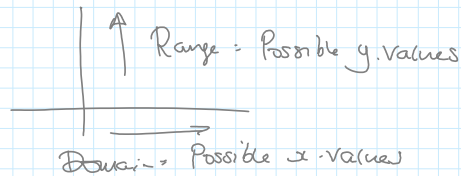


One-to-One Functions and Implied domains

Wednesday, 28 March 2018 6:18 pm



★ Work to be complete by the end of the teaching:

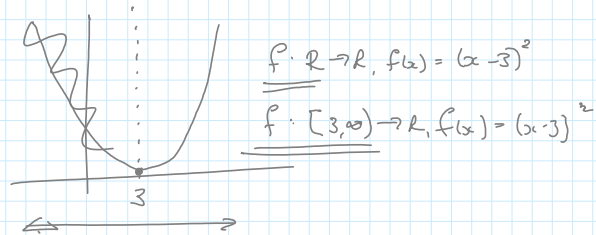
5D: Implied domains 190 1abcg 2all 3bcdehjk 4bcd 5bcd 6 8 10

RECAP:

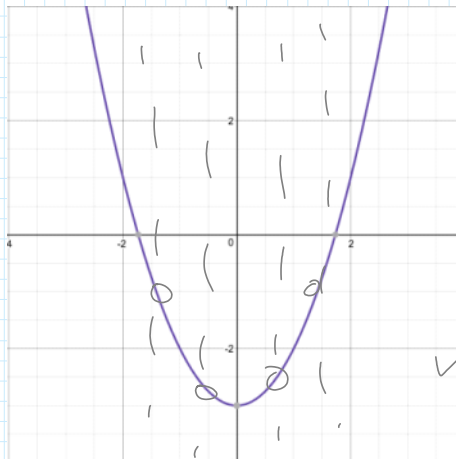
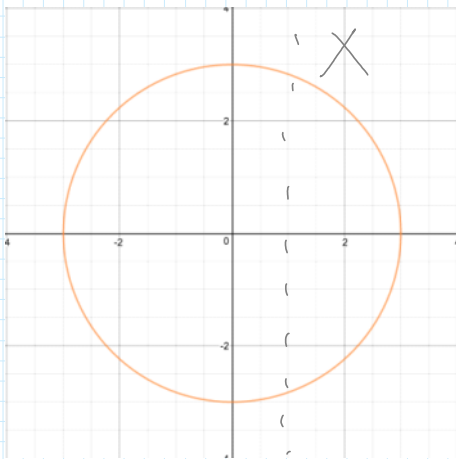
We have already talked extensively about functions. We have covered:

- What a function is
- How to write a function using function notation
- Domains
- Ranges
- Real Numbers and Integers
- Vertical line tests (to test for a function)
- How to restrict a function

Function:



Vertical line test



There is a difference between a function and a relation.

All equations are relations:

$$x^2 + y^2 = 9$$

The above is not a function as it doesn't pass the "vertical line test".

Quick Test:

The Vertical Line Test: A graph is a function if you can draw a vertical line through it and it only cuts the graph once.

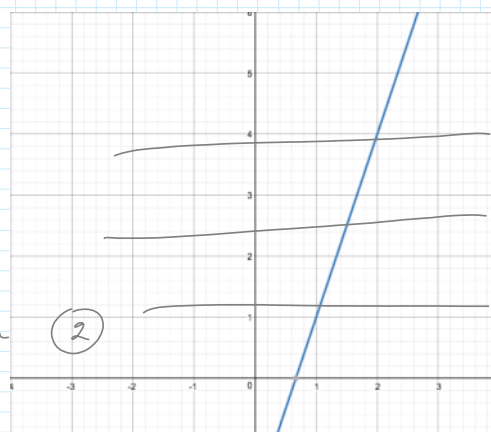
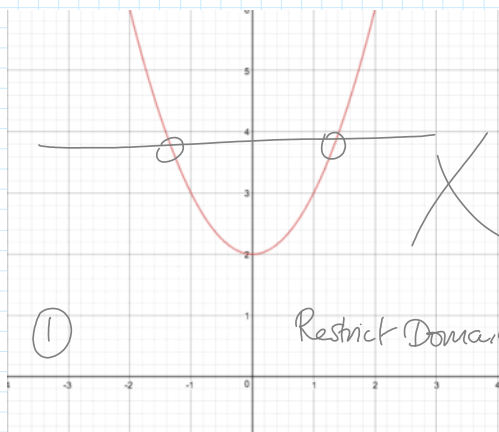
One-to-one functions

A function is a particular type of relation (as said above!).

