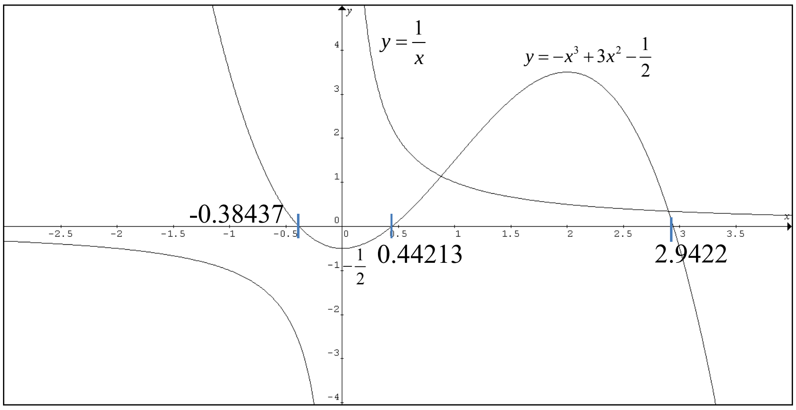
JJC H1 Math Prelim 2014 Solutions

No	Solution
1(i)	$\log_5 y = \log_5 25^q = 2q$
1(ii)	$\log_5 xy = \log_5 x + \log_5 y$ $\log_5 x = p - \log_5 y$ $= p - 2q$
1(iii)	$xy = 5^p$ and $y = 5^{2q}$ $x^3y^2 = \dfrac{(xy)^3}{y} = \dfrac{5^{3p}}{5^{2q}} = 5^{3p-2q}$ Alternative: $x^3y^2 = \left(5^{p-2q}\right)^3\left(5^{2q}\right)^2 = 5^{3p-2q}$
2(i)	$x^2l = 500 \Rightarrow l = \dfrac{500}{x^2}$ $A = 2x^2 + 4xl$ $= 2x^2 + 4x\left(\dfrac{500}{x^2}\right)$ $= 2x^2 + \dfrac{2000}{x}$
2(ii)	$\dfrac{dA}{dx} = 4x - \dfrac{2000}{x^2}$ $\dfrac{dA}{dx} = 0$ $4x - \dfrac{2000}{x^2} = 0$ $x = 500^{\frac{1}{3}} = 7.94$
	$\dfrac{d^2A}{dx^2} = 4 + \dfrac{4000}{x^3} > 0$ Therefore A is minimum.

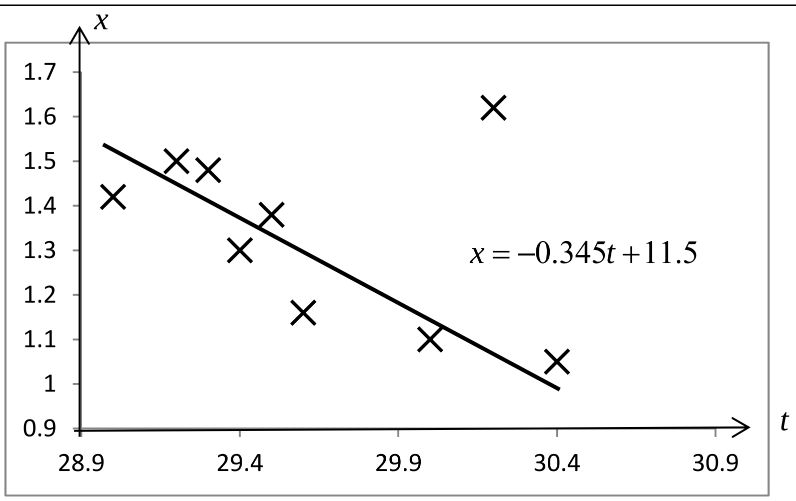
Table within 2(ii):

