Applications to maximum and minimum problems

Year 11

Mathematical Methods

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 Mathematical Methods course.

- Understand how to apply the previous theory to maximum and minimum problems

Recap of past learning

In previous topics we have looked at a number of basic concepts which can be used separately and combined in application style problems. In every case, students get confused when to apply these things or what they are actually doing.

For example:

- Tangents and normal
- Rates of change
- Stationary points
- Types of stationary points

This lesson is going to look at how we can apply all the theory from before into application style problems.

Examples have been extracted, with permission, from the Cambridge Mathematical Methods Units 1 and 2 Textbook

Example 1

A loop of string of length 100 cm is to be formed into a rectangle. Find the maximum area of this rectangle.

$A = xy$

$A = x(50 - x)$

$A = 50x - x^2$

$\frac{dA}{dx} = 50 - 2x$

$0 = 50 - 2x$

$2x = 50$

$x = 25$

$\therefore A_{max} = xy$

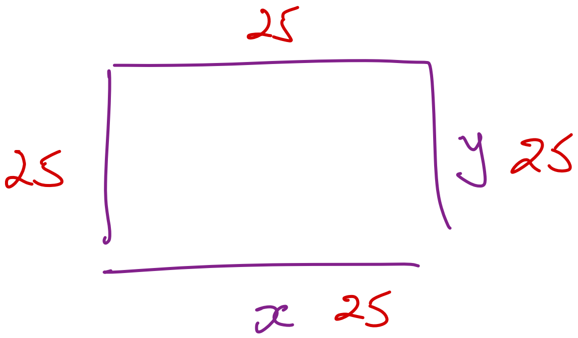
$= 25 \times 25$

$= 625 \text{ cm}^2$

$x(50 - x) = 0$

$\therefore x = 0 \quad 50 - x = 0$

$x = 50$

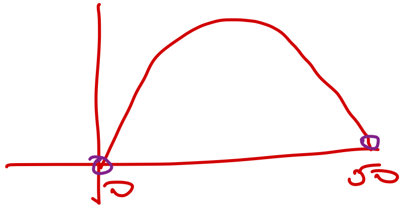


$P = 2x + 2y$

$100 = 2x + 2y$

$x + y = 50$

$y = 50 - x$



Example 2

Given that $x + 2y = 4$, calculate the minimum value of $x^2 + xy - y^2$.

$x + 2y = 4$

$x = 4 - 2y$

$Z = x^2 + xy - y^2$

$Z = (4-2y)^2 + (4-2y) \cdot y - y^2$

$= 16 - 16y + 4y^2 + 4y - 2y^2 - y^2$

$Z = 16 - 12y + y^2$

$\frac{dZ}{dy} = -12 + 2y$

$0 = -12 + 2y$

$2y = 12$

$y = \underline{\underline{6}}$

$x = 4 - 2y$

$x = 4 - 12$

$x = \underline{\underline{-8}}$

Example 3

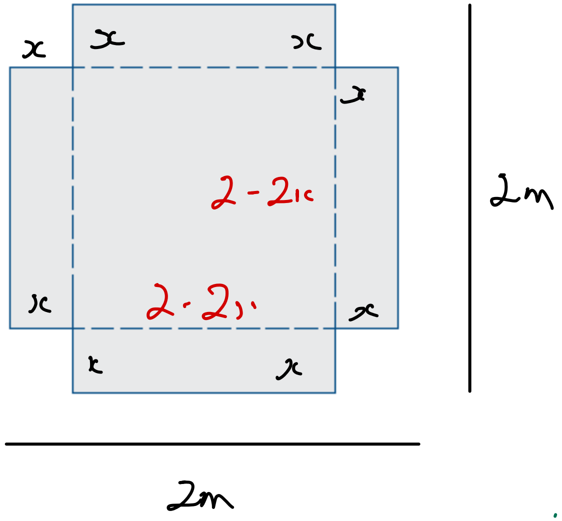
From a square piece of metal of side length 2 m, four squares are removed as shown in the diagram. The metal is then folded along the dashed lines to form an open box with height x m.

