

Antidifferentiation of polynomial functions

Year 11
Mathematical Methods

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 Mathematical Methods course.

- Know what it means to antidifferentiate
- Understand what the notation is for antidifferentiation
- Know what the rules are for antidifferentiation
- Know how to use the CAS to antidifferentiate



Recap of past learning

We have, in the previous lessons, been looking at how to differentiate functions.

Differentiation is the process of finding the gradient of the tangent to a point on a graph. This becomes incredibly powerful as the course progresses. But what if we are given the gradient function and are asked to find the original function? Easy?

Well, let's have a look and see what this lesson holds …



Differentiating the constant

When we differentiate the following functions, we notice something rather interesting:

$$y = x^2 + 2$$

$$y = x^2 - 6$$

$y = x^2 + 2$

$y' = \frac{dy}{dx} = 2x^1$

$2x^0$

$0 \cdot x$

$y = x^2 - 6$

$y' = 2x^1$



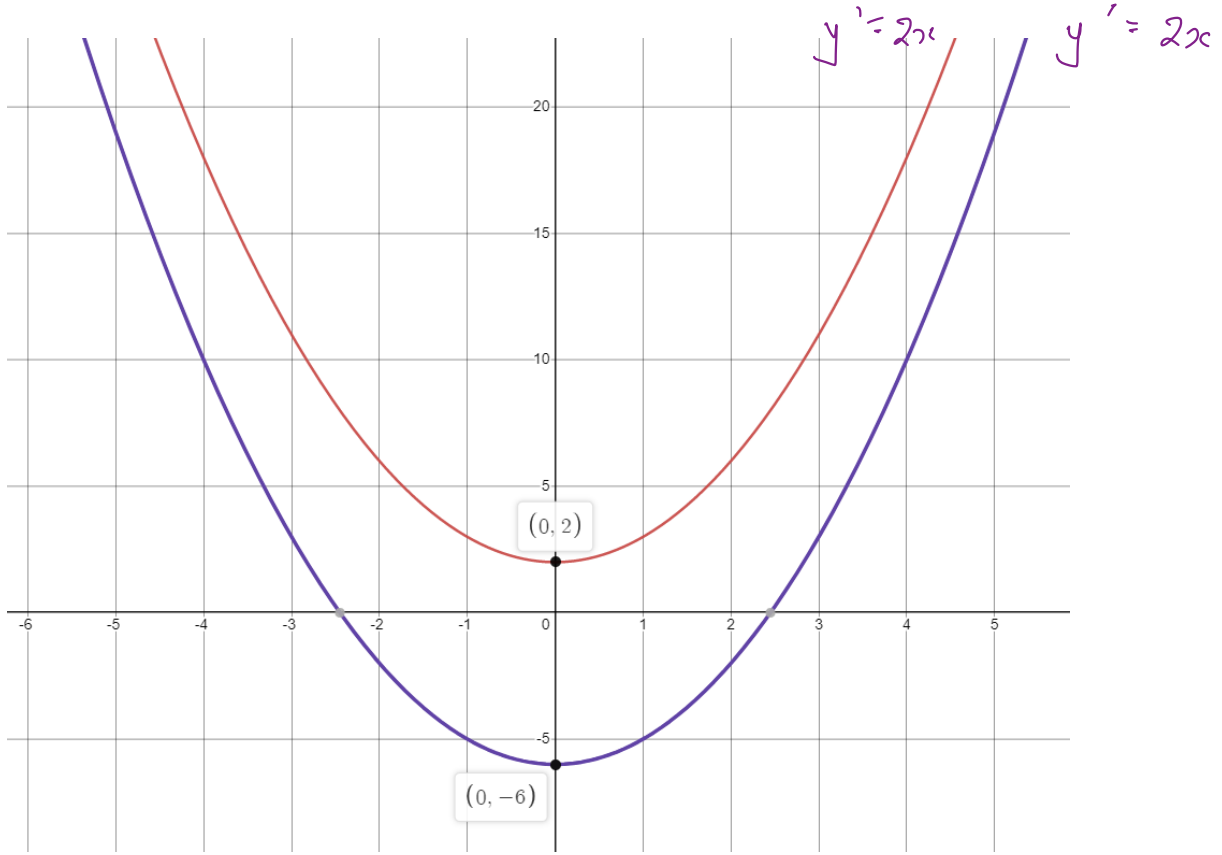
Differentiating the constant

Both of the following graphs have the same gradient function. This makes sense when you consider the graphs.

