# The Truncus

Wednesday, 25 April 2018 5:44 pm

Work to be completed at the end of teaching:

| | | |
|---|---|---|
| 4B: The truncus | 151 | 1abcefgh, 2abcefgh, 3ac |

**RECAP**

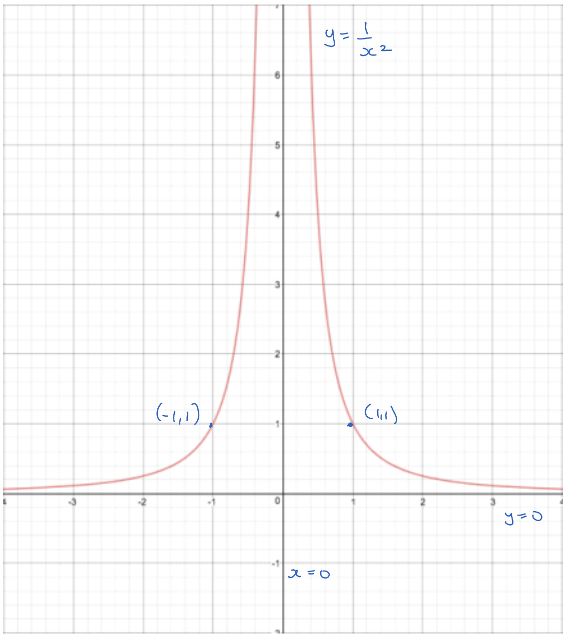
We have been working on all the work for Functions and Relations.
Now we look at some of the most used Functions in Mathematical Methods.

- Rectangular Hyperbola
- The Truncus
- The Square Root Graph
- Circles

This lesson will be used to look at ...

**The Truncus**

Always nice to start with what it looks like yes?!




**Base Equation:** $\boxed{y = \frac{1}{x^2}}$

$y = \cancel{\frac{1}{0^2}}$

$0 = \frac{1}{x^2}$ $\cancel{x^2 = \frac{1}{0}}$

**Question**: Why are there no negative y values?

Important information about the graph:

**Asymptotes:**
$x = 0$
$y = 0$

Helpful points:
- (1, 1)
- (-1, 1)

Plotting points on the graph will help the examiner know that you have understood transformations and have calculated things correctly.

**Domain and Range:**
- Domain: $\mathbb{R}\backslash\{0\}$
- Range: $\mathbb{R}^+$ or $(0, \infty)$

$x \in \mathbb{R} \setminus \{0\}$
$f(x) \in \mathbb{R}^+$

**Shape**
It is important that you memorise the shape.
This is the base shape from which all other trunci are born.

Note: The plural of truncus is trunci

As with all functions, the base truncus can be transformed.
It is vital you understand what the form of a transformed truncus can look like

**Examples of how functions can be made to look:**

1. $\boxed{y = \frac{1}{x^2}}$ BASE

2. $\boxed{y = \frac{3}{x^2}}$ 3. $\frac{1}{x^2}$

3. $y = \frac{1}{(x-3)^2}$ Translation → 3

4. $y = \frac{2}{(x+1)^2} + 4$ ← 1

5. $y = \frac{4}{(2x+3)^2} - 3$

6. $y = -\frac{2}{(x-1)^2} - \frac{1}{2}$

7. $y = \frac{3}{(1-x)^2} + 2$

As will all functions, you need to be able to look at them and understand what each number stands for and how they can be applied as a transformation of a truncus.

**RECAP: Order of Transformations**

Remember ... Unless they give you an order of transformations ... you must use Dr T

Not Mr T ... like from the 1980's TV show called The A Team.

Dr T stands for:
- Dilations
- Reflections
- Translations

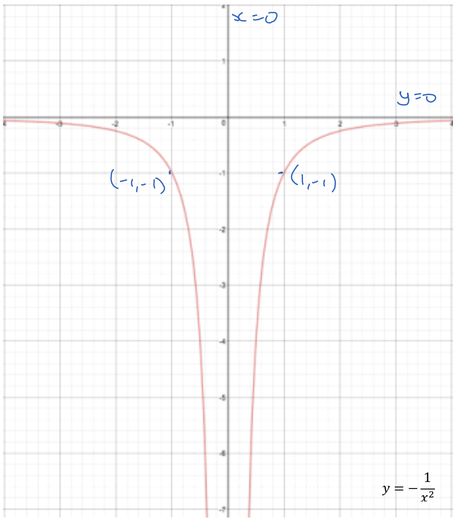
D
R
T

Always identify which are which in the equation and then construct them in that order!