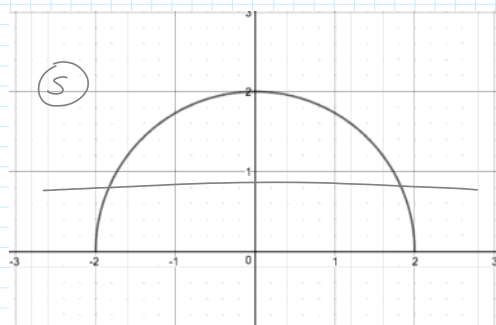
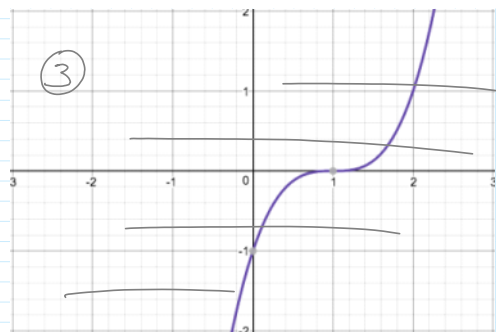
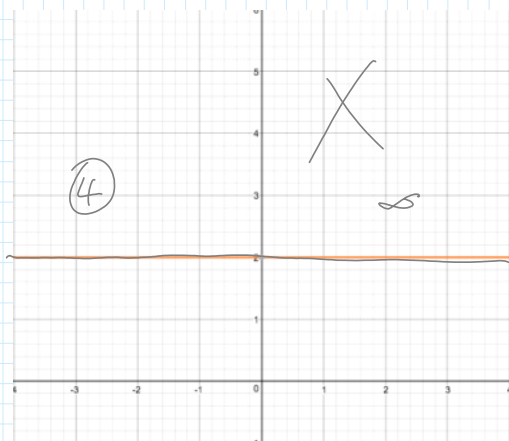
We can now sub-divide functions - much like we sub-divide humanity

A particularly useful type of function is a **one-to-one** function

Inverse functions



Two of these graphs are one-to-one functions.
But which two?



To be a one-to-one function you have to pass the horizontal line test.
i.e. if you can draw a horizontal line through a relation and it only cuts the relation once, it is a **one-to-one** function.

A function is said to be one-to-one if different X-values map to different Y-values.

Another way of saying this is: one-to-one function cannot contain two different ordered pairs with the same second coordinate

(x, y)

Implied Domains (again)

The following functions all give us a domain as part of the function notation:

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 3x + 4$$

$$f: [0, 4) \rightarrow \mathbb{R}, f(x) = x^2 - 3$$

$$f: (-\infty, 0) \rightarrow \mathbb{R}, f(x) = x^3 + 2x + 3$$

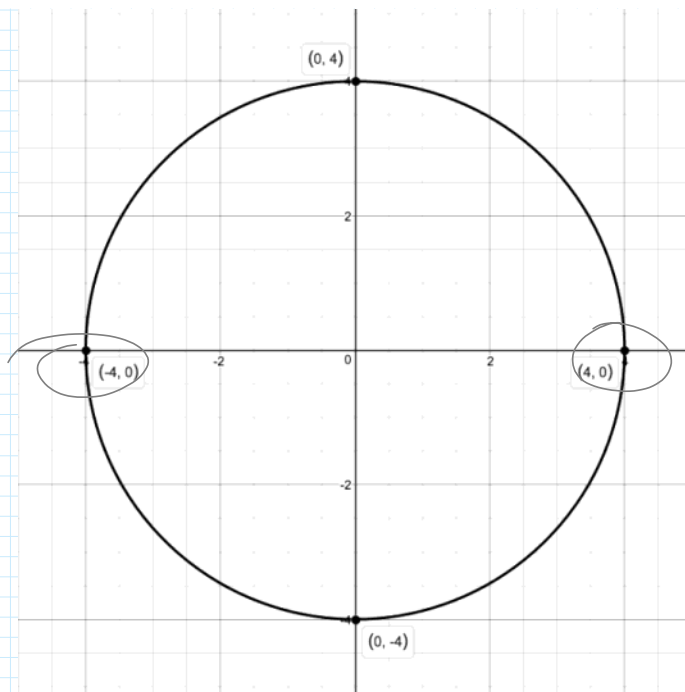
$$f: [4, 7] \rightarrow \mathbb{R}, f(x) = \frac{1}{x^2}$$

***repeat
repeat**

What happens when they don't give us a domain?

