

(ii) Show that the probability that the participant has not drawn from box C from the first draw to the fourth draw is $\frac{2}{15}$. [1]

(iii) Given that the participant draws from box B on the fourth draw, find the probability that he has not drawn from box C from the first draw to the fourth draw. [4]

8 In a particular year, a Household Expenditure Survey was conducted and the table below presents the average monthly household expenditure, $\$y$ in thousands, for the respective monthly household income group, $\$x$ in thousands.

Income group, x	1	2	3	4	5	6
Expenditure, y	1.11	1.42	2.25	3.40	3.55	4.85

(i) Sketch a scatter diagram to illustrate these values. [1]

(ii) Find the equations of the regression line of y on x and x on y. [2]

(iii) Interpret the gradient and the y-intercept of the regression line of y on x. [2]

(iv) Calculate the product moment correlation coefficient, and comment on its value in relation to your scatter diagram. [2]

(v) It is required to estimate the monthly household expenditure based on an income of \$9 350. Find the required estimate by using a suitable regression line. [1]

(vi) Comment on the reliability of the estimate in part **(v)**. [1]

9 A baby drinks either BabyGrow or InfanGrow milk powder. The sleeping hours of babies on a particular night follow independent normal distributions with means and standard deviations as given in the table, according to the brand of milk powder they drink.

	Mean	Standard deviation
BabyGrow	8.0	σ
InfanGrow	6.5	0.795

(i) Given that 85% of babies who drink BabyGrow slept less than 9 hours, show that $\sigma = 0.965$. [2]

(ii) Find the probability that the total sleeping hours of three randomly chosen babies who drink BabyGrow exceeds four times the sleeping hours of a randomly chosen baby who drinks InfanGrow, by more than one hour. [3]

In a random sample of twelve babies who drink BabyGrow, the number of babies who slept more than 7 hours was recorded.

(iii) Find the probability that at least ten babies slept more than 7 hours. [3]

(iv) Given that 50 such samples are taken and the probability that the mean number of babies who slept more than 7 hours exceeds α is less than 0.9, find the range of α. [3]

10 In Factory A, it is claimed that the mean mass of each bag of beans produced is 22 kg. To investigate this claim, the mass, x kg, of a random sample of 50 bags of beans are obtained and summarised below:

$$\sum(x-18)=165.3 \text{ and } \sum(x-18)^2=876.5.$$

(i) What do you understand by the term "unbiased estimate"? [1]

(ii) Find the unbiased estimates of the population mean and variance. [2]

(iii) Test, at the 5% level of significance, whether this claim is valid. [4]

(iv) State the meaning of the p-value obtained in part **(iii)**. [1]

In Factory B, masses of bags of beans produced follow a normal distribution, with standard deviation 4 kg. It is claimed that the mean mass of each bag of beans produced is at least μ_0 kg. A random sample of 50 bags of beans yielded a sample mean of 8 kg. Given that there is sufficient evidence at the 5% level of significance to conclude that this claim is valid, find the set of possible values of μ_0. [4]

11 In a school with 2400 students, an average of 1 out of 5 students is sick in a month.

(i) State, in this context, two conditions that must be met for the number of students who are sick in a month to be well modelled by a binomial distribution. Explain why each of these conditions may not be met. [3]

For the remainder of this question, assume that these conditions are met.

(ii) In a sample of 25 randomly selected students, find the probability that the last student selected is the fifth student who is sick in a month. [3]

(iii) Find the least value of n such that in a sample of n randomly selected students, the probability that at least 6 are sick in a month is more than 0.95. [3]

(iv) Using a suitable approximation, find the probability that less than 21% of the student population is sick in a month. [4]

(v) Find the expected number of months in which less than 21% of the student population is sick in a month over a period of 6 years. [1]

END OF PAPER