x	7.94^-	7.94	7.94^+
$\dfrac{dA}{dx}$	$-$	0	+
	\	_	/

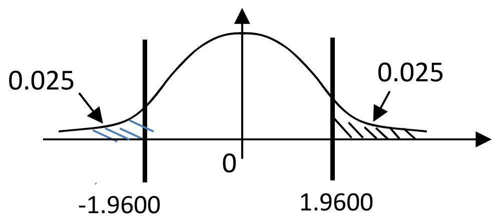
No	Solution
3(i)	Gradient of tangent is $\frac{1}{2}$. $y-\left(\frac{11}{4}-\frac{3}{4}a\right)=\frac{1}{2}(x-4)$ $y=\frac{1}{2}x-\frac{3}{4}a+\frac{3}{4}$
3(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4a^3}{(2x-1)^3}=\frac{1}{2}$ $\frac{4a^3}{(2x-1)^3}=\frac{1}{2}$ $x=a+\frac{1}{2}$ $y=1-\frac{1}{4}a$ Therefore coordinates of P are $\left(a+\frac{1}{2},1-\frac{1}{4}a\right)$
3(iii)	Let $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4a^3}{(2x-1)^3}=0$ Since there is no solution to the above equation, C has no stationary points.
4(i)	Sub $y=x+q$ into $x^2-2x+2y^2=3$, $x^2-2x+2(x+q)^2=3$ $3x^2+(4q-2)x+2q^2-3=0$ Since line intersects curve at 2 distinct points, Discriminant > 0 $(4q-2)^2-4(3)(2q^2-3)>0$ $16q^2-16q+4-24q^2+36>0$ $-8q^2-16q+40>0$ $q^2+2q-5<0$ Using GC, $-3.45<q<1.45$ Since $q>0$, $0<q<1.45$
4(ii)	$9x+5y+4=120$ $9x+5y=116$ ....(1) $5x+10y+5=120$ $5x+10y=115$ ....(2) Solving the simultaneous equations, we get $x=9$ and $y=7$. Therefore I can make maximum 13 Lawrence knots from a ribbon.

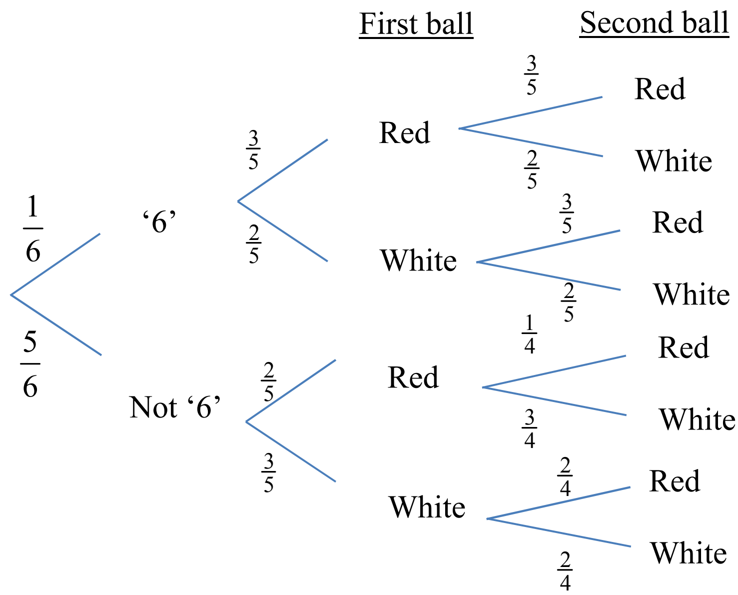
No	Solution
5(i)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2 + \dfrac{3}{1-x}$ $y = 2x - 3\ln(1-x) + C$ Since the curve passes through $(0,1)$, $1 = 2(0) - 3\ln(1-0) + C$ $C = 1$ $\therefore\ y = 2x - 3\ln(1-x) + 1$
5(ii)(a)	 x-intercepts for $y = -x^3 + 3x^2 - \dfrac{1}{2} = -0.384,\ 0.442,\ 2.94$ y-intercept for $y = -x^3 + 3x^2 - \dfrac{1}{2} = -0.5$ Points of intersections = (0.87874, −1.1380) and (2.8995, $-0.3448\dot{9}$) = (0.879, −1.14) and (2.90, −0.345) (3sf)
5(ii)(b)	$x^3 - 3x^2 + \dfrac{1}{2} < -\dfrac{1}{x}$ $-x^3 + 3x^2 - \dfrac{1}{2} > \dfrac{1}{x}$ $x < 0$ or $0.879 < x < 2.90$
5(ii)(c)	Area bounded $= \int_{0.87874}^{2.89950} \left(-x^3 + 3x^2 - \dfrac{1}{2} - \dfrac{1}{x} \right) \mathrm{d}x = 3.973$ (to 3 d.p.)

