

Solution of literal equations and systems of equations (4H)

Sunday, 4 March 2018 7:34 pm

★ Work to be completed by the end of teaching:

Solutions and systems of equations	4H	1-21(s)	←
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RECAP:

What is a literal equation?

A literal equation is one where you have an equation for x where the coefficients and the constants are all pronumerals.

Examples:

$$\begin{aligned} x^2 - ax + a &= 0 \\ x^3 + 2ax^2 - 3a^2x &= 0 \end{aligned} \quad \parallel \quad x = 4 \quad x = 7 \quad \boxed{x = \dots a}$$

How do you solve literal equations?

Basically, you are being asked to re-arrange for x and solve in exactly the same way as you have been doing for the previous exercises. The only difference is, you will end up with answers which will include letters and not numbers.

You will be expected to use the same tools as you have done previously to solve polynomials:

- Quadratic formula
 - T Method
 - Cross Method
 - Completing the Square
 - Polynomial Division
 - Synthetic Division
- quadratics

Examples:

Solve the following literal equation for x :

$$x^2 + x + k = 0$$

Quadratic

$$\begin{aligned} a &= k \\ b &= 1 \\ c &= 0 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1 - 4k \cdot k}}{2k}$$

$$x = \frac{-1 \pm \sqrt{1 - 4k^2}}{2k}$$

$$x \in [-\frac{1}{2}, \frac{1}{2}] \setminus \{0\}$$

$$1 - 4k^2 \geq 0$$

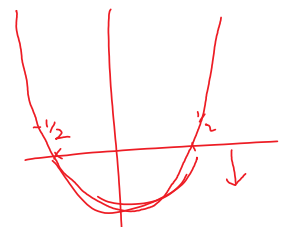
$$-4k^2 \geq -1$$

$$4k^2 \leq 1$$

$$k^2 \leq \frac{1}{4}$$

$$k \leq \pm \frac{1}{2}$$

$$x - 1$$



$$x^3 - 7ax^2 + 12a^2x = 0$$

$$x(x^2 - 7ax + 12a^2) = 0$$

$$x = 0$$

$$x(x - 3a)(x - 4a) = 0$$

$$\begin{array}{r} 12a^2 \\ -3a \quad -4a \end{array}$$

$$\underline{x = 0} \quad \text{or} \quad \underline{x = 3a} \quad \text{or} \quad \underline{x = 4a}$$

$$ax^{\frac{1}{3}} = b$$

$$x^{\frac{1}{3}} = \frac{b}{a}$$

$$\sqrt[3]{x} = \frac{b}{a}$$

$$\underline{\underline{x = \left(\frac{b}{a}\right)^3}}$$

RECAP 2:

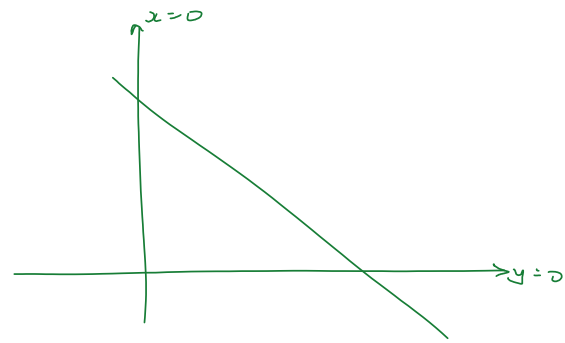
Remember, way back to Year 10 and 11 when you needed to solve simultaneous equations. If we remember anything we remember that the equations are written in intercept form:

$$\begin{cases} 3x + 2y = 17 \\ 2x - y = 2 \end{cases}$$

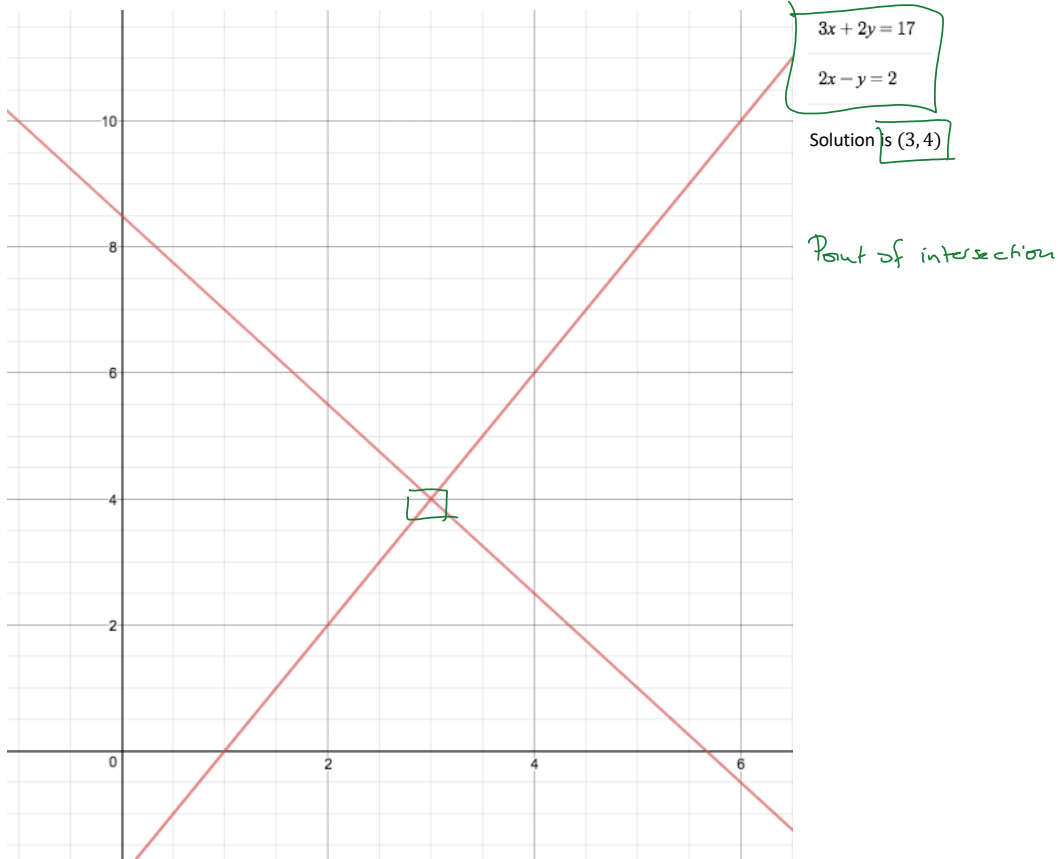
Substitution!

