Rectangular Hyperbolas

Friday, 20 April 2018 11:05 AM

Work to be completed at the end of teaching:

4A: Rectangular hyperbolas	149	1aceghjkl ,2aceghijkl, 3bd, 4befh, 5, 7

RECAP

We have been working on all the work for Functions and Relations.
Now we look at some of the most used Functions in Mathematical Methods.

- Rectangular Hyperbola
- The Truncus
- The Square Root Graph
- Circles

This lesson will be used to look at …

Rectangular Hyperbolas

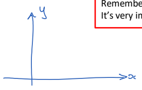
First things first … there are different versions of Hyperbolas!
It's important to note that we are looking at Rectangular Hyperbolas!

D
R
T

Equation of the function:

$$y = \frac{1}{x}$$

This is what I call the base function.
Remember that, at some point, you are going to be transforming this!
It's very important to ensure that you can spot a hyperbola and it's transformations



$x = 0$ $y = \frac{1}{0}$

← Undefined!

Examples of transformed hyperbolas (rushing the gun!)

① $y = \frac{1}{x-1}$ ② $y = \frac{1}{x+3}$ ③ $y = \frac{3}{x}$

④ $y = \frac{1}{x} + 3$ ⑤ $y = \frac{3}{2x-1} + 3$ ⑥ $y = \frac{1}{4x+2}$

RECAP: Order of Transformations

Remember … Unless they give you an order of transformations … you must use Dr T

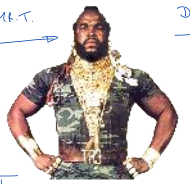
Not Mr T … like from the 1980's TV show called The A Team.

Dr T stands for:

- Dilations
- Reflections
- Translations

D
R
T

Mr.T



DR.T

Always identify which are which in the equation and then construct them in that order!

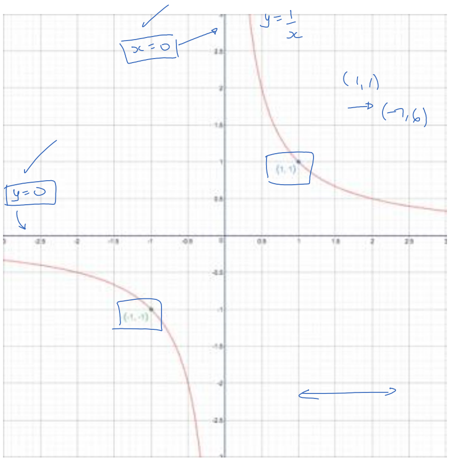
$y = \frac{-3}{2(x+1)} - 6$

$\boxed{y = \frac{1}{x}}$

The BASE graph for a Rectangular Hyperbola

You must memorise the following shape a important points for all graphs in this chapter.
You will be expected to show all the relevant information on all graphs!
This will include:

- All intercepts on the x- and y-axes.
- All turning points
- All asymptotes
- The equation of the curve.



$y = \frac{1}{x}$

$\boxed{x = 0}$

$\boxed{y = 0}$

$(1,1)$

$(-1,6)$

Important information about the graph:

Asymptotes:
$x = 0$
$y = 0$

$y = \frac{1}{\boxed{x}}$ $0 = \frac{1}{x}$

$x = \frac{1}{0}$

Helpful points:

- (1, 1)
- (-1, -1)

Plotting points on the graph will help the examiner know that you have understood transformations and have calculated things correctly.

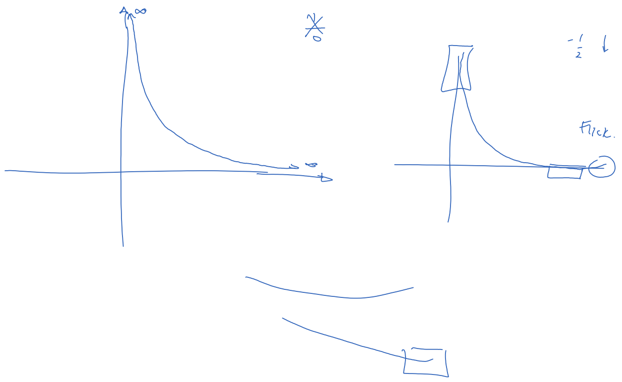
Domain and Range:

- Domain: $\mathbb{R}\{0\}$
- Range: $\mathbb{R}\{0\}$

Shape
It is important that you memorise the shape.
This is the base shape from which all other hyperbolas are born.

