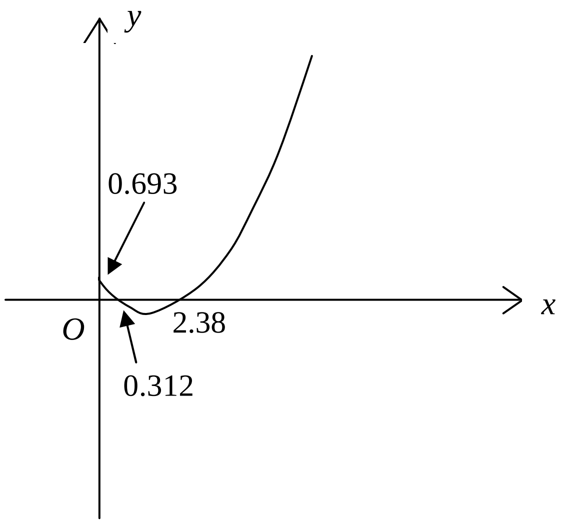
2014 SH2 H1 Math Preliminary Exam Suggested Solutions

1	$\frac{2}{e^{2x}}=3(3e^{2x}-1)$ $2u=3\left(\frac{3}{u}-1\right)$ $2u^2=3(3-u)$ $2u^2+3u-9=0$ $(2u-3)(u+3)=0$ $u=1.5$ or $u=-3$ $e^{-2x}=1.5$ or $e^{-2x}=-3$ $-2x=\ln(1.5)$ (No solution since $e^{-2x}>0$ for all x) $x=-\frac{1}{2}\ln(1.5)$
2(a)	By considering the area of the frame, $(6+2x)(4+2x)-24=15$ $24+20x+4x^2-24=15$ $4x^2+20x-15=0$ $4\left(x^2+5x-\frac{15}{4}\right)=0$ $x^2+5x-\frac{15}{4}=0$ $\left(x+\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2-\frac{15}{4}=0$ $\left(x+\frac{5}{2}\right)^2=10$ $x+\frac{5}{2}=\pm\sqrt{10}$ $x=\sqrt{10}-\frac{5}{2}$ or $x=-\sqrt{10}-\frac{5}{2}$ (reject since $x>0$)
2(b)	$2x^2+kx+2=0$ Discriminant $=k^2-4(2)(2)$ $=k^2-16$ $=(k+4)(k-4)\geq 0$ $k\leq -4$ or $k\geq 4$
3(i)	$\frac{d}{dx}e^{2x^3+\ln x}=\left(6x^2+\frac{1}{x}\right)e^{2x^3+\ln x}$

3(ii)	$\int \left(\frac{6x^3+1}{x}\right)e^{2x^3+\ln x+1}+6x^2+\frac{1}{x}\ \mathrm{d}x$ $=e\int \left(\frac{6x^3+1}{x}\right)e^{2x^3+\ln x}\ \mathrm{d}x+\int 6x^2+\frac{1}{x}\ \mathrm{d}x$ $=e\cdot e^{2x^3+\ln x}+2x^3+\ln\lvert x\rvert+c$ $=e^{2x^3+\ln x+1}+2x^3+\ln\lvert x\rvert+c$
4(i)	$k-e^{3x+1}=k-3$ $e^{3x+1}=3$ $3x+1=\ln 3$ $x=\frac{\ln(3)-1}{3}$
4(ii)	$\int_0^{\frac{\ln 3-1}{3}} k-e^{3x+1}-(k-3)\ \mathrm{d}x$ $=\int_0^{\frac{\ln 3-1}{3}} 3-e^{3x+1}\ \mathrm{d}x$ $=\left[3x-\frac{1}{3}e^{3x+1}\right]_0^{\frac{\ln 3-1}{3}}=\ln 3-1-\frac{1}{3}e^{\ln 3-1+1}-\left(0-\frac{1}{3}e^{1}\right)$ $=\ln 3-1-\frac{1}{3}e^{\ln 3}+\frac{e}{3}=\ln 3-1-1+\frac{e}{3}$ $=\ln 3-2+\frac{e}{3}$ $\therefore\ p=3, q=-2, r=\frac{1}{3}$
5(i)	$y=\ln(x+2)+x^2-3x$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x+2}+2x-3$ $\frac{1}{x+2}+2x-3=0$ $1+(2x-3)(x+2)=0$ $2x^2+x-5=0$ $x=\frac{-1\pm\sqrt{1^2-4(2)(-5)}}{2(2)}$ $x=\frac{-1+\sqrt{41}}{4}$ or $x=\frac{-1-\sqrt{41}}{4}<0(\text{reject})$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{1}{(x+2)^2}+2$ When $x=\frac{-1+\sqrt{41}}{4}, \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=1.9109>0$ $\therefore$ minimum point
5(ii)	Since $f'(x)>0$ in this interval, the graph of $y=f(x)$ is increasing in this interval.