**Reflections of Trunci**

Let's look at what happens with a reflection in the $x$- and $y$-axis

### ✱ Reflection in the $x$-axis




Important information about the graph:

**Asymptotes:**

$x = 0$
$y = 0$

Helpful points:

- (1,- 1)
- (-1,- 1)

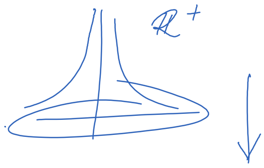
Plotting points on the graph will help the examiner know that you have understood transformations and have calculated things correctly.

**Domain and Range:**

- Domain: $\mathbb{R}\backslash\{0\}$
- Range: $\mathbb{R}^-$ or $(-\infty, 0)$

**Shape**

It is important that you memorise the shape.



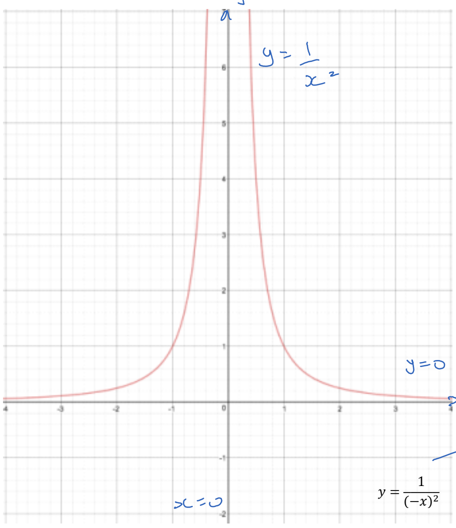
$\mathbb{R}^+$

$\mathbb{R}^-$ $\quad \frac{-1}{x^2}$

$x \in \mathbb{R}\backslash\{0\}$

$f(x) \in \mathbb{R}^-$

### Reflection in the $y$-axis




This is the same as the base graph.

**Asymptotes:**

$x = 0$
$y = 0$

Helpful points:

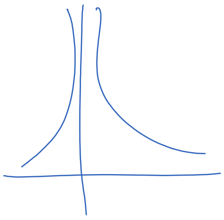
- (1, 1)
- (-1, 1)

**Domain and Range:**

- Domain: $\mathbb{R}\backslash\{0\}$
- Range: $\mathbb{R}^+$ or $(0, \infty)$

**Shape**

It is important that you memorise the shape.



$y = \frac{1}{x^2}$ $\quad x \to -x$

$y = \frac{1}{(-x)^2}$

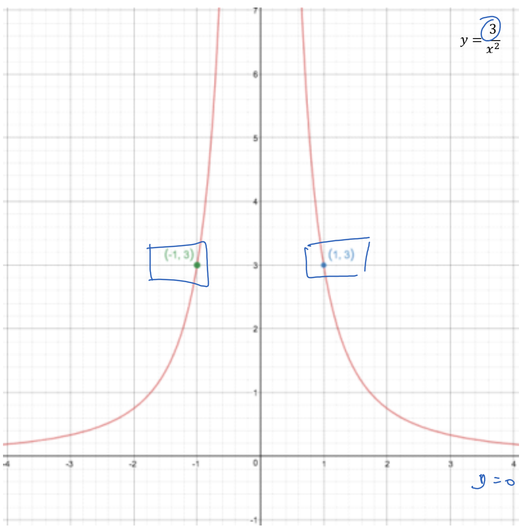
$y = \frac{1}{(-x)^2}$

$= \frac{1}{x^2}$

## Dilations of the Trunci

Again, we need to remember that dilations can take place away from the $x$-axis and away from the $y$-axis.

