

Signed Area

Year 11
Mathematical Methods

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Learning Objectives

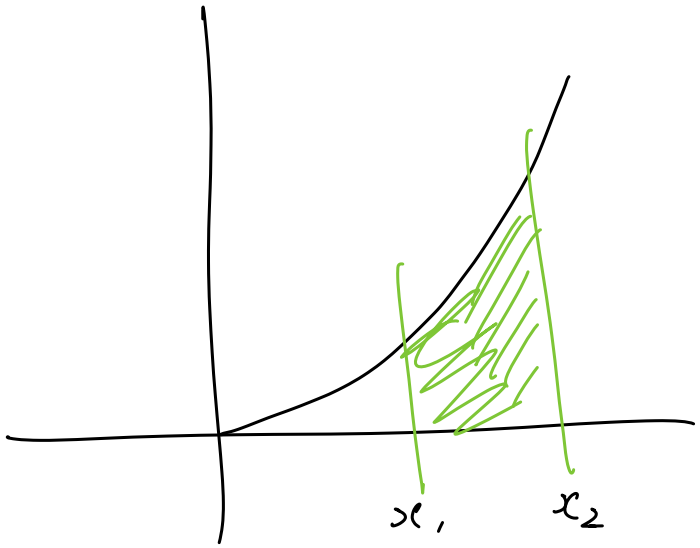
By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 Mathematical Methods course.

- Understand what it means to be a signed area
- Apply this information to find the area under a range of functions which sit both above and below the x-axis.



Recap of past learning

In the last lesson we looked at how we can use the definite integral to find the area under a continuous function and between two limits and the x-axis. This has many, many applications. But a deeper understanding is going to be needed as there are a number of tricks which can be employed to test your understanding over your ability to regurgitate.



$$\int_{x_1}^{x_2} f(x)\,dx$$

$$x^2 \qquad \frac{x^3}{3}$$



Recap: Definition of the integral

We can use the following notation to find the area under a continuous function. It is called the **definite integral.**

The symbol is called the "integral sign" and really means "sum"

$$\int_a^b f(x)\ dx$$

This is stating, "by considering really small widths"

Limits of the integral denote the start and end of the area we are trying to find

