Differentiation = Finding the gradient of the tangent to a point *

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

BASICALLY ...

$f(x) = y = 3x^2$

$f'(x) = y' = 6x$

This is the same as

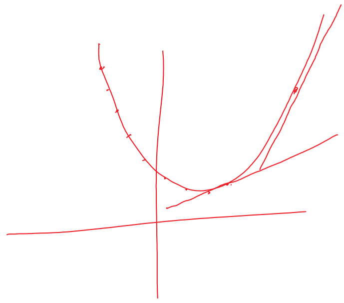
$3x^0 = (3 \times 0)x^{-1}$

$= 0$

$y = 2x^1$

$y' = 2x^0$

$y' = 2$



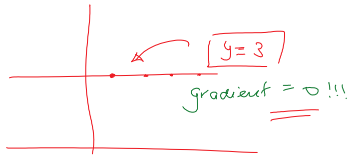
notice the different notations

$\frac{dy}{dx}$

$f(x) = y = 5x^3 + 3x^2 + 2x + 3$

$f'(x) = \frac{dy}{dx} = 15x^2 + 6x + 2$

This makes sense too ...



$y = 3$

$y' =$ gradient of tangent

Differentiate 'y' with respect to 'x' $\frac{dy}{dx}$

We differentiate to find the gradient of the tangent to a point *

$y = 3x^2 + 4x - 3$

Find gradient of tangent to the given curve at the point $(1, 4)$

$y' = 6x + 4$

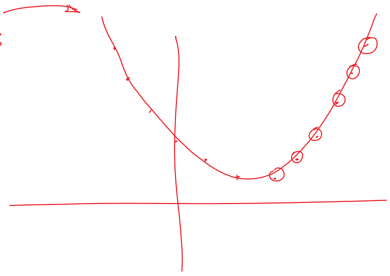
Substitute

$y' = 6(1) + 4$

$= 10$

$f(x) = 3x^2 + 4x - 3$

$f'(x) = 6x + 4$



Rule *

$y = ax^n$

$y' = \frac{dy}{dx} = f'(x) = anx^{n-1}$

i.e. multiply power and subtract one from the power

$y = 3x^4$

At this time:

$y = (x+3)^2$ → Multiply out

$y = x^2 + 6x + 9$

$y' = 2x + 6$

$y = (2t-1)^2(t+2)$ → Don't get confused by change in letters

$y = \frac{x^2 + 3x}{x}$ → Don't be tricked by division .. cancel down

$y = \frac{x^2}{x} + \frac{3x}{x}$

$y = x + 3$

$y' = 1$

Doing things backwards

Find points where gradient $= 8$ for $y = x^3$

Must be co-ordinates (x, y)

given gradient so work backwards

$y' = 3x^2$

$y' =$ gradient

$8 = 3x^2$

$\frac{8}{3} = x^2$

$x = \pm\sqrt{\frac{8}{3}}$

$x = \pm\left(\frac{8}{3}\right)^{1/2}$

put back into equation and find y values

$y = x^3$

$x = -\left(\frac{8}{3}\right)^{1/2}$ and $\left(\frac{8}{3}\right)^{1/2}$

$y = -\left(\frac{8}{3}\right)^{3/2}$

$y = x^3$

$x = -\left(\frac{8}{3}\right)^{1/2}$ and $\left(\frac{8}{3}\right)^{1/2}$

$y = -\left(\frac{8}{3}\right)^{3/2}$

or $y = \left(\frac{8}{3}\right)^{3/2}$

$\therefore$ Co-ords $= \left(-\left(\frac{8}{3}\right)^{1/2}, -\left(\frac{8}{3}\right)^{3/2}\right)$ UGLY

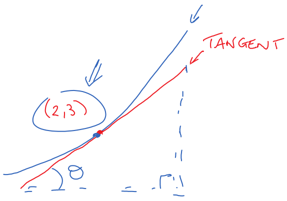
or $\left(\left(\frac{8}{3}\right)^{1/2}, \left(\frac{8}{3}\right)^{3/2}\right)$

TRICKS

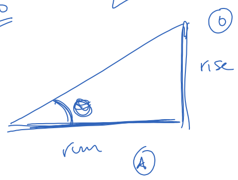
Differentiation = gradient of tangent

This can also be found using

$m = \frac{\text{Rise}}{\text{Run}}$



Also

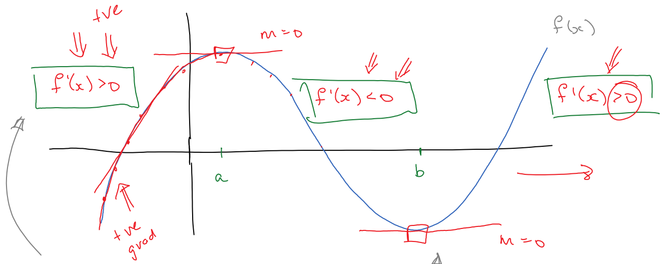


$\tan\theta = \frac{\text{rise}}{\text{run}}$

$\therefore \tan\theta = m$ ← gradient & angle are related

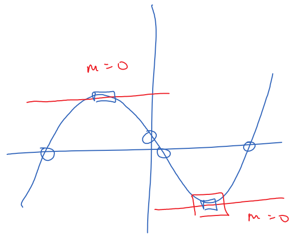
$m = \tan\theta$

NOTATION, NOTATION, NOTATION!



simply means a positive gradient

Zero gradient / Turning point



Eg. Remember Maths is a trick:

$\{x : f'(x) > 0\} = \{x : -1 < x < 5\} = (-1, 5)$

$\{x : -1 < x < 5\} = (-1, 5)$

$f'(x) = m$