Name	( )	Class	

RIVER VALLEY HIGH SCHOOL
2014 Year 6 Prelim 2
Higher 1

MATHEMATICS 8864/01

Paper 1 **19 Sept 2014**

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)
Cover Page

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.
Write your name, class and index number in the space at the top of this page.

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, place the cover page on top of your answer paper and fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [35 marks]

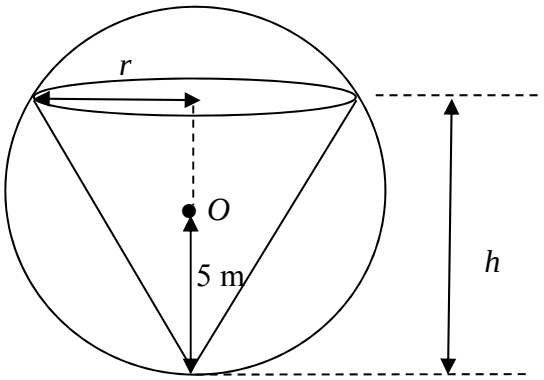
1 Given that $e^{2x} - 3e^{x} + 12e^{-x} = 4$, use the substitution $u = e^{x}$ to find the exact value(s) of x. [3]

2 If $y = \ln\left(x\sqrt{x+1}\right)$ and $x > 0$, find $\frac{dy}{dx}$. Hence find the exact value of $\int_{3}^{8} \frac{3x+2}{x(1+x)}\,dx$. [6]

3 (i) The normal to the curve $y = \frac{a}{\sqrt{x}} - \frac{1}{x}$ at the point (4, b) is parallel to the line $2y = 10 - x$. Find the values of the constants a and b. [4]

(ii) Using the value of a found in part (i), sketch $y = \frac{a}{\sqrt{x}} - \frac{1}{x}$, showing clearly any asymptotes. By sketching a suitable graph on the same diagram, state the number of real roots of the equation $10 = \frac{1 - a\sqrt{x}}{x}$. [4]

4



The diagram shows an inverted right cone of radius r and height h inscribed in a sphere of centre O and fixed radius 5 metres. The circumference of the base of the cone is in contact with the inner surface of the sphere. The vertex of the cone touches the lowest point of the sphere. The volume of the cone is denoted by V.

(i) Show that $V = \frac{\pi h^2}{3}(10 - h)$. [3]

(ii) Hence find the exact maximum value of V as h varies. [5]

[The volume of a cone of radius r and height h is $\frac{1}{3}\pi r^2 h$.]

5 (i) Sketch the graphs of $y = x^2 - k^2$ and $y = 3k^2$ on the same axes, where k is a positive constant. Indicate clearly any turning points and axial intercepts. [2]

(ii) Find in terms of k, the x-coordinates of the points where the two graphs intersect. [1]

(iii) Given $k = 1$, calculate the exact area of the finite region enclosed by the curve $y = x^2 - k^2$ and the line $y = 3k^2$. [3]

(iv) Find the range of values of k for which the area of the finite region enclosed by the curve $y = x^2 - k^2$ and the line $y = 3k^2$ is more than 200 units^2. [4]

Section B: Statistics [60 marks]

6 In a shopping complex, the manager has decided to introduce a new promotion and wishes to know the views of the shoppers in the complex.

(i) Give a reason why it would be difficult to use stratified sampling. [1]

(ii) Explain how a systematic sample could be obtained from 10% of the shoppers. [2]

7 (a) Given that events A and B are such that $\mathrm{P}(A) = 0.5$, $\mathrm{P}(A \cup B) = 0.6$ and $\mathrm{P}(B') = 0.5$, find $\mathrm{P}(A \cap B)$. [3]

(b) Events X and Y are such that $\mathrm{P}(X) = 0.2$ and $\mathrm{P}(Y) = 0.3$. Find the values of $\mathrm{P}(X \cup Y)$ if X and Y are

(i) mutually exclusive;

(ii) independent. [3]

8 Everyday, Alex has a choice of 2 routes to get to school. The probability that he chooses route A is 0.75. If he chooses route A, the probability that he will be late for school is 0.7. If he chooses route B, the probability that he will not be late for school is 0.9.