This is the function we are trying to find the area under



An interesting example

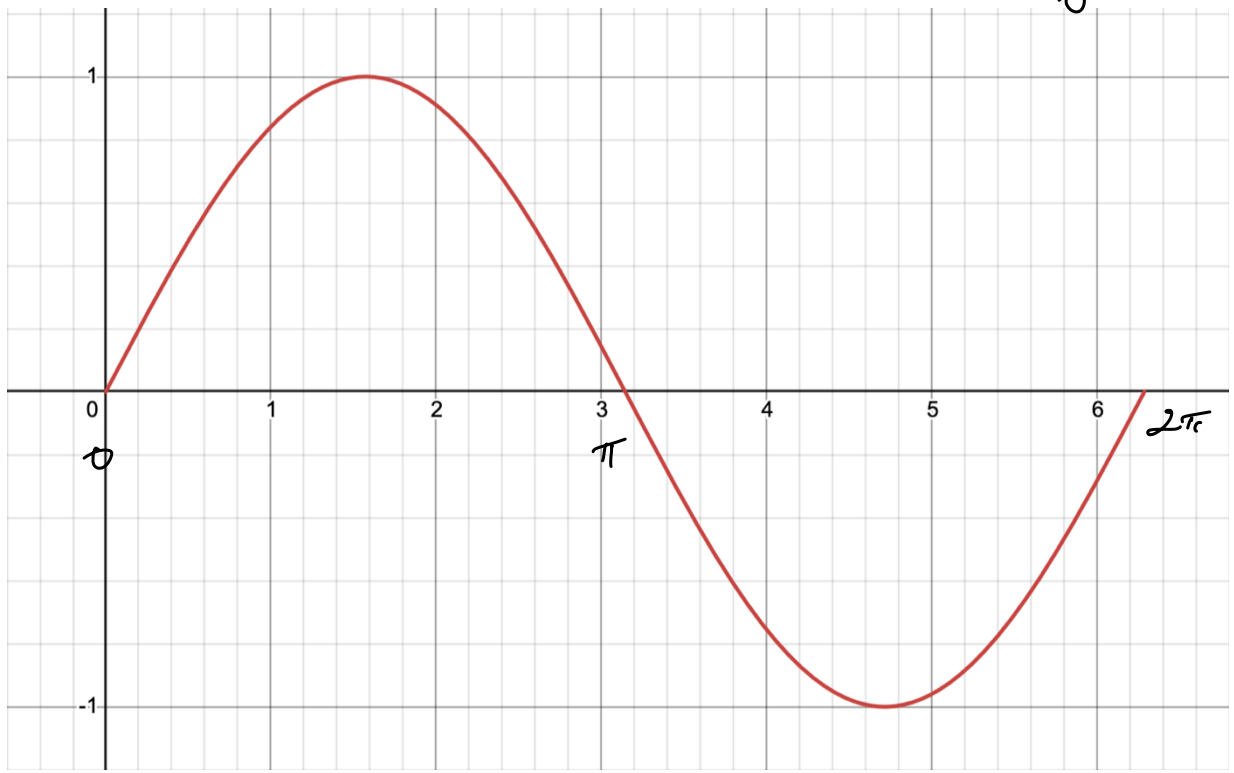
$$\int_0^{2\pi} \sin x \, dx$$

Whilst integrating Trig functions isn't part of this section of the course, it provides some interesting teaching.

Let's look at one cycle of a sine curve where $x \in [0, 2\pi]$.

We can now find the area between 0 and π quite easily i.e. the top part of the curve.

For this, we will need to use a CAS!

