Types of stationary points

Year 11

Mathematical Methods

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 Mathematical Methods course.

- Understand that there are different types of stationary points
  - Local maximum
  - Local minimum
  - Stationary point of inflection
- Understand what it means to be a turning point

Recap of past learning

In the previous lesson I looked at what it meant to be a stationary point.

In essence: It’s a point on a function which has a zero gradient.

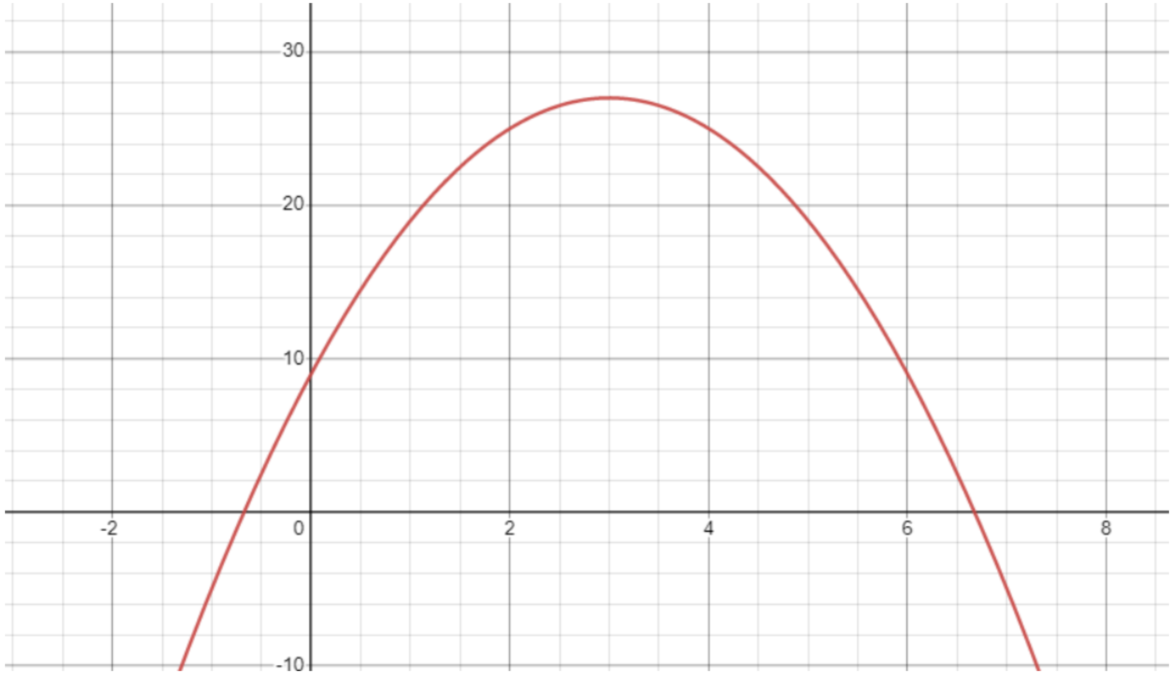
We also discovered that there were three main places on a function where there might be a zero gradient.

This lesson looks, once again, at the different types of stationary point.

Recap: What is a stationary point?

A stationary point is a point on a function where the gradient of the tangent at that point is equal to zero.

Remember: A gradient of zero means that there is no vertical rise for any amount of horizontal distance moved.