### Dilation factor 'a' from the $x$-axis




**Important information**

**Asymptotes:**

$x = 0$
$y = 0$

Helpful points:

- (1, 3)
- (-1, 3)

**Domain and Range:**

- Domain: $\mathbb{R}\backslash\{0\}$
- Range: $\mathbb{R}^+$ or $(0, \infty)$

This graph has been stretched by a factor of 3 away from the $x$-axis.

It is vital that you know which number relates to a dilation from the $x$-axis.

$f(x) = \frac{1}{x^2}$ $\quad \frac{3}{x^2}$

$a f(x)$ $\quad \boxed{3.f(x)}$

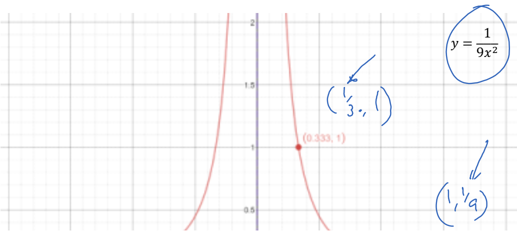
(1, 1)

(1, 3)

$\frac{9}{x^2}$

$\frac{4}{3x^2}$ $\quad \frac{4}{3} \times \frac{1}{x^2}$

### Dilation factor 'a' from the $y$-axis




**Important information**

**Dilation:** Factor $\frac{1}{3}$ from $y$-axis

**Asymptotes:**

$x = 0$
$y = 0$

Helpful points:

- $(1, \frac{1}{9})$
- $(-1, \frac{1}{9})$

$x \to \frac{x}{a}$

$y = \frac{1}{x^2} = \frac{1}{\left(\frac{x}{1/3}\right)^2}$

$\frac{x}{1/3} = x \div \frac{1}{3}$

$= \frac{1}{(3x)^2}$ $\quad \frac{1}{(3x^2)}$


$y = 0$

Helpful points:
- (1, $\frac{1}{9}$)
- (-1, $\frac{1}{9}$)

**Domain and Range:**
- Domain: $\mathbb{R}\backslash\{0\}$
- Range: $\mathbb{R}^+$ or $(0, \infty)$

This graph has been stretched by a factor of $\frac{1}{3}$ away from the $y$-axis.

It is vital that you know which number relates to a dilation from the $y$-axis.

$\frac{x}{1/3} = x \div \frac{1}{3}$

$= x \times \frac{3}{1}$

$= \frac{1}{(3x)^2}$

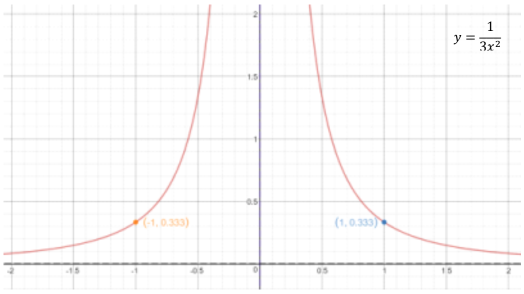
$= \frac{1}{9x^2}$

$= \frac{1}{9} \cdot \frac{1}{x^2}$

$\frac{1}{(3x^2)}$

$\frac{1}{3}$ $\frac{1}{9x^2}$

**Side note:** It's important to note the difference between the graph of $y = \frac{1}{3x^2}$ and $y = \frac{1}{(3x)^2}$




**Important information**

**Asymptotes:**
$x = 0$
$y = 0$

Helpful points:
- (1, $\frac{1}{9}$)
- (-1, $\frac{1}{9}$)

**Domain and Range:**
- Domain: $\mathbb{R}\backslash\{0\}$
- Range: $\mathbb{R}^+$ or $(0, \infty)$

This graph has been stretched by a factor of $\frac{1}{3}$ away from the $x$-axis.

It is vital that you know which number relates to a dilation from the $x$-axis.

