The Tangent Function

Wednesday, 25 April 2018 6:42 pm

At the end of the teaching I would like you to complete the following questions:

tangent graph and equations	6J	2,3,5,6,11,13,15

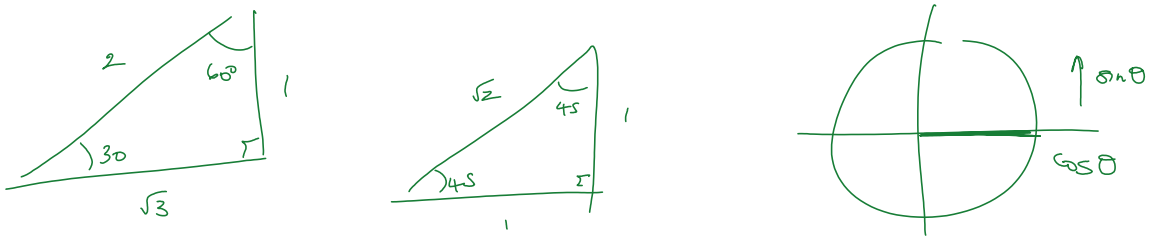
RECAP:

We have spent a lot of time looking at the graphs of sine and cosine.
These are certainly very important functions, but there is one other which might occasionally need our consideration.
This graph is very different in shape and period from it's cousins so we need to ensure we are not tricked.

There are some important things to note for this chapter:

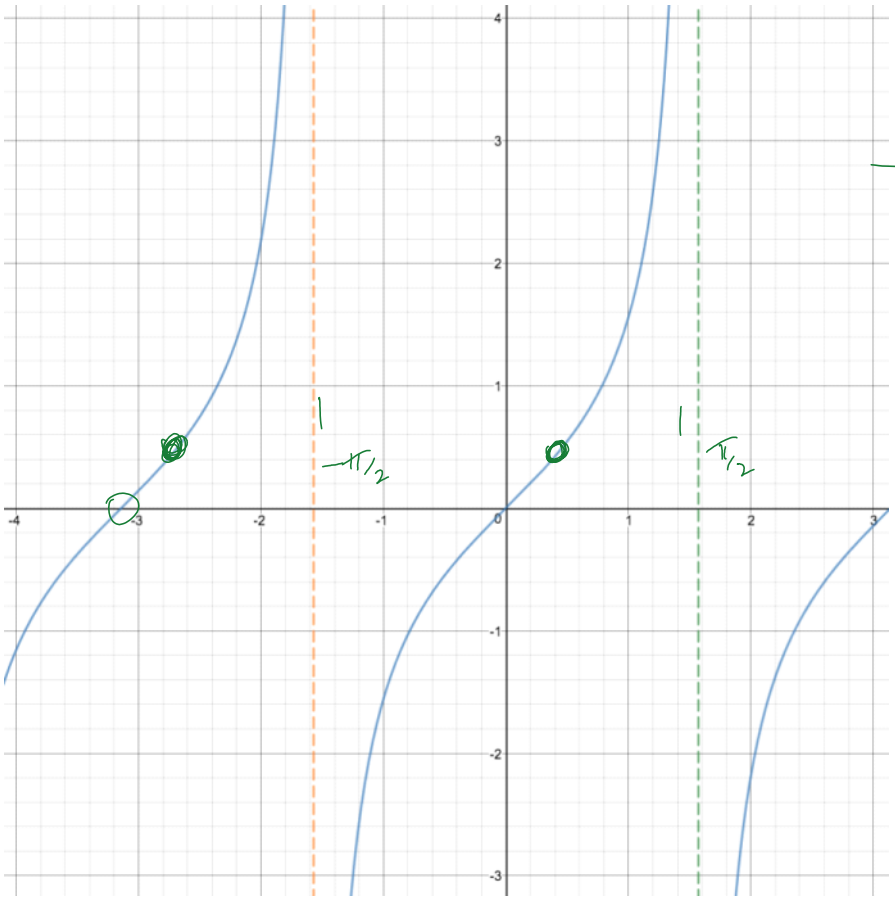
$$tan\theta = \frac{sin\theta}{cos\theta}$$

We can work out the tangent for any of our "typical values for θ by using the two triangles we have been using to date:



So, here it is:

The Tan Graph



Important Information

Asymptotes:

$-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$ etc ...