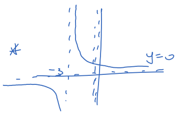
$x \in \mathbb{R} \setminus \{0\}$

$f(x) \in \mathbb{R} \setminus \{0\}$



Asymptotes

An Asymptote is a line such as the value of x or y increases (or decreases) the graph gets closer and closer to the line **but never crosses it.**



3

Transformations of Hyperbolas

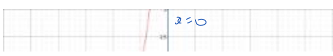
Knowing the base graph we can now use it to transform.
Examples are shown below.
We will look at Reflections (as they are easiest), Dilations and then translations before looking at the all together!

D
R
T

Reflections of Rectangular Hyperbolas

We can reflect in the x-axis and y-axis

Reflection in the x- and y-axis



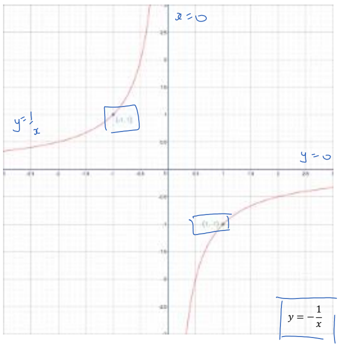
Asymptotes:
$x = 0$
$y = 0$

What do you notice?
They are both the same!

$y = \frac{1}{x}$

We can reflect in the x-axis and y-axis

Reflection in the x- and y-axis



Asymptotes:
$x = 0$
$y = 0$

Helpful points:
- (-1, 1)
- (1, -1)

Domain and Range:
- Domain: $\mathbb{R}\backslash\{0\}$
- Range: $\mathbb{R}\backslash\{0\}$

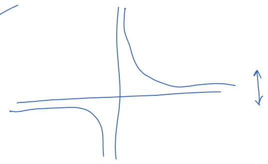
What do you notice?
They are both the same!

Remember:
When you reflect in the y-axis, replace x with $-x$
When you reflect in the x-axis, replace y with $-y$

$y = \frac{1}{x}$

$y = \frac{1}{-x} = \frac{-1}{x}$

$-y = \frac{1}{x} \Rightarrow \frac{-1}{x}$

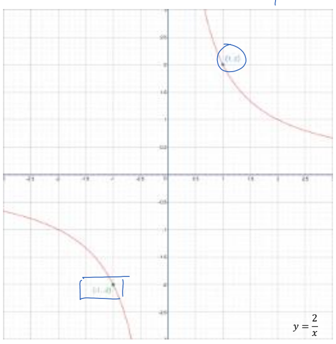


Dilations of Rectangular Hyperbolas

We can dilate away from the x-axis and y-axis.
It's really important to know the difference!

Dilation from the x-axis factor a

factor 2

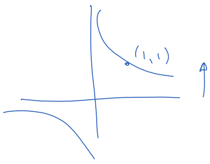


Asymptotes:
$x = 0$
$y = 0$

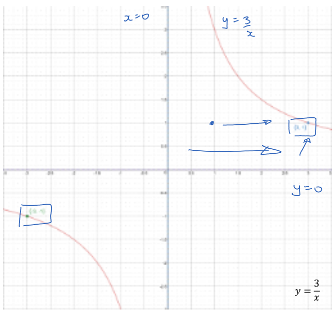
Helpful points:
- (1, a)
- (1, -a)

Domain and Range:
- Domain: $\mathbb{R}\backslash\{0\}$
- Range: $\mathbb{R}\backslash\{0\}$

Remember:
When you dilate away from the y-axis, replace x with $\frac{x}{a}$
When you dilate away from the x-axis, replace y with $\frac{y}{a}$



Dilation from the y-axis factor a



Asymptotes:
$x = 0$
$y = 0$

Helpful points:
- (a, 1)
- (-a, -1)

Domain and Range:
- Domain: $\mathbb{R}\backslash\{0\}$
- Range: $\mathbb{R}\backslash\{0\}$

Important note:

You might notice that the following graph can actually describe **both** a dilation from the x-axis and the y-axis.