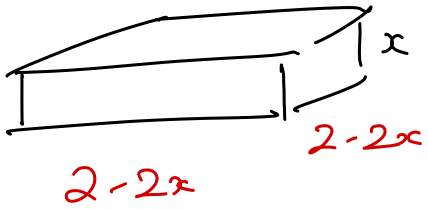
a Show that the volume of the box, V m^3, is given by $V = 4x^3 - 8x^2 + 4x$.

b Find the value of x that gives the box its maximum volume and show that the volume is a maximum for this value.

c Sketch the graph of V against x for a suitable domain.

d Find the value(s) of x for which $V = 0.5$ m^3.





$x = \frac{1}{3}$ or $\cancel{x = 1}$

$\therefore x = \frac{1}{3}$ max

$V = (2-2x)(2-2x)x$

$V = 4x^3 - 8x^2 + 4x$

$\frac{dV}{dx} = 12x^2 - 16x + 4$

$0 = 12x^2 - 16x + 4$

x	0.3	$\frac{1}{3}$	0.4
x'	+	0	−
	/	—	\

$\therefore$

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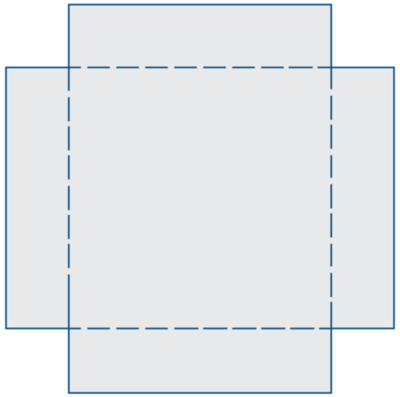

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c Sketch the graph of V against x for a suitable domain.

d Find the value(s) of x for which $V = 0.5$ m^3.

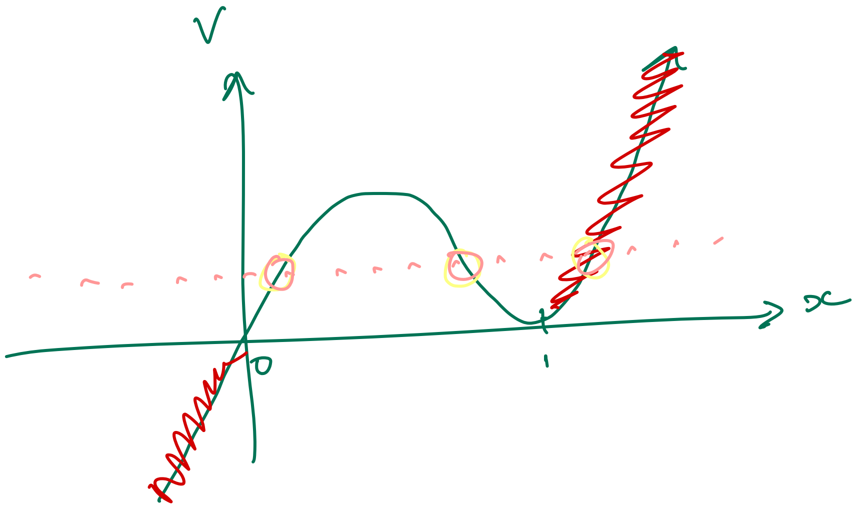
$V = x(2-2x)(2-2x)$

$0 = x(2-2x)(2-2x)$

$x = 0 \qquad 2-2x = 0$

$2x = 2$

$x = 1$



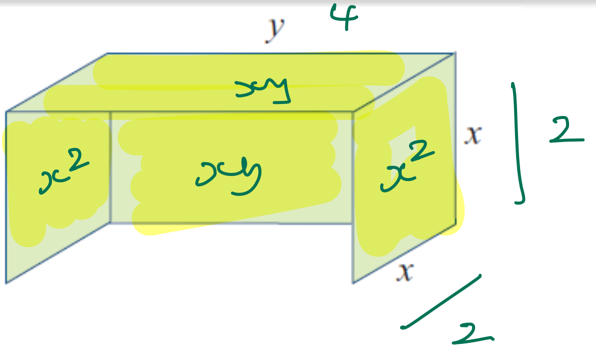
Example 4

A canvas shelter is made up with a back, two square sides and a top. The area of canvas available is 24 m^2. Let V m^3 be the volume enclosed by the shelter.

a Find the dimensions of the shelter that will create the largest possible enclosed volume.

b Sketch the graph of V against x for a suitable domain.

c Find the values of x and y for which $V = 10$ m^3.



