Tangents and normals

Year 11

Mathematical Methods

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 Mathematical Methods course.

- Know how to find the equations of the tangent to a point of a curve
- Know how to find the equations of the normal to a point of a curve
- Use the CAS effectively to find the tangents and normal.

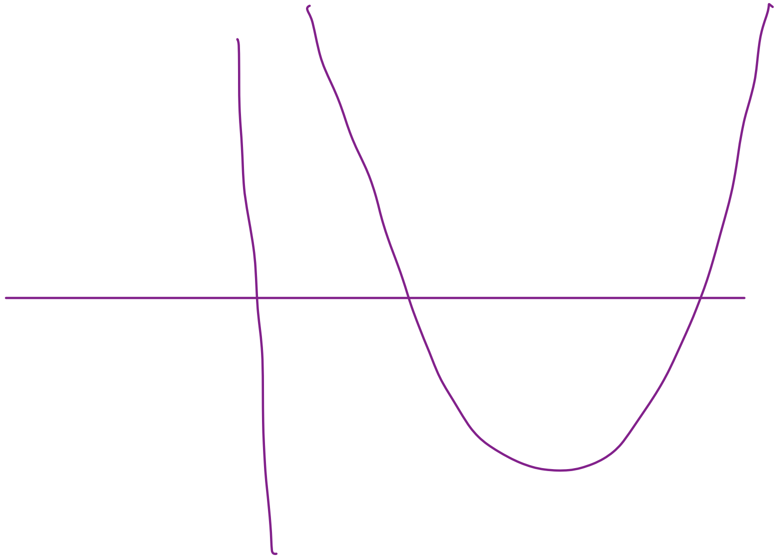
Recap of past learning

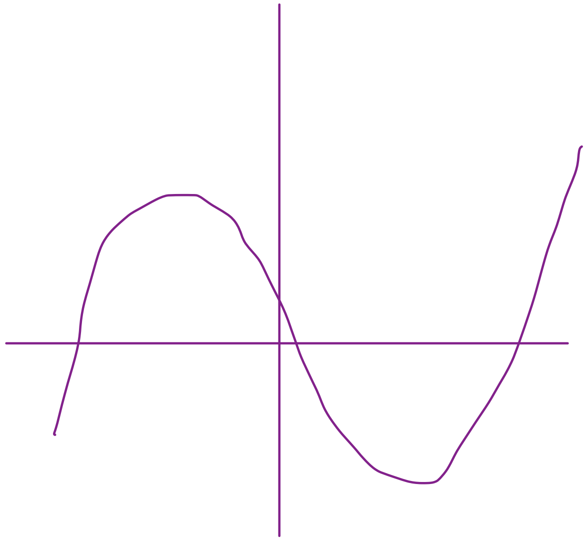
This is the first lesson of a new section of the course and it’s where the real fun starts.

Being able to sketch curves from their equations is really important in Methods. Whilst you have, in previous years, been asked to sketch straight lines and quadratics, there are many, many more graphs which can be drawn. The ability to be able to sketch a large range of curves is going to be really important.

This section starts to look at how we might find the **key points** of a graph to enable us to draw a pretty accurate sketch.

Firstly, let’s recap what a straight line is!





Equation of a straight line

Earlier in the course you would have spent some time finding the equation of a straight line.

You might use the form below:

$$y = mx + c$$

This requires you to know the gradient of the line and one point to help you find 'c'.

Alternatively you could use the equation on the right.

$$y - y_1 = m(x - x_1)$$

Where

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

This supposes you have been given two coordinates to help find the gradient.

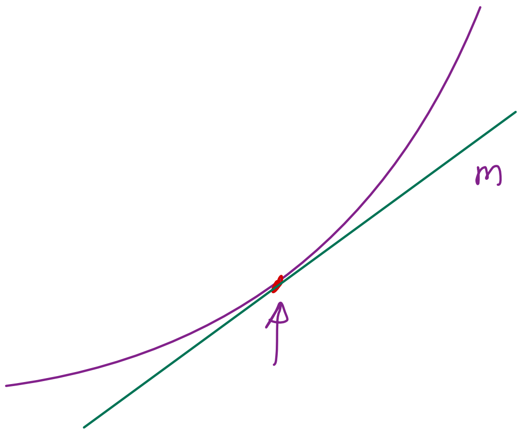
Differentiation helps you find the gradient of the tangent to a point

When we don’t have a nice straight line, we know that we can now use differentiation to help us find the **gradient of the tangent to a point**.