5(iii)	Using GC, $\dfrac{dy}{dx} = -4.97$ at $x = 1$.
5(iv)	
5(v)	Notice that from the graph, the required region lies below the *x*-axis. Therefore the area of the required region $= \left\| \int_1^2 \ln(x+2) + x^2 - 3x \, dx \right\|$ $= 0.9173$ (4 d.p)
6	Number the calls from 1 to 3600 Randomly select a call from the first $\dfrac{3600}{50} = 72$ calls Select every 72^{th} call thereafter until a sample of 50 calls have been selected. Stratified random sampling is more appropriate for this situation, because the sample of calls collected will be more representative according to the proportion of the calls pertaining to each of the services (mobile, Internet and TV). OR Quota sampling is more appropriate because it does not require a sampling frame (a list of all the calls in a day).
7i	Required probability $= \dfrac{6+10}{100}$ $= 0.16$
7ii	Required probability $= \dfrac{\text{n}(\text{people whose favourite creature is lion and favourite colour is blue})}{\text{n}(\text{people whose favourite colour is blue})}$ $= \dfrac{8}{11+16+8} = \dfrac{8}{35}$
7iii	$\text{P}(C \cap D) = \dfrac{9}{100} \neq 0$ Therefore not mutually exclusive.

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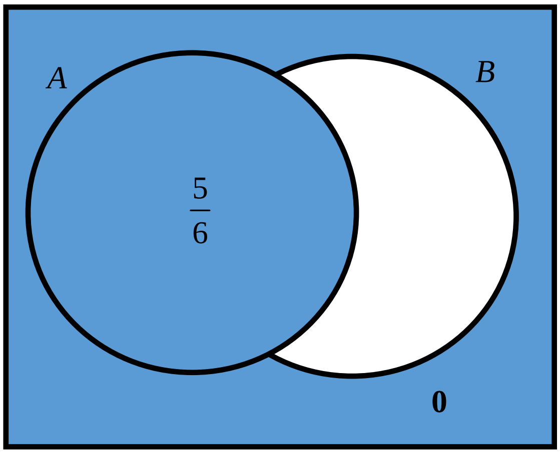
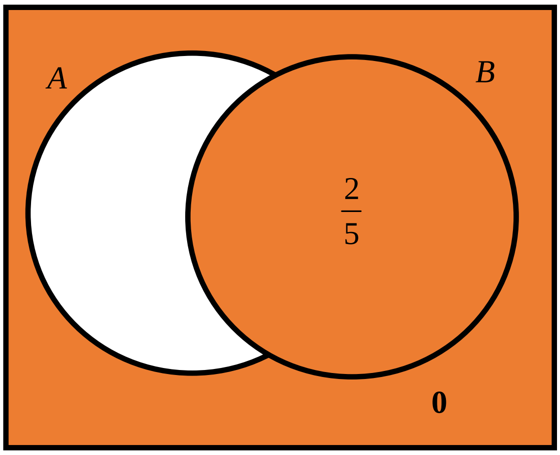
There are 9 people whose favourite colour is green and favourite creature is the penguin therefore not mutually exclusive.</td></tr>
<tr><td>8i</td><td>

$(A\cup B)'=\varnothing \Rightarrow \mathrm{P}(A\cup B)'=0 \Rightarrow \mathrm{P}(A\cup B)=1$

$A\cup B'$:



$A'\cup B$:



Since $\mathrm{P}(A\cup B)'=0$,

$\mathrm{P}(A\cup B')=\mathrm{P}(A)=\frac{5}{6}$

$\mathrm{P}(A'\cup B)=\mathrm{P}(B)=\frac{2}{5}$

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$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$

$1=\frac{5}{6}+\frac{2}{5}-\mathrm{P}(A\cap B)$

$\mathrm{P}(A\cap B)=\frac{7}{30}$

$\mathrm{P}(A)\times\mathrm{P}(B)=\frac{5}{6}\times\frac{2}{5}=\frac{1}{3}\neq\frac{7}{30}$

Therefore A and B are not independent events.

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Let X be the number of days on which Maxihard receives more than 10 calls to the hotline out of 365

