

The discriminant

Thursday, 1 March 2018 6:10 pm

Work to be completed once teaching has taken place:

Discriminant: MM Ex 3: 2017 4061 Sat: 7 8 10 12 15 16

RECAP:

In the previous lesson we were looking at the general quadratic equation which had the form:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We looked at what is under the square root sign and said that it made us nervous!

I looked at how this thing under the square root can have three possible states ...

The discriminant

For a parabola with the form $y = ax^2 + bx + c$ the discriminant is $b^2 - 4ac$.

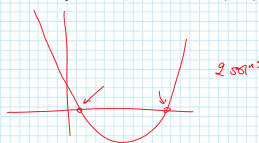
The discriminant has a certain symbol ... Δ

We know the following for the discriminant:

Case 1: $b^2 - 4ac > 0$ pos

When the value of this is greater than zero, we know that we will always end up with two solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



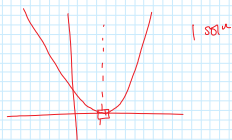
Case 2: $b^2 - 4ac = 0$

When the value is equal to zero then we only have one solution. This is known as a repeated root. It's where the graph JUST touches the x-axis.

$$x = \frac{-b \pm \sqrt{0}}{2a}$$

$$= \frac{-b}{2a}$$

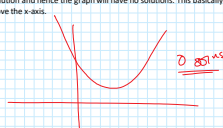
Aus of symmetry



Case 3: $b^2 - 4ac < 0$ neg

This cannot have a solution and hence the graph will have no solutions. This basically means the minimum point is above the x-axis.

$$\sqrt{b^2 - 4ac}$$



Using the discriminant

We can do things forwards ...

Example: Find the discriminant of $y = 2x^2 - 3x - 5$ and state whether the graph of each crosses the x-axis, touches the x-axis or does not intersect the x-axis.

$$y = ax^2 + bx + c$$

$$a = 2$$

$$b = -3$$

$$c = -5$$

$$\Delta = b^2 - 4ac$$

$$= (-3)^2 - 4 \cdot 2 \cdot (-5)$$

$$= 9 + 40$$

$$= 49$$

$$\Delta > 0 \quad 2 \text{ solutions}$$

$$\Delta = 0$$

$$\Delta < 0$$

Or backwards ...

Example: Find the values of p for which the equation $4x^2 - 2px + 1 = 0$ has:

- One solution
- No solution
- Two distinct solutions

One solution (make it equal to zero):

$$\Delta = b^2 - 4ac = 0$$

$$(-2p)^2 - 4 \cdot 4 \cdot 1 = 0$$

$$\Rightarrow 4p^2 - 16 = 0$$

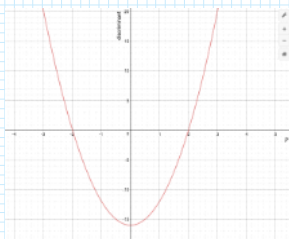
$$\Rightarrow p = \pm 2$$

$$\therefore p = 2 \text{ or } p = -2$$

No solution (make discriminant less than zero):

$$\Delta < 0$$

$$-2 < p < 2$$

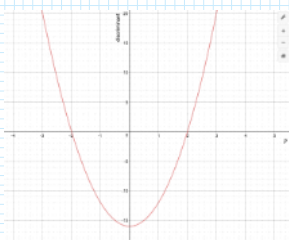


Two distinct solutions (make discriminant greater than zero):

$$\Delta > 0$$

$$x < -2$$

$$x > 2$$



Rational Numbers and Perfect Squares

Remember that a perfect square can be expressed in the form: $y = (x - b)^2$

A rational number is one which can be expressed as a fraction (with integer numerator and denominator).

An irrational number is one which cannot be expressed as a fraction e.g. $\sqrt{2}$

For the equation $ax^2 + bx + c = 0$ where a , b and c are rational numbers:

If Δ is a perfect square and $\Delta \neq 0$, then the equation has two rational solutions.

If $\Delta = 0$, then the equation has one rational solution.

If Δ is not a perfect square and $\Delta > 0$, then the equation has two irrational solutions.

$$y = (x - 1)^2$$

$$y = (x - 2)^2$$

$$y = (x - 3)^2$$

$$-\frac{b}{2a}$$

$$\sqrt{9}$$

$$\sqrt{5}$$

Thing to remember with all the questions ...

The discriminant is easy ... it's $b^2 - 4ac$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = -b \pm \sqrt{1}$$

$$-b \pm \sqrt{4}$$

$$-b \pm \sqrt{9}$$

discriminant

$$\Delta = 0$$

$$\Delta > 0$$

$$b^2 - 4ac = 1$$

$$\Delta = 0$$

$$\Delta / 4p^2 - 16 = 0$$

$$4p^2 = 16$$

$$p^2 = 4$$

$$p = \pm \sqrt{4}$$

$$p = \pm 2$$

$$4p^2 - 16 = 0$$

$$p = 2$$

$$p = -2$$

$$\Delta > 0$$

$$\Delta < 0$$

$$\Delta = 0$$

$$\Delta > 0$$

$$\Delta < 0$$

$$\Delta = 0$$

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$$\Delta < 0$$

- If Δ is a perfect square and $\Delta \neq 0$, then the equation has two rational solutions.
 If $\Delta = 0$, then the equation has one rational solution.
 If Δ is not a perfect square and $\Delta > 0$, then the equation has two irrational solutions.

Thing to remember with all the questions ...

The discriminant is easy ... $b^2 - 4ac$

You need to look at all the ways the questions are written and the code they use. There are LOTS of different ways to write a question which is effectively asking for the same thing.

Examples of wording which are effectively all asking you the same thing:

- Determine the discriminant of each of the following quadratics:
- Without sketching the graphs of the following quadratics, determine whether they cross or touch the x-axis:
- By examining the discriminant, find the number of distinct solutions of:
- By examining the discriminant, state the nature and number of distinct solutions for each of the following:
- For m and n rational numbers show that $mx^2 + (2m + n)x + 2n = 0$ has rational solutions.
- Find the discriminant of the equation $4x^2 + (m - 4)x - m = 0$, where m is a rational number, and hence show that the equation has rational solution(s).

$$x = \frac{-b \pm \sqrt{1}}{2a}$$

$$x = \frac{-b \pm \sqrt{4}}{2a}$$

$$x = \frac{-b \pm \sqrt{9}}{2a}$$

$$x = \frac{-b \pm 1}{2a}$$

$$x = \frac{-b \pm 2}{2a}$$

$$x = \frac{-b \pm 3}{2a}$$

$$x = \frac{-b \pm \sqrt{2}}{2a}$$

discriminant

$\Delta = 0$
 $\Delta > 0$

$$b^2 - 4ac = 1$$

$$b^2 - 4ac = 4$$

$$b^2 - 4ac = 9$$

$$b^2 - 4ac = 16$$

$$b^2 - 4ac = 25$$

$$b^2 - 4ac = 36$$

Rational