As this point has a coordinate, we have all we need to use the single equation on the right.

$$y - y_1 = m(x - x_1)$$

$m \quad (x_1, y_1)$



Example

Find the equation of the tangent to the curve $y = x^3 + \frac{1}{2}x^2$ at the point where $x = 1$.

$y = x^3 + \frac{1}{2}x^2$

1. $y' = 3x^2 + x$

m @ $x = 1$ $\quad m = 3 + 1 = \underline{\underline{4}}$

$x = 1 \quad y = 1^3 + \frac{1}{2} \cdot 1^2$

$= 1 + \frac{1}{2}$

$= \frac{3}{2}$

$(1, 3/2)$

$x_1 \quad y_1$

$y - y_1 = m(x - x_1)$

$y - \frac{3}{2} = 4(x - 1)$

$2y - 3 = 8(x - 1)$

$2y - 3 = 8x - 8$

$2y = 8x - 5$

$\underline{\underline{y = 4x - \frac{5}{2}}}$

Example: Using the CAS

Note: The tangentLine() function is one of the most useful ones on the CAS.

Find the equation of the tangent to the curve $y = x^3 + \frac{1}{2}x^2$ at the point where $x = 1$.

Finding the equation of the normal

The normal is a line which is perpendicular to the tangent.

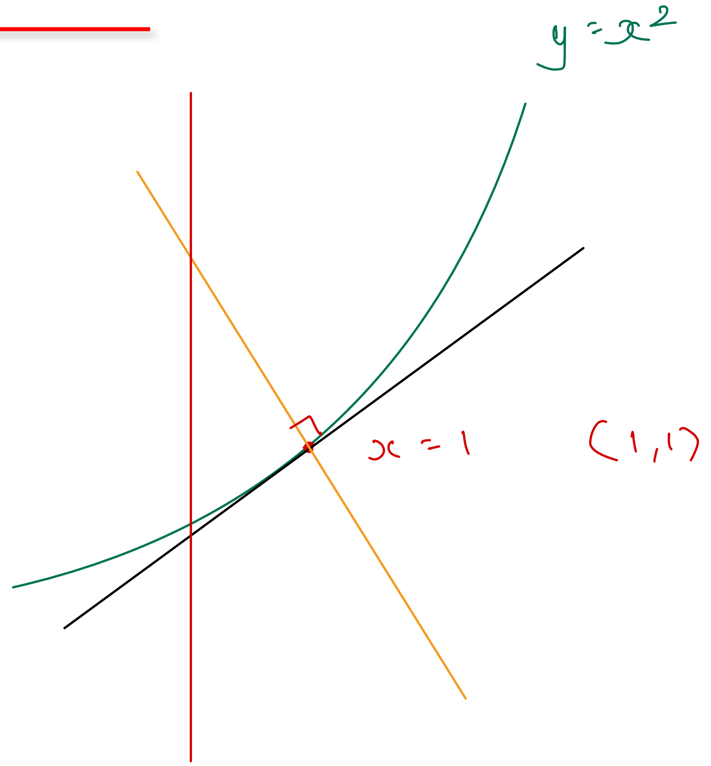
The minute I see the word perpendicular I know the gradients of the normal and tangent are connected by the following:

$$m_1 \times m_2 = -1$$

Or,

$$m_2 = -\frac{1}{m_1}$$

Or, **take the gradient of the tangent, reciprocate it and reverse the sign.**



Example

Find the equation of the normal to the curve with equation $y = x^3 - 2x^2$ at the point $(1, -1)$.

$y' = m = 3x^2 - 4x$

m @ $x = 1$ $\quad m = 3 - 4$

$= -1$

$m_T = -1 \quad m_n = 1$

$y - y_1 = m(x - x_1)$

$y - (-1) = 1(x - 1)$

$y + 1 = x - 1$

$y = x - 2$

Example

Note: With the following example, we used the following steps to solve it:

1. Differentiate the function
2. Find the gradient of the tangent at the given point.
3. Find the gradient of the normal by reciprocating the gradient of the tangent and reversing the sign
4. Use $y - y_1 = m(x - x_1)$ as the normal is a straight line.

Find the equation of the normal to the curve with equation $y = x^3 - 2x^2$ at the point $(1, -1)$.

Example: Using the CAS

Note: The normalLine() function is also really important!

Find the equation of the normal to the curve with equation $y = x^3 - 2x^2$ at the point $(1, -1)$.

Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 18A
Questions: TBA

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