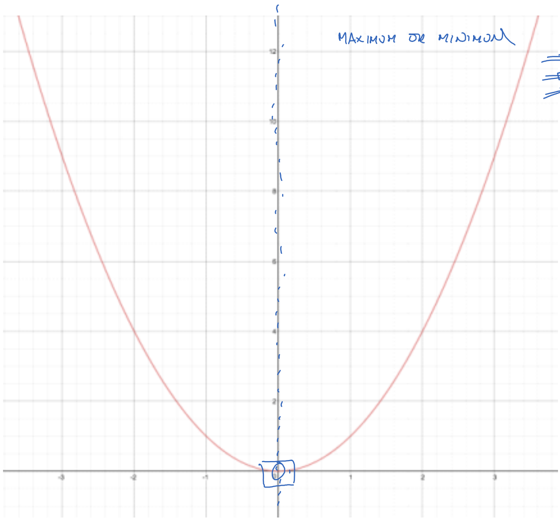
Quadratics Functions (4A)

Thursday, 22 February 2018 12:32 PM

Quadratics properties (important and vital)	4A	1-18(s)

Work to be completed at the end of teaching:

Moving quadratics … more transformations!



$y = x^2$

- All quadratics have an axis of symmetry
- They are **even** functions
- They have a turning point (max or min)

Turning point form.

Dilation
Reflection
Translation

$-\frac{1}{2}$ ↓ ①

Axis of symmetry

We have already seen that quadratics can be written in a number of different ways:

This one equation can be written in lots of different ways, but they are all the same equation.

$x^2 + 5x - 2 = 0$

solving $y=0$

$x^2 + 5x = 2$

$y = x^2 + 5x - \boxed{2}$

$x^2 - 2 = -5x$

$x(x+5) = 2$

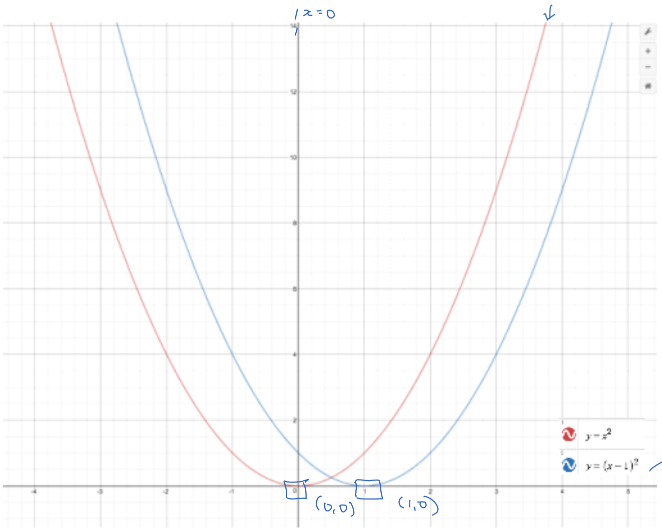
$\boxed{x = \frac{2}{x+5}}$

$\boxed{x + 5 = \frac{2}{x}}$

$\boxed{x = \frac{2}{x} - 5}$

The above, simply shows the same quadratics expressed in different ways through algebra tricks.
It can be argued than NONE of them are particularly useful.
That's not true of all way of expressing quadratics …

$x=0$



(0,0) (1,0)

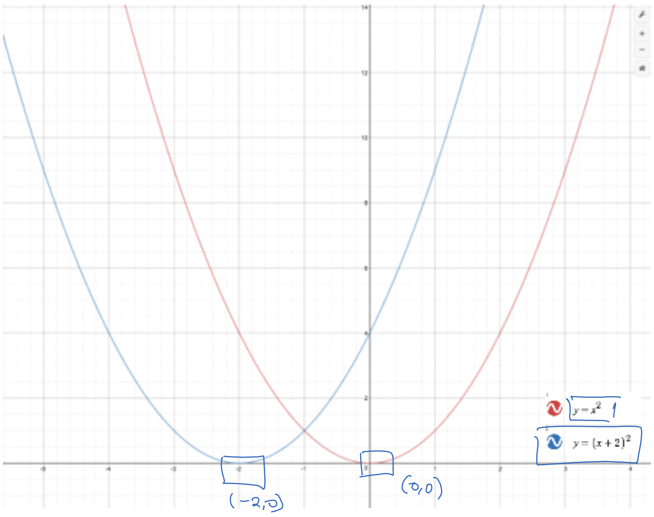
$2 + \sqrt{2}$

$2 - \sqrt{2}$

(1,0)

