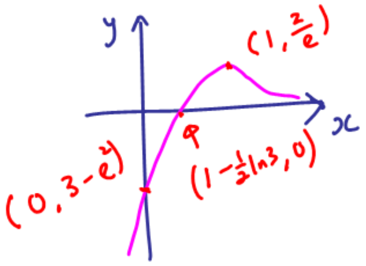
2014 H1 Maths Prelim Solution

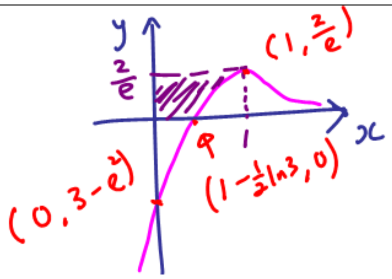
S/N	Solution	
1	$\left(5m-3x\right)^2 > 10-x^2$ $25m^2-30mx+9x^2 > 10-x^2$ $10x^2-30mx+25m^2-10>0$ $a=10, b=-30m, c=25m^2-10$ $\left(-30m\right)^2-4\left(10\right)\left(25m^2-10\right)<0$ $900m^2-1000m^2+400<0$ $-100m^2+400<0$ $m^2>4$ $m>2$ or $m<-2$	
2	Let $y=\ln\left(\dfrac{e^{x-ax^2}}{\sqrt{1+4x^2}}\right)=\ln e^{x-ax^2}-\dfrac{1}{2}\ln\left(1+4x^2\right)$ $=x-ax^2-\dfrac{1}{2}\ln\left(1+4x^2\right)$ $\dfrac{dy}{dx}=1-2ax-\dfrac{1}{2\left(1+4x^2\right)}\times 8x$ $=1-2ax-\dfrac{4x}{1+4x^2}$	
3(i)	At the points of intersection, $\sqrt{3(6)+p}=\dfrac{6+q}{3}$ …(1) $\sqrt{3(9)+p}=\dfrac{9+q}{3}$ ….(2) From (1) $18+p=\dfrac{1}{9}\left(q^2+12q+36\right)$…(3) From (2) $27+p=\dfrac{1}{9}\left(q^2+18q+81\right)$…(4) Eqn (4) – Eqn(3) $9=\dfrac{1}{9}\left(6q+45\right)$ therefore $q=6$, hence $p=-2$.	
(ii)	$\int_6^9 \sqrt{3x-2}-\dfrac{1}{3}\left(x+6\right)\,dx$ $=\left[\dfrac{\left(3x-2\right)^{\frac{3}{2}}}{3\left(\frac{3}{2}\right)}-\dfrac{x^2}{6}-2x\right]_6^9$	

	$=\left[\left(\frac{(3(9)-2)^{\frac{3}{2}}}{3\left(\frac{3}{2}\right)}-\frac{9^2}{6}-2(9)\right)-\left(\frac{(3(6)-2)^{\frac{3}{2}}}{3\left(\frac{3}{2}\right)}-\frac{6^2}{6}-2(6)\right)\right]$ $=\frac{1}{18}$	
(iii)	$\frac{dy}{dx}=\frac{1}{2}(3x-2)^{-\frac{1}{2}}(3)$ $\frac{3}{2\sqrt{3x-2}}=\frac{1}{3}$ $\sqrt{3x-2}=\frac{9}{2} \quad \Rightarrow x=\frac{89}{12}$ At R, $y=\frac{9}{2}$, Equation of tangent at R : $\frac{y-4.5}{x-\frac{89}{12}}=\frac{1}{3}$ At intersection with $y=-x$, Using G.C. point of intersection $\left(-\frac{73}{48},\frac{73}{48}\right)$ or $(-1.52, 1.52)$	
4(i)	$3e^{-x}-e^{2-3x}=0$ $e^{-x}\left(3-e^{2-2x}\right)=0$ $e^{-x}=0$ (rejected since $e^{-x}>0$) or $3-e^{2-2x}=0$ $\ln 3=2-2x \quad \Rightarrow x=\frac{2-\ln 3}{2}=1-\frac{1}{2}\ln 3$ Alternatively, $3=e^{2-2x}$ $\ln 3=2-2x$ $x=\frac{2-\ln 3}{2}$ or $1-\ln\sqrt{3}$	
4(ii)	$y=3e^{-x}-e^{2-3x}$ $\frac{dy}{dx}=-3e^{-x}+3e^{2-3x}$ $\frac{dy}{dx}=0 \quad \Rightarrow e^{-x}\left(-3+3e^{2-2x}\right)=0$ $e^{2-2x}=1 \quad \Rightarrow x=1$ coordinates $\left(1, 2e^{-1}\right)$ or $\left(1, \frac{2}{e}\right)$	