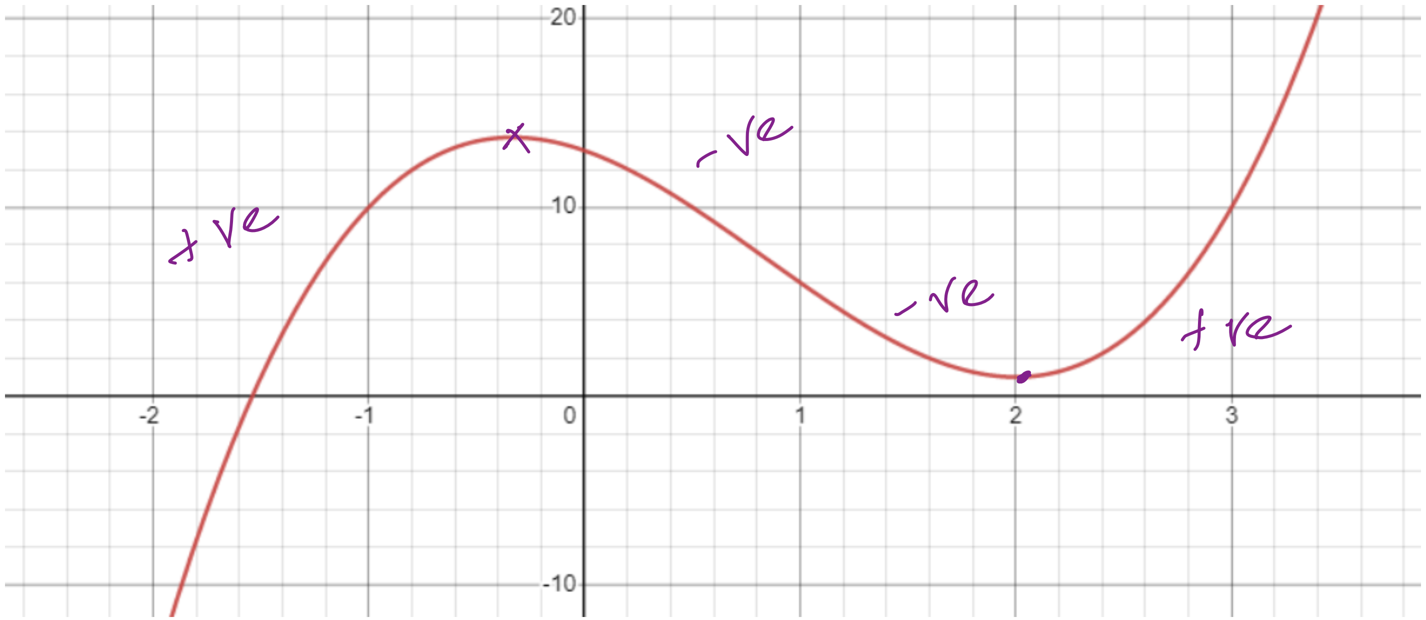
Maximum and minimums

We looked, in the previous lesson, at the following function which showed an obvious local maximum and local minimum.

A local maximum is really a way of describing a maximum at a particular point on the graph.

Remember: a function can have more than one maximum. When it has only one, we would normally just call it a maximum.

The local maximum is a point within an interval at which the function has a maximum value. The absolute maxima is also called the global maxima and is the point across the entire domain of the given function, which gives the maximum value of the function.



Stationary points of inflection

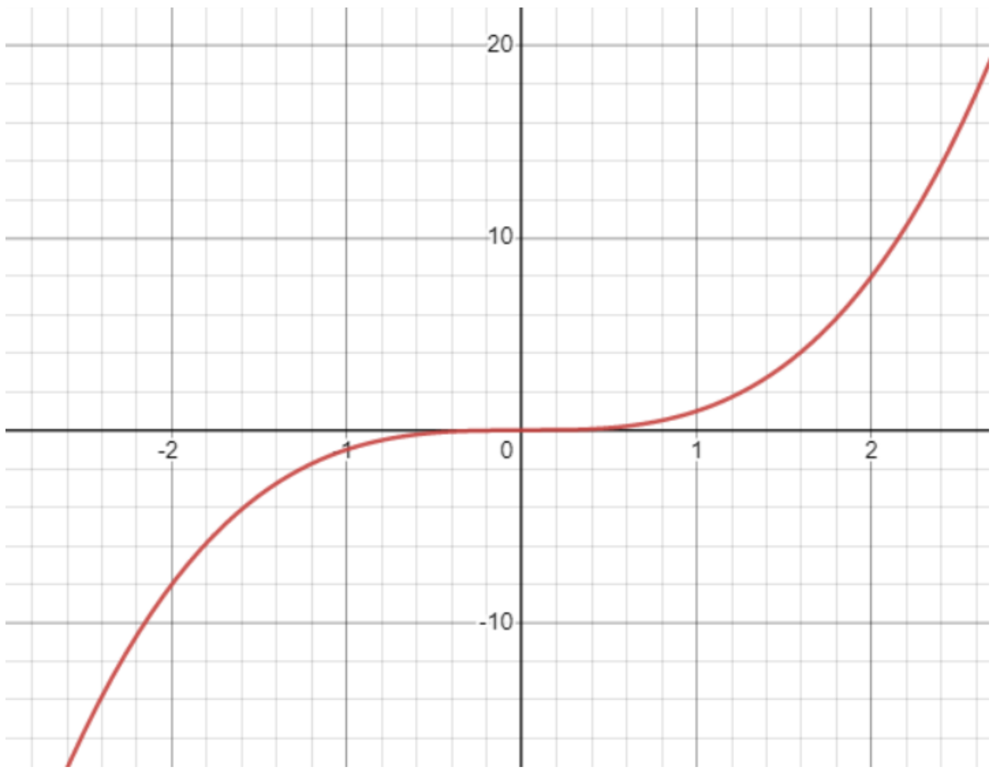
In the previous lesson we looked the graph on the right which is the graph of $y = x^3$

When we differentiate this and find its stationary point, we see it occurs at the point (0, 0).