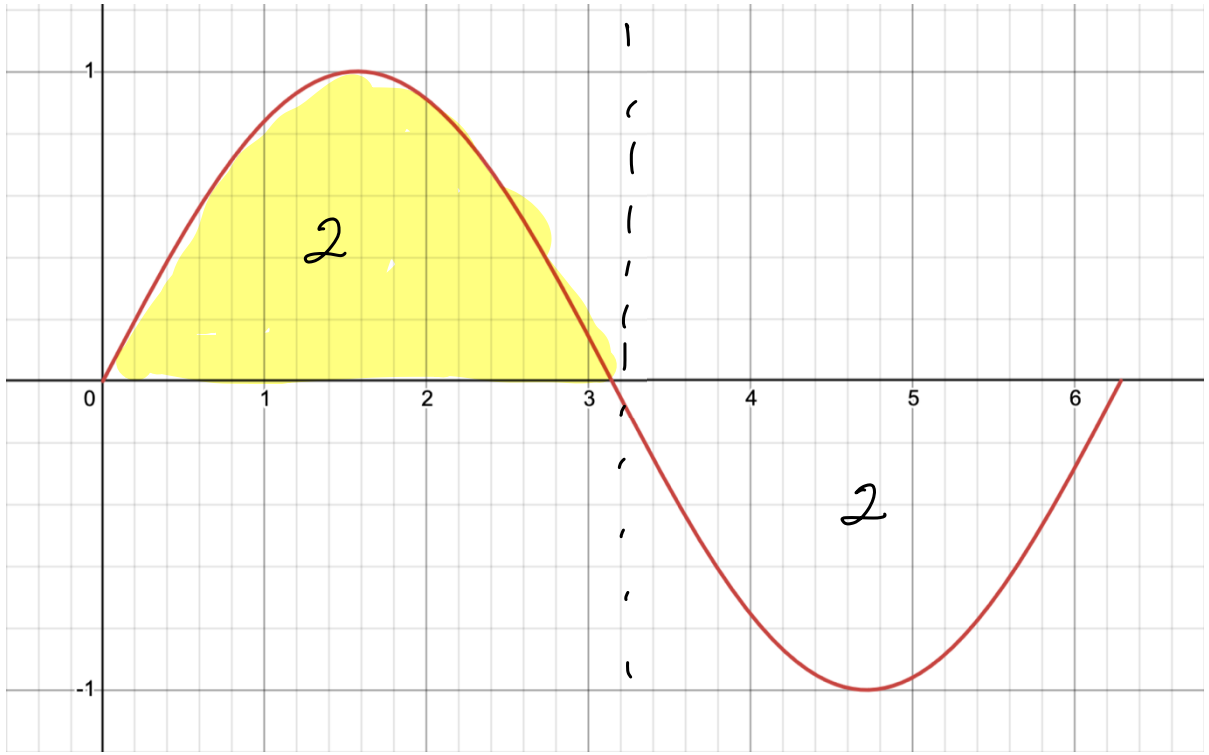


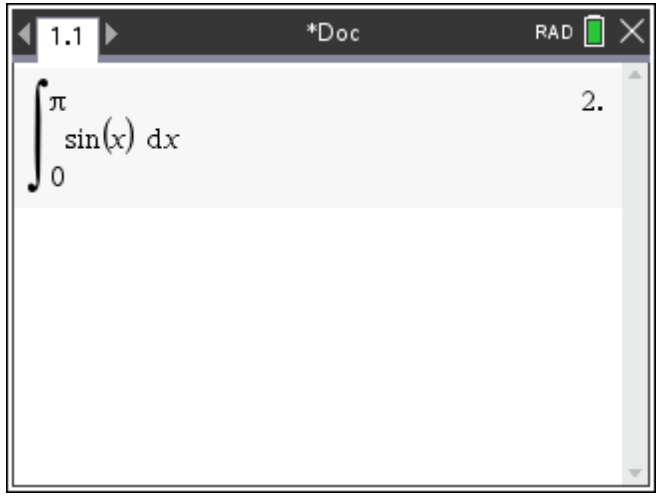


An interesting example

We can see the area is equal to 2 units.

So, the area between 0 and 2π should equal 4 units.