$V = x^2 \cdot y$

$24 = 2x^2 + 2xy$

$12 = x^2 + xy$

$12 - x^2 = xy$

$y = \dfrac{12 - x^2}{x}$

$V = x^{\cancel{2}} \cdot \left(\dfrac{12 - x^2}{\cancel{x}}\right)$

$= 12x - x^3$

$V = 12x - x^3$

$\dfrac{dV}{dx} = 12 - 3x^2$

$0 = 12 - 3x^2$

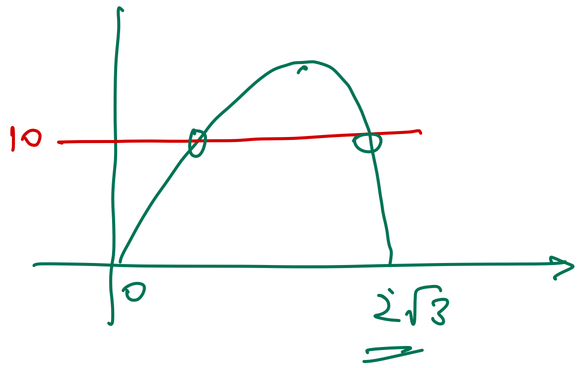
$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

$\therefore x = 2$

$\therefore$ dim $2, 2, 4$

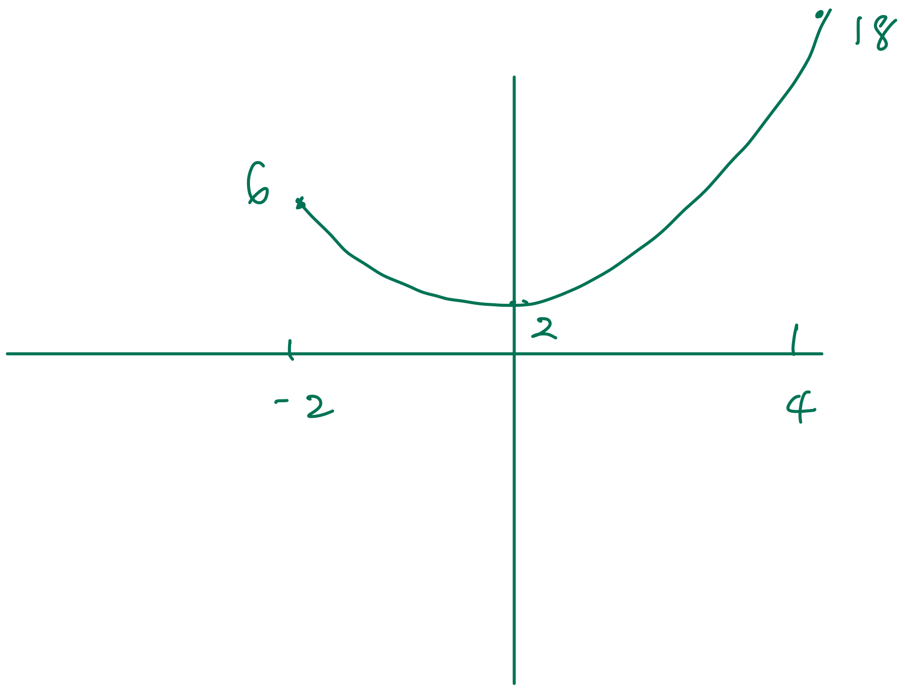


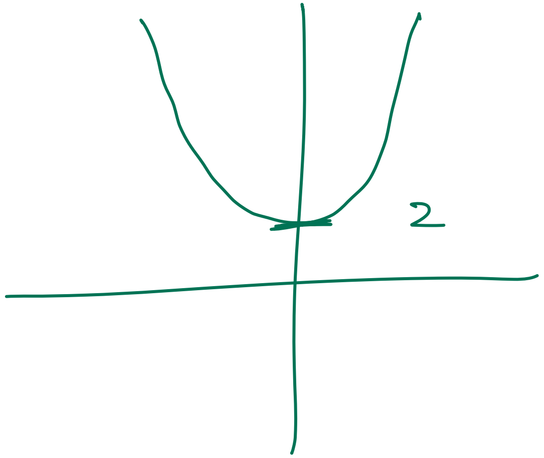
$x = 0.893$

$x = 2.93$

Example 5

Let $f: [-2, 4] \to \mathbb{R}$, $f(x) = x^2 + 2$. Find the absolute maximum value and the absolute minimum value of the function.





