SAINT ANDREW'S JUNIOR COLLEGE

Preliminary Examinations

MATHEMATICS
Higher 1 **8864**

Friday **29th August 2014** **3 hours**

Additional materials : Answer paper
List of Formulae(MF15)
Cover Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.

Answer **all** the questions. Total marks : **95**
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically state otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematic steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of 7 printed pages including this page.

[Turn over]

Section A: Pure Mathematics [35 marks]

1 (i) Show that $-x^2+x-1$ is always negative for all real values of x. [2]

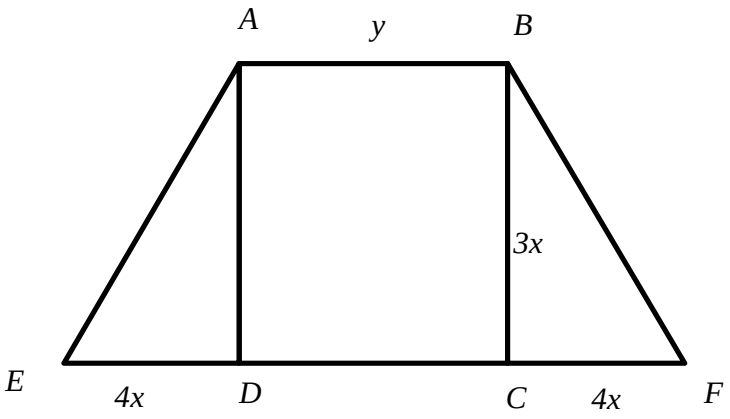
(ii) Hence, solve the inequality $\dfrac{x^2-x+1}{2x^2+5x-3}>0$. [2]

2 It is given that $y=\dfrac{e^x+e^{-x}}{e^x-e^{-x}}$.

(i) Show that $e^{2x}=\dfrac{y+1}{y-1}$. [2]

(ii) Find the value of x when $y=-\dfrac{5}{4}$. [2]

3



The diagram above shows a rectangle ABCD with two triangles beside it. It is given that $AB=y$ cm, $BC=3x$ cm, $DE=4x$ and $CF=4x$ cm. The total area of the rectangle and triangles is $A\,\text{cm}^2$. Given that the total perimeter of the diagram, ABFCDE is 30 cm,

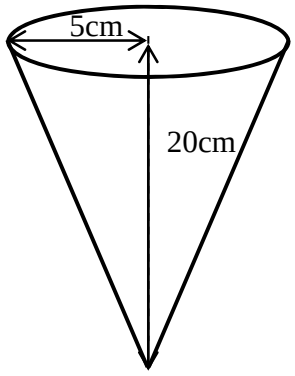
(i) Show that $A=45x-15x^2$ [3]

(ii) Find the maximum value of A. [3]

4 (a) Find $\int \frac{\left(2+\sqrt{x}\right)^2}{\sqrt{x}}\,\mathrm{d}x$. [2]

(b) Find the exact value of $\int_0^2 \left(\frac{1}{(8-5x)^3} - \mathrm{e}^{7-3x}\right)\mathrm{d}x$. [3]

5 A container is in the shape of an inverted right circular cone with radius 5 cm and depth 20 cm as shown in the diagram below. Given that the container is initially empty and water is poured into it at a constant rate of 5.5 $\mathrm{cm}^3\mathrm{s}^{-1}$, find



(i) the rate of increase of the depth of water at the instant when the depth of water is 4 cm, giving your answer in exact form. [5]

(ii) the time taken for the depth of water to increase from 4 cm to 10 cm. [2]

6(a) The curve C has equation $y = \frac{px+q}{x-1}$, where p and q are constants. The point $A(2,4)$ lies on C.

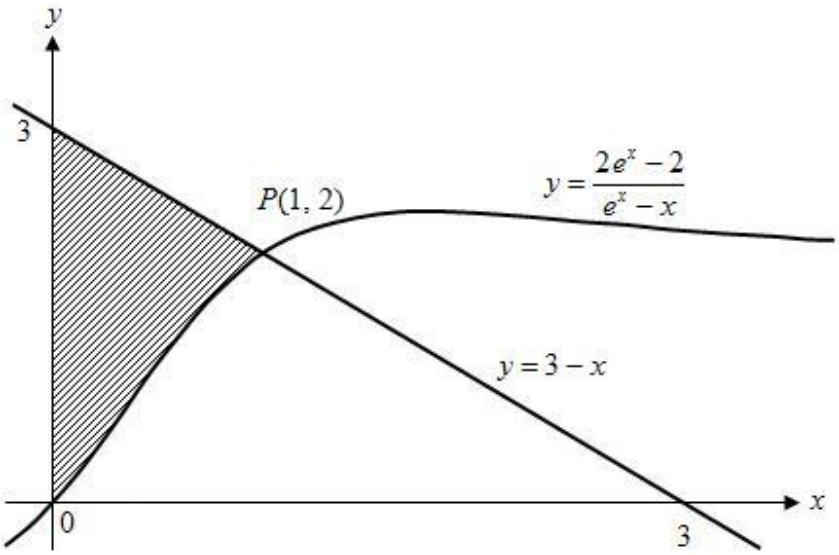
(i) Given further an asymptote of C has equation $y = 3$, find the values of p and q. [2]

(ii) Using the values of p and q found in part (i), sketch C. Indicate clearly the coordinates of any axial intercepts and the equations of the asymptotes. [3]

(b) (i) Given that $y = \ln(e^x - x)$, find $\frac{dy}{dx}$., [1]

(ii) In the diagram below $P(1, 2)$ is the point of intersection of the curve $y = \frac{2e^x - 2}{e^x - x}$ and the line $y = 3 - x$.