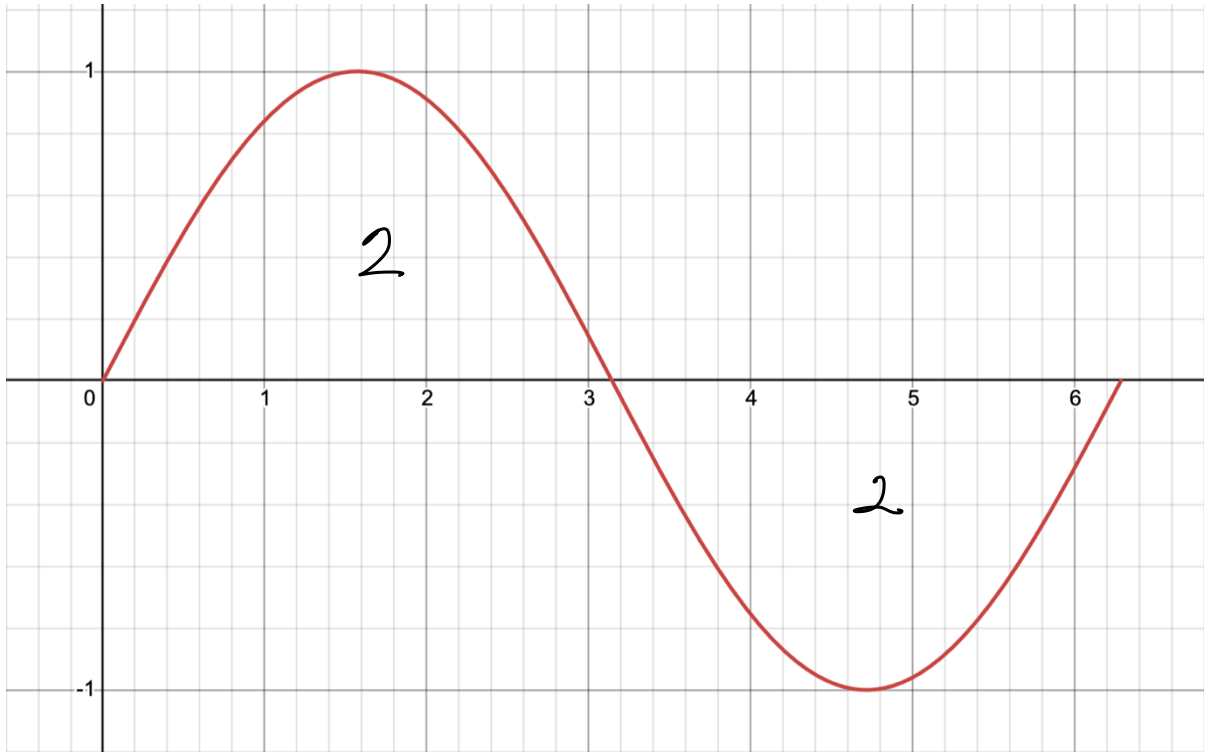


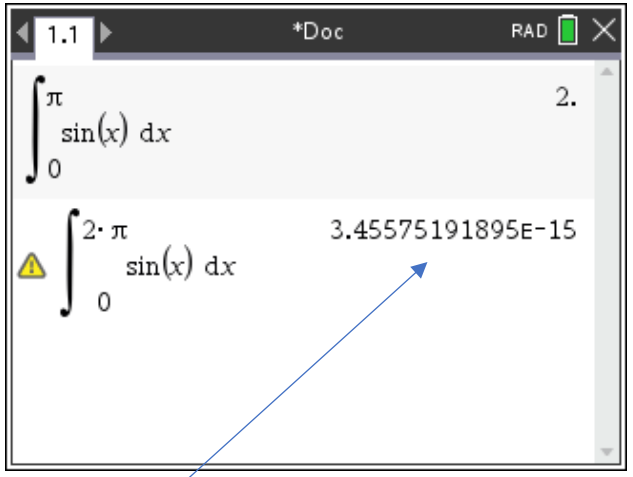


An interesting example

We can see the area is equal to 2 units.

So, the area between 0 and 2π should equal 4 units but it isn't! Why?





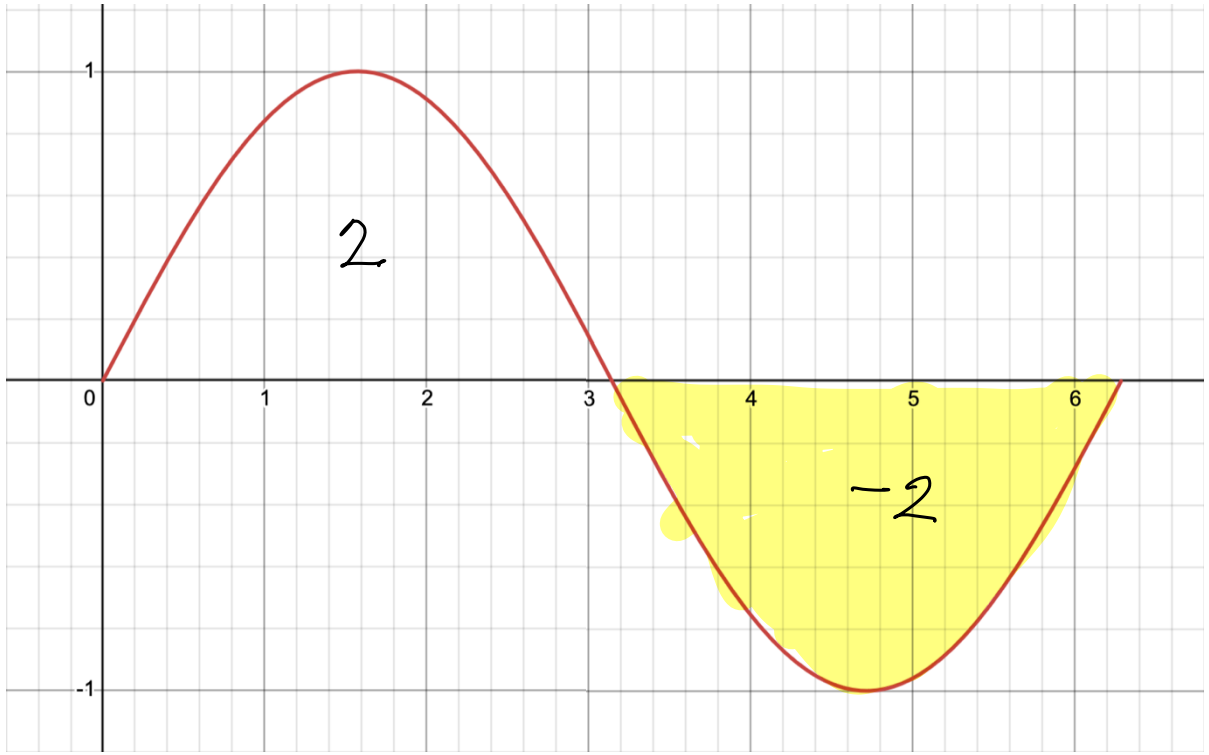
This is effectively zero.

