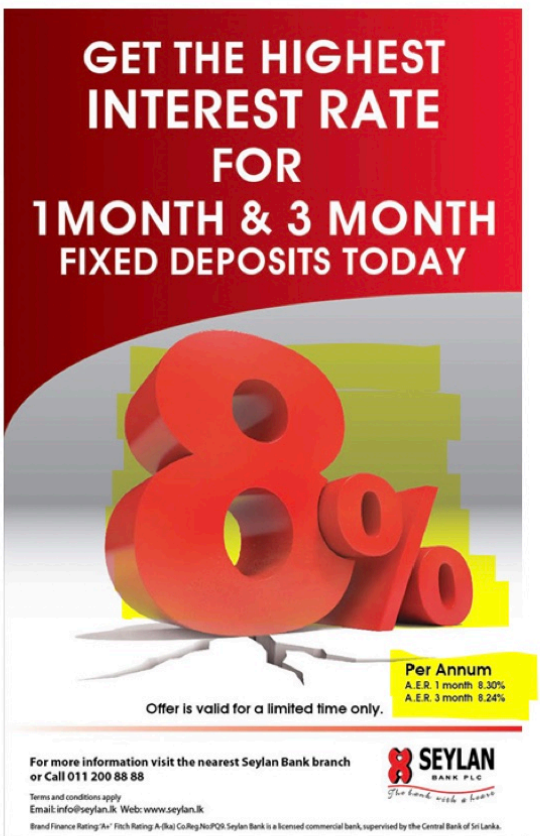
★ By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- Know what a nominal interest rate is
- Know what an effective interest rate is
- Know how to convert nominal interest rates into compounding interest rates

RECAP:

In a previous lesson we looked at how to calculate compound interest.
We learned that compounding meant that we were getting interest on the interest.
This was much better than simple interest.

Interest Rates: An Explanation



Notice the VERY LARGE 8% interest rate.
Notice that in much smaller print there are other interest rates given.
What do all these rates mean?
What does A.E.R mean?

Nominal Interest Rates

Compound interest rates are usually quoted as annual rates (or interest per annum).
This will be true in pretty much all of the questions you are asked to complete.

This annual rate is also called the nominal interest rate.

Sometimes we might need to know how much interest we are being paid monthly, or weekly.
We can convert nominal interest rates into compounding interest rates.

Example

The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

An investment account will pay interest at the rate of 3.6% per annum. Convert this interest rate to 3·6% p.a

- *a monthly rate* $\frac{3.6}{12} = 0.3\%$
- *a fortnightly rate* $\frac{3.6}{26} = 0.14\%$
- *a quarterly rate.* $\frac{3.6}{4} = 0.9\%$

Important Information

We must know the following information:

- *12 months in year*
- *4 quarters in each year*
- *26 fortnights in a year*
- *52 weeks in a year*
- *365 days in a year*

Effective Interest Rates

Whilst we might feel that we are paying 8% per annum, we might actually find that we are paying much more. This depends on when the interest is being added to the load (or the investment).

$\frac{5.2}{4}$

With a **loan** we would be looking at paying interest as few times as possible over the course of the loan. With an **investment** we would be looking at having interest paid as often as possible.

Here is an example which I've created using Excel

Principal Investment	6000		
Nominal Interest Rate	5.2 %		
	Per year	**Per Quarter**	**Per Month**
Month	5.2	1.3	0.43
0	6000.00	6000.00	6000.00
1			6026.00
2			6052.11
3		6078.00	6078.34
4			6104.68
5			6131.13
6		6157.01	6157.70
7			6184.38
8			6211.18
9		6237.06	6238.10
10			6265.13
11			6292.28
12	6312.00	6318.14	6319.54
Total Interest Earned	312.00	318.14	319.54
Effective Annual Interest Rate	5.2	5.30	5.33

6000

5.2%

6000×1.052

$R = 1 + \frac{r}{100}$

There is quite a lot of Maths here ... so let's explain where I got each of the values from.