But the gradients either side of the point are both positive.

Where this occurs, we call the point a **stationary point of inflection**.

Note: For those doing specialist mathematics, this is different from a point of inflection.

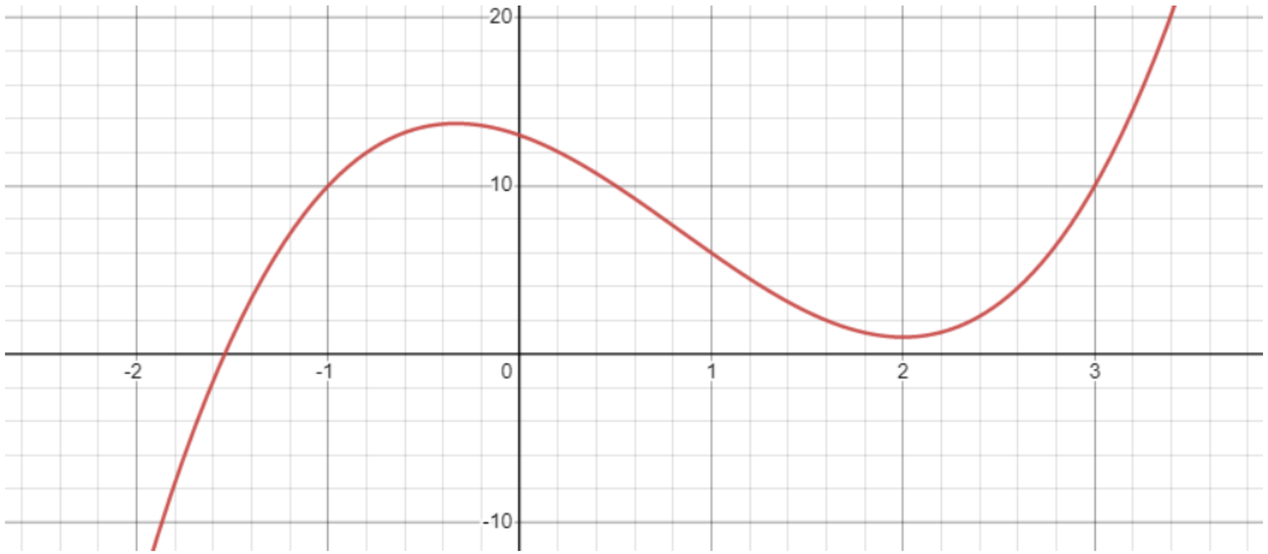


Stationary points and Turning Points

Barry has been at it again and decided that we can call points where there is a local maximum and local minimum as “**turning points**”.

Sigh.

I suppose this makes sense as it’s a point where the function seems to turn.



Stating the nature of a turning point

In Methods we may be asked to prove why something is maximum or minimum. We can do this by testing the values of gradient either side of a point.

We are only interested in seeing whether the gradient is positive or negative either side of the point.

If we look at the equation $y = x^3$ we can test to find where there are stationary points and test their nature.

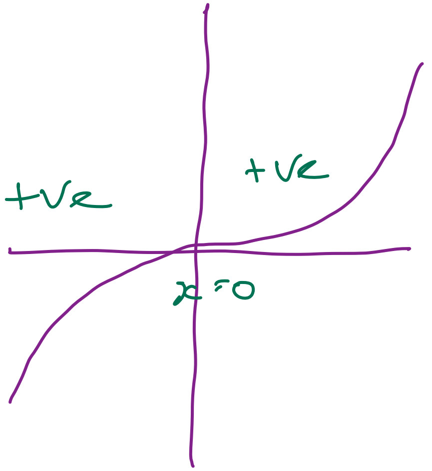
$y = x^3$

$y' = 3x^2$

$3x^2 = 0$

$x^2 = 0$

$x = 0$



$x = -0.1$ $\qquad$ $x = 0.1$

$y' = 3x^2$

$y' = 3 \times (-0.1)^2$

$= +ve$

$y' = 3(0.1)^2$

$= +ve$

$\therefore$ S.P.I.

Example

For the function $f: \mathbb{R} \to \mathbb{R}, f(x) = 3x^3 - 4x + 1$:

a) Find the stationary points and state their nature

b) Sketch the graph

Note: State their nature means you have to prove they are a max or min. This can be done by testing either side of the point to see what the gradient is.