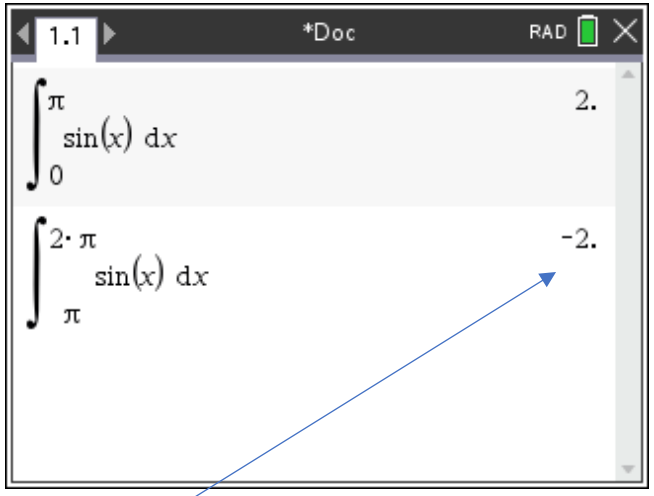


An interesting example

As it turns out the areas under the x-axis are negative. The areas above the x-axis are positive. When we add these together it becomes zero. Which is absolutely nonsensical as there is definitely area there!

$$\int_0^{2\pi} \sin x \, dx = 0$$





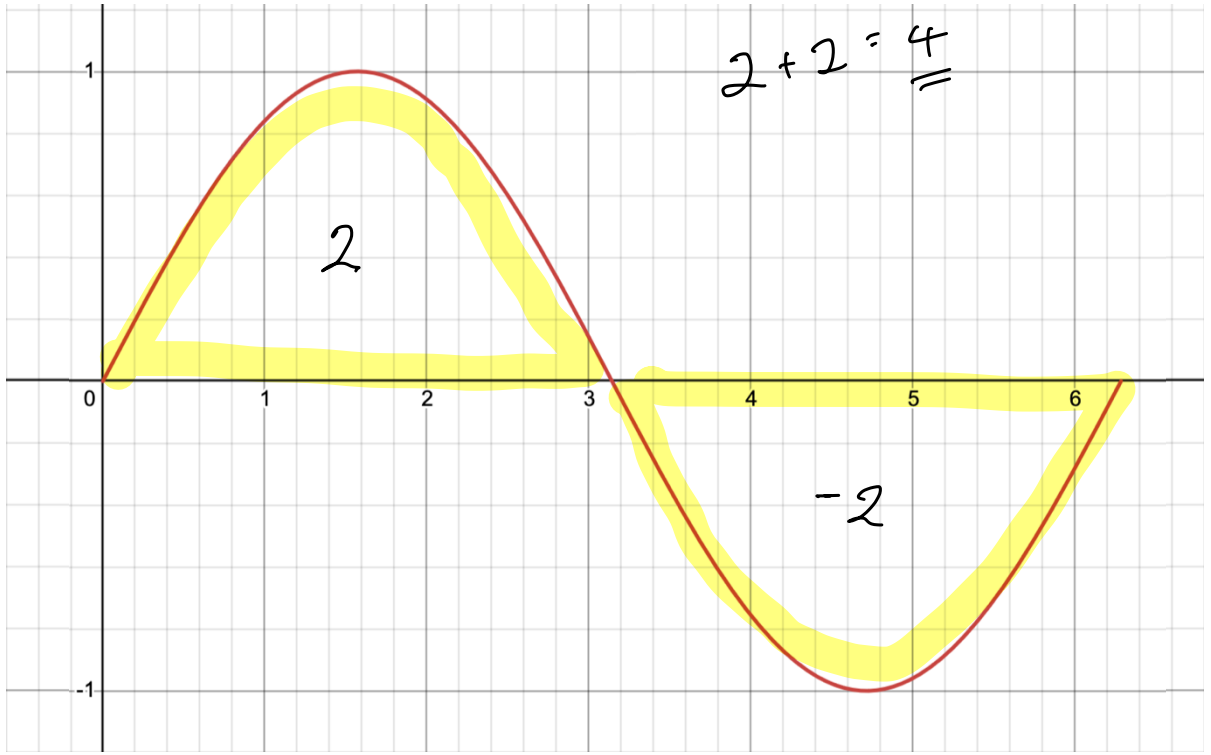
Notice the negative sign