E.g. $x^2 + y^2 = 16$

We simply imply the domain. We assume that the domain of the function is all Real Numbers for which the function has meaning.



Hence the implied domain of the above function is: $x \in [-4, 4]$

$$x \in [-4, 4]$$

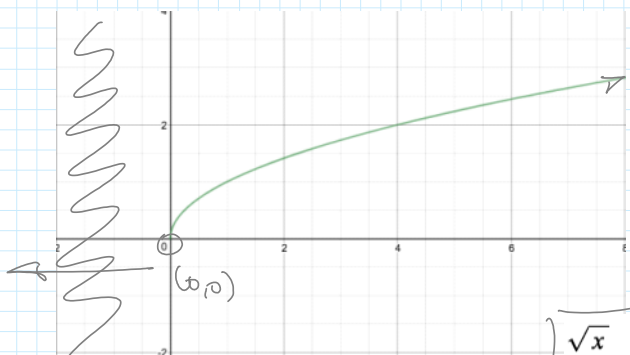
An important graph to know before doing this exercise

One of the most used graphs in Mathematics is the graph $y = \sqrt{x}$

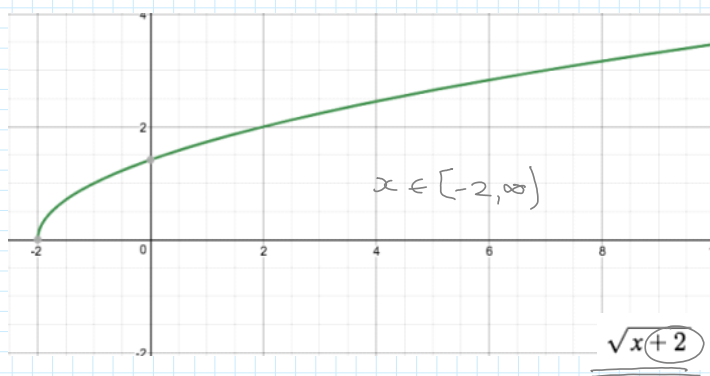
Let's look at this graph for a moment!