You must also know that for many graphs, a dilation from the $x$-axis can also be described as a dilation from the $y$-axis

$\frac{1}{(3x)^2}$ $\frac{1}{3x^2}$

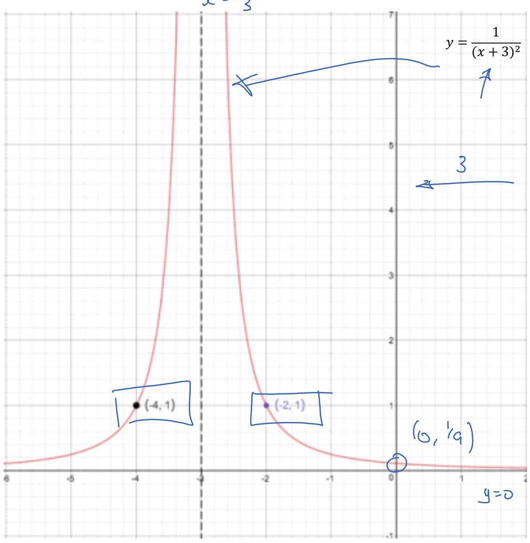
$\frac{1}{9x^2}$ $\frac{1}{3x^2}$

**Translations for Trunci**

Finally, we can move the truncus both vertically and horizontally in the same way we can move other graphs.

**Translation parallel to the $x$-axis**

$x = -3$




**Important information**

**Asymptotes:**
$x = -3$
$y = 0$

Helpful points:
- (-4, 1)
- (-2, 1)

**Domain and Range:**
- Domain: $\mathbb{R}\backslash\{-3\}$
- Range: $\mathbb{R}^+$ or $(0, \infty)$

This graph has been moved 3 units in the negative direction parallel to the $x$-axis.

(0, 1) (-2, 1)

(-1, 1) (-4, 1)

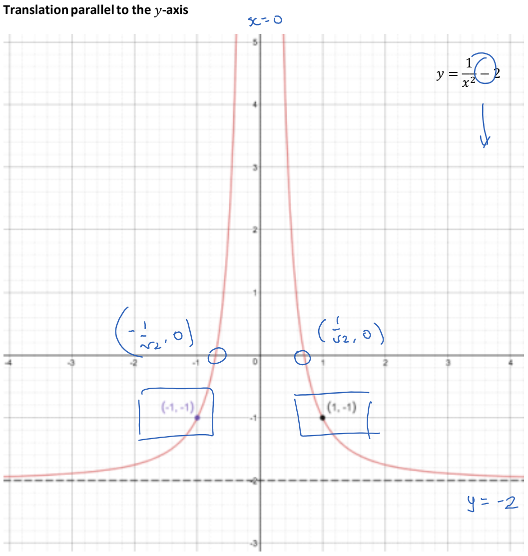
$y = \frac{1}{(x+3)^2}$

$y = \frac{1}{(3)^2}$

$= \frac{1}{9}$

**Translation parallel to the $y$-axis**

$x = 0$




**Important information**

**Asymptotes:**
$x = 0$
$y = -2$

Helpful points:
- (-1, -1)
- (1, -1)

**Domain and Range:**
- Domain: $\mathbb{R}\backslash\{0\}$
- Range: $(-2, \infty)$

This graph has been moved 2 units in the negative direction parallel to the $y$-axis.

2

(1, 1) (1, -1)

(-1, 1) (-1, -1)

$y = \frac{1}{x^2} - 2$

$0 = \frac{1}{x^2} - 2$

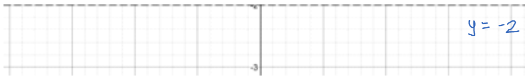
$2 = \frac{1}{x^2}$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}}$

