When is a function differentiable?

Sunday, 25 March 2018 7:13 PM

By the end of the teaching I am asking that you complete the following work:

Differentiability	9M	1(s),2,4,5

RECAP:

We have pretty much completed all the work on Differentiation and, as such, all of the work can now be assessed on your SACs.

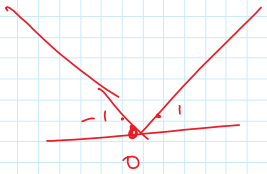
Last time we looked at the ideas behind continuity and what makes a function continuous at a point.
This lesson we look at differentiability of points.

Hybrid Functions: A recap

We can draw a number of different functions on one graph by using the idea of a hybrid function.
We met one previously:

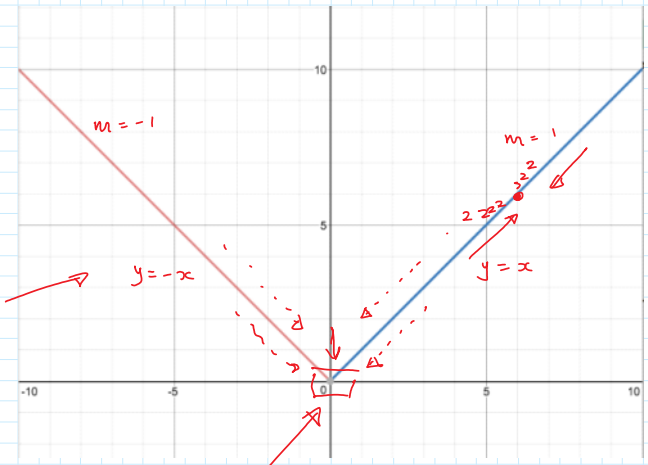
$$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$f(0) = 0$



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$y = mx + c$



At the point (0,0) they share a common value

* REALLY IMPORTANT *

Remember, to be continuous,

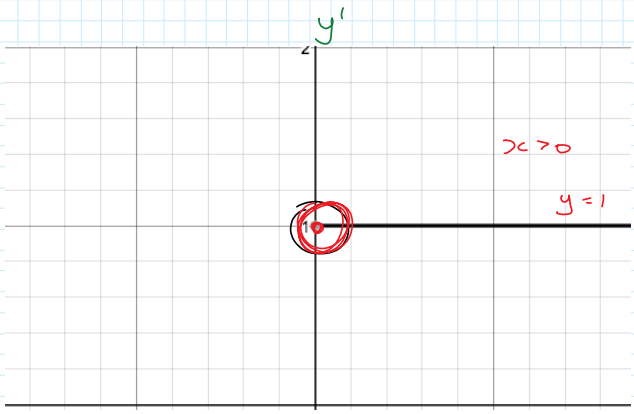
$f(x)$ is defined at $x = a$

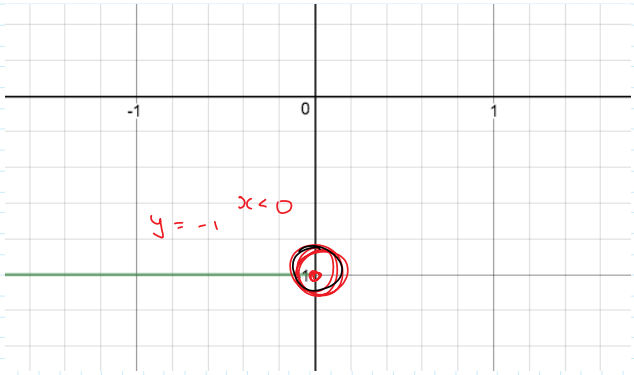
$\lim_{x \to a} f(x) = f(a)$

If we were to draw a graph of the gradient function we would notice the following:

The gradient at x=0 is not differentiable as the gradient to the left of 0 is -1 and to the right of 0 is 1.

$$f(x) = \begin{cases} x, & x \geq 0 \quad (m = 1) \\ -x, & x < 0 \quad (m = -1) \end{cases}$$





You don't always have to draw the graphs of the gradient function to find if a point is differentiable