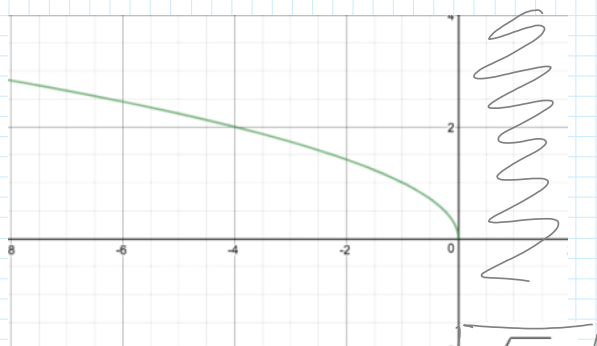
The basic graph



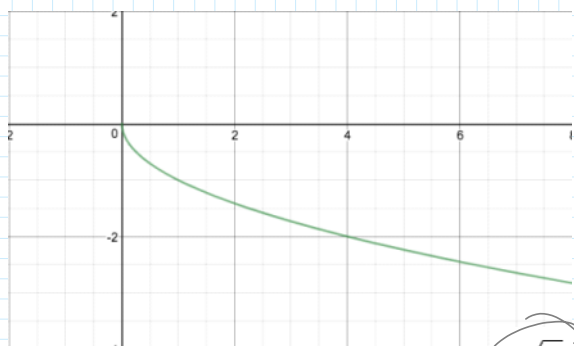
A translated graph

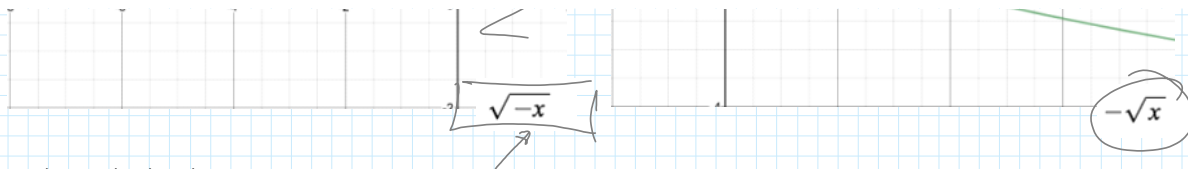


Another translated graph

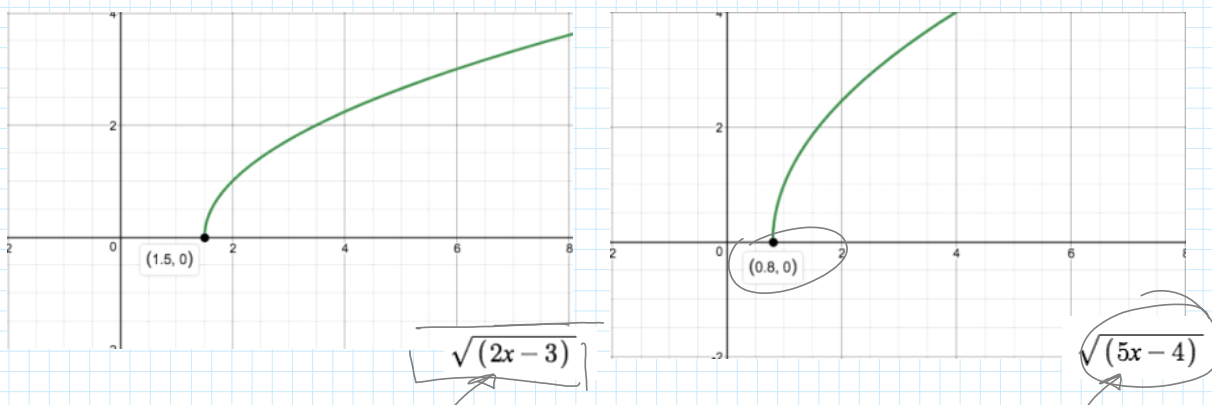


Another translated graph





Another translated graph



We note, from all the graphs above, that we have a section of the graph for which the values of x are **undefined**. How do we find this?

As we can't have a negative number under the square root we are always looking for this to be positive.

Sort of like the work we have been doing with determinants $b^2 - 4ac \geq 0$ for example!

Finding the implied domain for a $\sqrt{\quad}$ graph

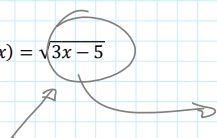
You simply place the function under the square root equal to zero.

Solve for x .

This will give you the lowest (or highest!) value the graph can be.

Examples:

Find the implied domain of the function $f(x) = \sqrt{3x-5}$



$$3x - 5 = 0$$

$$3x = 5$$

$$x = 5/3$$

$$x \in [5/3, \infty)$$

$$2x - 3 = 0$$

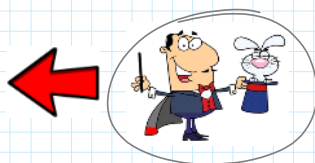
$$2x = 3$$

$$x = 3/2$$

$$\frac{3}{2} \rightarrow \infty$$

$$[3/2, \infty)$$

Find the implied domain of the function $f(x) = \sqrt{3-x}$



$$\sqrt{3-x}$$

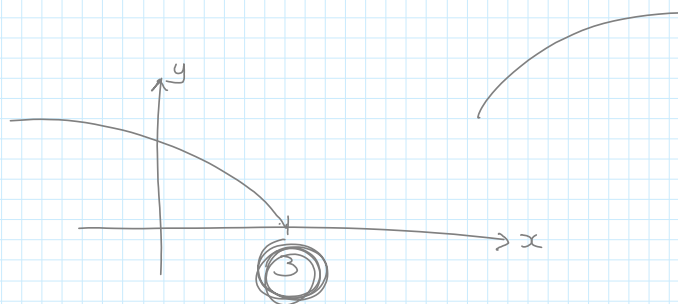
$$= \sqrt{-x+3}$$

$$= \sqrt{-(x-3)}$$

$$3-x=0$$

$$-x=-3$$

$$x=3$$



$$(-\infty, 3]$$