The language of polynomials (4C)

Sunday, 25 February 2018 4:42 pm

RECAP

Previously we have looked at how quadratics are a really useful part of Mathematics.
We looked at how to solve them:
- By putting a quadratic equation equal to zero

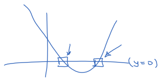
How to factorise them:
- T method
- Cross Method
- Quadratic Formula
- Completing the square

What the discriminant means:
- How many roots to expect from a quadratic equation
  - 2,1 or none.

Yr 10, 11 $\boxed{12}$

$b^2-4ac \;\begin{matrix}<0\\=0\\>0\end{matrix}$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$



$y = x^2+3x-6$

Polynomials

A quadratic is one example of a polynomial.

Leading term tells you the **degree** of the polynomial

degree 2 $\leftarrow$ x^{2}

$y = 3x^2 + 4x - 2$

Constant Term

Leading term

Finding the value of a Polynomial

Assuming we have the following Polynomial, we can find the value of the polynomial for any value of x

$$\boxed{P(x) = 8x^3 - 3x^2 + 4x - 3}$$

$P(x) = 8x^3 - 3x^2 + 4x - 3$

$y =$

Hence $P(1)$ means substitute 1 into the polynomial.

$8(1)^3 - 3(1)^2 + 4(1) - 3$

$$P(1) = 8 - 3 + 4 - 3 = 6$$

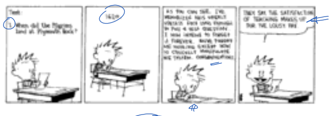
$P(1) = P(x) = P(1)$

$F(2)$

$Z(3)$ $\leftarrow$ $Z(x) = \text{sub } 3 \text{ for } x.$

Forwards and backwards

Remember, in Mathematics, what we can do forwards we can also do backwards.
This is testing your understanding of the concepts and NOT whether you can regurgitate facts.



They may give you the value of a Polynomial for a given value of x but then ask you to find the value of a co -efficient.

Example: Find the value of r given that $P(2) = 11$ and $P(x) = 2x^3 - rx^2 + x + 5$

$P(2) = 11$

$11 = 2(2)^3 - r(2)^2 + (2) + 5$

$11 = 16 - 4r + 2 + 5$

$11 = 23 - 4r$

$4r = 23 - 11$

$4r = 12$

$r = 3$

Polynomial operations

Polynomials can be added, subtracted, multiplied and divided.
Just make sure you collect like terms.

Example:
If $h(x) = 2x^3 + 3x^2 + 2x - 3$ and $g(x) = 5x^2 - 4x + 8$, find $h(x) + g(x)$

$h(x) + g(x) = (2x^3 + 3x^2 + 2x - 3) + (5x^2 - 4x + 8)$
$= \cancel{2x^3} + \cancel{3x^2} + \cancel{2x} \cancel{-3} + \cancel{5x^2} - \cancel{4x} + \cancel{8}$
$= 2x^3 + 8x^2 - 2x + 5$

$h(x) - g(x)$

$(2x^3 + 3x^2 + 2x - 3) - (\boxed{5x^2} - 4x + 8)$

$2x^3 + 3x^2 + 2x - 3 - 5x^2 + 4x - 8$

$h(x).g(x)$ $h(x) \div g(x)$

(CAS)

Equating coefficients

This is an area of Mathematics which I was taught at school which was pretty much the only thing I got "first time".
This made so much sense.

Basically, we know that an equals sign (=) means that what is on the left of an equation must be identical to what's on the r ight.

This can be used to find missing values from polynomials.

LHS = RHS

Example:
It is known that $3x^3 - 36x^2 + 144x - 198 = a(x-4)^3 + b$. Find the values of a and b.

x, x^2, x^3

$3x^3 - 36x^2 + 144x \boxed{-198} = ax^3 - 12ax^2 + 48ax \boxed{-64a + b}$

LHS = RHS

$3x^3 = ax^3$

$a = 3$

$-198 = -64a + b$

$-198 = -192 + b$

$b = -6$

$-64 \times 3 = 192$

$3(x-4)^3 - 6$

CAS!