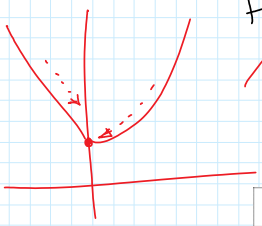
Example

$$f(x) = \begin{cases} x^2 - 3x + 2, & x \geq 0 \\ -x + 2, & x < 0 \end{cases}$$

Quad

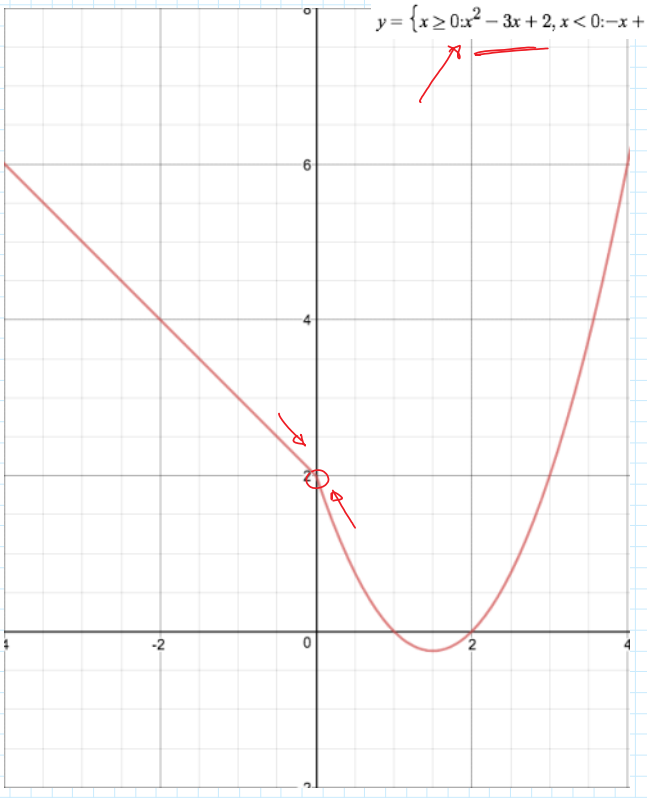
S.L.

1. Check where the functions meet.
2. In the above example, they need at x=0
3. Find the value of f(x) at the point they meet



$$f(0) = \begin{cases} 2 \\ 2 \end{cases}$$

They are both the same value so the function is continuous!



→ DESMOS NOTATION

$\{x \geq 0 : x^2 - 3x + 2, x < 0\}$

4. Now we can check to see if the point they "meet" is differentiable.
5. Does the gradient from the left, get closer to the same value as the gradient coming from the right.

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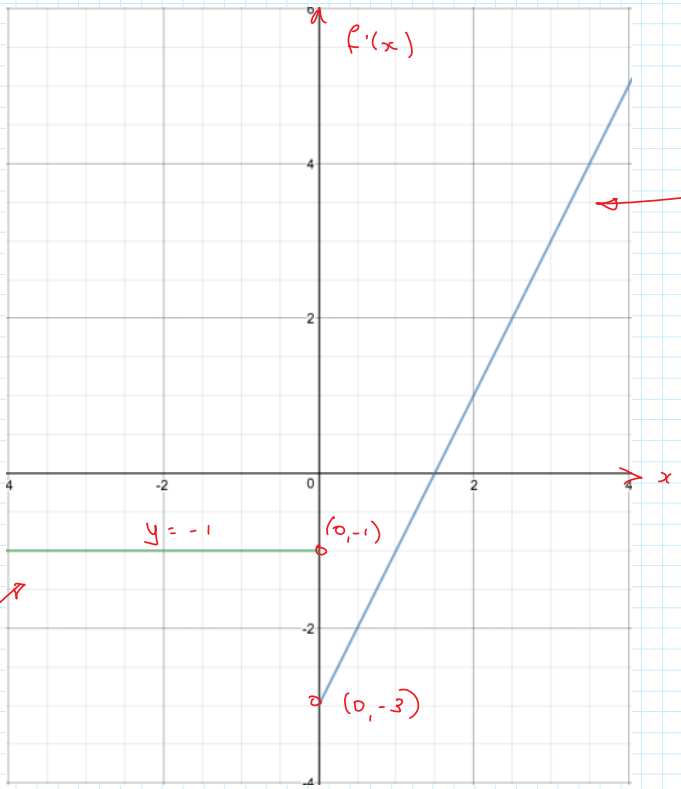
$$f'(x) = \begin{cases} 2x - 3, & x \geq 0 \\ -1 & x < 0 \end{cases}$$

Substitute in the point where they "meet" i.e. x=0 and we see:

$$f'(0) = \begin{cases} -3 \\ -1 \end{cases}$$

Not the same so... @ x=0 the point is NOT differentiable

And a graph just to prove it!



IMPORTANT NOTE:

If a function is differentiable at x=a, then it is also continuous at x=a.
But the converse is not true.