Interest rates:

	Per year	**Per Quarter**	**Per Month**
Month	5.2	1.3	0.43

5.2% $\frac{5.2}{4}$ $\frac{5.2}{12}$

Values in the columns:

Per Quarter
1.3
6000.00
6078.00
6157.01
6237.06
6318.14

$6000 \times 1.013 = 6078$

$6078 \times 1.013 = 6157.01$

$r = 1.3$

$R = 1 + \frac{1.3}{100} = 1.013$

Interest Values

Principal Investment	6000		
Nominal Interest Rate	5.2 %		
	Per year	**Per Quarter**	**Per Month**
Month	5.2	1.3	0.43
0	6000.00	6000.00	6000.00
1			6026.00
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3		6078.00	6078.34
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Effective Annual Interest Rate

Total Interest Earned	312.00	318.14	319.54
Effective Annual Interest Rate	5.2	5.30	5.33

5.2%.

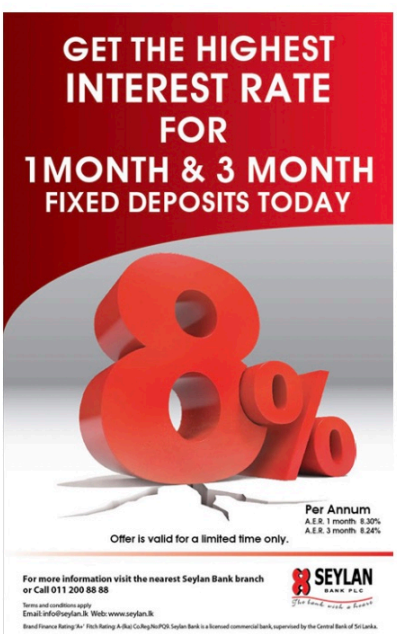
5.33%.

$\frac{312}{6000} \times 100 = 5.2$

$\frac{318.14}{6000} = 5.30$

$\frac{319.54}{6000} = 5.33$

Back to the image we started the lesson with



Principal Investment	1000		
Nominal Interest Rate	8 %		
	Per year	**Per Quarter**	**Per Month**
Month	8	2	0.67
0	1000.00	1000.00	1000.00
1			1006.67
2			1013.38
3		1020.00	1020.13
4			1026.93
5			1033.78
6		1040.40	1040.67
7			1047.61
8			1054.59
9		1061.21	1061.63
10			1068.70
11			1075.83
12	1080.00	1082.43	1083.00
Total Interest Earned	80.00	82.43	83.00
Effective Annual Interest Rate	8	8.24	8.30

Note:
The effective interest rate of a loan or investment is the interest earned after one year expressed as a percentage of the amount borrowed or invested.

There is a formula for this!

$$r_{effective} = \left(\left(1 + \frac{\frac{r}{n}}{100}\right)^{n} - 1\right) \times 100\%$$

Example
The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

Brooke would like to borrow $20000. She is deciding between two loan options:

- option A: 5.95% per annum compounding weekly
- option B: 6% per annum compounding quarterly.

Calculate the effective interest rate for each investment.

$$A: \quad r_e = \left(\left(1 + \frac{\frac{5.95}{52}}{100}\right)^{52} - 1\right) \times 100 = 6.13\%$$

$$B: \quad r_e = \left(\left(1 + \frac{\frac{6}{4}}{100}\right)^{4} - 1\right) \times 100 = 6.14\%$$

Which investment option is the best and why?

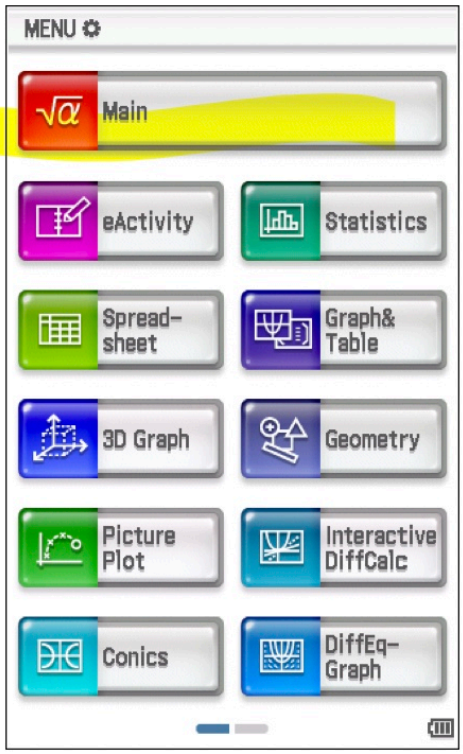
Remember:
- Borrowing money you want the lowest AER
- Saving money you want the highest AER