4(iii)



4(iv)



Area = $1\left(\frac{2}{\mathrm{e}}\right)-\int_{1-0.5\ln 3}^{1} 3\mathrm{e}^{-x}-\mathrm{e}^{2-3x}\,\mathrm{d}x=0.442$

5(i)

Area, $A=\frac{1}{2}(6x)(4x)+6xy+\frac{1}{2}\pi(3x)^2$

$=12x^2+6xy+\frac{9}{2}\pi x^2$

$10x+2y+\frac{1}{2}\left(2\pi(3x)\right)=300$

$10x+2y+3\pi x=300$

$y=150-5x-\frac{3\pi}{2}x$

$A=12x^2+6x\left(150-5x-\frac{3\pi}{2}x\right)+\frac{9}{2}\pi x^2$

$=900x-18x^2-\frac{9}{2}\pi x^2$

$=900x-\frac{9}{2}(4+\pi)x^2$

5(ii)

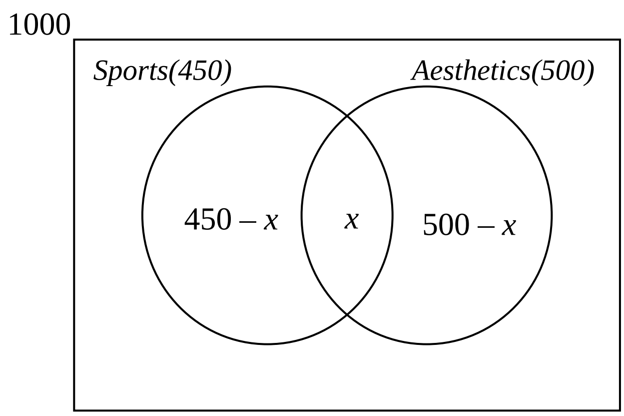
$\frac{\mathrm{d}A}{\mathrm{d}x}=900-9(4+\pi)x$

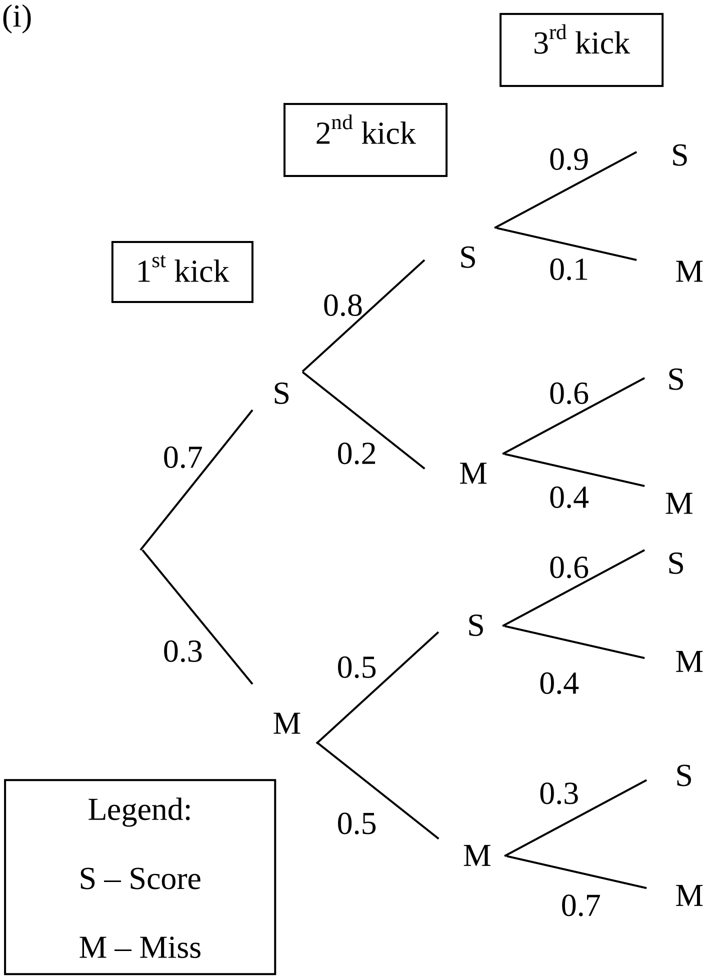
Let $\frac{\mathrm{d}A}{\mathrm{d}x}=0,\quad x=\frac{100}{4+\pi}$