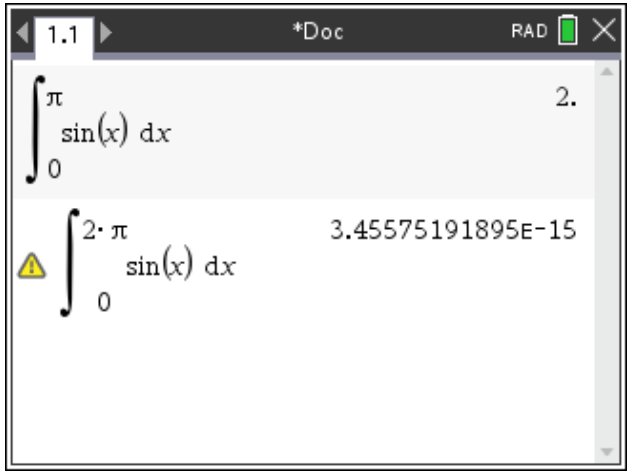


The signed area

It's really important to note that the signed area of the below is actually zero.

When being asked for the signed area, it isn't asking for the total area.







Sketch, sketch and sketch again!

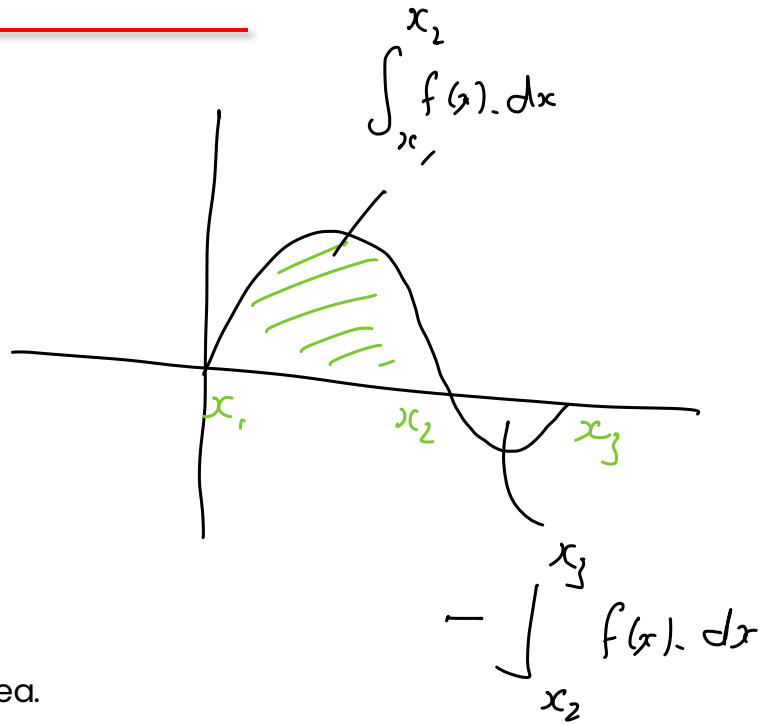
It is critically important, when doing these questions, that you sketch the graphs to see what they look like.