$f(x) = x^2 + 2$

$f(-2) = 4 + 2 = \underline{\underline{6}}$

$f(4) = 18$

max $= \underline{\underline{18}}$

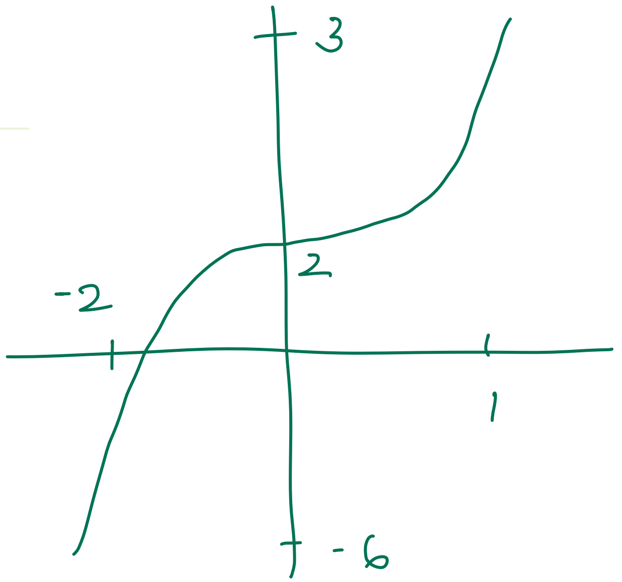
min $= \underline{\underline{2}}$

Example 6

Let $f: [-2, 1] \to \mathbb{R}$, $f(x) = x^3 + 2$. Find the maximum and minimum values of the function.

$f(-2) = -6$

$f(1) = 3$



max = 3

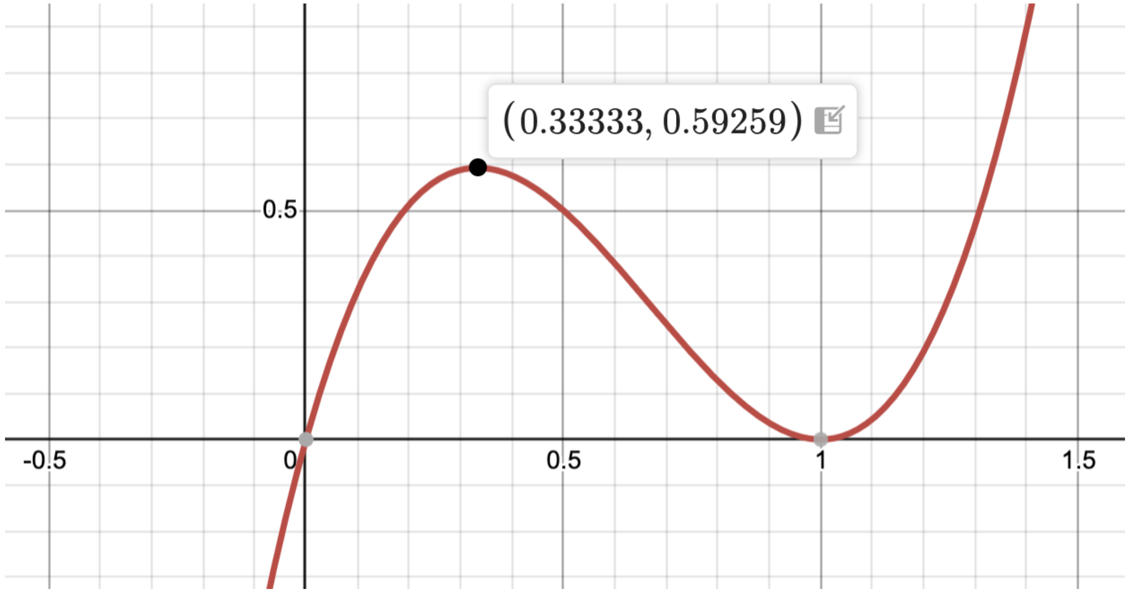
min = -6

Example 7

In Example 10, the maximum volume of a box was found. The maximum value corresponded to a local maximum of the graph of $V = 4x^3 - 8x^2 + 4x$. This was also the absolute maximum value.

If the height of the box must be at most 0.3 m (i.e. $x \leq 0.3$), what will be the maximum volume of the box?

$x = \frac{1}{3}$



$x = 0.3$

$V = 0.588 \text{ m}^3$

Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 18E
Questions: TBA

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