The Quotient Rule (9K)

Monday, 19 March 2018 6:49 PM

Work to be completed by the end of teaching:

Exercise 9K: Questions 1, 2 and 3

RECAP:

We are building tools to allow us to differentiate a range of different expressions and equations.
In a previous lesson we looked at the product rule.
This is now building on the range of tools we have already covered:

- Positive whole number powers
- Negative integer powers
- Using the chain rule to differentiate complex expressions
- Rational powers
- The Chain Rule
- The Product Rule

$y = (x^2+3)^3$

$y' = (x)(x^2+3)^3$

The Quotient Rule

This is used when we need to divide two x-terms.
Note: We don't have to use the Quotient Rule.
We might choose to make life much harder for ourselves and use the Product Rule

For example:

$y = \dfrac{x-2}{(x^2+4x+1)}$

$y = (x-2)(x^2+4x-1)^{-1}$ (u, v)

$y' = uv' + vu'$

The Quotient Rule might be easier!
The rule states:

$y' = \dfrac{vu' - uv'}{v^2}$

$y' = \dfrac{vu' - uv'}{v^2}$

$y = x(x^2+3)^3$ (u, v)

Example

$y = \dfrac{x-2}{x^2+4x+1}$ (u, v) BREAK

$u = x-2$ $\quad v = x^2+4x+1$

$u' = 1$ $\quad v' = 2x+4$

$y' = \dfrac{vu' - uv'}{v^2}$

$y' = \dfrac{(x^2+4x+1).1 - (x-2).(2x+4)}{(x^2+4x+1)^2}$

$= \dfrac{x^2+4x+1 - (2x^2+4x-4x-8)}{(x^2+4x+1)^2}$

$= \dfrac{x^2+4x+1-2x^2+8}{(x^2+4x+1)^2}$

$= \dfrac{-x^2+4x+9}{(x^2+4x+1)^2}$

EXAMPLE

$y = \dfrac{(3x^2+2)^3}{x+1}$ (u, v)

$u = (3x^2+2)^3$ $\quad v = x+1$

$u' = 3(3x^2+2)^2.6x$ $\quad v' = 1$

$= 18x(3x^2+2)^2$

$x+1$

v

$u' = 3(3x^2+2).6x$

$= 18x(3x^2+2)^2$

$v' = 1$

$y' = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{(x+1).18x.(3x^2+2)^2 - (3x^2+2)^3.1}{(x+1)^2}$

$= \dfrac{18x(x+1)(3x^2+2)^2 - (3x^2+2)^3}{(x+1)^2}$

$= \dfrac{(3x^2+2)^2\left[18x(x+1) - (3x^2+2)'\right]}{(x+1)^2}$

$= \dfrac{(3x^2+2)^2\left[18x^2 + 18x - 3x^2 - 2\right]}{(x+1)^2}$

$= \dfrac{(3x^2+2)^2(15x^2+18x-2)}{(x+1)^2}$

These types of questions test:

- Chain Rule ✓
- Quotient Rule ✓
- Algebra ✓
- Fractions ✓
- Differentiation ✓