

Name: _____

Class: _____



JURONG JUNIOR COLLEGE

J2 Preliminary Examination 2014

MATHEMATICS
Higher 1

8864
2 September 2014

3 hours

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 95.

At the end of the examination, fasten all your work together securely, with the cover page in front.

This question paper consists of **8** printed pages.

[Turn over

Section A: Pure Mathematics [35 marks]

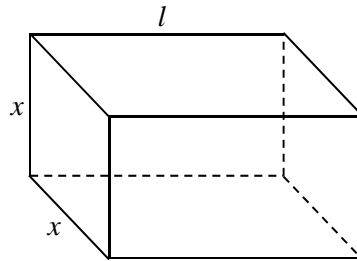
- 1** Given that $\log_5 xy = p$ and $y = 25^q$, express the following in terms of p and/or q . Simplify your answers.

(i) $\log_5 y$; [1]

(ii) $\log_5 x$; [2]

(iii) $x^3 y^2$. [2]

- 2** A metal container with square ends of sides x cm and length l cm is shown in the diagram below.



The volume of the container is fixed at 500 cm^3 .

- (i) Show that the total surface area of the container, $A \text{ cm}^2$, is given by

$$A = 2x^2 + \frac{2000}{x}. \quad [2]$$

- (ii) Using differentiation, find the value of x when the total surface area of the container is a minimum. [3]

- 3** The curve C has equation $y = 1 - \frac{a^3}{(2x-1)^2}$, where a is a positive constant. The

tangent to C at point P passes through $\left(4, \frac{11}{4} - \frac{3}{4}a\right)$ and is parallel to the line

$$2y - x = 3.$$

- (i) Find the equation of this tangent. [2]

- (ii) Find the coordinates of P in terms of a . [3]

- (iii) Explain why C has no stationary points. [1]

- 4 (i)** The line $y = x + q$ intersects the curve $x^2 - 2x + 2y^2 = 3$ at two distinct points. Given that q is a positive constant, find the set of values of q . [4]
- (ii)** A ribbon, 120 cm long, is used to make two types of knots, the Lawrence knot and the Shakespeare knot. A Lawrence knot and a Shakespeare knot requires x cm and y cm of ribbon respectively. If I make 9 Lawrence knots and 5 Shakespeare knots, I will have 4 cm of ribbon left. If I make 5 Lawrence knots and 10 Shakespeare knots, I will have 5 cm of ribbon left. What is the maximum number of Lawrence knots I can make from a ribbon? [3]
- 5 (i)** Given that the gradient function of a curve is $2 + \frac{3}{1-x}$ and that the curve passes through the point $(0, 1)$, determine the equation of the curve. [3]
- (ii)(a)** Sketch the graphs of $y = -x^3 + 3x^2 - \frac{1}{2}$ and $y = \frac{1}{x}$ on the same diagram, indicating clearly the coordinates of the axial intercepts and any intersection points. [4]
- (b)** Hence solve the inequality
- $$x^3 - 3x^2 + \frac{1}{2} < -\frac{1}{x}. \quad [3]$$
- (c)** Find the area bounded by the curves $y = \frac{1}{x}$ and $y = -x^3 + 3x^2 - \frac{1}{2}$, giving your answer in 3 decimal places. [2]

Section B: Statistics [60 marks]

- 6** An established Union in Singapore has a membership size of 550 000. The Director of the Union would like to boost their membership growth. Hence, he wishes to interview 50 members to improve on their current recruitment drives which are held at various companies.

- (i) Explain how random sampling could be carried out to choose the 50 members. Explain briefly why this may not provide the representative sample that the Director hopes to achieve. [2]
- (ii) Name a more appropriate sampling method, and explain how this could be carried out to provide the representative sample that the Director wants. [2]

- 7** The weekly mean temperature, t $^{\circ}\text{C}$, and the number of dengue cases, x , in hundreds, in a country is recorded over 9 weeks as shown in the following table.

t	30.0	29.6	29.4	29.5	30.2	29.3	29.2	29.0	30.4
x	1.10	1.16	1.30	1.38	1.62	1.49	1.50	1.42	1.05

- (i) Give a sketch of the scatter diagram for the data, as shown in your calculator. Identify a data pair which should be regarded as suspect. [3]
- (ii) Remove the suspect data pair, and calculate the product moment correlation coefficient and equation of the regression line of x on t , for the revised data. Sketch this line on your scatter diagram. [3]
- (iii) Use the line of regression obtained in (ii) to estimate the weekly mean temperature when the number of dengue cases is 140. State with a reason whether it would be a good practice to regard this value as a valid prediction. [3]

- 8** Machine Alpha produces red pens and the probability that a red pen is faulty is constant. The red pens are then packed into packs, each containing 20 pens.