We could solve these in one of 4 ways:

- Graphically
- By substitution $x = y =$
- By elimination
- Using the CAS



Whichever way you solve them ... you need to remember you are trying to find a point of intersection.



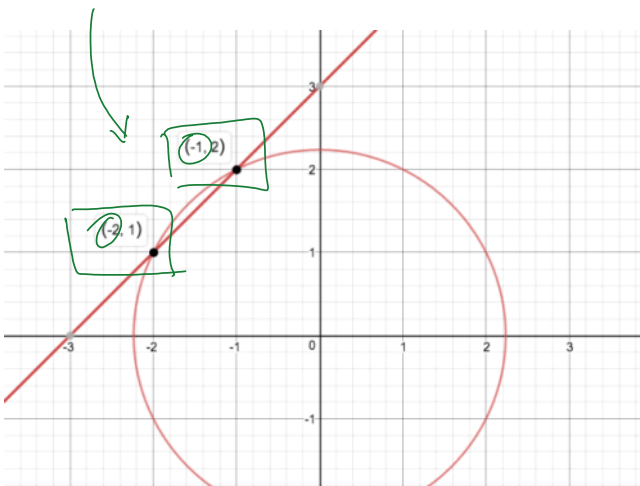
When we are given complex questions which are asking for points of intersection, we can use the same methods as above! Whilst we wouldn't normally choose to do it by hand graphically, we might opt to use the CAS. More normally we would use substitution to solve these equations.

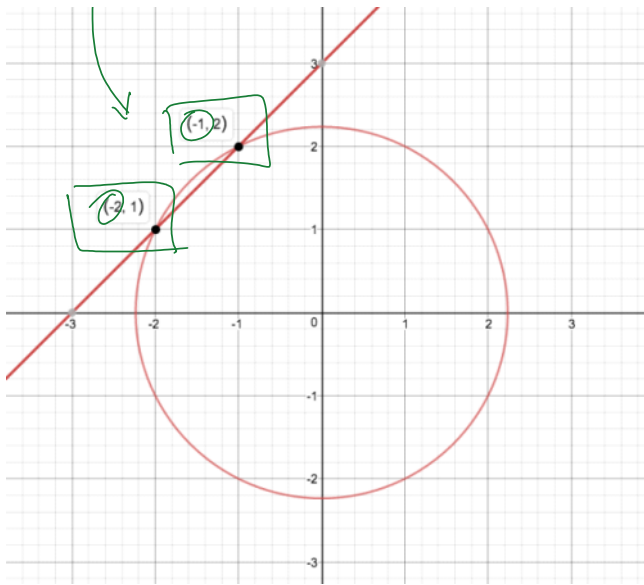
Example:

$$x^2 + y^2 = r^2$$

- Find the points of intersection of the circle with equation $x^2 + y^2 = 5$ and the line $x - y = -3$

Graphically this looks like:





Algebraically, we know that, at the points of intersection we can see that both the x- and y-values are the same in both equations. Hence we can use substitution to allow us to find the solutions

$y = x + 3$ can then be put into $x^2 + y^2 = 5$ to become:

$$x^2 + y^2 = 5 \quad y = x + 3$$

$$x^2 + (x + 3)^2 = 5$$

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We then have a range of tools open to us to solve this problem ...

$$x^2 + (x + 3)^2 = 5$$

$$x^2 + x^2 + 6x + 9 = 5$$

$$2x^2 + 6x + 4 = 0$$

$$\div 2 \quad x^2 + 3x + 2 = 0$$

$$\underline{x^2 + x} + \underline{2x + 2} = 0$$

$$x(x + 1) + 2(x + 1) = 0$$

$$(x + 1)(x + 2) = 0$$

$$x = -1 \quad \text{or} \quad x = -2$$

solutions $\Rightarrow$ words

$$x = -1$$

$$y = 2$$

$$(-1, 2)$$

$$\begin{array}{r} 2 \\ 1 \overline{) 2} \end{array}$$

$$x = -2$$

$$y = 1$$

$$(-2, 1)$$

o

$$(x + 3)^2 = (x + 3)(x + 3)$$

$$= x^2 + 6x + 9$$

$$\cancel{x^2 + 9}$$

$$\underline{\underline{x^2 + y^2 = 5}}$$

$$\underline{\underline{y = x + 3}}$$