Rational Powers (9F)

Wednesday, 14 March 2018 6:05 pm

Work to be completed by the end of teaching:

Rational Powers	9F	2,3,4,5-6(e),7

RECAP:

We have now looked at being able to differentiate the following:

- Positive whole number powers x^3
- Negative integer powers x^{-2}
- Using the chain rule to differentiate complex expressions

We now look at extending the idea to rational powers which can (and will) be used with the chain rule!

Rational and Irrational

Remember:

- rational numbers are those which can be expressed as a fraction.
- Irrational numbers are those which cannot be expressed as a fraction e.g. π

π $\sqrt{2}$ $\sqrt{3}$

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$

1.6 $\frac{16}{10}$ $\frac{8}{5}$

Basic Examples

Differentiate the following:

① $y = x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}\boxed{x^{-1/2}}$

$= \frac{1}{2} \times \frac{1}{x^{1/2}}$

$= \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$

② $y = x^{\frac{1}{3}}$

$y' = \frac{1}{3}x^{-2/3}$

$y = x^{\frac{3}{5}}$

$y' = \frac{3}{5}.x^{-2/5}$

Tricks, tricks and more tricks ... otherwise known as notation

Rational powers tend to trick people over and over again.
It is really important you can see rational powers in all equations.

Example: $\sqrt{x^2+4}$ $= (x^2+4)^{1/2}$

Example: $\sqrt[3]{x^3 - 3x + 2}$ $= (x^3 - 3x + 2)^{1/3}$

Example: $\sqrt[5]{x^2 - 3x + 2}$ $= (x^2 - 3x + 2)^{1/5}$

Example: $(x^2 + 3x - 2)^{\frac{1}{3}}$

The Chain Rule: A recap

Now we can use the chain rule to solve these wonderful questions!

Example: $\sqrt{x^2 + 4}$ $= (x^2+4)^{1/2}$

$$f'(x) = \frac{1}{2}(x^2+4)^{-1/2} \cdot 2x$$

$$= \cancel{\tfrac{1}{2}} \cdot \cancel{2}x\,(x^2+4)^{-1/2}$$

$$= \underline{x\,(x^2+4)^{-1/2}} = \frac{x}{(x^2+4)^{1/2}} = \frac{x}{\sqrt{x^2+4}}$$

Example: $\sqrt[3]{x^3 - 3x + 2}$ $= (x^3 - 3x + 2)^{1/3}$

$$f'(x) = \frac{1}{3}(x^3 - 3x + 2)^{-2/3} \cdot (3x^2 - 3)$$

$$= \frac{1}{3}(3x^2 - 3)(x^3 - 3x + 2)^{-2/3}$$

$$= \frac{1}{\cancel{3}} \cdot \cancel{3}(x^2 - 1)(x^3 - 3x + 2)^{-2/3} = (x^2-1)\underline{(x^3 - 3x + 2)^{-2/3}}$$

Example: $\sqrt[5]{x^2 - 3x + 2}$

Example: $\sqrt[5]{x^2 - 3x + 2}$

$$f(x) = (x^2 - 3x + 2)^{1/5}$$

$$f'(x) = \frac{1}{5}(x^2 - 3x + 2)^{-4/5} \cdot (2x - 3)$$

$$= \frac{1}{5}(2x - 3)(x^2 - 3x + 2)^{-4/5}$$