$y = (x-1)^2$

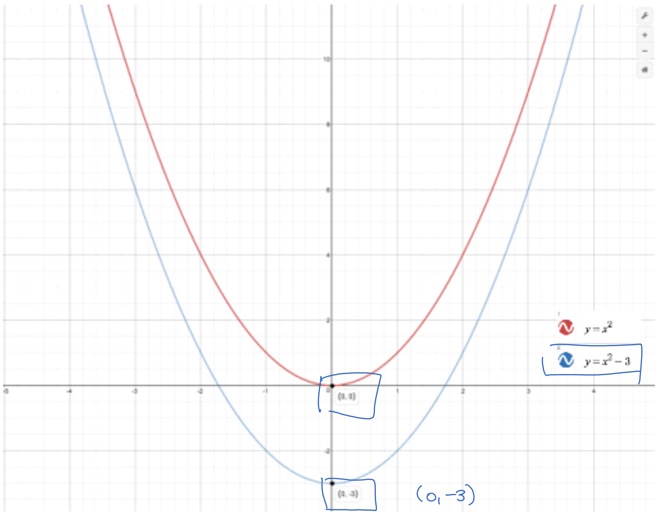
→ 1



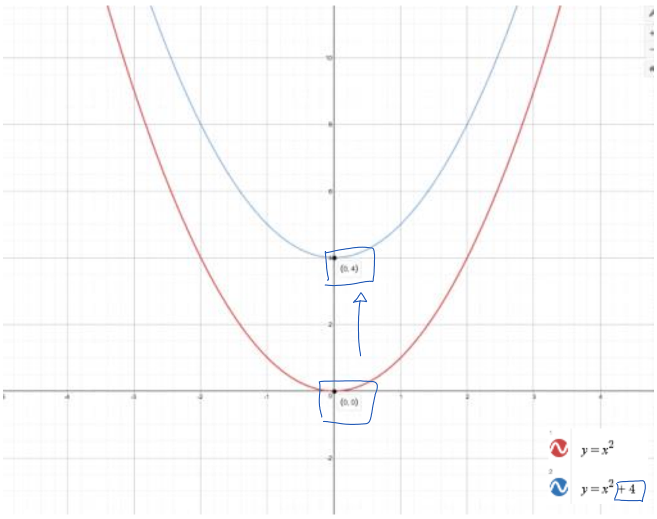
(-2,0)

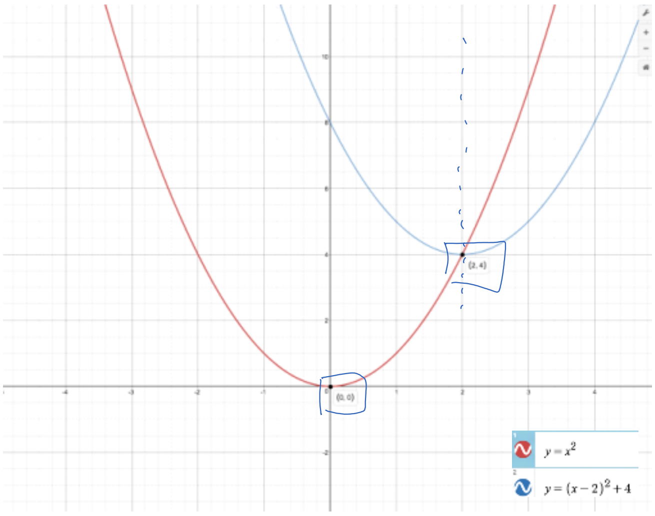
$y = (x+2)^2$

2



$y = x^2 \boxed{-3}$





MAX / MIN

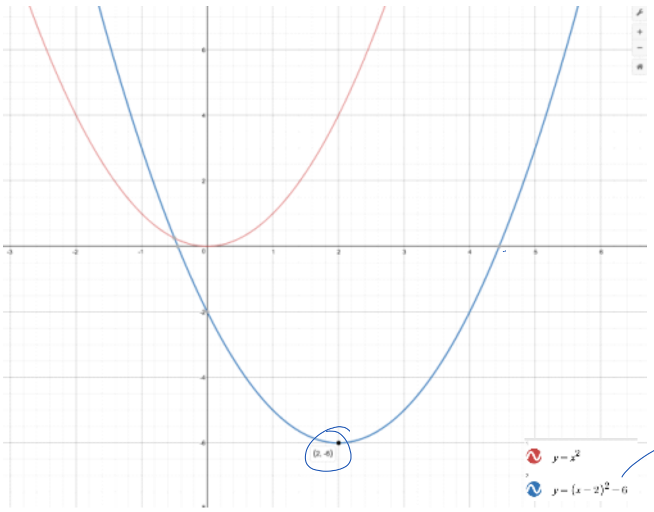
$y = x^2$ MIN

$y = -x^2$ MAX

$y = (x-2)^2 + 4$

$(2, 4)$

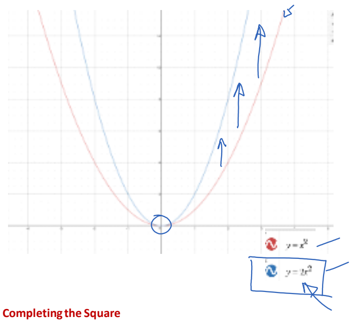
$x = 2$

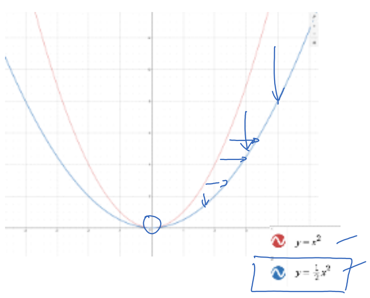


$y = (x-2)^2 - 6$

$(2, -6)$

The above are called **transformations**.
When we move horizontally or vertically we call then **translations**.
When we stretch the graph we call it a **dilation** which can happen from the x-axis and the y-axis