(i) Draw a probability tree diagram to represent the information above. [2]

Find the probability that Alex

(ii) will be late for school. [1]

(iii) chooses route A given that he is not late for school. [2]

Given that there are 5 school days in a week, find the probability that Alex is late at most 2 times in 2 weeks. [2]

9 A newly developed type of sponge aims to reduce the amount of water loss due to evaporation. In an experiment, a piece of the sponge is emerged in water and then placed in open air. The amount of water it retained, x litres, is measured against time, t days.

t (days)	2	4	6	8	10
x (litres)	3.2	2	1.68	0.46	0.2

(i) Draw a sketch of the scatter diagram for the data, as shown in your calculator. [1]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the question. [2]

(iii) Find the equation of the regression line of x on t, in the form $x = mt + c$. Sketch this line on your scatter diagram. [2]

(iv) Explain also the meaning of the constants m and c in the context of the question. [2]

(v) Calculate an estimate of the amount of water it retained in litres after 5 days by using an appropriate regression line. Comment on the reliability of this estimate. [3]

10 On a remote island, a zoologist measures the tail lengths of a random sample of 100 monkeys. The length of tails, x mm are summarised by $\sum (x - 300) = -60$ and $\sum (x - 300)^2 = 1240$. In a species of monkeys known to her, the tail lengths have mean 300 mm.

(i) Find the unbiased estimates of the population mean and variance. [2]

(ii) Test at 5% significance level, whether monkeys on the island have tails of the different mean length as the species known to her. State the meaning of 5% significance level in the context of this question. [5]

(iii) Using the p-value from (ii), deduce whether the conclusion will be the same as that in (ii) if a 1-tailed test is conducted at 5% significance level. [1]

Another sample of 100 monkeys of the same species from another island is tested and the new population standard deviation is known to be 3.48 mm. The mean of this sample is k mm. A test at the 10% significance level indicates that the tail length is longer than expected.

(iv) Find the least possible integer value of k. [3]

11 It is known that $100p\%$ of the population of a large city uses the social networking platform InstaFace. A researcher wishes to interview these users and he selects n people at random. Let F denote the number of people who use InstaFace.

(i) State, in the context of the question, an assumption needed for F to be well-modelled by a binomial distribution. [1]

Assume now that F has the distribution B(n, p).

(ii) Given that $n = 20$ and $p = 0.4$, find the probability that there is at least 4 people but fewer than 8 people use InstaFace. [2]

(iii) Given that $n = 200$ and $p = 0.3$, use a suitable approximation to estimate the probability that there is more than 60 people who use InstaFace. You should state the mean and variance of the distribution used in the approximation. [4]

(iv) Given that $n = 20$ and $\mathrm{P}(F \leq 1) = 0.15$, write down an equation for p, and find this value numerically. [2]

(v) Given that $n = 40$ and $p = 0.6$, find the probability that the mean number of people using InstaFace in 50 cities is more than 25. [2]

12 In a supermarket, the mass of a randomly chosen apple of variety A follows a normal distribution of mean 120 g and standard deviation 25 g. It is also observed that 10% of the apple of variety B has mass less than 100 g while 90% of the apple of variety B has mass not exceeding 150 g.

(i) Find the probability that a randomly chosen apple of variety A is of mass more than 150 g. [1]

(ii) Assuming a normal distribution for the mass of an apple of variety B, find the mean and standard deviation of this distribution. [4]

(iii) Find the probability that the total mass of 2 randomly chosen apples of variety A is more than the total mass of 3 randomly chosen apple of variety B. [3]

(iv) Find the probability that out of 10 randomly chosen apples of variety A, at least 8 of them will each have mass more than 80 g. [4]

– End of Paper –

COVER PAGE

RIVER VALLEY HIGH SCHOOL EXAMINATIONS

CANDIDATE NAME	:	_____
CLASS / INDEX NO	:	☐☐ / ☐☐
SUBJECT TITLE / PAPER NO	:	_____
DATE	:	☐☐ / ☐☐ / ☐☐

Instructions to Candidates

1 Write the information required in the box above.

2 Write the number of the questions attempted in the left hand margin of the writing paper and also in the left hand column of the box on the cover.

3
* Place this cover page on top of your answer sheets.
* Use the string provided to tie this cover page to your answer sheets securely.
* Answer sheets should not be tied so tightly that the examiners cannot easily turn the pages.
* Do not staple your answer sheets.

For Candidate's Use	For Examiner's Use
Question Number	Marks Obtained
Total Marks	

Calculator Model:
(if applicable)