- (i) State, in the above context, a necessary assumption needed for the number of faulty red pens in a pack to be well modelled by a binomial distribution. [1]

The probability that a red pen is faulty is 0.01.

- (ii) Find the probability that there are at least 2 faulty pens in a pack. [2]

The packs of red pens are then packed into boxes, each containing 30 packs.

Shop A purchased a box of the red pens and sold in packs in their shop.

- (iii) A teacher bought 3 packs of red pens. She is satisfied if a pack has less than 2 faulty pens. Find the probability that she is not satisfied with the first pack she took, but is satisfied with the second and third pack. [2]

Shop B also purchased a box of the red pens but unpacked all the pens and placed them on a display shelf to sell individually.

- (iv) Find the expected number of faulty red pens on the display shelf. [1]
(v) Using a suitable approximation, find the probability that more than 5 pens are faulty. [4]

- 9** It is claimed that it took at most 40 minutes to travel on JJ Expressway from Town *A* to Town *B*. The Land Transport Authority would like to verify if this is true in order to decide if they want to build another lane on the expressway to shorten the travelling time. The travelling time, x minutes, from Town *A* to Town *B* was measured on 80 random trips and the results are summarised by

$$\Sigma(x - 40) = 113 \quad \text{and} \quad \Sigma(x - 40)^2 = 2985.$$

- (i) Test, at 2% significance level, whether the claim is valid. [6]
- (ii) Without repeating the test in (i), explain if there is sufficient evidence to conclude that the claim is valid if we test at 1% significance level instead. [2]

The Land Transport Authority finally decided to build another lane on the expressway. After it was completed, the Land Transport Authority claims that it now takes an average of 30 minutes to travel from Town *A* to Town *B*. The new population variance is now known to be 9 minutes². The travelling time was again taken on another 80 random trips and the mean of this sample is k minutes. A test at 5% significance level indicates that the Land Transport Authority's claim is valid.

- (iii) Find the range of values of k , giving your answer correct to 2 decimal places. [3]

- 10 (a)** The events A and B are such that $P(A) = 0.3$, $P(B) = p$ and $P(B \cap A') = 0.105$.

(i) Given that A and B are independent events, find p . [2]

Find the probability that

(ii) both A and B occur, [1]

(iii) A occurs but not B . [1]

- (b)** Amy is playing a game which involves tossing a fair die and picking balls. Box A has 3 red balls and 2 white balls. Box B has 2 red balls and 3 white balls.

If the result of the die is a '6', then she will pick two balls from box A, at random with replacement.

If the result of the die is not a '6', then she will pick two balls from box B, at random without replacement.

(i) Draw a tree diagram to represent the possible outcomes. [2]

(ii) Find the probability that the second ball chosen is red. [2]

(iii) Given that the second ball chosen is white, find the probability that it came from box A. [3]

- 11** The masses, in grams, of a randomly selected lettuce and a randomly selected cabbage produced by a vegetable farm are said to be normally distributed with mean and standard deviation given in the following table.

	Mean (grams)	Standard deviation (grams)
Lettuce	415	15
Cabbage	350	10

Find the probability that

- (i) the mass of a randomly chosen lettuce is between 400 g and 450 g, [1]
- (ii) the total mass of 3 randomly chosen lettuces exceed 3 times the mass of a randomly chosen cabbage by more than 220 g, [3]
- (iii) the average mass of 8 randomly chosen lettuces less the average mass of 12 randomly chosen cabbages is less than 60 g. [4]

The lettuces and cabbages are packed into boxes to be sent to supermarkets for sale. The lettuces are packed into boxes of 12 while the cabbages are packed into boxes of 15. The mass of a randomly chosen empty box is normally distributed with mean 20 g and standard deviation 2 g.

- (iv) Find the probability that the total mass of a box of lettuces and a box of cabbages is at least 10.2 kg. [4]

The vegetable farm also produces cauliflowers. The mass of a randomly chosen cauliflower has a mean of 400 g and standard deviation of σ g. Given that the probability that the average mass of 100 randomly chosen cauliflowers is at most 405 g is 0.8, find the value of σ . [3]