1
1 1
1 2 1
1 3 3 1

$(x-4)^3$

	x	-4
1	x^0	$(-4)^3$
3	x^1	$(-4)^2$
3	x^2	$(-4)^1$
1	x^3	$(-4)^0$

$4 \times 4 = 16$
$16 \times 4 = 64$

$-64 + 3x(16) + 3x^2(-4) + x^3$

$-64 + 48x - 12x^2 + x^3$

$16 \times 3 = 48$

$a(x^3 - 12x^2 + 48x - 64) + b$

$a(x-4)^3 + b$

Work to be completed at the end of teaching:
Methods 3&4 Textbook
Exercise 4C
Questions 1-9(s)

$(x+1)(x^2 - 2x - 4)$

-4	
1	4
2	2

$x^3 + (bx^2 + x^2) + (bx + cx) + c$

$x^3 - x^2 - 6x - 4$

$x^3 + x^2(b+1) + x(b+c) + c$

$x^3 - x^2 \quad -6x \quad -4$

$b + c = -6$
$b - 4 = -6$
$c = -4$

$b = -2$

$-1 = b + 1$
$-1 = -2 + 1$

$a(x+c)^3 + b = ax^3 + ac^3 + 3acx^2 + 3ac^2x + b$

$x^3 - 5x^2 - 2x + 24$

$b + ac^3 = 24$

$a = 1$

$3ac = -5$

$(x-1)^2$

$(x-1)^2 - 1 - 4$

$(x-1)^2 - 5 = 0$

$(x + \sqrt{5} - 1)(x - \sqrt{5} - 1)$

$(x-1)^2 = 5$

$x = \pm\sqrt{5}$

$x = 1 \pm \sqrt{5}$

$x = (1 + \sqrt{5})$

$x = 1 - \sqrt{5}$

$x = 4$

$x = 2$

$x - 4 = 0$

$x - 2 = 0$

$(x-2)(x-4)$

$(x - 1 - \sqrt{5})$
$(x - 1 + \sqrt{5})$

$x = 2$

$(x - 2)$

$x = 1 + \sqrt{5}$
$(x - 1 - \sqrt{5})$

$x = 1 - \sqrt{5}$
$(x - 1 + \sqrt{5})$

$(x-1)(x^2 - 2x - 4)$

-4	
1	-4
2	-2

$(x - 1)(x - 1 - \sqrt{5})(x - 1 + \sqrt{5})$

abc 2 X

$b = 1$ $b = 3$

a) _ _ _ _ $\therefore$ Failed

b) $b = -2$ $c = -4$

$(x+1)(1x^2 - 2x - 4)$

-4	
1	-4
$+2$	-2

c) $(x+1)(x - 1 - \sqrt{5})(x - 1 + \sqrt{5})$

$(x-1)^2 - 5 = 0$

$(x-1)^2 = 5$

$x - 1 = \pm\sqrt{5}$

$x = 1 \pm \sqrt{5}$

$\boxed{x = 1 + \sqrt{5}}$ $\boxed{x = 1 - \sqrt{5}}$

$\boxed{(x - 1 - \sqrt{5}) = 0}$ $\boxed{(x - 1 + \sqrt{5}) = 0}$

$(x-2)(x+4)=0$

$x-2=0 \qquad x+4=0$

$x=2 \qquad x=-4$

$a(x+c)^3+b$

EXPAND

$ax^3 - 3acx^2 + 3ac^2x - ac^3 + b$

$x^3 - 5x^2 - 2x + 24$

$\boxed{a=1}$

$-3ac=-5$

$-3c=-5$

$c=\frac{5}{3}$

$\boxed{3ac^2=-2}$

$3.1.\left(\frac{5}{3}\right)^2 \neq -2$

∴ Not possible

$-ac^3+b=24$