Period: π

Range: $\mathbb{R}$

Vertical asymptotes can be found using the following equation:

$$\theta = \frac{((2k+1)\pi)}{2}$$

k = 0
k = 1

Axis intercepts at $\theta = k\pi$

k = 0 $\frac{(2k+1)\pi}{2} = \frac{\pi}{2}$

k = 1 $\frac{3\pi}{2}$

k = −1 $-\frac{\pi}{2}$

k = −2 $-\frac{3\pi}{2}$

As with all other graphs, the tan graph can be transformed!

$$y = 3\tan(2x - \frac{\pi}{3})$$

$y = 3\tan 2(x - \frac{\pi}{6})$

$y = 3\tan(2x - \frac{\pi}{3})$

Remember, this is a trick.
Always try and get the co-efficient of x to be 1.

$y = 3\tan 2(x - \frac{\pi}{6})$



Period = $\frac{\pi}{2}$

Dilation: Factor 3 from x-axis
Dilation: Factor $\frac{1}{2}$ from y-axis

Translation: $\frac{\pi}{6}$ in the positive direction of the x-axis

Period = $\frac{\pi}{n}$

Period = $\frac{\pi}{2}$

We can now start to use the Mathematics we've done before to now solve tan equations to a number of solutions.

Solve the following equation for $x \in (0, 2\pi)$

$3\tan(2x) = \sqrt{3}$

$\tan(2x) = \frac{\sqrt{3}}{3}$

$2x = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$\frac{6\pi}{6}$

$2x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$

Now we use the result:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Solve $\sin(2x) = \cos(2x)$ for $x \in [0, 2\pi]$

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$$\frac{\sin 2x}{\cos 2x} = \frac{\cos 2x}{\cos 2x}$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\tan 2x = 1$$

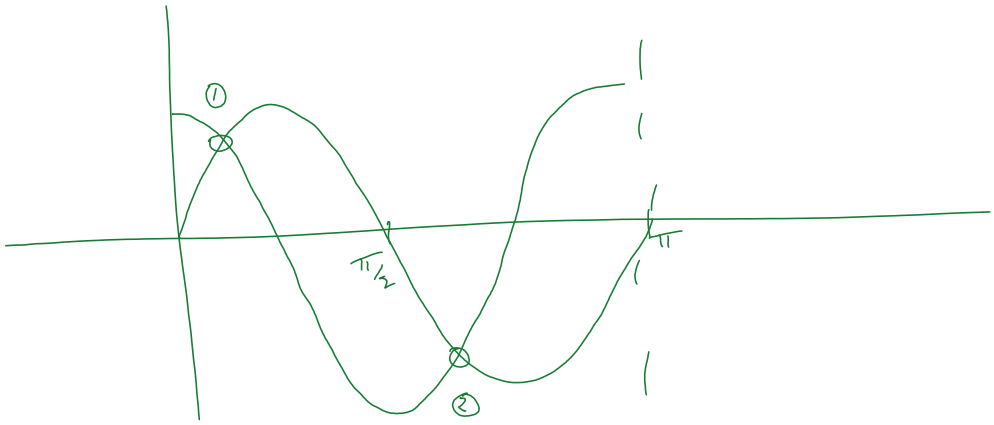
$$\frac{\sin 2x}{\cos 2x} = \tan 2x$$

$$2x = \tan^{-1}(1)$$

$$2x = \frac{\pi}{4} \quad \frac{5\pi}{4} \quad \frac{9\pi}{4} \quad \frac{13\pi}{4}$$

$$x = \frac{\pi}{8} \quad \frac{5\pi}{8} \quad \frac{9\pi}{8} \quad \frac{13\pi}{8}$$

$\sin 2x = \cos 2x \rightarrow \tan 2x = 1$



Important note:
When you equate equations and then go on to solve, you are, in effect, finding the points of intersection of the graphs