$f(x) = 3x^3 - 4x + 1$

$f'(x) = 9x^2 - 4$

$9x^2 - 4 = 0$

$(3x)^2 - (2)^2 = 0$

$(3x - 2)(3x + 2) = 0$

$\therefore 3x - 2 = 0$

$3x = 2$

$x = 2/3$

min

$x = -\frac{2}{3}$

max

x	-0.7	$-\frac{2}{3}$	-0.6
$f'(x)$	+ve	0	-ve

x	0.6	$\frac{2}{3}$	0.7
$f'(x)$	-ve	0	+ve

Example

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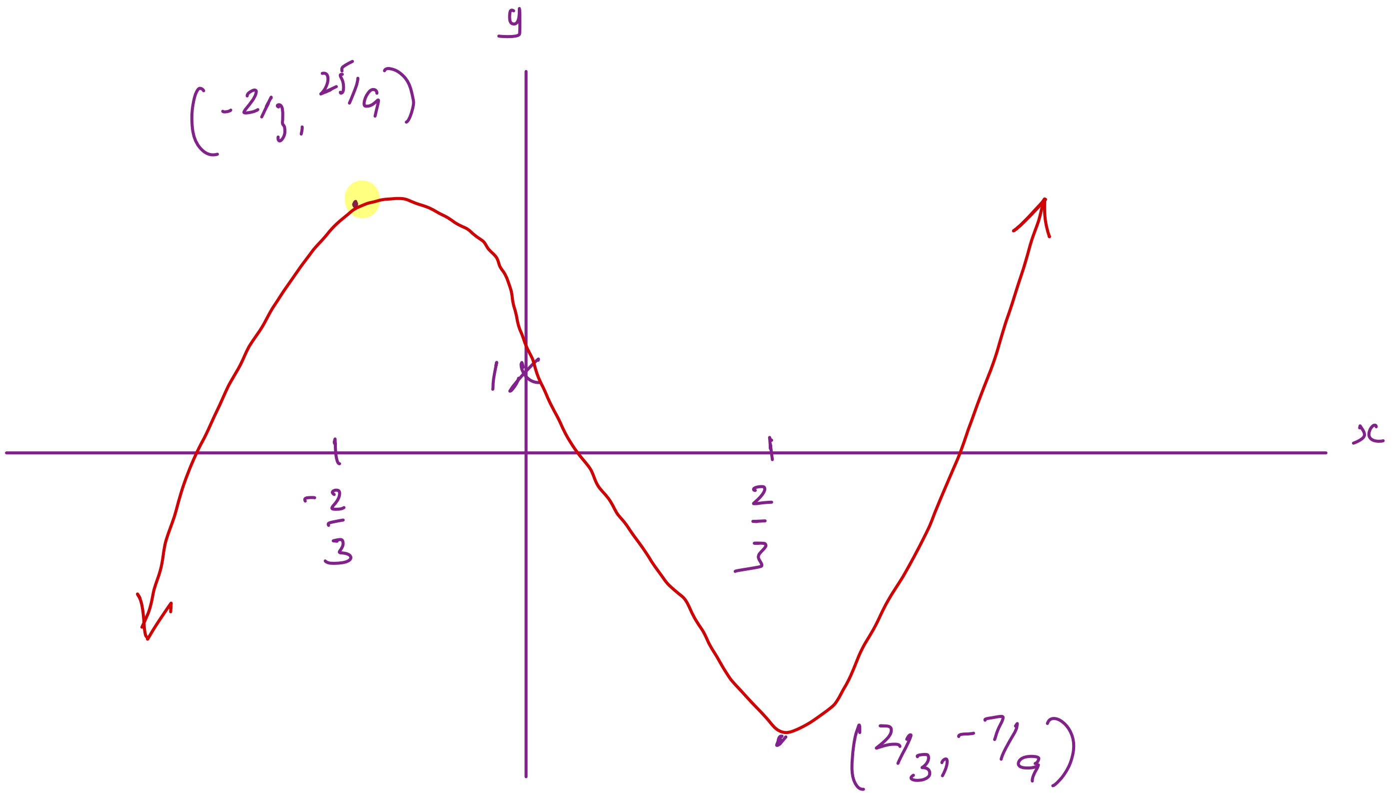

Example: Using the CAS

For the function $f: \mathbb{R} \to \mathbb{R}, f(x) = 3x^3 - 4x + 1$:

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Another way to find the nature of a stationary point

For the function $f: \mathbb{R} \to \mathbb{R}, f(x) = 3x^3 - 4x + 1$:

a) Find the stationary points and state their nature

$f'(x) = 9x^2 - 4$

$f''(x) = 18x$

$x = -\frac{2}{3}$ $\quad x = \frac{2}{3}$

$f''(x) = 18 \times -\frac{2}{3}$ $\quad 18 \times \frac{2}{3}$

$= -\text{ve}$ $\quad = +\text{ve}$

max $\quad$ min

$f'(x)$

$f''(x) > 0$ min

$=$

$f''(x) < 0$ max

$f''(x) = 0$ S.P.I.

Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 18D
Questions: TBA

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