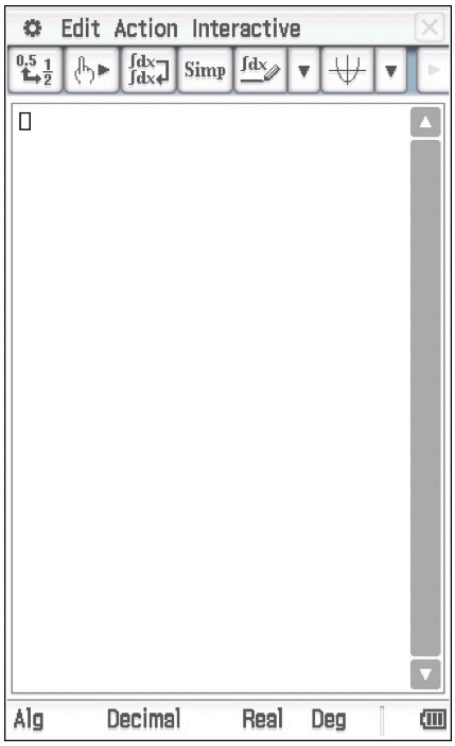
Example

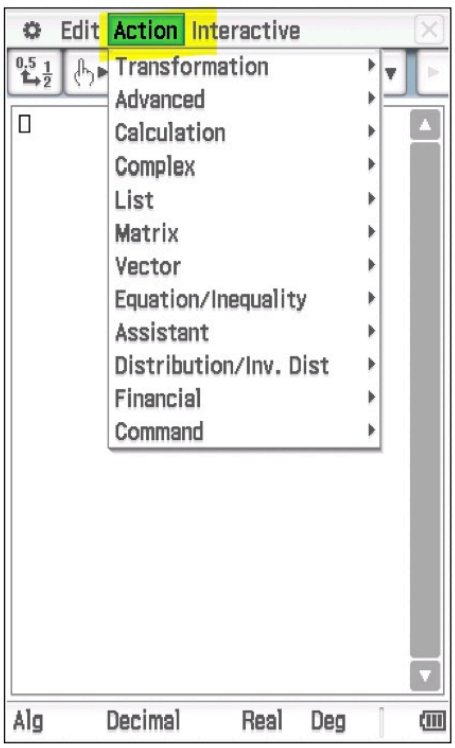
The following example has been extracted, with permission, from the Cambridge Further Maths Units 3 and 4 Textbook.

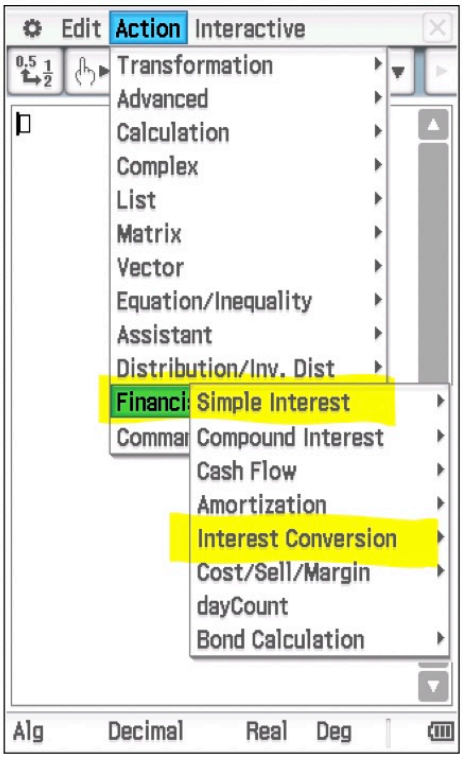
Marissa has $10000 to invest. She chooses an account that will earn compounding interest at the rate of 4.5% per annum, compounding monthly.

