Stationary points

Year 11

Mathematical Methods

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 Mathematical Methods course.

- Understand what it means to be a stationary point
- Know how to find the stationary points of a function

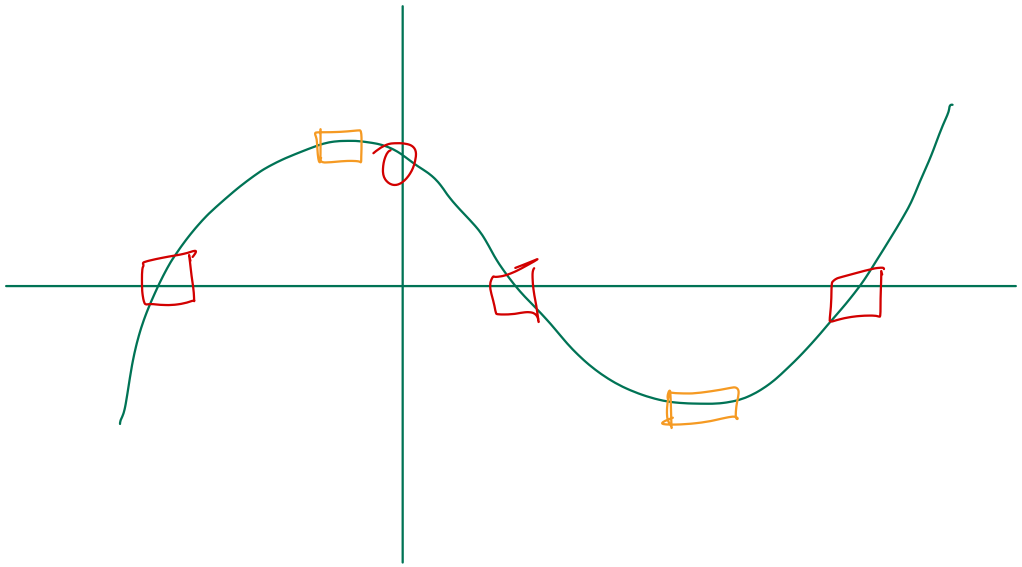
Recap of past learning

In the previous chapter and sections we have looked at how to find tangents, normal and rates of change. These are all important, but one of the most used sections of the course is the ability to find stationary points.

We know how to find the gradient of points along the function using the process of differentiation.

We saw that gradients could be positive or negative. This would also imply that they could be zero (as this is the state of being between a positive and negative).

This video looks at how we can find points with a zero gradient and what it means.



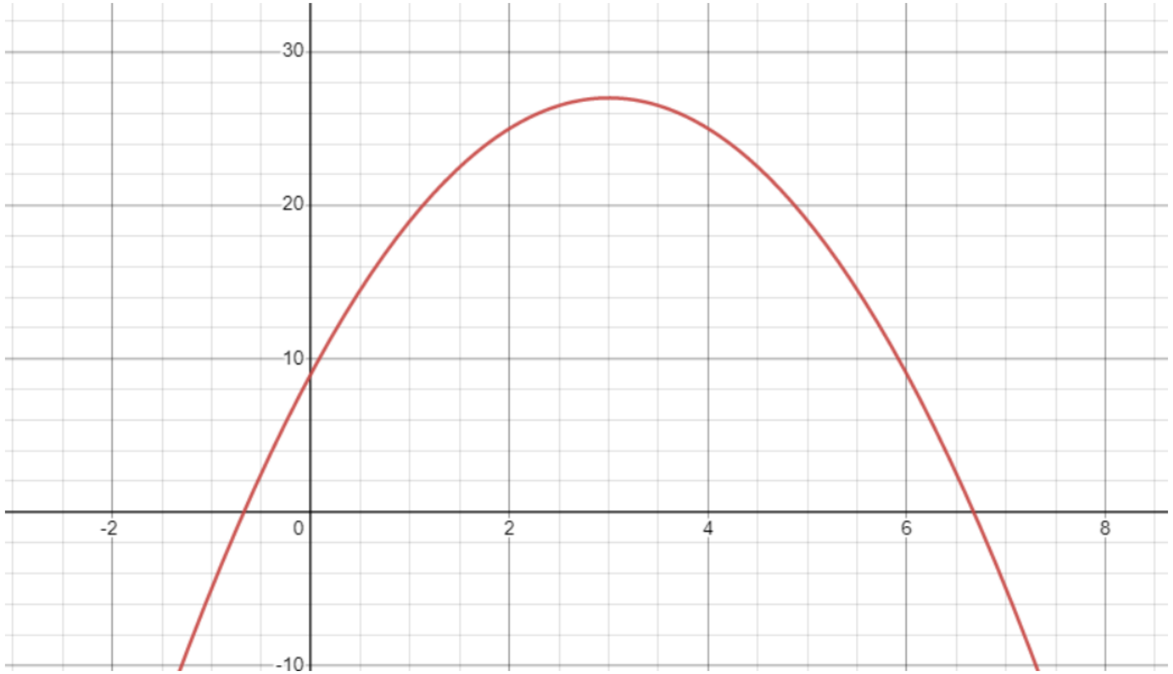
What is a stationary point?