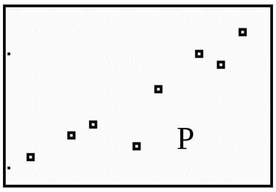
x	$\left(\frac{100}{4+\pi}\right)^-$	$\left(\frac{100}{4+\pi}\right)$	$\left(\frac{100}{4+\pi}\right)^+$
$\frac{\mathrm{d}A}{\mathrm{d}x}$	+ve	0	-ve
Slope	/	–	\

Therefore max at $x=\frac{100}{4+\pi}$

Max A = $=900\left(\frac{100}{4+\pi}\right)-\frac{9}{2}(4+\pi)\left(\frac{100}{4+\pi}\right)^2=45000/(4+\pi)$

5(iii)	 $3.87 \le x \le 7.64$ or $20.4 \le x \le 24.1$	
6(i)	Obtain an alphabetical list of the subscribers with the subscribed numbers corresponding to the list in the database and number them from 1 to 2000. By method of counting off, divide the members into 80 intervals with 25 members in each interval. Using a random number generator, obtain a random start by randomly generating a number between 1 to 25 inclusive, and every **25**$^{\text{th}}$ number thereafter. The members corresponding to the numbers selected would then form the sample of 80 members to be surveyed.	
(ii)	A member may have more than 1 phone number. Or Not all subscribers chosen will use the new mobile service. Or It is difficult to obtain a most updated list as the numbers of subscribers can change from day to day. Or Any other accepted answers	
8	 (i) Let x be the number of students with both CCA $(450 - x) + (500 - x) = 650$ $x = 150$ $\text{P(a student with 2 CCA)} = \dfrac{150}{1000} = 0.15$	

	(ii) $\text{P(Sports given Aesthetics)} = \dfrac{\text{P(Sports and Aesthetics)}}{\text{P(Aesthetics)}}$ $= \dfrac{150}{500}$ $= 0.3$	
7	(i)  (ii) $\text{P(scores 2)} = \text{P}(SSM) + \text{P}(SMS) + \text{P}(MSS)$ $= (0.7)(0.8)(0.1) + (0.7)(0.2)(0.6) + (0.3)(0.5)(0.6) = 0.23$ (iii) $\text{P(scores 3}^{\text{rd}}\text{ kick} \mid \text{scores 2)} = \dfrac{\text{P(scores 3}^{\text{rd}}\text{ kick and scores 2)}}{\text{P(scores 2)}}$ $= \dfrac{(0.7)(0.2)(0.6) + (0.3)(0.5)(0.6)}{0.23} = 0.757$ (to 3 sf)	
9	Let X denote the number of juicy kiwis out of 80 kiwis harvested. $X \sim \text{B}\left(80, \dfrac{5}{6}\right)$ (i) Expected number of juicy kiwis = $80 \times \dfrac{5}{6} = 66.7$ (3 s.f.) (ii) Required probability $= \text{P}\left(X > 70\right)$	

	$= 1 - \mathrm{P}(X \le 70)$ $= 0.122144$ $= 0.122\ (3\text{ s.f.})$ (iii) Required probability $= \mathrm{P}(X > 70)\left[\mathrm{P}(X \le 70)\right]^2 \times 3$ $= (0.12214)(0.87786)^2 \times 3$ $= 0.282\ (3\text{ s.f.})$ (iv) Let Y denote the number of months where more than 70 juicy kiwis are harvested out of 120 months. $Y \sim \mathrm{B}(120, 0.12214)$ Since n = 120 is large, np = 14.6568 > 5, nq = 105.3432 > 5, $Y \sim \mathrm{N}(14.6568, 12.8666)$ approximately $\mathrm{P}(12 < Y \le 24) \xrightarrow{\text{c.c.}} \mathrm{P}(12.5 < Y < 24.5) = 0.723$ (3 s.f.)	
10	(i) r = 0.921 It suggests there is a high positive linear correlation between the heights of father and the height of their 10 years old son. (ii)  The wrongly recorded pair was P(1.75, 1.52) (iii) With the removal of P, the rest of the points lie close to a straight line with product moment correlation coefficient r = 0.986. (iv) $(\bar{x}, \bar{y}) = (1.75, 1.57)$ (v) $y = 1.04x - 0.26$ $x = 0.934y + 0.29$ (vi) When $y = 1.65$, $x = 0.934(1.65) + 0.29 = 1.83$ The height of his father is estimated to be 1.83 m This estimate is unreliable as it is an extrapolation.	
11	(i) Let X denote the time taken by a person to complete the test A $X \sim \mathrm{N}(\mu, 4^2)$	