Completing the Square

Completing the square is a funky process which turns (most) quadratics into a form we can use to find the axis of symmetry, t he turning points, the x-axis and y-axis crossing points.

$$y = a(x \pm k) \pm h$$

$y = 2(x-3)^2 + 6$

MAX or MIN

$(3, 6)$

E.g. Factorise the following function by completing the square: $y = x^2 + 4x - 3$

$(x+2)^2 = x^2 + 4x + 4 - 7$

$(x+2)^2 - 7 = y$

$\therefore$ TP$(-2, -7)$

$\boxed{(x+2)}^2 = x^2 + 4$

$(x+2)(x+2)$

E.g. Factorise the following function by completing the square: $y = x^2 - 6x - 10$

$(x^2 - 6x - 10)$

$(x-3)^2 = x^2 - 6x + 9$

$(x-3)^2 - 19$

E.g. Factorise the following function by completing the square: $y = 3x^2 + 9x - 4$

$y = 3\left[x^2 + 3x - \frac{4}{3}\right]$

$3\left[\left(x + \frac{3}{2}\right)^2 - \frac{43}{12}\right]$

TP $= \left(-\frac{3}{2}, -\frac{43}{4}\right)$

YCE

(1.5)

$y = 3\left[x^2 + 3x - \frac{4}{3}\right]$

$3\left[\left(x + \frac{3}{2}\right)^2 - \frac{43}{12}\right]$

$= 3\left(x + \frac{3}{2}\right)^2 - \frac{43}{4}$

$TP = \left(-\frac{3}{2}, -\frac{43}{4}\right)$

$AoS \Rightarrow x = -\frac{3}{2}$

VCE

(1.5)

$\left(x + \frac{3}{2}\right)^2 = x^2 + 3x + \frac{9}{4}$

$\frac{9}{4} + \frac{4}{3} = \frac{16 + 27}{12} = \frac{43}{12}$

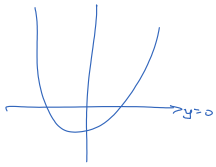
Finding solutions

E.g. $y = (x-3)^2 + 4$

$(x-3)^2 + 4 = 0$

$(x-3)^2 = -4$

$x - 3 = \pm\cancel{\sqrt{-4}}$



E.g. $y = (x+4)^2 - 3$

$(x+4)^2 - 3 = 0 \quad (+3)$

$(x+4)^2 = 3$

$x + 4 = \pm\sqrt{3}$

$x = -4 \pm \sqrt{3}$

$\pm$

E.g. $y = 2(x-3)^2 + 6$

Axis of Symmetry

We already know that a quadratic has a line of symmetry down the centre.
The x-value happens to coincide with the mid-point of the two solutions to the quadratic equation.
When we find the x-value, we can find the y-value and hence the maximum or minimum of the quadratic

$\boxed{x = -\frac{b}{2a}}$

$ax^2 + bx + c = 0$

The Quadratic Equation

Not all quadratics can be solved using the T-methods, Cross Method or Completing the square.
We have one more way to be able to do this … and it's using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where the values a, b and c come from the equation below:

$$ax^2 + bx + c = 0$$

$\sqrt{-4}$

The square root troubles me!!!

Unless we are doing Spesh … we have been taught that we cannot solve a square root where there is a negative number.

In Spesh we know about **imaginary numbers** … but that's not real maths so we ignore it in Methods.

With the quadratic equation we find something pretty awesome happening … which actually gives me a hint as to whether a quadratic will have solutions or not!

roots

Case 1: $\boxed{b^2 - 4ac > 0}$

discriminant

When the value of this is greater than zero, we know that we will always end up with two solutions

$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

$ax^2 + bx + c = 0$

Case 2: $b^2 - 4ac = 0$

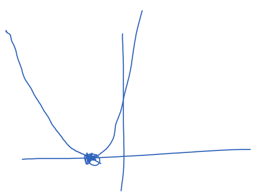
When the value is equal to zero then we only have one solution. This is known as a repeated root.
It's where the graph JUST touches the x-axis

$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

$= \frac{-b \pm \sqrt{0}}{2a}$

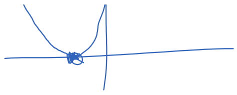
$= \frac{-b \pm 0}{2a}$

$x = \frac{-b}{2a}$



$= \frac{-b \pm 0}{2a}$

$x = \frac{-b}{2a}$



Case 3: $\boxed{b^2 - 4ac < 0}$

negative

This cannot have a solution and hence the graph will have no solutions. This basically means the minimum point is above the x-axis.

$\cancel{\sqrt{neg}}$