A stationary point is a point on a function where the gradient is equal to zero.

Remember: A gradient of zero means that there is no vertical rise for any amount of horizontal distance moved.

If I was to draw a tangent to a point with zero gradient, it would be a horizontal line.

So, looking at the graph on the right, we can see there is one stationary point. With the function shown this occurs at the **maximum** of the function.

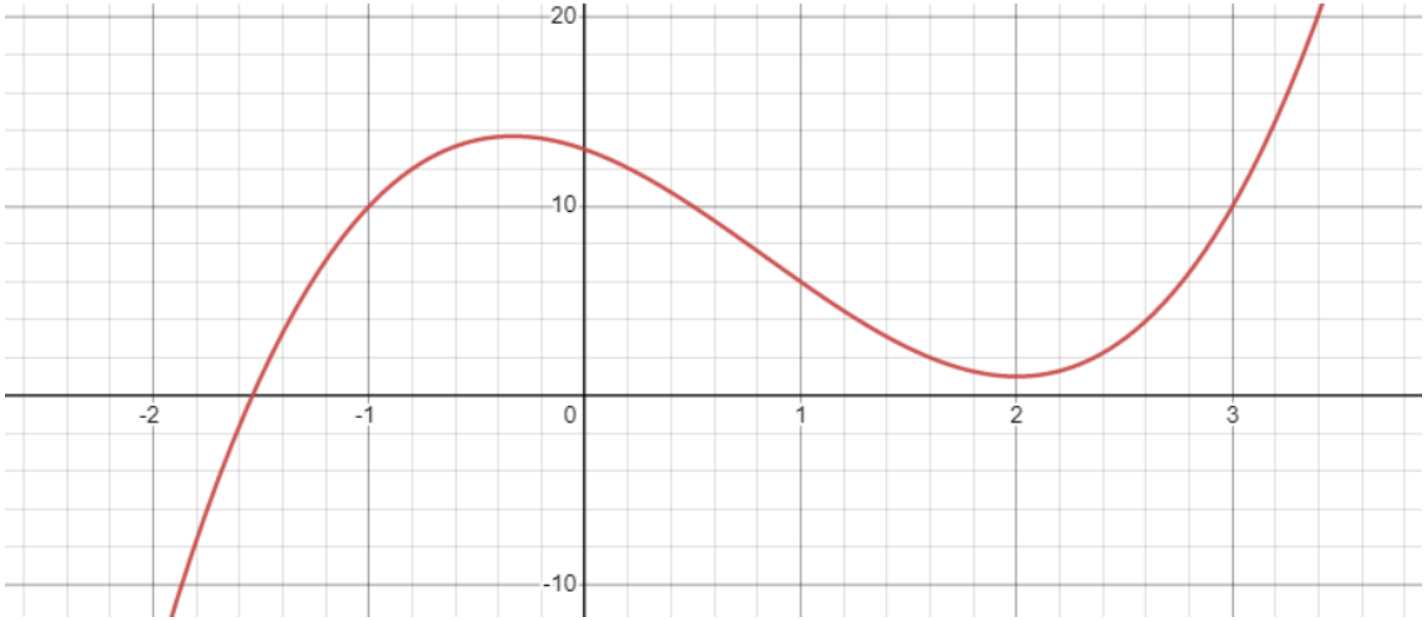


Maximum and minimums

It would see logical then that any points which are a local maximum or local minimum would have a gradient of zero. This would mean they are **stationary points**.

Looking at the function below, we can see there is a maximum and minimum and hence two stationary points.

Note: When given a function we can find the stationary points by differentiating the function and solving for where it equals zero.