$X \sim \mathrm{B}(365, p)$

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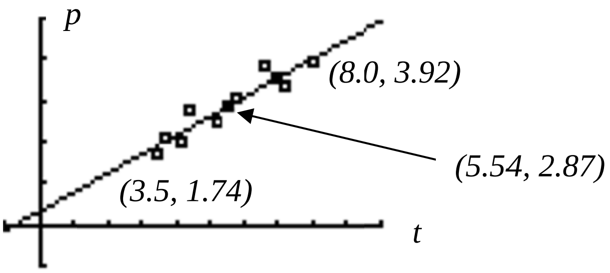
	$\mathrm{P}(X \geq 182.5) = \mathrm{P}(X \geq 183) = 0.015$ $\mathrm{P}(X \leq 182) = 0.985$ Using GC, $p = 0.44343$ (5 s.f.) $= 0.443$ (3 s.f.) (Sketch graph of binomcdf(365,X,182) and y = 0.985, find intersection)
9ii	$\mathrm{P}(X \leq 150) = 0.119$ (3 s.f.)
9iii	$\mathrm{P}(X = 180 \mid X > 150) = \dfrac{\mathrm{P}(X = 180)}{\mathrm{P}(X > 150)}$ $= \dfrac{0.0065689}{1 - 0.11889}$ $= 0.0746$ (3 s.f.)
9iv	Since $np = 161.695 > 5$ $nq = 203.305 > 5$ $n = 365 > 30$ $X \sim \mathrm{N}(161.695, 90.064115)$ approximately $\mathrm{P}(140 < X < 190) \overset{cc}{=} \mathrm{P}(140.5 < X < 189.5)$ $= 0.98554$ (5 s.f.)
10a	$\mathrm{H}_0 : \mu = 8.5$ $\mathrm{H}_1 : \mu \neq 8.5$ Under H_0, Test statistic: $Z = \dfrac{\overline{X} - 8.5}{0.55/\sqrt{50}} \sim \mathrm{N}(0,1)$ approximately by Central Limit Theorem. Level of significance: 10% Not rejected therefore $-z_{crit} < z_{calc} < z_{crit}$ $z_{calc} = \dfrac{C - 8.5}{0.55/\sqrt{50}}$ $-1.645 < \dfrac{C - 8.5}{0.55/\sqrt{50}} < 1.645$ $-0.12795 < C - 8.5 < 0.12795$ $8.38 < C < 8.62$
10bi	$\sum(x - 8.5) = -3 \Rightarrow \overline{x} = \dfrac{-3}{60} + 8.5 = 8.45$ $s^2 = \dfrac{18.46 - \dfrac{(-3)^2}{60}}{59} = 0.310$ (3 s.f.)
10bii	$\mathrm{H}_0 : \mu = 8.5$ $\mathrm{H}_1 : \mu < 8.5$ Under H_0,

	Test statistic: $Z=\dfrac{\bar{X}-8.5}{0.55/\sqrt{60}} \sim \mathrm{N}\left(0,1\right)$ approximately by Central Limit Theorem. Level of significance: 5% p – value: 0.241 > 0.05 Therefore H_0 is not rejected at 5% level of significance. There is insufficient evidence to conclude that expenditure has decreased.
10biii	5% level of significance means that… there is a probability of 0.05 of concluding that expenditure per customer has decreased when in fact it has not. OR the probability of wrongly claiming that the mean expenditure has reduced is 0.05.
11i	Let C be the random variable for the mass (in g) of a carrot. $C \sim \mathrm{N}(85,\sigma^2)$ Let T be the random variable for the mass (in g) of a tomato. $T \sim \mathrm{N}(112,9^2)$ $\mathrm{P}(C>80)=0.9$ $\mathrm{P}(C<80)=0.1$ $\mathrm{P}\left(\mathrm{Z}<\dfrac{80-85}{\sigma}\right)=0.1$ $\dfrac{80-85}{\sigma}=-1.28155$ $\sigma \approx 3.9015$ $\sigma = 3.9$ (1d.p.)
11ii	$\mathrm{P}\left(T_1>116\right)\mathrm{P}\left(105<T_2<110\right)+\mathrm{P}\left(T_2>116\right)\mathrm{P}\left(105<T_1<110\right)$ $=\mathrm{P}\left(T>116\right)\mathrm{P}\left(105<T<110\right)\times 2!$ $=(0.32836)(0.19372)\times 2$ $=0.127$
11iii	To investigate the difference between $\left(T_1+T_2\right)\&\,3C$, consider $\left\|3C-\left(T_1+T_2\right)\right\|$. $3C-T_1-T_2 \sim \mathrm{N}\left(3(85)-2(112), 3^2(3.9^2)+9^2+9^2\right)$ $3C-T_1-T_2 \sim \mathrm{N}\left(31, 298.89\right)$ $\mathrm{P}\left(\left\|3C-T_1-T_2\right\|>10\right)$ $=\mathrm{P}\left(3C-T_1-T_2>10\right)+\mathrm{P}\left(3C-T_1-T_2<-10\right)$ $=0.887757+0.00885735$ $=0.897$
11iv	The mass of a randomly chosen carrot is independent of the mass of a randomly chosen tomato.
11v	$\bar{C} \sim \mathrm{N}\left(85, \dfrac{3.9^2}{60}\right)$ $\mathrm{P}\left(\bar{C} \le 85.6\right)$ $=0.883$

12i	$\dfrac{6.9+8+5.2+3.7+x+7.2+6.6+4.4+4.1+5.8}{10}=5.54$ $51.9+x=55.4$ $x=3.5$
12ii	
12iii	Using GC, r = 0.945 (3s.f.) There is a strong positive **linear** correlation between students' average number of hours of sleep per day and students' GPA. i.e. As the average number of hours of sleep increases, the students' GPA increases **linearly.**
12iv	$p=0.4622t+0.30940$ $p=0.462t+0.309$ (3s.f.)
12v	$p=0.46220(7.6)+0.30940$ $p=3.82$ The estimate is reliable since t = 7.6 is within the data range and r = 0.945 is very close to 1.
12vi	The error of estimate of t is not minimized when using the regression line of p on t. (The regression line of t on p in more suitable as it minimizes the error of estimate of t.)
12vii	$p=0.462t+0.329$ (3s.f.)