Rates of Change

Year 11

Mathematical Methods

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 Mathematical Methods course.

- Understand what it means by average rate of change
- Understand what it means to be an instantaneous rate of change
- Know how to find both of the above for a range of questions

Recap of past learning

In the previous lesson we looked at how we can find the equations of tangents and normal to a point on a line (or graph). This is very important as the course progresses.

Another important section is being able to find average and instantaneous rates of change.

Average rate of change

In a previous section of the course, we found the following:

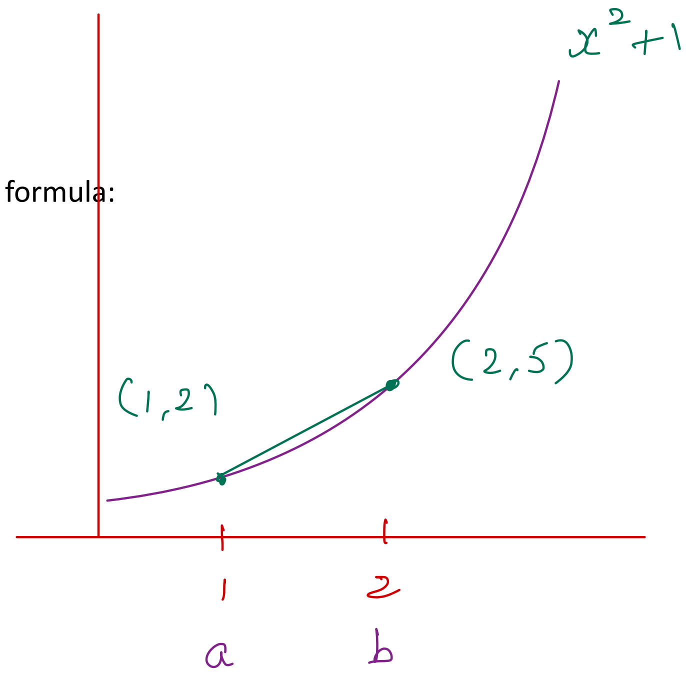
A rate of change is defined as **the gradient of a straight line joining two points.**

So, to find the average rate of change, all I need to know is two coordinates on the curve and use the following formula:

$$Average\ rate\ of\ change = \frac{f(b) - f(a)}{b - a}$$

Which is really just the same as:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



Instantaneous rate of change

In a previous section of the course, we found the following:

The instantaneous rate of change is defined as **the value of the gradient of the tangent to a graph at one point.**

So, to find the instantaneous rate of change, all we need to do is differentiate the equation and find the value of the gradient at the given point.

Example

For the function with rule $f(x) = x^2 + 2x$, find:

- the average rate of change for $x \in [2,3]$
- the average rate of change for the interval $[2, 2+h]$
- the instantaneous rate of change of f with respect to x when $x = 2$.

$$m = \frac{f(b) - f(a)}{b - a}$$

$b = 3$

$a = 2$

$$= \frac{15 - 8}{3 - 2}$$

$$m = \frac{7}{1} = 7$$

Example

For the function with rule $f(x) = x^2 + 2x$, find:

- the average rate of change for $x \in [2,3]$
- the average rate of change for the interval $[2, 2+h]$
- the instantaneous rate of change of f with respect to x when $x = 2$.

$$f(x) = x^2 + 2x$$

$$f'(x) = 2x + 2$$

$$f'(2) = 2(2) + 2$$

$$= \underline{\underline{6}}$$

$b = 2 + h$

$a = 2$

$$m = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{(2+h)^2 + 2(2+h) - 8}{\cancel{2} + h - \cancel{2}}$$

$$= \frac{\cancel{4} + 4h + h^2 + \cancel{4} + 2h - \cancel{8}}{h}$$

$$= \frac{6\cancel{h} + h^{\cancel{2}}}{\cancel{h}} = \underline{\underline{6 + h}}$$

Example

A balloon develops a microscopic leak and gradually decreases in volume.

Its volume, V (cm^3), at time t (seconds) is $V = 600 - 10t - \frac{1}{100}t^2, t > 0.$

a) Find the rate of change of volume after:
   i. 10 seconds
   ii. 20 seconds

b) For how long could the model be valid?

$V = \cancel{600} - 10t - \frac{1}{100}.t^2 \quad t > 0$

$\frac{dV}{dt} = -10 - \frac{2}{100}.t$

$= -10 - \frac{1}{50}.t$

$\frac{dV}{dt} = -10 - \frac{1}{5}$

$= -10\frac{1}{5} \ cm^3/s$

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Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 18B
Questions: TBA

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