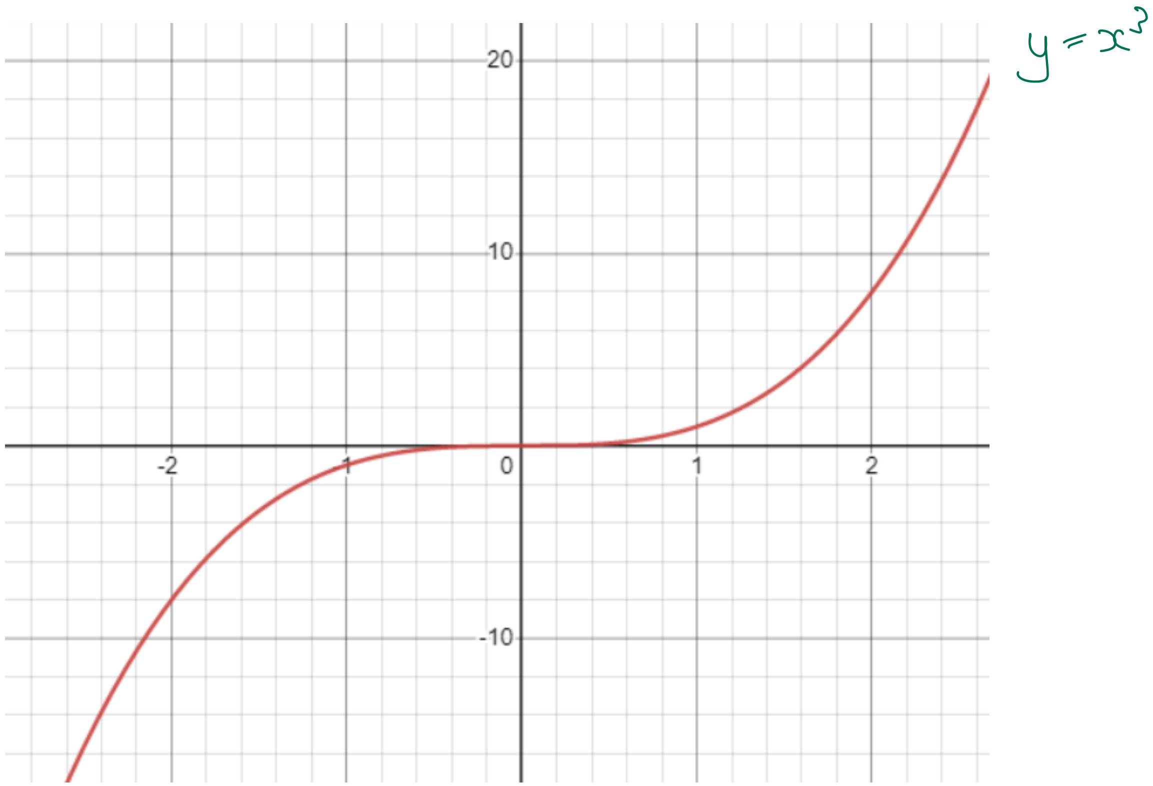
Stationary points of inflection

There is one other point on a graph which needs consideration.

These are called stationary points of inflection.

For maximums and minimums we notice that there is a change of sign of the gradient either side of the maximum or minimum. Positive to negative, or negative to positive.

But there are points with a zero gradient where the gradient either side is either positive to positive or negative to negative. These are called **stationary points of inflection.**



Example

Find the stationary points for the following function:

$$y = 9 - 12x - 2x^2$$

$y' = -12 - 4x$

$m = 0$

$-12 - 4x = 0$

$-4x = 12$

$x = -3$

$y = 9 - 12(-3) - 2(-3)^2$

$= 9 + 36 - 18$

$= 27$

$\therefore$ s.p $(-3, 27)$

Example

Find the stationary points for the following function:

$$p = 2t^3 - 5t^2 - 4t + 13, t > 0$$

$p' = 6t^2 - 10t - 4$

$p' = 0 \quad 6t^2 - 10t - 4 = 0$

$3t^2 - 5t - 2 = 0$

$(3t+1)(t-2) = 0$

$\therefore \; 3t + 1 = 0 \qquad t - 2 = 0$

$3t = -1 \qquad t = 2$

$\cancel{t = -\frac{1}{3}}$

$t > 0$

$p = 2(2)^3 - 5(2)^2 - 4(2) + 13$

$= 16 - 20 - 8 + 13$

$= 1$

$\therefore$ sp $(2, 1)$

Example

Find the stationary points for the following function:

$$y = 4 + 3x - x^3$$

$y' = 3 - 3x^2$

$y' = 0$ $\quad 3 - 3x^2 = 0$

$-3x^2 = -3$

$x^2 = 1$

$\underline{\underline{x = \pm 1}}$

$x = 1$ $\quad\quad x = -1$

$y = 4 + 3 - 1$ $\quad\quad y = 4 - 3 + 1$

$= \underline{\underline{6}}$ $\quad\quad = \underline{\underline{2}}$

SP $(1, 6)$ and $\underline{\underline{(-1, 2)}}$

Example

The curve with equation $y = x^3 + ax^2 + bx + c$ passes through the point $(0, 5)$ and has a stationary point at $(2, 7)$. Find the values of a, b and c.

Note: This is a common type of question which I have seen a number of times on exams.

$y' = 3x^2 + 2ax + b$ $\quad (2,7)$ $\quad y' = 0$

$0 = 3(2)^2 + 2.a(2) + b$

$= 12 + 4a + b = 0$

$a = -3.5 \quad b = 2 \quad c = 5$

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Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 18C
Questions: TBA

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