Section B

No	**Solution**
6(i)	To carry out random sampling (or simple random sampling), obtain **a sampling frame** such as a list of the members name in alphbetical order. Number all the members from 1 to 550 000, and generate 50 **random** numbers to get a sample of 50 numbers. This sample may not be representative as the sample may consist of members with same occupation.
6(ii)	A more appropriate sampling method is **stratified sampling**. This can be done by categorising the 550 000 members into **non-overlapping stratas** based on different types of occupation e.g. taxi drivers, cleaners. Then select **random** samples with sample size proportional to the relative size of each occupation.
7(i)	 (30.2, 1.62) is the suspect data pair.
7(ii)	$r = -0.88502 \approx -0.885$ (to 3s.f.) $x = -0.34514t + 11.499$ $x = -0.345t + 11.5$ (to 3 s.f.)
7(iii)	When $x = 1.4$, $1.4 = -0.34514t + 11.499$ $t = 29.261 \approx 29.3$ When the number of dengue cases is 140, the weekly mean temperature is 29.3 ^{o}C. No. Since both *t* and *x* are random values, so it is better to use the line of regression of *t* on *x*.

No	Solution
8(i)	A red pen being faulty is independent of another red pen being faulty.
8(ii)	Let X be the number of faulty red pens in a pack of 20 pens $X \sim \mathrm{B}(20, 0.01)$ $\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X \leq 1)$ $= 0.016859$ ≈ 0.0169
8(iii)	P(not satisfied, satisfied, satisfied) = $0.016859 \times (1 - 0.016859)^2 = 0.016296$ ≈ 0.0163
8(iv)	30 × 20 = 600 pens Let W be the number of faulty pens on the shelf out of 600 pens $W \sim \mathrm{B}(600, 0.01)$ Expected number of faulty pens = np = 600 × 0.01 = 6
8(v)	$W \sim \mathrm{B}(600, 0.01)$ Since n is large, np = 6 > 5, nq = 594 > 5, $W \sim \mathrm{N}(6, 5.94)$ approximately $\mathrm{P}(W > 5) \xrightarrow{c.c} \mathrm{P}(W > 5.5)$ $= 0.58127$ ≈ 0.581
9(i)	Let X be the travelling time from town A to town B $n = 80$, $\sum(x-40) = 113$ and $\sum(x-40)^2 = 2985$ $\bar{x} = \dfrac{\sum(x-40)}{n} + 40 = \dfrac{113}{80} + 40 = 41.4125$ (exact) $s^2 = \dfrac{1}{n-1}\left[\sum(x-40)^2 - \dfrac{\left(\sum(x-40)\right)^2}{n}\right]$ $= \dfrac{1}{79}\left[2985 - \dfrac{(113)^2}{80}\right] = \dfrac{226031}{6320}$ or 35.764

No	Solution
	$H_0: \mu = 40$ $H_1: \mu > 40$ Since $n = 80$ is large, by Central Limit Theorem, $\bar{X} \sim N\left(40, \dfrac{\left(\frac{226031}{6320}\right)}{80}\right)$ approximately or $\bar{X} \sim N\left(40, \dfrac{226031}{505600}\right)$ or $\bar{X} \sim N(40, 0.44705)$ approximately. Test statistic $Z = \dfrac{\bar{X} - 40}{\sqrt{\frac{226031}{505600}}} \sim N(0,1)$ approximately $\alpha = 0.02$ From GC, p-value $= 0.017319 \approx 0.0173$ $\therefore$ Since p-value = 0.0173 < 0.02 = α, we **reject H_0** at 2% level of significance and conclude there is **sufficient evidence** to conclude that the population mean travelling time is more than 40 minutes.
9(ii)	Since p-value is 0.0173 > 0.01 = α, we can conclude that there is insufficient evidence to conclude that the population mean travelling time is more than 40 minutes.
9(iii)	$H_0: \mu = 30$ $H_1: \mu \neq 30$ Since $n = 80$ is large, by Central Limit Theorem, $\bar{X} \sim N\left(30, \dfrac{9}{80}\right)$ approximately. Test statistic $Z = \dfrac{\bar{X} - 30}{\sqrt{\frac{9}{80}}} \sim N(0,1)$ approximately $\alpha = 0.05$  Critical Region: $z \le -1.9600$ or $z \ge 1.9600$ Claim is valid means we do not reject H_0, Hence, $-1.9600 < \dfrac{k - 30}{\sqrt{\frac{9}{80}}} < 1.9600$ $29.3426 < k < 30.6574$ $29.34 < k < 30.66$