$$\mathrm{P}(X \geq 61.6) = 0.8$$
$$\mathrm{P}(X < 61.6) = 0.2$$
$$\mathrm{P}\left(Z < \frac{61.6-\mu}{4}\right) = 0.2$$
$$\frac{61.6-\mu}{4} = -0.84162$$
$$\mu = 64.966 \approx 65.0 \text{ (3 s.f.)}$$

(ii) Let W be the number of people out of 10, who take at least 61.6 minutes to complete the test $W \sim \mathrm{B}(10, 0.8)$

$$\mathrm{P}(W < 7) = \mathrm{P}(W \leq 6) = 0.121$$

(iii) Let $T = X_1 + \cdots + X_N \sim N(64.966N, 16N)$

$$\mathrm{P}(T > 900) = 0.05$$
$$\mathrm{P}(T < 900) = 0.95$$
$$\mathrm{P}\left(Z < \frac{900-64.966N}{\sqrt{16N}}\right) = 0.95$$

Using GC, $\dfrac{900-64.966N}{\sqrt{16N}} = 1.64485$

$$64.966N + 6.5794\sqrt{N} - 900 = 0$$
$$N + 0.10127\sqrt{N} - 13.8534 = 0$$

Using GC, $\sqrt{N} = -3.772997$ (NA) or 3.671725

Hence N = 13 (nearest integer)

(iv) Let Y denote the time taken by a person to complete the test B. $Y \sim \mathrm{N}(30, 9)$

$$X - 2Y \sim \mathrm{N}(4.966, 52)$$
$$\mathrm{P}(X < 2Y) = \mathrm{P}(X - 2Y < 0) = 0.246$$

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(a) $\bar{x} = \dfrac{-38}{80} + 78 = 77.525 = 77.5$

$$s^2 = \frac{1}{80-1}\left[482 - \frac{(-38)^2}{80}\right]^2 = 5.8728 = 5.87$$

(i) $\mathrm{H_o} : \mu = 78$

$\mathrm{H_1} : \mu < 78$

Where μ is the mean Mathematics score of the school

Under $\mathrm{H_o}$, $\overline{X} \sim N(78, \dfrac{5.8728}{80})$, by CLT

At 5% significance level, reject $\mathrm{H_o}$ if $p < 0.05$

$\bar{x} = 77.525$, $\mu_o = 78$, $n = 80$, $s = \sqrt{5.8728}$

		From GC, p = 0.039789 < 0.05 Reject H_o and conclude at 5% significant level that there is sufficient evident that the principal has overstated their mean score.	
	(ii)	It is not necessary to assume normal distribution since n = 80 is large and so Central Limit Theorem applies.	
	(iii)	There is a probability of 0.05 of wrongly concluding that the mean Mathematics score of the school is less than 78 when in fact it is 78.	
	Or	There of a probability of 0.05 of wrongly concluding that the principal has overstated the maths score when he did not.	
	(b)	$H_o : \mu = 78$ $H_1 : \mu > 78$ Where μ is the mean Mathematics score of the school Under H_o, $\overline{X} \sim N(78, \frac{2.42^2}{n})$, by CLT At 3% significant level, reject H_o if p < 0.03 Principal's claim is understated => to reject H_o $P(\overline{X} > \overline{x}) < 0.03$ $P(\overline{X} > 78.6) < 0.03$ $P(\overline{X} < 78.6) > 0.97$ $P(Z < \frac{78.6-78}{\frac{2.42}{\sqrt{n}}}) > 0.97$ $P(Z < \frac{0.6\sqrt{n}}{2.42}) > 0.97$ $\frac{0.6\sqrt{n}}{2.42} > 1.8808$ $\frac{0.6\sqrt{n}}{2.42} > 1.8808$ $n > 57.545$ Smallest n is 58	