Any areas which are below the axis need to be negated to ensure they are added on.

So … for our previous example we should do this:

$$\int_0^{\pi} \sin(x)\,dx - \int_{\pi}^{2\pi} \sin(x)\,dx$$

The negative sign will negate the negative area.



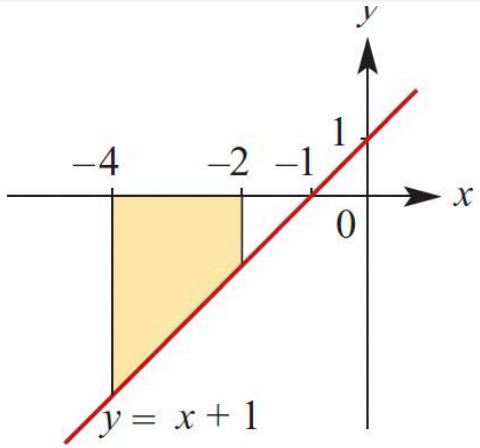
There is another way and that is to swap the terminals, but this can be hard to read and easy to forget:

$$\int_0^{\pi} \sin(x)\,dx + \int_{2\pi}^{\pi} \sin(x)\,dx$$



An example

Find the area of the shaded region.



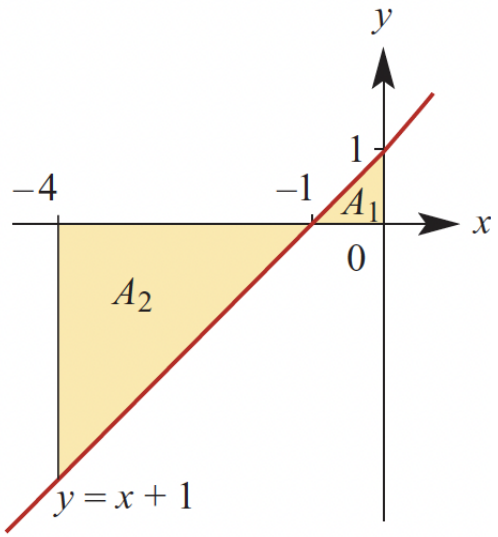
$$\int_{-4}^{-2} (x+1)\,dx = 4 \text{ units}^2$$



An example

a Find the total area of the shaded regions.

b Find the signed area of the shaded regions.



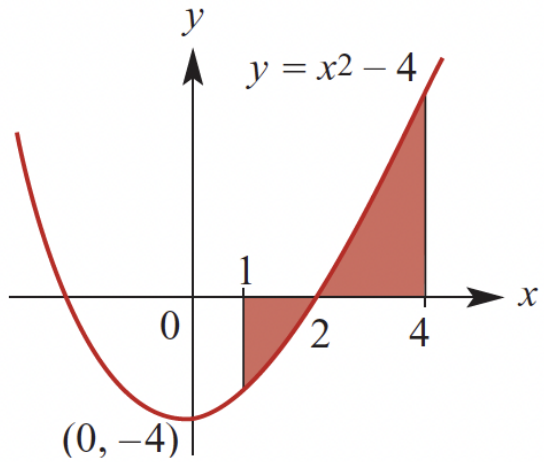
a. $\int_{-1}^{-4} (x+1)\,dx + \int_{-1}^{0} (x+1)\,dx = 5$

b. $-4.5 + 0.5 = -4$



An example

Find the area of the shaded region.