No	Solution
10(a) (i)	Given that A and B are independent events, $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$ $\mathrm{P}(B \cap A') = 0.105$ $\mathrm{P}(B) - \mathrm{P}(A \cap B) = 0.105$ $\mathrm{P}(B) - \mathrm{P}(A)P(B) = 0.105$ $p - 0.3p = 0.105$ $p = 0.15$
10(a) (ii)	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B) = 0.3(0.15) = 0.045$
10(a) (iii)	$\mathrm{P}(A \cap B') = \mathrm{P}(A) - \mathrm{P}(A \cap B) = 0.3 - 0.045 = 0.255$
10(b) (i)	
10(b) (ii)	P(the second ball chosen is red) $= \frac{1}{6}\left(\frac{3}{5}\right)\left(\frac{3}{5}\right) + \frac{1}{6}\left(\frac{2}{5}\right)\left(\frac{3}{5}\right) + \frac{5}{6}\left(\frac{2}{5}\right)\left(\frac{1}{4}\right) + \frac{5}{6}\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)$ $= \frac{13}{30}$
10(b) (iii)	P(second ball came from box A | second ball chosen is white) $= \frac{\mathrm{P}\left(\begin{array}{l}\text{second ball came from box A and}\\ \text{second ball chosen is white}\end{array}\right)}{\mathrm{P(second\ ball\ chosen\ is\ white)}}$ $= \frac{\frac{1}{6}\left(\frac{3}{5}\right)\left(\frac{2}{5}\right) + \frac{1}{6}\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)}{1 - \frac{13}{30}} = \frac{\frac{1}{15}}{\frac{17}{30}} = \frac{2}{17}$

No	Solution
11	Let L be the mass of a randomly chosen lettuce $L \sim \text{N}(415, 15^2)$ Let C be the mass of a randomly chosen cabbage $C \sim \text{N}(350, 10^2)$
11(i)	$\text{P}(400 < L < 450) = 0.83153$ ≈ 0.832
11(ii)	Consider $L_1 + L_2 + L_3 - 3C$, $\text{E}(L_1 + L_2 + L_3 - 3C) = (3 \times 415) - (3 \times 350) = 195$ $\text{Var}(L_1 + L_2 + L_3 - 3C) = (3 \times 15^2) + (3^2 \times 10^2) = 1575$ $L_1 + L_2 + L_3 - 3C \sim \text{N}(195, 1575)$ $\text{P}(L_1 + L_2 + L_3 - 3C > 220) = 0.26437$ ≈ 0.264
11(iii)	$\overline{L} \sim N\left(415, \frac{15^2}{8}\right)$ and $\overline{C} \sim N\left(350, \frac{10^2}{12}\right)$ Consider $\overline{L} - \overline{C}$, $\text{E}(\overline{L} - \overline{C}) = 415 - 350 = 65$ $\text{Var}(\overline{L} - \overline{C}) = \frac{15^2}{8} + \frac{10^2}{12} = \frac{875}{24}$ or 36.458 $\overline{L} - \overline{C} \sim \text{N}(65, 36.458)$ $\text{P}(\overline{L} - \overline{C} < 60) = 0.20381$ ≈ 0.204
11(iv)	Let X be the mass of an empty box $X \sim \text{N}(20, 2^2)$ $W = X + L_1 + L_2 + \ldots + L_{12}$ $\text{E}(X + L_1 + L_2 + \ldots + L_{12}) = 20 + (12 \times 415) = 5000$ $\text{Var}(X + L_1 + L_2 + \ldots + L_{12}) = 2^2 + (12 \times 15^2) = 2704$ $W \sim \text{N}(5000, 2704)$ $Y = X + C_1 + C_2 + \ldots + C_{15}$ $\text{E}(X + C_1 + C_2 + \ldots + C_{15}) = 20 + (15 \times 350) = 5270$ $\text{Var}(X + C_1 + C_2 + \ldots + C_{15}) = 2^2 + (15 \times 10^2) = 1504$ $Y \sim \text{N}(5270, 1504)$ $\text{E}(W + Y) = 5000 + 5270 = 10270$ $\text{Var}(W + Y) = 2704 + 1504 = 4208$ $W + Y \sim \text{N}(10270, 4208)$ $\text{P}(W + Y \geq 10200) = 0.85973$ ≈ 0.860

No	Solution
11(v)	Let F be the mass of a randomly chosen cauliflower Since n is large, by central limit theorem, $\bar{F} \sim \mathrm{N}\left(400, \dfrac{\sigma^2}{100}\right)$ $\mathrm{P}(\bar{F} \le 405) = 0.8$ $\mathrm{P}\left(Z \le \dfrac{405-400}{\frac{\sigma}{10}}\right) = 0.8$ $\dfrac{50}{\sigma} = 0.84162$ $\sigma = 59.409$ ≈ 59.4