$= \pm\frac{\sqrt{1}}{\sqrt{2}}$

$= \pm\frac{1}{\sqrt{2}}$




$= \pm \frac{\sqrt{1}}{\sqrt{2}}$

$= \pm \frac{1}{\sqrt{2}}$

## Combining Transformations of Rectangular Hyperbolas

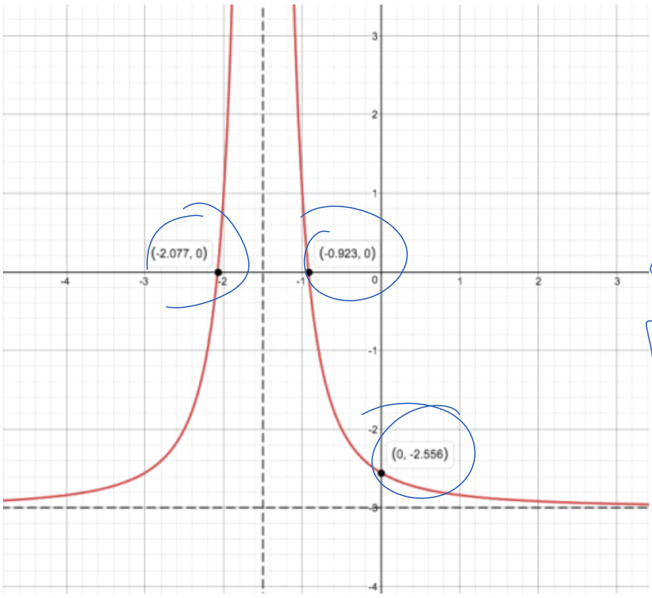
Let's ramp up the complexity just a little bit!
Let's consider the following function.

$$\boxed{y = \frac{4}{(2x+3)^2} - 3}$$

$1x$

$$\frac{4}{[2(x+3/2)]^2} - 3$$

$\frac{1}{2}$ $\frac{3}{2}$




**Important information**

**Asymptotes:**
$x = -1.5$
$y = -3$

**Domain and Range:**
- Domain: $\mathbb{R}\backslash\{-1.5\}$
- Range: $(-3, \infty)$

**Transformations:**

**Dilations:**
From $y$-axis: $\frac{1}{2}$
From $x$-axis: 4

**Reflections:**
None

**Translations:**
Parallel to $x$-axis: $-\frac{3}{2}$
Parallel to $y$-axis: $-3$

Remember, it's always important to show any crossing points on the $x$- and $y$-axes.

$(ab)^2$
$= a^2b^2$

$[2(x+3/2)]^2$
$4(x+3/2)^2$

$$\frac{\cancel{4}\,1}{1\,\cancel{4}(x+3/2)^2} - 3$$

$$\frac{1}{(x+3/2)^2} - 3$$

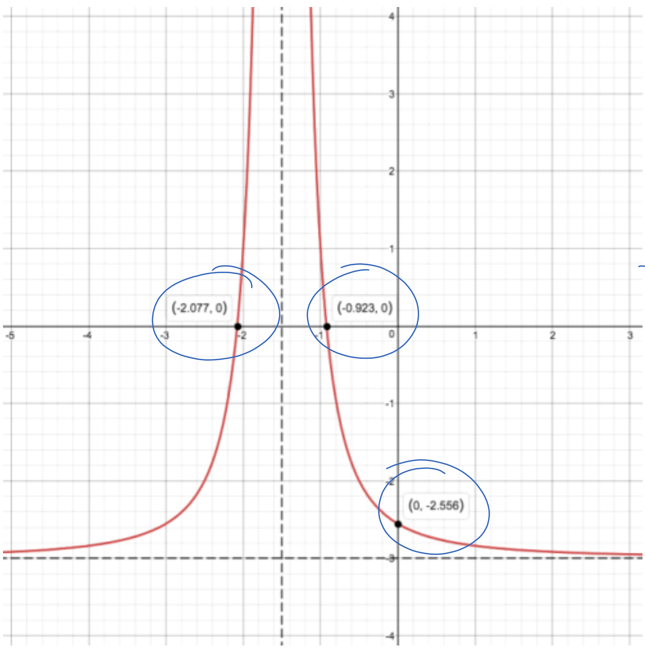
$\downarrow 3$

$3/2$

$\boxed{a.f(x)}$

Compare this with:

$$y = \frac{1}{\left(x+\frac{3}{2}\right)^2} - 3$$




It's the same graph as above!

Trying to have the co-efficient of x as 1 makes the transformations easier to identify!

From the above equation it's much easier to see:

**Transformations:**

**Dilations:**
None

**Reflections:**
None

**Translations:**
Parallel to $x$-axis: $-\frac{3}{2}$
Parallel to $y$-axis: $-3$

Remember, it's always important to show any crossing points on the $x$- and $y$-axes.