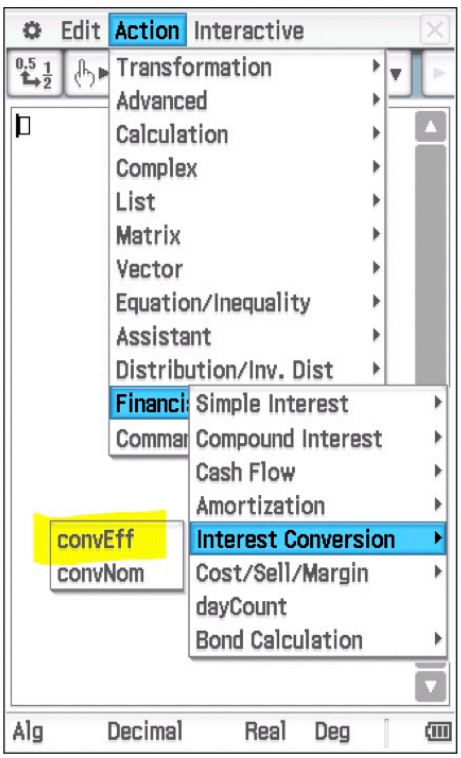
Use a CAS calculator to find the effective rate for this investment, correct to three decimal places.

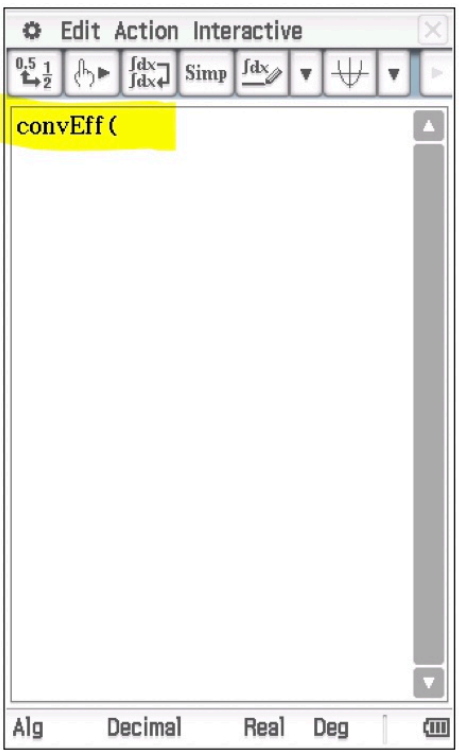












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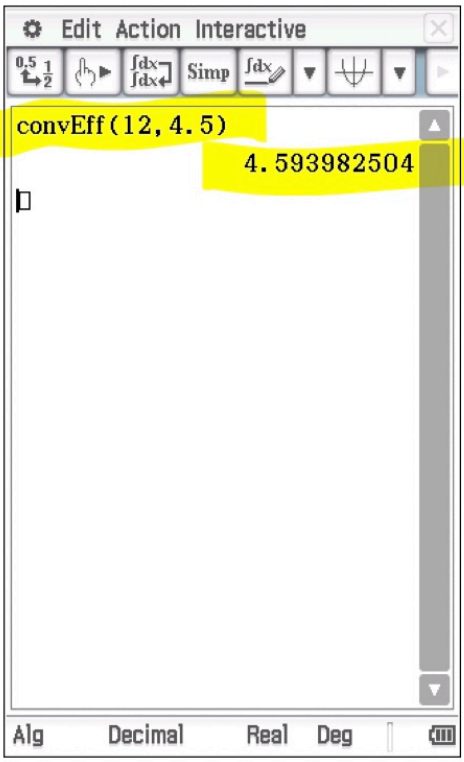

convEff(number of compounds per year, nominal rate).

4.59

Instructions for the Ti-Nspire