

Candidate Name: _____

Class: _____



JC2 PRELIMINARY EXAMINATION
Higher 1

MATHEMATICS

Paper 1

8864/01
16 September 2014
3 hours

Additional Materials: Cover page
 Answer papers
 List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your full name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **5** printed pages and **1** blank page.

Section A: Pure Mathematics [35 marks]

- 1** Without using a calculator, solve $1 + \log_3(x+2) = 2\log_3(4-x)$. [4]
- 2** (a) Differentiate $\ln \sqrt{\frac{e^{2x}}{4x+1}}$ with respect to x . [3]
- (b) Without using a graphic calculator, find the exact value of $\int_1^4 \frac{(x+1)^2}{3\sqrt{x}} dx$, simplifying your answer. [3]
- 3** $OABC$ is a square of side 1 m. A point P on AB and a point Q on BC are such that $AP = x$ m, $BQ = kx$ m, where k is a positive constant. Show that the minimum area of triangle OPQ can be expressed in the form $\frac{1}{2} - \frac{1}{ak}$, where a is a constant to be determined. [6]
- 4** A curve has equation $y = \frac{x+3}{x+1}$.
- (i) Sketch the curve, stating the equations of any asymptotes and the coordinates of any points of intersection with the x - and y - axes. [3]
- (ii) Using (i), solve the inequality $\frac{x+3}{x+1} \geq 0$. Hence, solve $\frac{3-x}{1-x} \geq 0$. [3]
- (iii) Find the area of the region bounded by the curve, the y -axis and the line $y = 2x$. [3]
- 5** The equation of a curve is $y = 4e^{-x} - 6e^{-2x}$.
- (i) Sketch the curve, stating the coordinates of any asymptote(s), turning point(s) and intersection(s) with the axes. [3]
- (ii) Find $\frac{dy}{dx}$. [1]
- (iii) Find the exact coordinates of the point P on the curve such that the tangent at P is parallel to the line $3y = 8x + 6$. [4]
- (iv) Hence show that the equation of the tangent at P can be written in the form $y = ax + a \ln b$, where a and b are exact constants to be found. [2]

Section B: Statistics [60 marks]

- 6 Events A and B are such that $P(A) = P(B)$, $P(A \cap B) = \frac{1}{5}$ and $P(A' \cap B') = \frac{2}{5}$. Find

- (i) $P(A)$, [3]
 (ii) $P(A' \cap B)$. [2]

- 7 The number of competitors in an international youth athletics competition held in Singapore is given in the table below.

	14-15 years	16-17 years	18-19 years
Male	750	700	800
Female	200	300	250

- (i) The organiser wishes to carry out a survey to find out the competitors' opinion on the accommodation and catering facilities provided. Describe how a stratified sample of 60 competitors can be obtained from the 3000 competitors. [2]

The organiser also desires to sample 1% of the spectators to find their opinions of the stadium facilities.

- (ii) Give a reason why it would be difficult to use a stratified sample. [1]
 (iii) Explain how a systematic sample could be carried out. [2]

- 8 Two players A and B regularly play each other at chess. When A has the first move in a game, the probability of A winning that game is 0.55 and the probability of B winning that game is 0.3. When B has the first move in a game, the probability of B winning that game is 0.4 and the probability of A winning that game is 0.35. Any game of chess that is not won by either players ends in a draw. A and B will toss a fair coin to decide who has the first move in a game.

- (i) Find the probability that A will win a particular game. [2]
 (ii) A and B plays 3 games. Find the probability that A wins exactly one game and B wins exactly one game. [3]

To make their game enjoyable, A and B agree to change the procedure for deciding who has the first move in the game. As a result of the new procedure, the probability of A having first move in any game is p . Find the value of p which gives A and B equal chance of winning each game. [2]

- 9** The systolic blood pressures, p mmHg, and the ages, t years, of 7 hospital patients are shown in the table below.

Patient	A	B	C	D	E	F	G
t	42	74	48	35	56	26	60
p	98	165	120	80	180	70	150

- (i) Draw a sketch of the scatter diagram for the data, as shown on your calculator. [2]
- (ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]
- (iii) Find the equation of the most suitable regression line, and justify your choice of line. [2]
- (iv) Use your regression line to estimate the blood pressure of a newborn baby. Comment on the relevance of this estimate in real life. [2]
- 10** A restaurant owner claims that a piece of steak sold in his restaurant has a mean mass of 250 g. The mass of a piece of steak is known to have a normal distribution with standard deviation 10 g. A random sample of n pieces of steak is weighed and the sample mean mass is 247.5 g. Find the largest value of n for the owner's claim to be accepted at the 5% level of significance. [4]

The restaurant owner decides to upsize his steaks and he now claims that the mean mass of a piece of steak is now 300 g. The mean mass of a random sample of 30 pieces of upsized steak is 297.5 g. The mass of a piece of steak still has a normal distribution with standard deviation 10 g. Test at the 5% significance level whether the owner's claim is an overestimation. [4]

Find the smallest level of significance at which the test would result in the rejection of the owner's claim, giving your answer correct to 4 decimal places. [1]

11 A factory manufactures white and dark chocolates in the ratio 3:2.

- (i) Chocolates are chosen at random. Find the maximum number of chocolates to be chosen so that there is at least 1% chance that there will be no dark chocolates. [3]

The chocolates are packed randomly and sold in packets of 10.

- (ii) Show that the probability that there are more white chocolates than dark chocolates in a randomly chosen packet of chocolates is 0.633. [2]
- (iii) It is given that half of the white chocolates contain almonds. Find the probability that, in a randomly chosen packet of chocolates, more than 3 are white chocolates that contain almonds. [3]
- (iv) Using a suitable approximation, find the probability that, in a box of 50 randomly chosen packets of chocolates, there are less than 30 with more white chocolates than dark chocolates. State the mean and variance of the distribution that you use. [4]

12 The mass of a cherry tomato is normally distributed with mean 7 g and standard deviation 0.9 g. The mass of a lime is normally distributed with mean 8 g and standard deviation 1.1 g. Stating clearly the mean and variance of all distributions that you use, find the probability that

- (i) the total mass of twenty randomly chosen cherry tomatoes is at least 145g, [2]
- (ii) twice the mass of a randomly chosen lime is more than the total mass of two randomly chosen cherry tomatoes. [3]

‘State an assumption needed for your calculations in (ii). [1]

Cherry tomatoes cost \$6 per kilogram and limes cost \$4 per kilogram. A local supermarket packs cherry tomatoes in packets of 25 and limes in packets of 10.

- (iii) Find the probability that a packet of limes costs at most 30 cents. State the mean and variance of any distribution that you use. [4]
- (iv) Tan buys three packets of limes and Lee buys a packet of cherry tomatoes. Find the probability that the amount paid by Lee differs from the amount paid by Tan by at most 10 cents. State the mean and variance of any distribution that you use. [4]

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