Using the answer obtained from part (i), find the exact area of the shaded region. [3]



Section B: Statistics [60 marks]

7 A and B are two events such that $\mathrm{P}(A) = \frac{1}{3}$, $\mathrm{P}(B) = \frac{1}{4}$

(i) Find $\mathrm{P}(A \cup B)$ if A and B are mutually exclusive. [2]

(ii) Find $\mathrm{P}(A \mid B)$ if A and B are independent. [1]

(iii) Using a Venn diagram or otherwise, find $\mathrm{P}(A' \cup B)$ if $\mathrm{P}(A \cap B) = \frac{1}{6}$. [2]

8 A sport club has 1450 members who are from 12 different branches. The club's committee wishes to seek members' views on where to hold the next annual meeting. A sample of 60 members is to be obtained and their views sought.
The following suggestion is made as to how to choose the sample:

"The names of all members are to be listed and numbered from 1 to 1450. Use a random number generator to generate 60 distinct numbers and corresponding members are included in the sample."

(i) For the above suggestion, name the method of sampling and identify one disadvantage of this method. [2]

(ii) State briefly how you would choose members by systematic random sampling method. [2]

9 Box *A* contains 3 red balls, 2 green balls and 1 yellow ball. Box *B* contains 5 red balls and 3 green balls.
One of the boxes is selected by rolling a fair die. If the die shows a '4' or '6', box *A* is selected and otherwise box *B* is selected.
One ball is chosen at random from the selected box and the colour of the ball is noted.

(i) Draw a tree diagram to represent this situation. [3]

(ii) Find the probability that a green ball is chosen. [2]

(iii) Given that that a red ball is chosen, find the probability that it came from box B. [3]

10 At a bakery shop, a machine is used to measure out quantities of dough for making loaves. For each batch, the mean amount of dough per loaf is set using a control on the machine. The weights of the resulting loaves are normally distributed with standard deviation 24 grams. The control is set to produce a batch of loaves with a mean weight of 1000 grams. The weights, in grams, of a random sample of nine loaves from this batch were

998 996 936 1002 957 968 920 943 1011

(i) Using the 5% significance level, examine whether the mean weight of this batch of loaves is 1000 grams. [4]

(ii) Explain what is meant by "5% level of significance" in the context of the question. [1]

(iii) Using the same above data, determine the set of values of μ_0 such that the null hypothesis is rejected in favour of the alternative hypothesis $\mu < \mu_0$ at 5% significance level given that $\mu = \mu_0$.

[5]

11 At Rice and Noodle Restaurant, the waiting time of a new customer depends on the number of customers who are already seated in the restaurant. Mr Lee is a customer who visited the restaurant on 7 different days. The table below shows, for each of these days, the number, *x*, of customers already seated and his waiting time, *t* minutes.

Day	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
x	6	9	10	7	4	12	8
t	7	11	*k*	9	5	14	9

(i) Given that the equation of the least square regression line of *t* on *x* is $t = 0.19 + 1.19x$, find the value of *k*, giving your answer to the nearest integer. [3]

(ii) Using the value of *k* found in part (i), draw a sketch of the scatter diagram for the data and find the product moment correlation coefficient. [3]

(iii) Using a suitable regression line and giving a reason for your choice, estimate

(a) Mr Lee's waiting time when the number of customers already seated in the restaurant is 14;

(b) the number of customers already seated in the restaurant if Mr Lee's waiting time is 8 minutes. [3]

For each of the above, comment on the reliability of the estimate. [2]

12 Students in ABC Junior College are taking the mid-year exam. There are 40 classes with 20 students in each class. The probability of a student getting an 'A' in the mid-year exam is 0.3.

(i) Find the probability that, in a class of 20,

(a) exactly 5 students getting 'A's in the exam. [2]

(b) more than 7 students getting 'A's in the exam. [2]

(ii) Find the probability that there are 25 classes with less than 8 students getting 'A's in the exam. [3]

(iii) Using a suitable approximation, find the probability that there are at least 250 students getting 'A's in the exam in the entire school. State the mean and variance of the approximation. [4]

13 (a) Given that $X \sim N(\mu, \sigma^2)$ and $P(12 < X < 15) = P(X > 15) = 0.4$, find the values of μ and σ. [4]

(b) Muffin House sells muffins. The weight, in grams, of a fruity muffin follows a normal distribution with a mean of 220 and a standard deviation of 25. The weight, in grams, of a chocolaty muffin follows a normal distribution with a mean of 112 and a standard deviation of 15.

(i) The fruity muffins are packed in boxes of *n* each. Given that the probability that the mean weight of fruity muffins in a box is less than 210 grams is 0.164, find *n*, giving your answer to nearest integer. [3]

(ii) Find the probability that the weight of three chocolaty muffins differs from twice the weight of a fruity muffin by less than 50 grams. [3]

State an assumption needed for your calculations in part (ii). [1]

End of Paper