The chain rule (9E)

Wednesday, 14 March 2018 6:05 pm

Work to be completed by the end of teaching:

Chain Rule	9E	1,3,5,6

RECAP:

We have, so far, learnt techniques to differentiate simple functions.

Example: $f(x) = (3x^3) + (5x^2) - (2x) + (8)$ $\quad f'(x) = 9x^2 + 10x - 2$

We do this by looking at each individual term and differentiating it.
This is great! Well it is when the function is written in fully expanded form.
Not so great when it's not!



She has a "short cut"!

We have also got techniques for differentiating negative powers.

Example: $f(x) = \frac{1}{x^2} + 3x - 2$ $\quad f(x) = x^{-2} + 3x - 2$

$f'(x) = -2x^{-3} + 3$

We can, once again, differentiate each term.

But what happens we have something more complex? $\quad = \frac{-2}{x^3} + 3$

Enter ... the chain rule

The chain rule

When we have something more complex, we need to find a way to either make it easier, or learn a rule to help us differentiate it.



"The missing link"

Example: $f(x) = (x^2+3)^3$

$y = u^3$ $\quad u = x^2 + 3$

$\frac{dy}{du} = 3u^2$ (1) $\quad \frac{du}{dx} = 2x$ (2)

$f'(x)$

$\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{dy}{dx}$

$3u^2$

$3(x^2+3)^2$

$\frac{dy}{dx} = 3(x^2+3)^2 \cdot 2x = 6x(x^2+3)^2$

Leibniz notation

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Example 2: $f(x) = (3x^2+2x)^{-3}$

$y = u^{-3}$ $\quad u = 3x^2 + 2x$

$\frac{dy}{du} = -3u^{-4}$ $\quad \frac{du}{dx} = 6x + 2$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= -3(3x^2+2x)^{-4} \cdot (6x+2)$

$= -3(6x+2)(3x^2+2x)^{-4}$

$= -6(3x+1)(3x^2+2x)^{-4}$

Example 3: $f(x) = \frac{10}{4x^2+2}$

$f(x) = 10(4x^2+2)^{-1}$

$y = 10u^{-1}$ $\quad u = 4x^2 + 2$

$\frac{dy}{du} = -10u^{-2}$ $\quad \frac{du}{dx} = 8x$

$\frac{dy}{dx} = -10(4x^2+2)^{-2} \cdot 8x$

$= -80x(4x^2+2)^{-2} = \frac{-80x}{(4x^2+2)^2}$

The "short cut"

As with most things in Mathematics, there is a short cut.
Be warned that, like most short cuts, you know when to use it.
Don't think it will work in all cases.
And you have to be sure that the questions are written to allow the short cut to work.

For example (x^2+3) will not work with the Chain Rule

$f(x) = (x^2+3)^2$

$f'(x) = 3(x^2+3)^2 \cdot 2x$

$= 6x(x^2+3)^2$

$f(x) = (3x^2+2x)^{-3}$

$f'(x) = -3(3x^2+2x)^{-4} \cdot (6x+2)$

$= -3(6x+2)(3x^2+2x)^{-4}$

$= -6(3x+1)(3x^2+2x)^{-4}$

$f(x) = \frac{10}{4x^2+2}$

$f(x) = 10(4x^2+2)^{-1}$

$f'(x) = -10(4x^2+2)^{-2} \cdot (8x)$

$= \frac{-80x}{(4x^2+2)^2}$

This is a tool to handle complex problems.

Remember, this is a tool which is there to help you solve complex problems.
Never lose sight of the fact that you are working with gradients of tangents to points on a curve.
The Chain Rule is allowing you to differentiate more complex functions without having to expand it to it's respective terms.

Example:



Find the gradient of the tangent to the curve with equation $y = \frac{16}{3x^3+x}$ at the point $(1, 4)$

$f(x) = 16(3x^3+x)^{-1}$

$f'(x) = -16(3x^3+x)^{-2} \cdot (9x^2+1)$ $\quad x = 1$

$= -16(3+1)^{-2}(9+1)$

$= -16(4)^{-2}(10)$

$= \frac{-160}{4^2} = \frac{-160}{16} = -10$

$y = (3x^2)(2x)(-2)$

$y = (x+5)^2 = (x^2)+(6x)+(9)$

$y = (x^3+3)^4$

$y = u^4$

$\frac{dy}{du} = 4u^3$

$u = x^3+3$

$\frac{du}{dx} = 3x^2$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = 4(x^3+3)^3 \cdot 3x^2$

$= 12x^2(x^3+3)^3$

$y = [(x^4+x^2)]^3$

$y = u^3$

$\frac{dy}{du} = 3u^2$

$u = x^4+x^2$

$\frac{du}{dx} = 4x^3+2x$

$\therefore \frac{dy}{dx} = 3(x^4+x^2)^2 \cdot (4x^3+2x)$

$y = (2x+3)^2$

$y' = 2(2x+3)^1 \cdot 2$

$y = \frac{1}{3x^2+4}$

$y = \frac{1}{u}$

$y = u^{-1}$

$u = 3x^2+4$

$(3x^2+4)^{-1}$

$-1(3x^2+4)^{-2} \cdot (6x)$

product quotient

$y = x^3+2x^2+x$ $\quad u \quad v$

$y = (x+3)^2(x-2)$

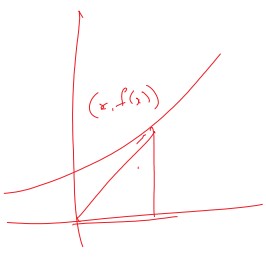
$y' = uv' + vu'$

$y = \frac{x^2+3x}{x+2}$

$y' = \frac{vu' - uv'}{v^2}$

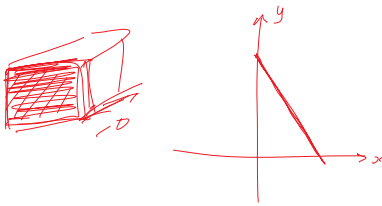
$u = (x+3)^2$ $v = x-2$

$u' = 2(x+3)$ $v' = 1$





$20 \times 2\pi r$



$f(x) = (x^2 - 2x)^2$