$$y = x^2 + 2$$

$$y = x^2 - 6$$

They are the same graph. They have simply been translated. Hence, each x-coordinate will have the same gradient.





Differentiating the constant

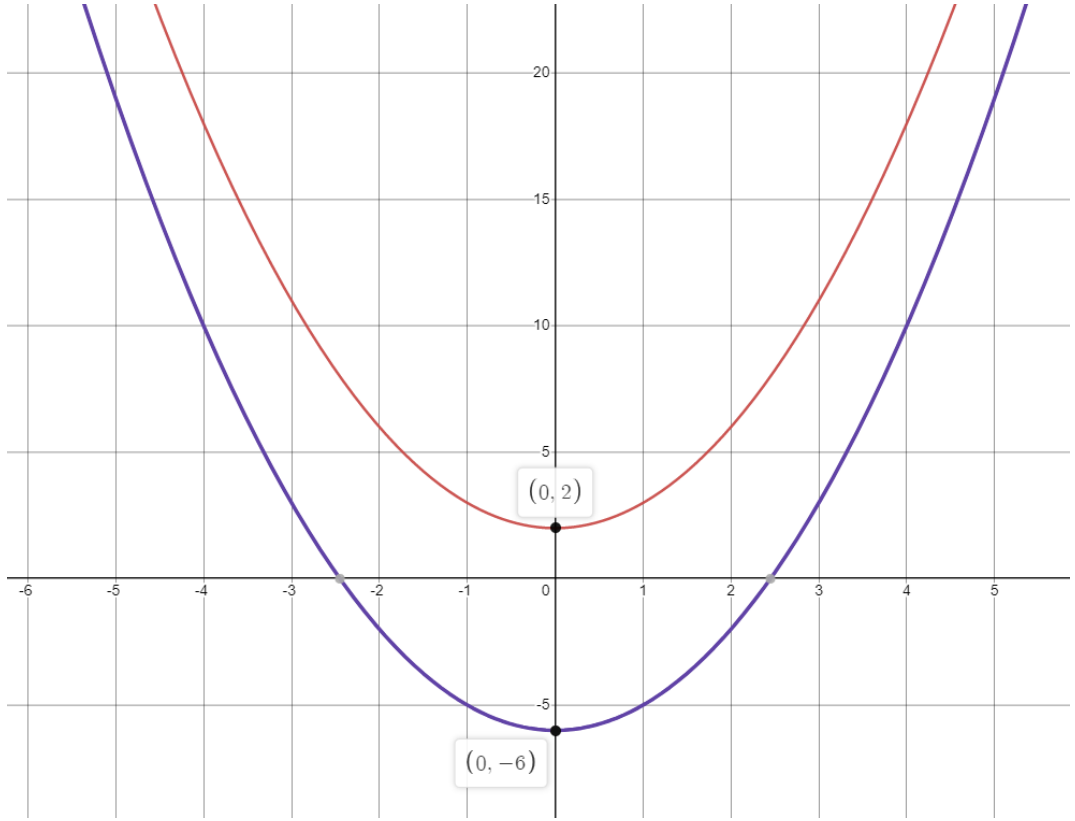
We therefore know that any graph of the form:

$$y = x^2 \pm b$$

Will always have a gradient function of $y = 2x$

But, as we can see, the graphs are different. **When we differentiate, the constant term becomes zero.**

So, how do we reverse the process and get the constant back?





Antidifferentiation (Integration)

Note: Being from the UK, I am more likely to use the term **integration.**

We know that all graphs of the form $y = x^2 \pm c$ will always have a differential of $f'(x) = 2x$

So, let's use some notation to show we can reverse the process.