$|x^2 - 4|$

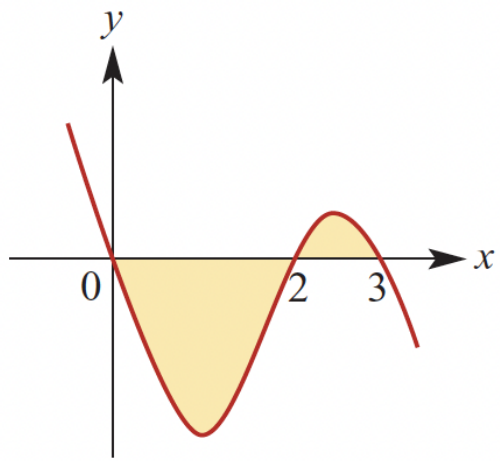
$$A = \int_2^1 (x^2-4)\,dx + \int_2^4 (x^2-4)\,dx$$

$$= \frac{37}{3}$$



An example

Find the area enclosed by the graph of $y = x(2 - x)(x - 3)$ and the x-axis.



$$A = \int_2^0 f(x).dx + \int_2^3 f(x).dx$$

$$= \frac{37}{12}$$



Interesting questions which are more logic based

Given that $\int_1^3 f(x)\,dx = 6$, evaluate:

a $\int_1^3 2f(x)\,dx$

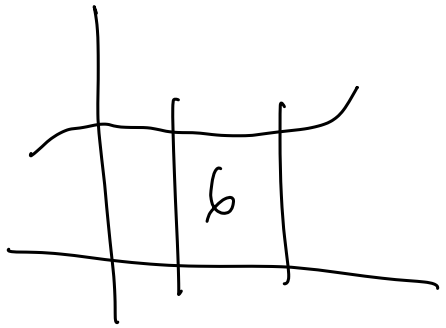
$= 12$

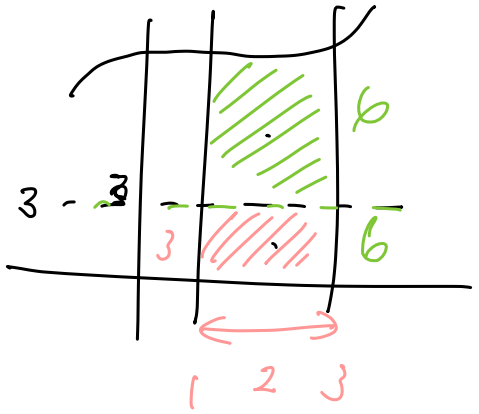
b $\int_1^3 f(x) + 3\,dx$

$= 12$

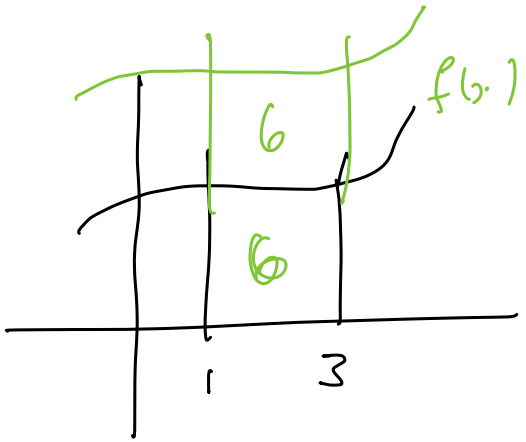
c $\int_3^1 f(x)\,dx$

$= -6$





Note: These questions are on exams ALL THE TIME as they test your understanding. If you draw a sketch it becomes hard not to answer these correctly!





Learning Objectives

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Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 21C
Questions: 11

Chapter Review (page 715):
Questions: 1, 2

Sheet MM231:
Set 1: Questions: 1, 8, 9, 10





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