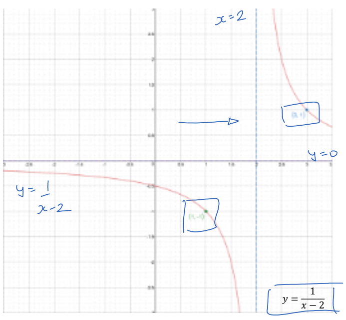
$y = \frac{2}{x}$

This is both a dilation factor 2 from the x-axis **AND** a dilation of factor $\frac{1}{2}$ from the y-axis.

Translations of Rectangular Hyperbolas

We can translate a function both parallel to the x-axis and the y-axis.
When we translate a function **the asymptotes move too.**

Translation parallel to the x-axis a units



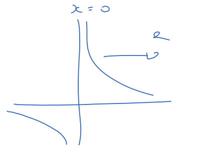
Asymptotes:
$\boxed{x = 2}$
$y = 0$

Helpful points:
- (1+a, 1)
- (-1+a, -1)

Domain and Range:
- Domain: $\mathbb{R}\backslash\{2\}$
- Range: $\mathbb{R}\backslash\{0\}$

2 2

Remember:
When you translate along the y-axis, replace y with $y - a$
When you translate along the x-axis, replace x with $x - a$



$y = \frac{1}{x-2}$

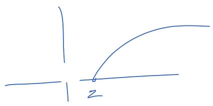
$x - 2 = 0$

$\boxed{x = 2}$

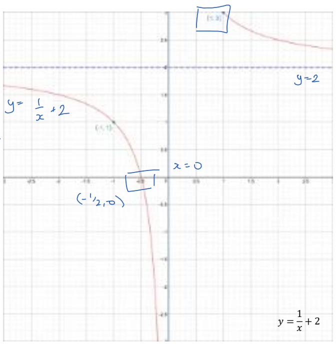
$y = \sqrt{x - 2}$

$x - 2 = 0$

$x = 2$



Translation parallel to the y-axis a units



Asymptotes:
$x = 0$
$y = 2$

Helpful points:
- (1, 1+a)
- (-1, -1+a)

Domain and Range:
- Domain: $\mathbb{R}\backslash\{0\}$
- Range: $\mathbb{R}\backslash\{2\}$

$y = 0$

$y = \frac{1}{x} + 2$

$0 = \frac{1}{x} + 2$

$-2 = \frac{1}{x}$

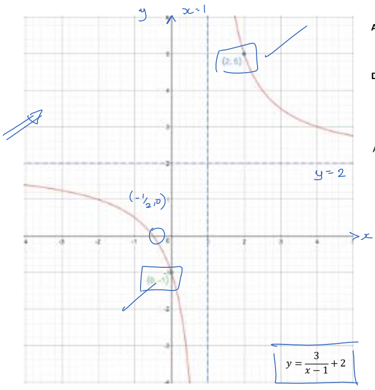
$x = -\frac{1}{2}$

~~x~~

2 (1, 1) (1, 3)

Combining Transformations of Rectangular Hyperbolas

Combining Transformations of Rectangular Hyperbolas



Asymptotes:

$x = 1$
$y = 2$

Domain and Range:

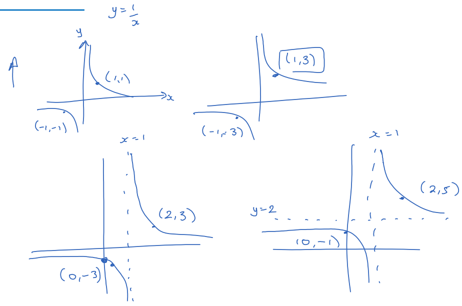
- Domain: $\mathbb{R}\backslash\{1\}$
- Range: $\mathbb{R}\backslash\{2\}$

Always make sure you look for Dilations, Reflections and then Translations

$$y = \frac{3}{x-1} + 2$$

D
R
T

↕ 3
↑ 2
→ 1



Solving for Intercepts

Remember that you need to find any x- and y-axis intercepts.
To solve for intercepts you need to:

- Put $x = 0$ to solve for y-intercepts
- Put $y = 0$ to solve for x-intercepts

$$y = \frac{3}{x-1} + 2$$

$$y = \frac{3}{0-1} + 2$$
$$= -3 + 2$$
$$= -1$$
$(0, -1)$

$$0 = \frac{3}{x-1} + 2$$
$$-2 = \frac{3}{x-1}$$
$$x - 1 = -\frac{3}{2}$$
$$x = -\frac{3}{2} + 1$$
$$x = -\frac{1}{2}$$
$(-1/2, 0)$