The Product Rule (9J)

Monday, 19 March 2018 6:30 PM

multiply

Previously we looked at differentiating basic terms:

$y = x^2 + 3x - 2 \longrightarrow y = 2x + 3$

We looked at negative powers:

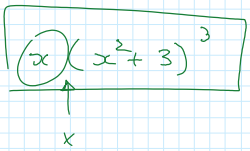
$y = x^{-2} + x^{-3} + 4 \longrightarrow y' = -2x^{-3} - 3x^{-4}$

And fractional, rational powers:

$y = x^{5/2} + x^{1/3} + 5 \longrightarrow y' = \frac{5}{2}x^{3/2} + \frac{1}{3}x^{-2/3}$

And finally, more complex questions which require the Chain Rule:

$y = (x^3 + 3x)^3 \longrightarrow y' = 3(x^3+3x)^2 \cdot (3x^2+3)$



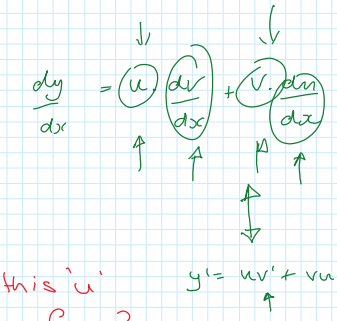
This lesson we look at more complex examples where we have two terms multiplied together:

The Product Rule

Example: $y = (2x^2+1)(5x^3+16)$ ← We could multiply this out... but that's too complex!

We could prefer to use the following rule:

$$\star \quad \frac{dy}{dx} = u\cdot\frac{dv}{dx} + v\cdot\frac{du}{dx}$$



Which can be written clearer as $y' = uv' + vu'$

Where did this 'u' and 'v' come from?

Back to the example

$y = (2x^2+1)(5x^3+16)$ — BREAK → $u = 2x^2+1$, $v = 5x^3+16$

RETURN ← $u' = 4x$, $v' = 15x^2$

$\star$ Hence, $y' = uv' + vu'$

$y' = (2x^2+1)15x^2 + (5x^3+16)\cdot 4x$

$= 15x^2(2x^2+1) + 4x(5x^3+16)$

Example 2

$y = x^{-3}(10x^2-5)^3 \longrightarrow u = x^{-3}$, $v = (10x^2-5)^3$

$u' = -3x^{-4}$, $v' = 3(10x^2-5)^2 \cdot 20x = 60x(10x^2-5)^2$

$y' = uv' + vu'$

$= x^{-3}\cdot 60x(10x^2-5)^2 + (10x^2-5)^3 \cdot -3x^{-4}$

FACTORISE $\star$ $= 60x^{-2}(10x^2-5)^2 - 3x^{-4}(10x^2-5)^3$ $\star$

(① $60x^{-2}(10x^2-5)^2$, ② $3x^{-4}(10x^2-5)^3$)

$= 3x^{-2}(10x^2-5)^2\left[20 - x^{-2}(10x^2-5)\right]$

$$= 3x^{-2}(10x^2-5)^2\left[20 - x^{-2}(10x^2-5)\right]$$

$$= 3x^{-2}(10x^2-5)^2(20 - 10 + 5x^{-2})$$

$$= 30x^{-2}(10x^2-5)^2(10 + 5x^{-2})$$

(4)

WORK TO BE DONE

→ Q1: Question 1 only

There are lots more questions but they deal with trigonometry and exponentials

↓

We will return to this ☺