Indefinite integral → $\int 2x.dx = x^2 + c$

General antiderivative of $2x$.

Note: Be careful of the wording with these questions. If it says find a solution you do NOT need to write the +c. As, a solutions can include a 'c' value of 0.

$\int 2x.dx = x^2 + c$



Rule for Antidifferentiation (Integration)

Where there were rules for differentiation there are rules for integration.

As the processes are the reverse of each other, then we can state the following:

$2x^3$

Differentiation: multiply by the power and subtract one from the power.

So, the reverse of this must be:

Integration: Add one to the power, divide by the new power.



Examples

Find the general antiderivative (indefinite integral) of each of the following:

a $3x^5$ **b** $3x^2 + 4x^3 + 3$

$\frac{x^6}{2} = \frac{6x^5}{2}$

a. $\int 3x^5 \, dx = \frac{3x^6}{6_2} + c$

$= \frac{x^6}{2} + c$

b. $\int (3x^2 + 4x^3 + 3) \, dx$ $\quad 3x^0$

$= \frac{3x^3}{3_1} + \frac{4x^4}{4_1} + \frac{3x^1}{1} + c$

$= x^3 + x^4 + 3x + c$



Examples

$(0,0)$ (x, y)

It is known that $f'(x) = x^3 + 4x^2$ and $f(0) = 0$. Find $f(x)$.

$$f(x) = \int f'(x)\,dx$$

$$f(x) = \int (x^3 + 4x^2)\,dx$$

$$f(x) = \frac{x^4}{4} + \frac{4x^3}{3} + c$$

$$0 = \frac{0^4}{4} + \frac{4(0)^3}{3} + c$$

$$c = 0$$

$$\therefore f(x) = \frac{x^4}{4} + \frac{4x^3}{3}$$



Examples

If the gradient of the tangent at a point (x, y) on a curve is given by $5x$ and the curve passes through $(0, 6)$, find the equation of the curve.

$x \quad y$

$(0, 6)$

$f'(x) = 5x$

$f(x) = \int f'(x)\,dx$

$= \int 5x\,dx$

$f(x) = \frac{5x^2}{2} + c$

$6 = \frac{5(0)^2}{2} + c$

$c = 6$

$f(x) = \frac{5x^2}{2} + 6$



Examples

Find y in terms of x if:

a $\frac{dy}{dx} = x^2 + 2x$, and $y = 1$ when $x = 1$

b $\frac{dy}{dx} = 3 - x$, and $y = 2$ when $x = 4$

$$\frac{dy}{dx} = x^2 + 2x$$

$$y = \int (x^2 + 2x)\,dx$$

$$y = \frac{x^3}{3} + \frac{2x^2}{2} + C$$

$$1 = \frac{1^3}{3} + 1^2 + C$$

$$1 = \frac{1}{3} + 1 + C$$

$$C = -\frac{1}{3}$$

$$\therefore y = \frac{x^3}{3} + x^2 - \frac{1}{3}$$



Examples

Find y in terms of x if:

a $\frac{dy}{dx} = x^2 + 2x$, and $y = 1$ when $x = 1$

b $\frac{dy}{dx} = 3 - x$, and $y = 2$ when $x = 4$

$3x^0$

$\frac{3x^1}{1}$

$\frac{dy}{dx} = 3 - x$

$y = \int (3 - x) \cdot dx$

$y = 3x - \frac{x^2}{2} + c$

$2 = 3(4) - \frac{(4)^2}{2} + c$

$2 = 12 - 8 + c$

$2 = 4 + c$

$c = -2$

$y = 3x - \frac{x^2}{2} - 2$



Using the CAS

Find y in terms of x if:

a $\frac{dy}{dx} = x^2 + 2x$, and $y = 1$ when $x = 1$

b $\frac{dy}{dx} = 3 - x$, and $y = 2$ when $x = 4$



Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 17E
Questions: TBA





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